Formal Language and Automata

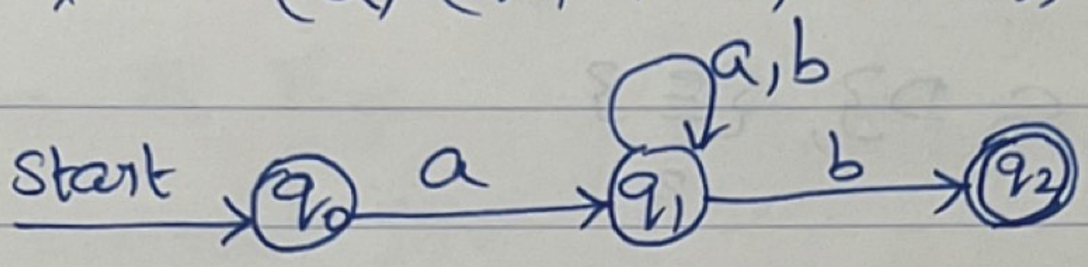
CT-I Answer Key. Part-A

1. c) 2^n
2. b) regular
3. b) NFA and DFA are equal in power.
4. a) $(0+1)^* 010 (0+1)^*$
5. c) n+1

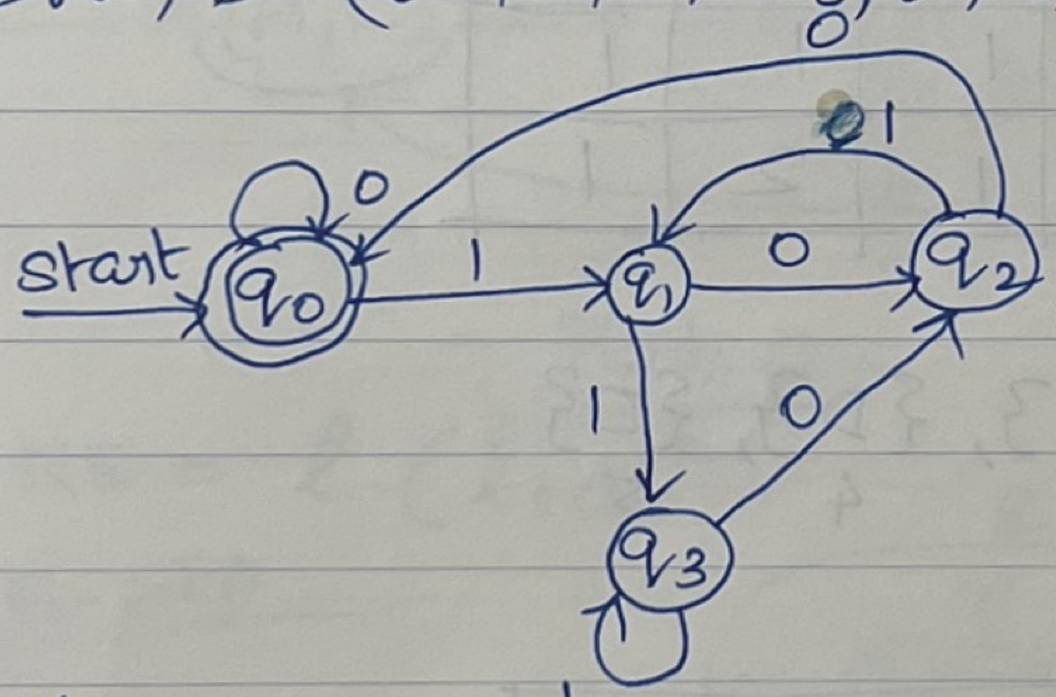
Part-B

6. RE = $a(a+b)^* b$ (2)

NFA; N = (~~Q~~, {q_0, q_1, q_2} {a, b}, δ, q_0, {q_2}) (3)



7. DFA, D = ({q_0, q_1, q_2, q_3}, {0,1}, δ, q_0, {q_0})



(3)

$\hat{\delta}(q_0, 1100) \Rightarrow$

$\hat{\delta}(q_0, \varepsilon) = q_0$

$\hat{\delta}(q_0, 1) = \delta(\hat{\delta}(q_0, \varepsilon), 1) = \delta(q_0, 1) = q_1$ (2)

