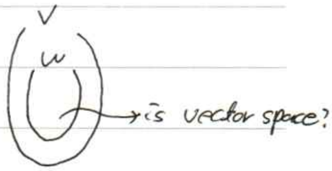
Subspaces.

* 집합 → 부분집합. ⇒ Vector space $\xrightarrow{\text{subset}}$ subspaces.



· W, subset of vector space V is Vectorspace too?

* Def 1). - $W \subset V$: subspace if itself is a vectorspace under the same v.a. and s.m.

* Thm 4.2.1). Subspace Test.

· Check Axioms 1 and 6 only.

↳ 서브셋 ... 이미 만족.

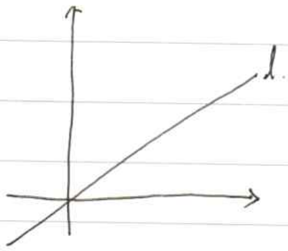
↳ 1 and 6 : closedness under v.a. and s.m.

* e.x. 1) $V = \{\underline{0}\}$ v.a. $\underline{0} + \underline{0} = \underline{0}$.

s.m. $k\underline{0} = \underline{0}$.

$W \subset V$ ... $W = \{\underline{0}\}$

* e.x. 2) Line $\mathbb{R}^2, \mathbb{R}^3 \ldots$

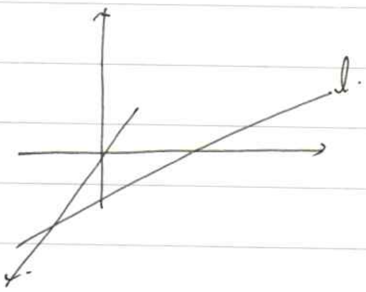


$V = \mathbb{R}^2$ .. 모든 2차원 벡터의 집합.

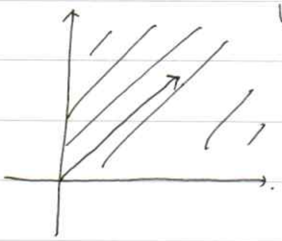
$l : W \subset V$ ... 직선 l은 V의 서브셋.

⇒ v.a, s.m. 에 닫혔다.

∴ W : S.S.
Sub Space.



* e.x. 4).



· W를 Subspace 이고, 1사분면이라 가정.

⇒ S.m ··· 음수배 : W 벗어남.

⇒ 부적절.

* e.x.5). $M_{nn} \supset W$. · W = Set of n×n Symmetric matrices.

(let n=2).

· $\underline{u}, \underline{v} \in W$. $\underline{u} = \begin{bmatrix} a_1 & b_1 \\ b_1 & c_1 \end{bmatrix}$ $\underline{v} = \begin{bmatrix} a_2 & b_2 \\ b_2 & c_2 \end{bmatrix}$

$\underline{u}+\underline{v} = \begin{bmatrix} a_1+a_2 & b_1+b_2 \\ b_1+b_2 & c_1+c_2 \end{bmatrix}$

$k\underline{u} = \begin{bmatrix} ka_1 & kb_1 \\ kb_1 & kc_1 \end{bmatrix} \in W$.

*e.x.6). · $W \subset M_{22}$. · Set of invertible matrices.

· $\underline{u}, \underline{v} \in W$ $\underline{u} = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$, $\underline{v} = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$

$\cancel{a_1 - d_1 \neq 0}$

$a_1d_1 - b_1c_1 \neq 0$ $a_2d_2 - b_2c_2 \neq 0$.

~~u+v~~

$\det(\underline{u}+\underline{v}) = (a_1+a_2)(d_1+d_2) - (b_1+b_2)(c_1+c_2)$.

* side case : $\underline{u} = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$, $\underline{v} = \begin{bmatrix} -1 & 2 \\ -2 & 5 \end{bmatrix}$.

⇒ 합성 속하지 않는다 ··· Not subspace.

*ex 7). · $F(-\infty, \infty)$. : V.S.

∪

Set of all continuous functions. ··· $W = \overset{\text{Continuous.}}{C(-\infty, \infty)}$. S.S.

NO.

* ex 9). $F(-\infty, \infty)$.

$\cup$

W = Set of all $\underbrace{\text{polynomial functions}}_{}$.
(다항 함수.)

→ S.S.

degree. 제한 없이 생각 가능.

$\boxed{P_\infty : P_{(\text{degree})} \ (\text{degree})\text{차 (이하!) 다항 함수 전체.} \quad \hookrightarrow \infty\text{차 다항함수 전체.}}$

* ex 10). W = Set of polynomials of degree n.

$= \{ a_0 + a_1 x + \cdots + a_n x^n \mid a_0, \cdots, a_n \in \mathbb{R} \}$.
$a_n \neq 0$.

~~S.S~~. n차의 subspace가 n차가 아닐 수 있음. (계수가 같다면 제거됨).

W = Set of polynomials of degree $\leq n$.

$\boxed{= P_n.}$

* ex 11). (a). $U = \left\{ \begin{bmatrix} x & 0 \\ 2x & y \end{bmatrix} \middle| x, y \in \mathbb{R} \right\}$. * $V = M_{22}$.

