Chap 4.1.

#2. a. $\underline{u}=(0,4)$, $\underline{v}=(1,-3)$.

$\underline{u}+\underline{v}=(2,2)$.

$2\underline{u}=(0,8)$.

b. $\underline{u}+(-\underline{u})=\underline{0}=(1,1)$.

$\neq(0,0)$.

c. $\underline{0}=k\underline{0}$

$k=-1 \Rightarrow -\underline{0}=\underline{0}=(-1,-1)$.

#8

$$\underline{u}=\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}, \quad \underline{v}=\begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \quad (\underline{u},\underline{v} \in V).$$

axiom 1. $\underline{u}+\underline{v} \in V$. $\quad \underline{u}+\underline{v}=\begin{bmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{bmatrix}$

axiom 2. $\underline{u}+\underline{v}=\underline{v}+\underline{u}$ $\quad \underline{u}+\underline{v}=\begin{bmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{bmatrix}=\underline{v}+\underline{u}=\begin{bmatrix} a_2+a_1 & b_2+b_1 \\ c_2+c_1 & d_2+d_1 \end{bmatrix}$

axiom 3. $\underline{u}+(\underline{v}+\underline{w})=(\underline{u}+\underline{v})+\underline{w}$

let $\underline{w}=\begin{bmatrix} a_3 & b_3 \\ c_3 & d_3 \end{bmatrix}$. $\quad \underline{u}+(\underline{v}+\underline{w})=\begin{bmatrix} a_1+(a_2+a_3) & b_1+(b_2+b_3) \\ c_1+(c_2+c_3) & d_1+(d_2+d_3) \end{bmatrix}$

$$(\underline{u}+\underline{v})+\underline{w}=\begin{bmatrix} (a_1+a_2)+a_3 & (b_1+b_2)+b_3 \\ (c_1+c_2)+c_3 & (d_1+d_2)+d_3 \end{bmatrix}$$

axiom 4,5. $\underline{u}+(-\underline{u})=(-\underline{u})+\underline{u}=\underline{0}$.

$\underline{0}+\underline{u}=\underline{u}+0=\underline{u}$

$$\underline{0}=\begin{bmatrix} a_1-a_1 & b_1-b_1 \\ c_1-c_1 & d_1-d_1 \end{bmatrix}=\underline{u}+(-\underline{u})=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\underline{0}+\underline{u}=\begin{bmatrix} 0+a_1 & 0+b_1 \\ 0+c_1 & 0+d_1 \end{bmatrix}=\underline{u}+\underline{0}=\underline{u}$$

#8 axiom 6.10. $\underline{u} \in V \Rightarrow k\underline{u} \in V.$ $k=1 \Rightarrow \underline{u} = 1 \cdot \underline{u} = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$

axiom 7. $k(\underline{u}+\underline{v}) = k\underline{u} + k\underline{v}$

$$k(\underline{u}+\underline{v}) = \begin{bmatrix} k(a_1+a_2) & k(b_1+b_2) \\ k(c_1+c_2) & k(d_1+d_2) \end{bmatrix}.$$

$$k\underline{u}+k\underline{v} = \begin{bmatrix} ka_1+ka_2 & kb_1+kb_2 \\ kc_1+kc_2 & kd_1+kd_2 \end{bmatrix}.$$

axiom 8. $(k+m)\underline{u} = k\underline{u} + m\underline{u}$

$$(k+m)\underline{u} = \begin{bmatrix} (k+m)a_1 & (k+m)b_1 \\ (k+m)c_1 & (k+m)d_1 \end{bmatrix}.$$

$$k\underline{u}+m\underline{u} = \begin{bmatrix} ka_1+ma_1 & kb_1+mb_1 \\ kc_1+mc_1 & kd_1+md_1 \end{bmatrix}.$$

axiom 9. $k(m\underline{u}) = (km)\underline{u}$

$$k\begin{bmatrix} ma_1 & mb_1 \\ mc_1 & md_1 \end{bmatrix} = \begin{bmatrix} kma_1 & kmb_1 \\ kmc_1 & kmd_1 \end{bmatrix}.$$

$$km\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} = \begin{bmatrix} kma_1 & kmb_1 \\ kmc_1 & kmd_1 \end{bmatrix}.$$

