선형대수학. - Ch.07. Diagonal, Triangular, and Symmetric Matrices.

Diagonal

· A Square matrix
in which all the entries off the main diagonal are zero.

· $\begin{bmatrix} ? & 0 & 0 \\ 0 & ? & 0 \\ 0 & 0 & ? \end{bmatrix}$ $D = \begin{bmatrix} d_1 & & 0 \\ & d_2 & \\ & \ddots & \\ 0 & & d_n \end{bmatrix}$

· D is invertible $\Leftrightarrow$ $d_i \neq 0$ for all i.

· $D^{-1} = \begin{bmatrix} \frac{1}{d_1} & & 0 \\ & \frac{1}{d_2} & \\ & & \ddots \\ 0 & & \frac{1}{d_n} \end{bmatrix}$, $D^k = \begin{bmatrix} d_1^k & & 0 \\ & d_2^k & \\ & & \ddots \\ 0 & & d_n^k \end{bmatrix}$

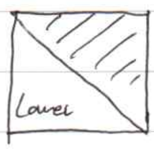
*Ex 1)

$$\begin{bmatrix} d_1 & & 0 \\ & d_2 & \\ 0 & & d_3 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} = \begin{bmatrix} d_1a_{11} & d_1a_{12} & d_1a_{13} & d_1a_{14} \\ d_2a_{21} & d_2a_{22} & d_2a_{23} & d_2a_{24} \\ d_3a_{31} & d_3a_{32} & d_3a_{33} & d_3a_{34} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix} \begin{bmatrix} d_1 & & 0 \\ & d_2 & \\ 0 & & d_3 \end{bmatrix} = \begin{bmatrix} d_1a_{11} & d_2a_{12} & d_3a_{13} \\ d_1a_{21} & d_2a_{22} & d_3a_{23} \\ d_1a_{31} & d_2a_{32} & d_3a_{33} \\ d_1a_{41} & d_2a_{42} & d_3a_{43} \end{bmatrix}$$

Triangular.

· (Upper triangular matrix.
(Lower triangular matrix.



Upper

· Let A is UT, B is LT

a) A^T is LT, B^T is UT.

b) AA is UT, BB is LT

(UT)(UT) is UT, (LT)(LT) is LT.

c) A, B is invertible $\Leftrightarrow$ The diagonal entries are all non-zero.

d) A^{-1} is UT, B^{-1} is LT.

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symmetric.

· A: Square.
$A = A^T$

"대칭" main diagonal에 대칭...

$$\begin{bmatrix} 1 & 4 & 5 \\ 4 & -3 & 0 \\ 5 & 0 & 7 \end{bmatrix}$$

· Symmetric Matrix is Upper triangle matrix and Lower triangle matrix.

*Thm 1.7.2.
· A^T is symmetric.
· $A+B$, $A-B$ are symmetric.
· kA is symmetric.

*Thm 1.7.3.
· (Symmetric)(Symmetric) is symmetric.

*Thm 1.7.4.
· A is invertible, Symmetric then
A^{-1} is symmetric.

*Thm. 1.7.5.
· A is invertible then
AA^T, A^TA are invertible.

Linear Transformations.

"Linear" ... 오직 1차식 (상수배, 덧셈...).

- $(s_1, s_2, \cdots s_n)$ → line vector. ⇒ $\in \mathbb{R}^n$
- $\mathbb{R}$: set of real numbers.

  $\mathbb{R}^2$: Set of 2-tuples of real numbers.

  ⋮

  $\mathbb{R}^n$: Set of n-tuples of real numbers.
- $\begin{bmatrix} s_1, & s_2, & \cdots, & s_n \end{bmatrix}$ → row vector.

  $\begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix}$ → column Vector. (default in this lecture).

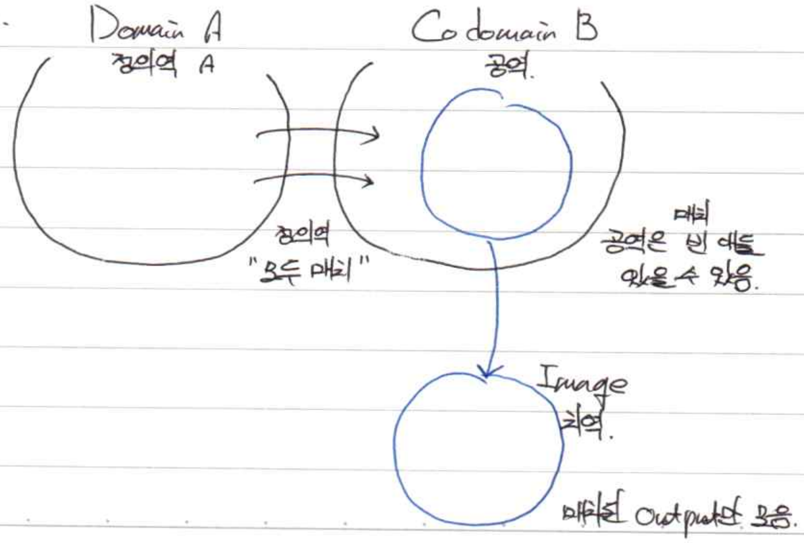
- $e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \ e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \ \cdots, \ e_n = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ n \end{bmatrix}$ → Standard basis Vector.

let $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = x_1 e_1 + x_2 e_2 + \cdots + x_n e_n$

(Linear combination of x and e)

Functions and transformation.

