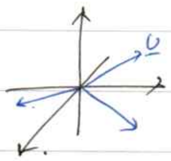
*Thm 4.6.6. W: Subspace of a f.d. v.s. V.

(a) W: finite-dimensional.

(b) $\dim(W) \le \dim(V)$.

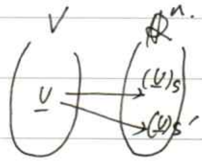
(c) $W = V \Leftrightarrow \dim(W) = \dim(V)$.

4.7. Change of Basis.



$(\underline{v})_S \neq (\underline{v})_{S'}$

$(\underline{v})_S = (c_1, c_2, \cdots, c_n) \in \mathbb{R}^n$.



$\begin{cases} S \to S' \\ (\underline{v})_S \to (\underline{v})_{S'} \text{ ?} \end{cases}$

$(\underline{u}_1', \cdots, \underline{u}_n')$

- $B \to B' \Rightarrow [\underline{v}]_B \to [\underline{v}]_{B'}$?

$B = (\underline{u}_1, \cdots \underline{u}_n)$, $B' = (\underline{u}_1', \cdots, \underline{u}_n')$

$[\underline{v}]_{B'} = P[\underline{v}]_B$. where $P = \underbrace{[\,[\underline{u}_1]_{B'}, [\underline{u}_2]_{B'}, \cdots, [\underline{u}_n]_{B'}\,]}_{}$. ($n \times n$.)

P: transition mat.

$\underset{\text{old}}{P}{}_{B \to \underset{\text{new}}{B'}} = [\ \text{———}.\ ]$.

*ex1). $B = \{(1,0), (0,1)\}$.
$B' = \{(1,1)\ (2,1)\}$

$P_{B \to B'} = [\,[\underline{u}_1]_B\ [\underline{u}_2]_B\,] = \begin{bmatrix} c_1 = -1 & c_3 = 2. \\ c_2 = 1 & c_4 = -1 \end{bmatrix}$.

$[\underline{u}_1]_B \to \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$.

$\begin{cases} (1,0) = \cancel{c_1}^{-1}(1,1) + \cancel{c_2}^{1}(2,1). \\ (0,1) = \cancel{c_3}^{2}(1,1) + \cancel{c_3}^{-1}(2,1). \end{cases}$

$P_{B' \to B} = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$

- $[\underline{v}]_{B'} = P_{B \to B'}[\underline{v}]_B$

Invertibility.

$[\underline{v}]_{B'} = \underbrace{P_{B\to B'}}_{\text{transformation.}} [\underline{v}]_B.$

$P_{B\to B'} P_{B'\to B} = I.$

$P_{B'\to B} = (P_{B\to B'})^{-1}.$

*Thm 4.7.1. "Efficient Method"

- Step 1. $\begin{array}{cc} \text{new} & \text{old.} \end{array}$ $[B' \mid B]$.
- Step 2. Elementary Row Operation.
- Step 3. $[I \mid \boxed{\ }]$. $\to P_{B\to B'}$

* ex 3).

$$\begin{array}{cc} \text{new} & \text{old.} \end{array}$$
$$\left[\begin{array}{cc:cc} 1 & 2 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cc:cc} 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & -1 \end{array}\right].$$

$\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix} \to P_{B\to B'}$

$$\begin{array}{cc} \text{old} & \text{new} \end{array}$$
$$\left[\begin{array}{cc:cc} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \end{array}\right]$$

$\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \to P_{B'\to B}.$

*Thm.4.7.2. $B = \{\underline{u}_1, \cdots \underline{u}_n\}.$
$S = \{\text{Standard Basis}\}.$ $\to P_{B\to S} = [\underline{u}_1\ \underline{u}_2 \cdots \underline{u}_n.]$