#12. axiom 1. $\underline{u}+\underline{v} \in V$ $\quad (a_0+a_1x)+(b_0+b_1x) = ((a_0+b_0)+(a_1+b_1)x)$

2. $\underline{u}+\underline{v} = \underline{v}+\underline{u}$ $\quad (a_0+b_0)+(a_1+b_1)x = (b_0+a_0)+(b_1+a_1)x.$

3. $\underline{u}+(\underline{v}+\underline{w}) = (\underline{u}+\underline{v})+\underline{w}$ $\quad$ Let $\underline{w} = (c_0+c_1x).$

$$(a_0+(b_0+c_0))+(a_1+(b_1+c_1))x = ((a_0+b_0)+c_0)+((a_1+b_1)+c_1)x.$$

axiom 4.5. $\underline{u}+(-\underline{u}) = (-\underline{u})+\underline{u} = \underline{0}$ $\quad (a_0+(-a_0))+(a_1+(-a_1))x = 0+0x.$

$\underline{0}+\underline{u} = \underline{u}+\underline{0} = \underline{u}$ $\quad ((-a_0)+a_0)+((-a_1)+a_1)x = 0+0x.$

$$(0+0x)+(a_0+a_1x) = a_0+a_1x.$$

axiom 6.10. $\underline{u} \in V \Rightarrow k\underline{u} \in V.$ $\quad$ Let $k=1.$

$1 \cdot \underline{u} = \underline{u}$ $\quad 1(a_0+a_1x) = a_0+a_1x.$

$$(ka_0+ka_1x) \subset V.$$

#12. axiom 7. $k(\underline{u}+\underline{v}) = k\underline{u}+k\underline{v}$ $\quad k((a_0+b_0)+(a_1+b_1)x)$

$= k(a_0+a_1x)+k(b_0+b_1x)$.

axiom 8. $(k+m)\underline{u} = k\underline{u}+m\underline{u}$ $\quad (k+m)(a_0+a_1x) = k(a_0+a_1x)+m(a_0+a_1x)$.

axiom 9. $k(m\underline{u}) = (km)\underline{u}$ $\quad k(m(a_0+a_1x)) = km(a_0+a_1x)$

$= kma_0+kma_1x$.

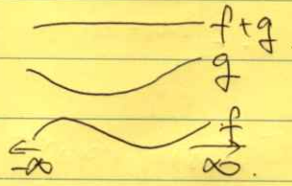
#14.

$f(x) = a_0+a_1x+\cdots+a_nx^n$.

$g(x) = b_0+b_1x+\cdots+b_nx^n$.

$k(x) = c_0+c_1x+\cdots+c_nx^n$.

axiom 1. $f+g \in C(-\infty,\infty)$ $\quad (a_0+b_0)+(a_1+b_1)x+\cdots+(a_n+b_n)x^n \in C(-\infty,\infty)$.



axiom 2. $f+g = g+f$ $\quad (a_0+b_0)+(a_1+b_1)x+\cdots+(a_n+b_n)x^n$

$= (b_0+a_0)+(b_1+a_1)x+\cdots+(b_n+a_n)x^n$.

axiom 3. $f+(g+k) = (f+g)+k$. $\quad (a_0+(b_0+c_0))+(a_1+(b_1+c_1))x+\cdots$

$+(a_n+(b_n+c_n))x^n$.