$\underline{u}, \underline{v} \in V$ ... $\underline{u} = \begin{bmatrix} x_1 & 0 \\ 2x_1 & y_1 \end{bmatrix}$, $\underline{v} = \begin{bmatrix} x_2 & 0 \\ 2x_2 & y_2 \end{bmatrix}$.

$$\begin{cases} \underline{u} + \underline{v} = \begin{bmatrix} x_1 + x_2 & 0 \\ 2(x_1 + x_2) & y_1 + y_2 \end{bmatrix} \in V. \\ k\underline{u} = \begin{bmatrix} kx_1 & 0 \\ 2kx_1 & ky_1 \end{bmatrix} \in V. \end{cases}$$

(b). $W = \left\{ A \middle| A \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$
$$\begin{cases} A, A_2 \in W. \\ kA \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} +k \\ -k \end{bmatrix}. \\ A_2 \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}. \end{cases}$$

↓

$A_1 + A_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$ ... Not subset.

* ex(2).

(a)

$U = \{ \underline{P} \mid \underline{P} = 1 + ax - ax^2 \}$

P_1, P_2 $\begin{cases} P_1 = 1 + a_1 x - a_1 x^2 \\ P_2 = 1 + a_2 x - a_2 x^2 \end{cases}$

V.a. $P_1 + P_2 = 2 + \underbrace{(a_1 + a_2)}_{\text{any}} x - \underbrace{(a_1 + a_2)}_{\text{any}} x^2$

must be 1. $\Rightarrow$ Not subspace.

(b).

$W = \{ P(x) \mid P(2) = 0 \}$ … 2를 해로 갖는 모든 다항함수.

P_1, P_2 $\begin{cases} P_1(2) = 0 \\ P_2(2) = 0 \end{cases}$

V.a. $(P_1 + P_2)(x) = P_1(\cancel{x}^{\,2}) + P_2(\cancel{x}^{\,2})$

$= 0 + 0 = 0.$

S.m. $(k\underline{P_1})(2) = kP_1(2) = 0.$

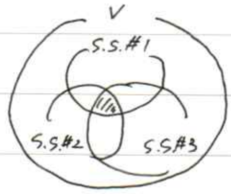
$\Uparrow$ W

$\Rightarrow$ Subspace.

* Thm 4.22).

$\overset{\text{subspace}}{W_1}, \overset{\text{subspace}}{W_2}, \cdots, \overset{\text{subspace}}{W_r} \subset V.$

$W_1 \cap W_2 \cap \cdots \cap W_r$: S.S.



* Thm 4.2.3).

$\mathbb{R}^n \supset W = \{ \underline{x} \mid A\underline{x} = \underline{0} \}$: S.S.

pf) $\underline{x_1}, \underline{x_2} \in W.$

$\begin{cases} A\underline{x_1} = \underline{0} \\ A\underline{x_2} = \underline{0} \end{cases}$ * $\underline{x_1}, \underline{x_2}$ is solution.

Solution space

· 해가 유일한 하나 (자명해). $(= \underline{0})$.

· 무수히 많은 해.

↓

vector spaces.

V.a. $A(\underline{x_1} + \underline{x_2}) = \underline{0}.$

S.m. $kA(\underline{x_1}) = \underline{0}.$ $\Rightarrow$ S.S.

* ex 13.

(a).

$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & -4 & 6 \\ 3 & -6 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$\rightsquigarrow x - 2y + 3z = 0.$

… 해는 무수히 많다.

… 법선벡터가 $(1, -2, 3)$인 평면.

$(0, 0, 0)$을 지나므로 Subspace.

(b)

$$\begin{bmatrix} 1 & -2 & 3 \\ -3 & 7 & -8 \\ -2 & 4 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow \begin{cases} x - 2y + 3z = 0. \\ -3x + 7y - 8z = 0. \end{cases}$$

$\downarrow$.

Sol : $t(-5, -1, 1)$.

$(-5, -1, 1)$ 벡터에 t배한 값의 집합.

= 직선.

+ $(0, 0, 0)$ 지남 $\therefore$ subspace.

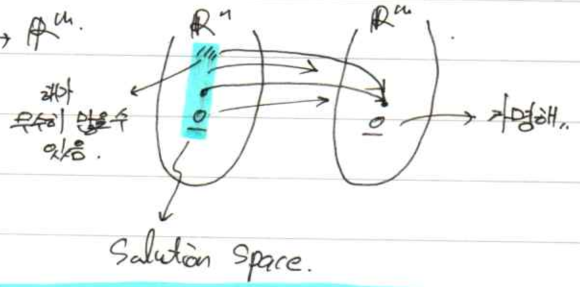
(c)

$$\begin{bmatrix} 1 & -2 & 3 \\ -3 & 7 & -8 \\ 4 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

(d)

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

* Thm 4.2.4). $T_A : R^n \longrightarrow R^m$.



* Definition. 공역의 $\underline{0}$으로 매핑되는 정의역의 집합을 Kernel 으로 정한다.
(= Kernel = solutionspace, Subspace of T).