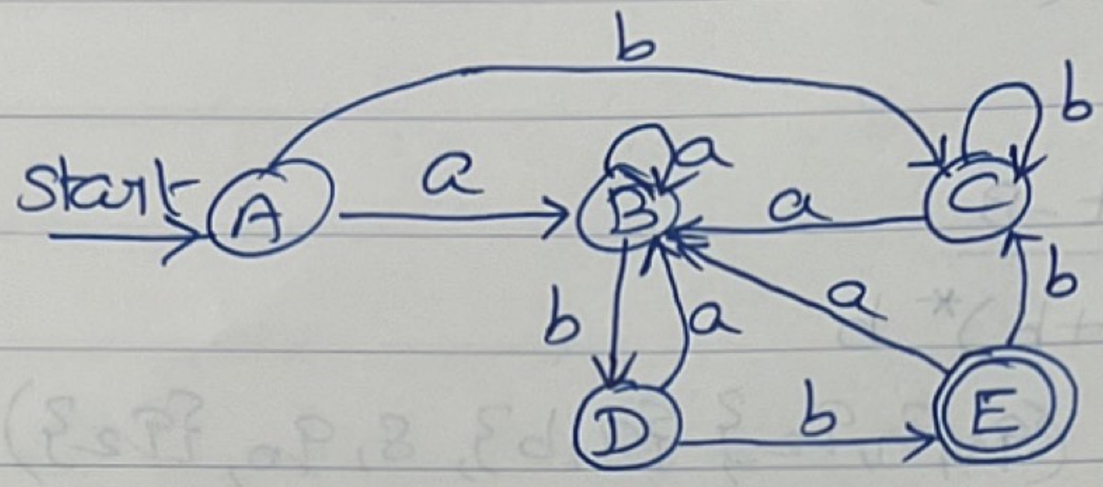
$\hat{\delta}(q_0, 11) = \delta(\hat{\delta}(q_0, 1), 1) = \delta(q_1, 1) = q_3$

$\hat{\delta}(q_0, 110) = \delta(q_3, 0) = q_2$

$\hat{\delta}(q_0, 1100) = \delta(q_2, 0) = q_0$

Since q_0 is a final state, "1100" is accepted.

8.



$\pi_0 = \{A, B, C, D\}, \{E\}$
1 2

	A	B	C	D	E
a	1	1	1	1	1
b	1	1	1	2	1

$\pi_1 = \{A, B, C\}, \{D\}, \{E\}$
3 4 5

	A	B	C	D	E
a	3	3	3	3	3
b	3	4	3	5	3

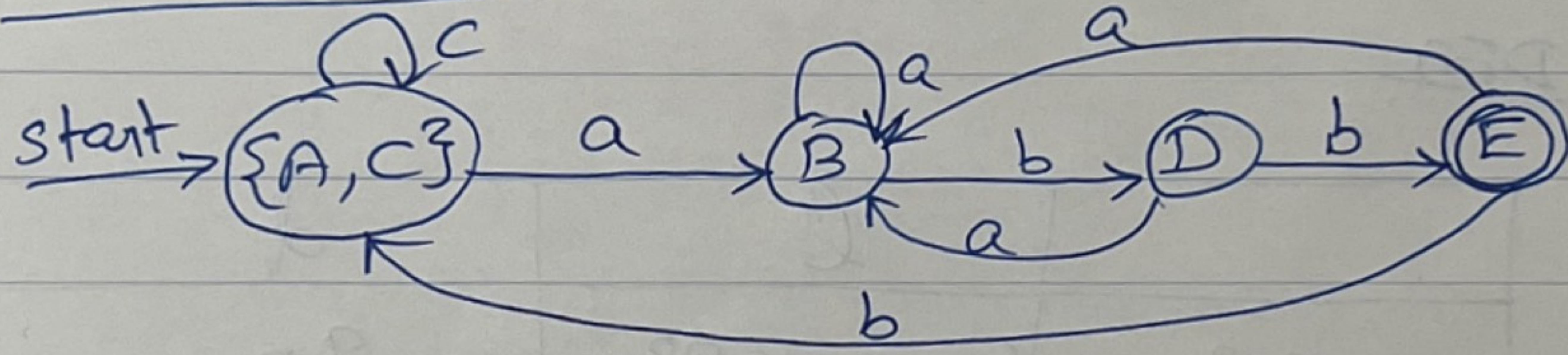
$\Pi_2 = \{A, C\}, \{B\}, \{D\}, \{E\}$
6 7 8 9