$= ((a_0+b_0)+c_0)+((a_1+b_1)+c_1)x+\cdots$

$+((a_n+b_n)+c_n)x^n$.

axiom 7. $k(f+g) = kf+kg$. $\quad k(a_0+b_0)+k(a_1+b_1)x+\cdots+k(a_n+b_n)x^n$

$= ka_0+ka_1x+\cdots+ka_nx^n+kb_0+kb_1x+\cdots+kb_nx^n$.

axiom 8. $(k+m)f = kf+mf$ $\quad (k+m)a_0+(k+m)a_1x+\cdots+(k+m)a_nx^n$

$= ka_0+ka_1x+\cdots+ka_nx^n+ma_0+ma_1x+\cdots+ma_nx^n$.

axiom 9. $k(mf) = (km)f$ $\quad k(ma_0)+k(ma_1x)+\cdots+k(ma_nx^n)$

$= (km)a_0+(km)a_1x+\cdots+(km)a_nx^n$.

axiom 10. $1\cdot f = f$ $\quad 1(a_0+a_1x+\cdots+a_nx^n) = a_0+a_1x+\cdots+a_nx^n$.

Chap 4.2.

#2.

a. $\underline{v} = (a_1, b_1, c_1)$, $\underline{w} = (a_2, b_2, c_2)$.

$\underline{v} + \underline{w} = (a_1 + a_2,\ b_1 + b_2,\ c_1 + c_2)$. or $(a_1 + a_2,\ (a_1 + c_1 + 1) + (a_2 + c_2 + 1),\ c_1 + c_2)$.

$= (a', \cancel{b'}, c') = (a_1 + a_2,\ a_1 + a_2 + c_1 + c_2 + 1,\ c_1 + c_2)$.

$a' + c'$

$(a_1 + a_2,\ a_1 + a_2 + c_1 + c_2 + 1,\ c_1 + c_2)$

$\neq$

$(a_1 + a_2,\ a_1 + a_2 + c_1 + c_2 + 2,\ c_1 + c_2)$.

$\therefore$ Not subspaces of $\mathbb{R}^3$

b. $\underline{v} = (a_1, b_1, 0)$, $\underline{w} = (a_2, b_2, 0)$.

$\underline{v} + \underline{w} = (a_1 + a_2,\ b_1 + b_2,\ 0)$.

$k\underline{v} = (ka_1,\ kb_1,\ k0)$

$= (ka,\ kb,\ 0)$

$\therefore$ Subspaces of $\mathbb{R}^3$.

c. $\underline{v} = (a_1, b_1, c_1)$, $w = (a_2, b_2, c_2)$.

$\underline{v} + \underline{w} = (a_1 + a_2,\ b_1 + b_2,\ c_1 + c_2)$

$= (a', b', c')$.

$a_1 + b_1 = 7$, $a_2 + b_2 = 7$. $\Rightarrow a_1 + b_1 + a_2 + b_2 = 7$

$= b'$

$b' \neq 7$

$\therefore$ Not subspaces of $\mathbb{R}^3$.

#4.

a. $A^T = -A$.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \quad kA = \begin{bmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ \vdots & & \ddots & \vdots \\ ka_{m1} & ka_{m2} & \cdots & ka_{mn} \end{bmatrix}$$

let $k = -1$ then $kA = \begin{bmatrix} -a_{11} & \cdots & -a_{1n} \\ \vdots & \ddots & \vdots \\ -a_{m1} & \cdots & -a_{mn} \end{bmatrix} \neq -A = A^T = \begin{bmatrix} a_{11} & \cdots & a_{m1} \\ \vdots & \ddots & \vdots \\ a_{1n} & \cdots & a_{mn} \end{bmatrix}$

$\therefore$ Not subspaces of M_{nn}.

#4

b. $\underline{u} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & 0 \\ \vdots & & & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$ $\quad \underline{v} = \begin{bmatrix} 4 & 0 & \cdots & 0 & 0 \\ 0 & 4 & \cdots & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & \cdots & 4 & 0 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$

$\underline{u}+\underline{v}$ is not invertible. = has not only trivial solution.

$\therefore$ Not subspaces of M_{nn}.

c. $a, b \in$ V.S.