- $y = f(x)$. ...



Transformation. (similiar with function).

(mapping)

- $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$
  - $\mathbb{R}^n$ ↓ domain, n-vector
  - $\mathbb{R}^m$ ↓ codomain, m-vector

⟷ function : scalar.
transformation : vector. ↓ expand.

- $m = n$ then T is operator. (연산자)

※ Ex.

- $w_1 = a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n$

$w_2 = a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n$

$\vdots \qquad \vdots$

$w_m = \cdots + a_{mn}x_n$

- $$\begin{bmatrix} w_1 \\ \vdots \\ w_m \end{bmatrix} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$ ⇒ Matrix Transformation.

$$w = A \quad x$$

$T_A : \mathbb{R}^n \to \mathbb{R}^n$.
$w = T_A(x)$.
$x \xrightarrow{T_A} w$

※ Ex1

$w_1 = 2x_1 - 3x_2 + x_3 - 5x_4$
$w_2 = 4x_1 + x_2 - 2x_3 + x_4$
$w_3 = 5x_1 - x_2 + 4x_3$,

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 1 & -5 \\ 4 & 1 & -2 & 1 \\ 5 & -1 & 4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Ex2. Zero Transformation : $T_0(x) = 0x = 0$.

Ex3. Identity Transformation : $T_I(x) = Ix = x$.

※ Thm 18.1. Properties. Let $T_A : \mathbb{R}^n \to \mathbb{R}^m$.

(a) $T_A(0) = (0)$.

(b) $T_A(ku) = kT_A(u)$.

$Aku = kAu$ … Scalar Product, 순서 상관 X.

*Thm 1.8.1 Properties. (c, d). $T_A(\underline{u}+\underline{v})$

$= T_A(\underline{u}) + T_A(\underline{v})$

$T_A(k_1 u_1 + \cdots + k_n u_n)$

$= k_1 T_A(\underline{u}_1) + \cdots + k_n T_A(\underline{u}_n)$

⇒ Matrix transform은 linear property를 갖는다.

· Matrix transformation ⟺ Linear transformation

*Thm. 1.8.4.

· $T_A : \mathbb{R}^n \to \mathbb{R}^m$.
$T_B : \mathbb{R}^n \to \mathbb{R}^m$.

· $T_A(\underline{x}) = T_B(\underline{x})$ for every $\underline{x}$ then
$A = B$. → Standard Matrix

all standard Matrix are unique.

Procedure for finding standard matrix.

· $T : \mathbb{R}^n \to \mathbb{R}^m$.

$A = [T(e_1) \mid T(e_2) \mid \cdots \mid T(e_n)]$

*ex 4.

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 + x_2 \\ x_1 - 3x_2 \\ -x_1 + x_2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & -3 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

$$A = \begin{bmatrix} 2 & 1 \\ 1 & -3 \\ -1 & 1 \end{bmatrix}$$

*Ex7.

$T\left(\begin{bmatrix}-1\\1\end{bmatrix}\right)=\begin{bmatrix}-5\\5\end{bmatrix}$ → $\begin{bmatrix}-5\\5\end{bmatrix}=\begin{bmatrix}a&b\\c&d\end{bmatrix}\begin{bmatrix}-1\\1\end{bmatrix}$.

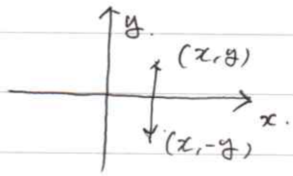
$T\left(\begin{bmatrix}2\\-1\end{bmatrix}\right)=\begin{bmatrix}7\\-6\end{bmatrix}$ → $\begin{bmatrix}7\\-6\end{bmatrix}=\begin{bmatrix}a&b\\c&d\end{bmatrix}\begin{bmatrix}2\\-1\end{bmatrix}$.

$$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right)=T\left(a\begin{bmatrix}-1\\1\end{bmatrix}+b\begin{bmatrix}2\\-1\end{bmatrix}\right) \qquad =\begin{bmatrix}2\\-1\end{bmatrix}$$

$$T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)=T\left(c\begin{bmatrix}-1\\1\end{bmatrix}+d\begin{bmatrix}2\\-1\end{bmatrix}\right)=2T\left(\begin{bmatrix}-1\\1\end{bmatrix}\right)+T\left(\begin{bmatrix}2\\-1\end{bmatrix}\right)=\begin{bmatrix}3\\4\end{bmatrix}$$

Reflection Operators. · using transformation.

- reflection about the x-axis.



$T(e_1)=T(1,0)=(1,0)$.

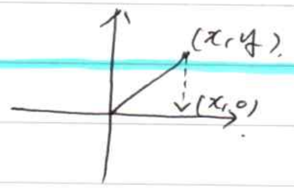
$T(e_2)=T(0,1)=(0,-1)$.

std mat $=\begin{bmatrix}1&0\\0&-1\end{bmatrix}$.

- operator — Images — Std Mat.

$T(x,y)=(x,0)$. $\quad T(e_1)=T(1,0)=(1,0)$, $\quad T(e_2)=T(0,1)=(0,0)$ $\quad \begin{bmatrix}1&0\\0&-1\end{bmatrix}$.

Projection Operators.
정사영



- $(x,y,z)\longrightarrow(0,y,z)$. $\Rightarrow \begin{bmatrix}0&0&0\\0&1&0\\0&0&1\end{bmatrix}$