	A	B	C	D	E
a	7	7	7	7	7
b	6	8	6	9	6

$\Pi_3 = \{A,C\}, \{B\}, \{D\}, \{E\}$

$\Pi_2 \equiv \Pi_3$

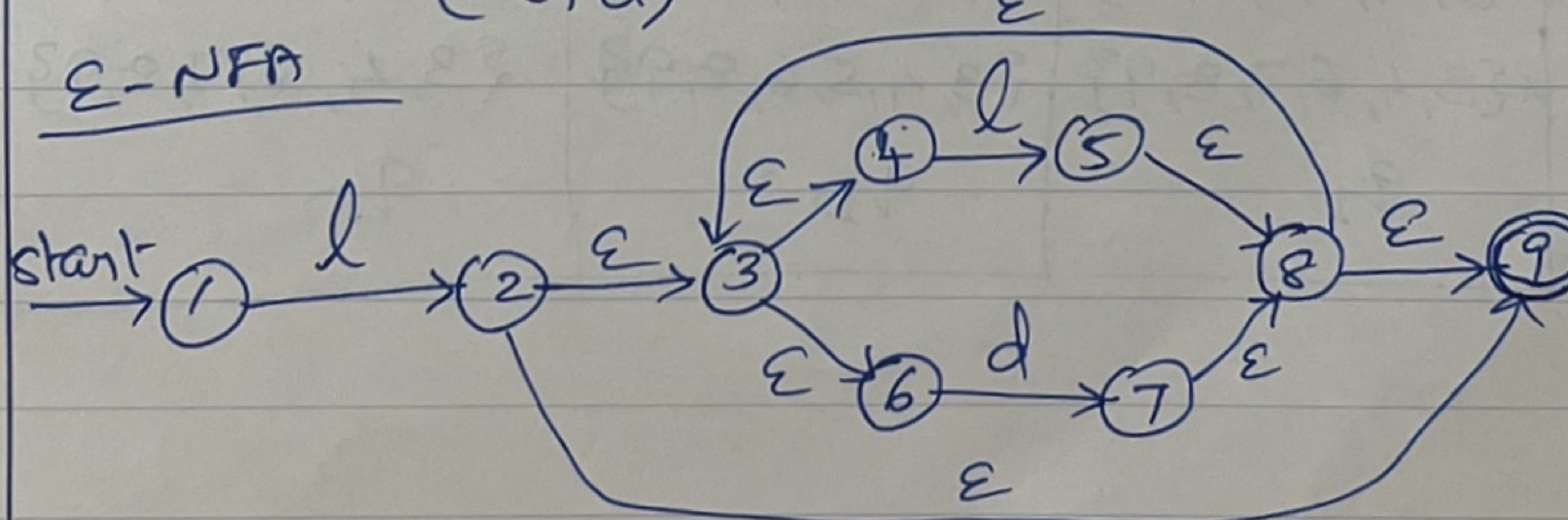
Minimized DFA.



Part-C

9. RE = $l\,(l+d)^*$ (2)

ε-NFA



(3)

DFA ⇒

ε-closure

ECLOSE(1) = {1}

" (2) = {2,3,4,6,9}

" (3) = {3,4}

" (4) = {4}

" (5) = {~~5,~~ 3,4,5,6,8,9}

" (6) = {6}

" (7) = {3,4,6,7,8,9}

" (8) = {3,4,6,8,9}

" (9) = {9}

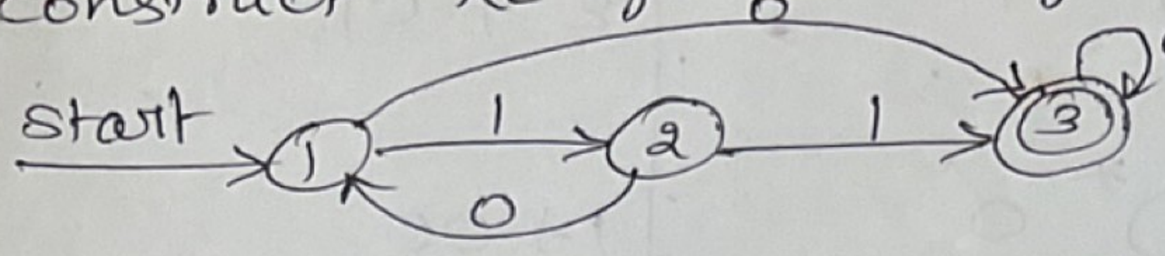
(1)

DFA

(4)

	l	d
→ {1}	{2,3,4,6,9}	q_D
*{2,3,4,6,9}	{3,4,5,6,8,9}	{3,4,6,7,8,9}
*{3,4,5,6,8,9}	{3,4,5,6,8,9}	{3,4,6,7,8,9}
*{3,4,6,7,8,9}	{3,4,5,6,8,9}	{3,4,6,7,8,9}
q_D	q_D	q_D

Construct RE for the following DFA.