$a = \begin{bmatrix} 2 & 0 & \cdots & 0 \\ 0 & 2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & & 2 \end{bmatrix}$, $b = \begin{bmatrix} 2^2 & 0 & \cdots & 0 \\ 0 & 2^2 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & 2^2 \end{bmatrix}$

$a+b = \begin{bmatrix} 2+2^2 & 0 & \cdots & 0 \\ 0 & 2+2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 2+2^2 \end{bmatrix} \neq \begin{bmatrix} 2^t & 0 & \cdots & 0 \\ 0 & 2^t & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 2^t \end{bmatrix}$

$\Rightarrow a(a \cdot b) = ac \rightarrow ac \neq ca.$

$\therefore$ Not subspaces of M_{nn}.

d. Same as b.

$\therefore$ Not subspaces of M_{nn}.

#8.

a. $f+g \Rightarrow (f+g)(x) = (f+g)(-x)$. $\quad f+g \subset (-\infty, \infty)$.

$kf \Rightarrow kf(x) = kf(-x)$. $\quad kf \subset (-\infty, \infty)$.

$\therefore$ Subspaces of $(-\infty, \infty)$.

b. let $f(x) = x^n + a_0 x^{n-1} + a_1 x^{n-2} + \cdots + a_{n-1}$

$g(x) = -x^n + b_0 x^{n-1} + b_1 x^{n-2} + \cdots + b_{n-1}$.

$f+g = (a_0+b_0)x^{n-1} + \text{———} \not\subset P^n$.

$n=2$. (degree is 2)

$\therefore$ Not subspaces of P^2.

$\Rightarrow f \subset P^2, g \subset P^2$

$f+g \not\subset P^2$

#12.

a. V.S. $= \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}$.

$\underline{u} = \begin{bmatrix} 0 & \alpha \\ \beta & 0 \end{bmatrix}$, $\underline{v} = \begin{bmatrix} 0 & x \\ y & 0 \end{bmatrix}$.

$\underline{u}+\underline{v} = \begin{bmatrix} 0 & \alpha+x \\ \beta+y & 0 \end{bmatrix} \in$ V.S.

$k\underline{u} = \begin{bmatrix} 0 & k\alpha \\ k\beta & 0 \end{bmatrix} \in$ V.S.

$\therefore$ Subspace of M_{22}

b. V.S. $= \left\{ E \text{ or } \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix}^n \right\}$

$\underline{u} = \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix}$, $\underline{v} = \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix}^2 = \begin{bmatrix} -4 & 2 \\ -2 & -3 \end{bmatrix}$

$\underline{u}+\underline{v} = \begin{bmatrix} -4 & 4 \\ -4 & -2 \end{bmatrix} \neq \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix}^n$.

$\therefore$ Not subspace of V.S.

c. let $\underline{u} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $\underline{v} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

$\underline{u}+\underline{v} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad \det(\underline{u}+\underline{v}) \neq 0$.

$\therefore$ Not subspace of M_{22}.

#16.

a. $f(x) = 2a_0 + 2a_1x + 2a_2x + \cdots + 2a_nx^n$ $(0 \le i \le n,\ a_i \in \mathbb{N})$.

$g(x) = 2b_0 + 2b_1x + 2b_2x + \cdots + 2b_nx^n$ $(0 \le i \le n,\ b_i \in \mathbb{N})$.

$f+g(x) = 2(a_0+b_0)x_0 + 2(a_1+b_1)x + \cdots + 2(a_n+b_n)x^n$. $\subset$ S.S.

$kf = k2a_0 + k2a_1x + \cdots + k2a_nx^n$. $\not\subset$ S.S.

($k=\pi$ then $2ka_0$ is not even).

$\therefore$ Not subspace of P_∞

b. $f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$ $(0 \le i \le n,\ \Sigma a_i = 0)$.

