*Def 1). A: $n \times n$ matrix. (A: standard matrix of an operation).

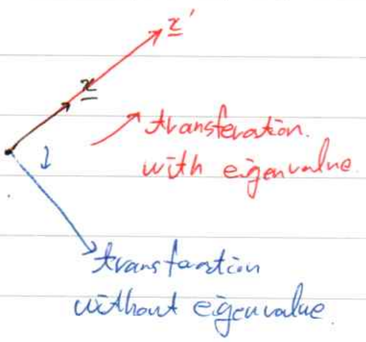
$\underline{x} \neq 0$, $\underline{x} \in \mathbb{R}^n$, $\underline{x}$: eigenvector. (corresponding to λ).

if $\boxed{A\underset{\text{vec}}{x} = \underset{\text{val}}{\lambda} x}$ for some λ (exists).

$\Rightarrow \lambda$: eigenvalue (Matrix) (Scalar)

x를 A로 변환. $A\underline{x} = \lambda \underline{x}$.

= T_A transferation의 eigen vector.



*eigen vector는 방향이 바뀌지 않는 "축이 되는" 벡터.

*ex1) $\underline{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. $A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$.

x는 A의 eigen vector인가?

$A\underline{x} = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix} = 3\underline{x}$. $\Rightarrow$ x는 A의 eigen vector이다. eigenvalue는 3이다.

Computing.

* thm 5.1.1.

$A\underline{x} = \lambda \underline{x} \longrightarrow \lambda \underline{x} - A\underline{x} = \underline{0}$.

$(\lambda - A)\underline{x} = \underline{0}$

불가능: scalar - Matrix

$(\lambda I - A)\underline{x} = \underline{0} \longrightarrow$ Consistent!!

homogeneous linear system. non-trivial linear system!!

= Not invertible.

$\det(\lambda I - A) = 0$. : characteristic equation of A.

* ex2).

$$A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}. \qquad \underline{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}. \quad \lambda = 3.$$

$$\det(\lambda I - A) = 0. \longrightarrow \begin{bmatrix} \lambda & \\ & \lambda \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}.$$

$$\det \begin{bmatrix} \lambda-3 & 0 \\ -8 & \lambda+1 \end{bmatrix}. \longleftarrow = \begin{bmatrix} \lambda-3 & 0 \\ -8 & \lambda+1 \end{bmatrix}.$$

$$= (\lambda-3)(\lambda+1) = 0.$$

$\therefore \lambda = 3, \ \lambda = -1.$ "A에 대한 λ는 3과 -1"

degree ~~at most n~~ → 사실 그냥 n이다.

$A : n \times n$ $\qquad \det(\lambda I - A) = \underline{p(\lambda)}$

$\downarrow$ $\qquad\qquad \downarrow$

at most n solutions!! $\qquad$ Characteristic polynomial.

↑
distinct

+) Complex solution을 ~~허용않다~~

일반적으로 복소수.

* ex3).

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}$$

sol)

$$\det(\lambda I - A) = \begin{vmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -4 & +17 & \lambda-8 \end{vmatrix}$$

$$= \lambda \begin{vmatrix} \lambda & -1 \\ 17 & \lambda-8 \end{vmatrix} + (-4) \begin{vmatrix} -1 & 0 \\ \lambda & -1 \end{vmatrix}$$

$$= \lambda(\lambda(\lambda-8)+17) + (-4)\cdot 1$$

$$= \lambda^2(\lambda-8) + 17\lambda - 4$$

$$= \lambda^3 - 8\lambda^2 + 17\lambda - 4$$

$$= \lambda(\lambda-4)^2 + \lambda - 4$$

$$= (\lambda-4)(\lambda^2-4\lambda+1). \quad = 0.$$

$$\therefore \lambda = 4, \ 2 \pm \sqrt{3}$$

NO.

*ex 4)

$$A = \begin{bmatrix} \lambda - a_{11} & -a_{12} & -a_{13} & -a_{14} \\ & \lambda - a_{22} & -a_{23} & -a_{24} \\ & & \lambda - a_{33} & -a_{34} \\ & & & \lambda - a_{44} \end{bmatrix}$$

sol) $\det(\lambda I - A) \Rightarrow (\lambda - a_{11})(\lambda - a_{22}) \cdots (\lambda - a_{44}) = 0$.