$k=0$

$R_{11}^{(0)}$	ε
$R_{12}^{(0)}$	1
$R_{13}^{(0)}$	0
$R_{21}^{(0)}$	0
$R_{22}^{(0)}$	ε
$R_{23}^{(0)}$	1
$R_{31}^{(0)}$	ϕ
$R_{32}^{(0)}$	ϕ
$R_{33}^{(0)}$	$\varepsilon + 0 + 1$

$k=1$

$$R_{ij}^{(1)} = R_{ij}^{(0)} + R_{i1}^{(0)} \left(R_{11}^{(0)}\right)^* R_{1j}^{(0)}$$

	By direct substitution	Simplification
$R_{11}^{(1)}$	$\varepsilon + \varepsilon(\varepsilon)^* \varepsilon$	ε
$R_{12}^{(1)}$	$1 + \varepsilon(\varepsilon)^* 1$	1
$R_{13}^{(1)}$	$0 + \varepsilon(\varepsilon)^* 0$	0
$R_{21}^{(1)}$	$0 + 0(\varepsilon)^* \varepsilon$ 1	0

$R_{22}^{(1)}$	$\varepsilon + 0(\varepsilon)^* 1$	$\varepsilon + 01$
$R_{23}^{(1)}$	$1 + 0(\varepsilon)^* 0$	$1 + 00$
$R_{31}^{(1)}$	$\phi + \phi\ (\varepsilon)^* \varepsilon$	ϕ
$R_{32}^{(1)}$	$\phi + \phi\ (\varepsilon)^* 1$	ϕ
$R_{33}^{(1)}$	$(\varepsilon + 0 + 1) + \phi\ (\varepsilon)^* 0$	$\varepsilon + 0 + 1$

$k = 2$

$$R_{ij}^{(2)} = R_{ij}^{(1)} + R_{i2}^{(1)} \left(R_{22}^{(1)}\right)^* R_{2j}^{(1)}$$

	By direct substitution	Simplification
$R_{11}^{(2)}$	$\varepsilon + 1(\varepsilon + 01)^* 0$	$\varepsilon + 1(01)^* 0 = (01)^*$
$R_{12}^{(2)}$	$1 + 1(\varepsilon + 01)^* (\varepsilon + 01)$	$1 + 1(01)^*$ $1(01)^*$
$R_{13}^{(2)}$	$0 + 1(\varepsilon + 01)^* (1 + 00)$	$0 + 1(01)^* (1 + 00)$
$R_{21}^{(2)}$	$0 + (\varepsilon + 01)(\varepsilon + 01)^* 0$	$0 + (01)^* 0$ $(01)^* 0$
$R_{22}^{(2)}$	$(\varepsilon + 01) + (\varepsilon + 01)(\varepsilon + 01)^* (\varepsilon + 01)$	$(\varepsilon + 01) + (01)^*$ $(01)^*$
$R_{23}^{(2)}$	$(1 + 00) + (\varepsilon + 01)(\varepsilon + 01)^* (1 + 00)$	$(1 + 00) + (01)^* (1 + 00)$ $(01)^* (1 + 00)$

$R_{31}^{(2)}$	$\phi + \phi\,(\varepsilon+01)^*\,0$	ϕ
$R_{32}^{(2)}$	$\phi + \phi\,(\varepsilon+01)^*\,(\varepsilon+01)$	ϕ
$R_{33}^{(2)}$	$(\varepsilon+0+1) + \phi\,(\varepsilon+01)^*\,(1+00)$	$(\varepsilon+0+1)$

$K=3$

$$R_{ij}^{(3)} = R_{ij}^{(2)} + R_{i3}^{(2)} \left(R_{33}^{(2)}\right)^* \left(R_{3j}^{(2)}\right)$$

$$RE = R_{13}^{(3)}$$

$$R_{13}^{(3)} = [0+1(01)^*(1+00)] + [0+1(01)^*(1+00)]\,(\varepsilon+0+1)^*(\varepsilon+0+1)$$

$$= [0+1(01)^*(1+00)] + [0+1(01)^*(1+00)](0+1)^*$$

$$RE = [0+1(01)^*(1+00)](0+1)^*$$