$g(x) = b_0 + b_1x + b_2x^2 + \cdots + b_nx^n$. $(0 \le i \le n,\ \Sigma b_i = 0)$.

$f+g = \Sigma a_i + \Sigma b_i = 0 + 0$. $\subset$ S.S.

$kf = k\Sigma a_i = k0 = 0$. $\subset$ S.S.

$\therefore$ Subspace of P_∞

c. $f(x) = a_0 + a_1x^{2\cdot1} + a_2x^{2\cdot2} + \cdots + a_nx^{2n}$

$g(x) = b_0 + b_1x^{2\cdot1} + \cdots + b_nx^{2\cdot n}$.

$f+g = (a_0+b_0) + (a_1+b_1)x^{2\cdot1} + \cdots + a_nx^{2n}$ $\subset$ S.S.

$kf = ka_0 + ka_1x^{2\cdot1} + \cdots + ka_nx^{2\cdot n}$ $\subset$ S.S.

$\therefore$ Subspace of P_∞

Chap 4.3.

#2.

$$\begin{bmatrix} 2 & 1 & 4 \\ 1 & -1 & 3 \\ 3 & 2 & 5 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}. \qquad \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -11/2 & 3/2 & 7/2 \\ 2 & -1 & -1 \\ 5/2 & -1/2 & -3/2 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$$

~~2 1 4 | 1 0 0~~

~~1 -1 3 | 0 1 0~~

~~3 2 5 | 0 0 1~~

~~1 1 1 -1 0 1~~

~~0 -2 2 1 1 -1~~

~~0 1 -1 -1/2 -1/2 1/2~~

~~1 0 2 -1/2 1/2 1/2~~

0 1 0 2 -1 -1

~~0 0 -1 -5/2 1/2 3/2~~.

0 0 1 5/2 -1/2 -3/2

1 0 0 -11/2 3/2 7/2.

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -11/2 & 3/2 & 7/2 \\ 0 & 1 & 0 & 2 & -1 & -1 \\ 0 & 0 & 1 & 5/2 & -1/2 & -3/2 \end{array}\right].$$

a. $(\alpha, \beta, \gamma) = \underline{u}$

$$\begin{bmatrix} -11/2 & 3/2 & 7/2 \\ 2 & -1 & -1 \\ 5/2 & -1/2 & -3/2 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 9/2 \\ -1 \\ -3/2 \end{bmatrix}$$

$\therefore (9/2, -1, -3/2)$

b. $(\alpha, \beta, \gamma) = \underline{v}$

$$\begin{bmatrix} -11/2 & 3/2 & 7/2 \\ 2 & -1 & -1 \\ 5/2 & -1/2 & -3/2 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 7/2 \\ 0 \\ -3/2 \end{bmatrix}$$

$\therefore (7/2, 0, -3/2)$

#2. C. $(\alpha, \beta, \gamma) = \underline{w}$

$$\begin{bmatrix} -11/2 & 3/2 & 7/2 \\ 2 & -1 & -1 \\ 5/2 & -1/2 & -3/2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore (0, 0, 0)$.

#8. a. $(2, 3, -7, 3) = k_1(2, 1, 0, 3) + k_2(3, -1, 5, 2) + k_3(-1, 0, 2, 1)$.

$$\begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & 0 \\ 0 & 5 & 2 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -7 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} \cancel{2} & \cancel{3} & \cancel{-1} & \cancel{2} \\ 1 & -1 & 0 & 3 \\ \cancel{0} & \cancel{5} & \cancel{2} & \cancel{-7} \\ \cancel{3} & \cancel{2} & \cancel{1} & \cancel{3} \end{bmatrix}$$

$$\begin{matrix} \cancel{1} & \cancel{-1} & \cancel{2} & \cancel{1} \\ \cancel{0} & \cancel{0} & \cancel{2} & \cancel{-2} \\ 0 & 5 & -1 & -4 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 3 & -3 \end{matrix}$$

$\therefore$ Not in span$\{v_1, v_2, v_3\}$

b.

$$\begin{matrix} 2 & 3 & -1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 5 & 2 & 0 \\ 3 & 2 & 1 & 0 \end{matrix} \longrightarrow \det = 0.$$

$\therefore$ Not in span$\{v_1, v_2, v_3\}$.