$\lambda = a_{11},\ \lambda = a_{22},\ \cdots,\ \lambda = a_{44}$.

↳ diagonal values are eigenvalues.

when A is (upper / lower triangular, diagonal) matrix.

*thm 5.1.3.) A : $n \times n$. Equivalents.

(a) λ : eigenvalue of A.

(b) λ : solution of $\det(\lambda I - A) = 0$.

(c) $(\lambda I - A)\underline{x} = \underline{0}$ has non trivial solutions.

(d) $\exists \underline{x} \neq \underline{0}$ such that $A\underline{x} = \lambda \underline{x}$.

"존재한다, x가"

0이 아닌 x가 존재

Finding Eigenvectors and Bases for Eigenspaces.

$\{\underline{x} \neq \underline{0} \mid (\lambda I - A)\underline{x} = \underline{0}\}$

1. Null space of $\lambda I - A$.
2. the kernel of $T_{\lambda I - A} : R^n \to R^n$
3. Set vectors for which $A\underline{x} = \lambda \underline{x}$

*ex6)

$$A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$$

sol) $(\lambda I - A)\underline{x} = \underline{0} \rightarrow \det(\lambda I - A) = 0$

$$\begin{vmatrix} \lambda+1 & -3 \\ -2 & \lambda \end{vmatrix} = \lambda(\lambda+1) - 6 = \underbrace{\lambda^2 + \lambda - 6 = 0}_{\text{Charactristic equation.}}$$

eigenspace는 저걸 만족하는 $\underline{x}$의 집합.

$$\therefore \overset{(i)}{\lambda = 2}, \quad \overset{(ii)}{\lambda = -3}.$$

$$\begin{bmatrix} \lambda+1 & -3 \\ -2 & \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$\lambda = 2$

$$\begin{bmatrix} 3 & -3 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad \begin{matrix} x_1 - x_2 = 0. \\ x_1 = x_2 = t. \end{matrix} \rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} t \\ t \end{bmatrix} = t \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{\text{salution space.}}$$

(1,1)에 t를 곱한 모든 vector.

basis = (1,1).

$\lambda = -3$

$$\begin{bmatrix} -2 & -3 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{matrix} 2x_1 + 3x_2 = 0 \\ x_1 = -3/2\, t. \\ x_2 = t. \end{matrix} \rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = t \begin{bmatrix} -3/2 \\ 1 \end{bmatrix}$$

∴ 2개의 eigen value에 대해

$\lambda = 2$	$\lambda = -3$
dim(e.s) = 1	dim(e.s) = 1.
basis = (1,1)	basis = $(-3/2, 1)$.

*ex 4)

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

sol) $\det(\lambda I - A) = 0$.

$$\begin{vmatrix} \lambda & -0 & +2 \\ -1 & \lambda & -1 \\ -1 & -0 & \lambda-3 \end{vmatrix} = (\lambda-2)\,(-1)^{2+2} \begin{vmatrix} \lambda & 2 \\ -1 & \lambda-3 \end{vmatrix}.$$

$$= (\lambda-2)(\lambda(\lambda-3)+2) = 0.$$

$$= (\lambda-2)\,(\lambda-2)(\lambda-1) = 0.$$

$\lambda = 1$, $\lambda = 2$.

(i) $\lambda = 1$. eigenspace?

$(\lambda I - A)\underline{x} = \underline{0}$
$\downarrow$
e.s.

$$\begin{bmatrix} 1 & 0 & 2 \\ -1 & -1 & -1 \\ -1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

(+) ~~param이 2개있다..!~~ → 줄어있고.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2s \\ s \\ s \end{bmatrix} = s\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \longrightarrow \text{eigenspace.}$$

(ii) $\lambda = 2$.

$$\begin{bmatrix} 2 & 0 & 2 \\ -1 & 0 & -1 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \rightarrow x_1 + x_3 = 0.$$

$x_1 = -t.$
$x_2 = s.$
$x_3 = t.$

$$\Downarrow$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = s\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}.$$

NO.

Eigenvalues and Invertibility.

* thm 5.1.4) A: invertible $\Leftrightarrow$ $\lambda = 0$ is not an eigenvalue of A.

∴ 행렬 A가 eigenvalue를 가지려면 가역할 수 없다
가역하면 eigenvalue를 갖지 못한다.

Equivalence theorem.

* Thm 5.1.5) (ㄴ)