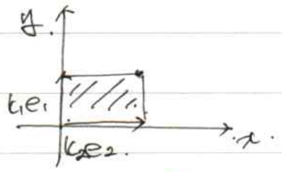
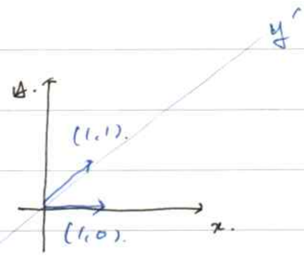
선형대수학 — 4.5 Coordinates and Basis. NO. 24.05.09.

~~[illegible]~~ # Retangle Coordinate System.





(4,3) $4e_1 + 3e_1$ → 기준 변경. ↳ $1 \cdot (1,0) + 3 \cdot (1,1)$.

*○ ; 기준으로 잡는 벡터. = 기저 ; Basis.

Basis를 어떻게 잡느냐? ⇒ 표현방법 변화.

Basis for a Vector Space.

* (1,0), (0,1). ⟶ 좌표 표현 유일. (= linearly indep.).
* (1,0), (0,1), $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ ⟶ 좌표 표현 유일 X. (= linearly dep.).

finite dimensional Vector space.) : Can be spanned by a finite set (of vector.).

유한 차원의 벡터 공간. 유한한 크기의 벡터 집합.

↕

infinite dimensional Vector space. : Cannot be ——"

*Def 1 finite-dimensional $V \supset S = \{v_1, v_2, \cdots, v_n\}$.

S: basis if (a) S spans V.
(b) S is linearly independent. } ⇒ S = basis of V (v.s.)

*ex 1). $\mathbb{R}^n$. standard basis vector : $e_1, e_2, \cdots, e_n$.

ex 2). $P_n \supset S = \{1, x, \cdots, x^n\}$. (단항식들의 모음). $a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$ … P_n Span함. … indep 함.

ex 3). $\mathbb{R}^3$, $v_1 = (1,2,1)$, $v_2 = (2,9,0)$, $v_3 = (3,3,4)$.

- spans $\mathbb{R}^3$? $\underline{v} = (v_1, v_2, v_3) = k_1 \underline{v_1} + k_2 \underline{v_2} + k_3 \underline{v_3}$
- linearly indep?

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 9 & 3 \\ 1 & 0 & 4 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} \overset{(1)}{=} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \quad \overset{(2)}{=} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

} needs invertible.

* ex4). M_{22}. $M_1 = \begin{matrix} 1 & 0 \\ 0 & 0 \end{matrix}$ $M_2 = \begin{matrix} 0 & 1 \\ 0 & 0 \end{matrix}$ $M_3 = \begin{matrix} 0 & 0 \\ 1 & 0 \end{matrix}$ $M_4 = \begin{matrix} 0 & 0 \\ 0 & 1 \end{matrix}$.

$M_1 \sim M_4$ spans M_{22}?

$B \in M_{22}$, $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = aM_1 + bM_2 + cM_3 + dM_4$ ... spans.

$M_1 \sim M_4$ indep? ✓.

basis!

* ex5). P_∞ $S = \{ \underline{\qquad\qquad} \}$. ~~유한한~~ Basis로 표현 x.

최고차수 ... n차..?

n을 넘고 같은 최고차수는 더 올라갈 것이므로. (∞).

* ex6). R^∞, $F(-\infty, \infty)$, $C(-\infty, \infty)$.

* Thm 4.5.1). S : basis of V.

$\underline{v}$: V is uniquely expressed by S.

* Def 2). $S = \{\underline{v_1}, \underline{v_2}, \cdots, \underline{v_n}\}$: basis of $\underline{V}$.

$\underline{V} = c_1\underline{v_1} + c_2\underline{v_2} + \cdots + c_n\underline{v_n}$.

$c_1, c_2, c_3 \cdots$ Coordinates of $\underline{V}$ relative to S.

$\Rightarrow$ S에 대한 좌표~~좌표~~

$(c_1, c_2, \cdots, c_n)$ = Coordinate vector relative to S

$(\underline{V})_S = (c_1, c_2, \cdots c_n)$.

* ex8). (a). P_n $\underline{p} = c_0 + c_1 x + \cdots + c_n x^n$

$\Rightarrow \boxed{(\underline{p})_S =} (c_0, c_1, \cdots, c_n)$. (* S : standard basis).

$(a)_b$. Relative from a to b.

(b). $B \in M_{22}$, $B = \begin{matrix} a & b \\ c & d \end{matrix}$.

$(B)_S = (a, b, c, d)$.

(a).

* ex9).

$S = \{\underline{v}_1, \underline{v}_2, \underline{v}_3\}$. $(\underline{v})_s = ?$

$v = (5, -1, 9)$.

$(5, -1, 9) = c_1(1,2,1) + c_2(2,9,0) + c_3(3,3,4)$.

$$\begin{bmatrix} & & \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \\ 9 \end{bmatrix}$$ (for solution).

and $= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (for check).

(b).

$(\underline{v})_s = (-1, 3, 2)$.

$\underline{v} = ?$

$\Rightarrow (-1)(1,2,1) + (3)(2,9,0) + (2)(3,3,4)$.