#8.

c. ~~2 3 -1 1~~

~~1 -1 0 1~~

~~0 5 2 1~~

~~3 2 1 1~~

1 -1 2 0

0 0 2 -1

~~0 5 -5 1~~

0 5 -3 0

0 0 -5 -1

4th column can be all zero.

∴ Not in span.

d. ~~2 3 -1 -4~~

~~1 -1 0 6~~

~~0 5 2 -19~~

~~3 2 1 4~~

~~1 -1 2 8~~

0 0 1 1

~~0 5 -1 -16~~

~~0 0 3 29~~

~~0 5 0 -15~~

0 1 0 -3

1 0 0 3

0 0 0 1

∴ In span.

#10.

$$\text{span}\{p_1, p_2, p_3, p_4\} \Rightarrow \begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \end{bmatrix}$$

~~1 1 1 2 P_0~~

~~1 -1 1 0 P_1~~.

0 0 1 -1 P_2

~~0 2 0 2 $P_0 - P_1$~~.

0 1 0 1 $(P_0 - P_1)/2$.

~~1 0 1 1 $(P_0+P_1)/2$~~

1 0 0 2 $(P_0+P_1-2P_2)/2$.

1 0 0 2 $(P_0+P_1-2P_2)/2$

0 1 0 1 $(P_0-P_1)/2$.

0 0 1 -1 P_2.

$\therefore$ Spans P_2.

#12.

a. $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\Rightarrow \text{span}\{(1,0), (2,1)\}$.

$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $k_1 + 2k_2 = 1$
$k_2 = 2$.

$\therefore$ in the span $\{T_A(e_1), T_A(e_2)\}$.

b. $\text{span}\{(1,1), (1,1)\}$.

$\left.\begin{matrix} k_1 + k_2 = 1 \\ k_1 + k_2 = 2. \end{matrix}\right\} \times$.

$\therefore$ Not in the span $\{T_A(e_1), T_A(e_2)\}$.

#14.

a. $\cos 2x = \cos^2 x - \sin^2 x$.

$\therefore$ lies in the span $\{f, g\}$.

b. Not lies in the span $\{f, g\}$.

c. lies in the span $\{f, g\}$.

d. Not lies in the span $\{f, g\}$.

e. lies in the span $\{f, g\}$.

Chap 4. 4.

#2. a. $\begin{vmatrix} -3 & 5 & 1 \\ 0 & -1 & 1 \\ 4 & 2 & 3 \end{vmatrix} = \begin{vmatrix} 1 & -2 & 0 \\ 0 & -1 & 1 \\ 5 & 0 & 3 \end{vmatrix} = \begin{vmatrix} 1 & -2 & 0 \\ 0 & -1 & 1 \\ 0 & 10 & 3 \end{vmatrix}$

$= 1 \begin{vmatrix} -1 & 1 \\ 10 & 3 \end{vmatrix} \neq 0.$

$\therefore$ linearly independent in R^3.

b. linearly dependent in R^3.

#4. a. $\begin{vmatrix} 2 & 3 & 2 \\ -1 & 6 & 10 \\ 4 & 2 & -4 \end{vmatrix} = \begin{vmatrix} 0 & 15 & 22 \\ -1 & 6 & 10 \\ 0 & 26 & 36 \end{vmatrix} = - \begin{vmatrix} 15 & 22 \\ 26 & 36 \end{vmatrix} \neq 0.$

$\therefore$ linearly independent in P_2.

b. linearly dependent in P_2.

#6. element at (2,2) is all 0 that gives matrix.

$\therefore$ linearly dependent in M_{22}.

#10. a. $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ 3 & 0 & 3 \\ 4 & -1 & 3 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}.$

1 0 1
~~2 1 3~~
3 0 3
~~4 1 3~~
1 -1 0
~~1 -1 0~~
0 0 0

$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ 3 & 0 & 3 \\ 4 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ \overset{0}{\cancel{1}} & \overset{0}{\cancel{0}} & \overset{0}{\cancel{1}} \\ 1 & -1 & 0 \\ \overset{0}{\cancel{1}} & \overset{0}{\cancel{-1}} & \overset{0}{\cancel{0}} \end{bmatrix}.$

$\therefore$ linearly dependent.

b. $V_3 = V_1 + V_2$.

#20.

$$W = \begin{vmatrix} e^x & xe^x & x^2e^x \\ e^x & e^x+xe^x & 2xe^x+x^2e^x \\ e^x & 2e^x+xe^x & 2e^x+4xe^x+x^2e^x \end{vmatrix}$$

$$= \begin{vmatrix} e^x & xe^x & x^2e^x \\ 0 & e^x & 2xe^x \\ 0 & 2e^x & 2e^x+4xe^x \end{vmatrix}.$$

$$= e^x(2e^x+4xe^x) - 2e^x \cdot 2xe^x \neq 0.$$

$\therefore$ linearly independent.

Chap 4.5.

\#2. $v_1 = (3, 1, -4)$. $v_2 = (2, 5, 6)$, $v_3 = (1, 4, 8)$.

$c_1\underline{v_1} + c_2\underline{v_2} + c_3\underline{v_3} = \underline{0}$

$c_1\underline{v_1} + c_2\underline{v_2} + c_3\underline{v_3} = (b_1, b_2, b_3)$. $\Big\}$ spans $\mathbb{R}^3$?

$$\begin{bmatrix} 3 & 2 & 1 \\ 1 & 5 & 4 \\ -4 & 6 & 8 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} \quad \cdots \quad \begin{bmatrix} 3 & 2 & 1 \\ 1 & 5 & 4 \\ -4 & 6 & 8 \end{bmatrix} \text{ is invertible?}$$

~~3 2 1~~

~~1 5 4~~

~~-4 6 8~~

~~-1 8 9~~

~~0 13 13~~

0 1 1

1 1 0 $\Rightarrow$ invertible.

1 0 1

$\therefore$ basis for $\mathbb{R}^3$.

\#4.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \Rightarrow \text{invertible.}$$

$\therefore$ basis for $\mathbb{R}^4$.

$$\begin{bmatrix} 0\cancel{1} & \cancel{1}0 & 0 & 1 \\ 0\cancel{1} & -1 & \cancel{1}0 & 0 \\ 0\cancel{1} & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \text{invertible.}$$

$\therefore$ basis for M_{22}.

#14

a. $(4, -3, 1)$

b. $$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$\cancel{1 \; 1 \; 0 \quad 1 \; 0 \; 0}$

$\cancel{1 \; 0 \; 1 \quad 0 \; 1 \; 0}$

$\cancel{0 \; 1 \; 1 \quad 0 \; 0 \; 1}$

$\cancel{0 \; 1 \; -1 \quad 1 \; -1 \; 0}$

$\cancel{0 \; 2 \; 0 \quad 1 \; -1 \; 1}$

$0 \; 1 \; 0 \quad \frac{1}{2} \; -\frac{1}{2} \; \frac{1}{2}$

$0 \; 0 \; +1 \quad -\frac{1}{2} \; +\frac{1}{2} \; +\frac{1}{2}$

$1 \; 0 \; 0 \quad \frac{1}{2} \; \frac{1}{2} \; -\frac{1}{2}$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$$

$\therefore (0, 2, -1).$

#22.

a.

$$T_A(\underline{u}) = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}.$$

$\therefore (2, -2, 0).$

22.

b. $$\left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 1 \\ 1 & -1 & 1 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$

$\therefore (1, 2, -3).$