Chap 1.6.

#2.

a. elementary

b. elementary.

c. elementary.

d. not elementary

4.

a. ~~$\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$~~

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$

c. $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 & \frac{1}{7} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

5

a. $\frac{1}{-16+15}\begin{bmatrix} -16 & 5 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 16 & -5 \\ 3 & -1 \end{bmatrix}$

b. not invertible.

#16.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 1 & 3 & 5 & 0 \\ 1 & 3 & 5 & 7 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 1 & 3 & 5 & 0 \\ 1 & 3 & 5 & 7 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -\frac{2}{7} & \frac{1}{7} \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ 3 & 5 & 0 & 0 \\ 3 & 5 & 7 & 0 \\ 3 & 5 & 7 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 5 & 7 & 0 \\ 0 & 5 & 7 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 5 & 7 & 0 \\ 0 & 5 & 7 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 5 & 7 & 0 \\ 0 & 5 & 7 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 7 & 0 \\ 0 & 5 & 7 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 7 & 0 \\ 0 & 5 & 7 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 7 & 0 \\ 0 & 0 & 7 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 7 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & -\frac{2}{7} & \frac{1}{7} \end{bmatrix}$$

#20

a. $\begin{bmatrix} 1/k_4 & 0 & 0 & 0 \\ 0 & 1/k_3 & 0 & 0 \\ 0 & 0 & 1/k_2 & 0 \\ 0 & 0 & 0 & 1/k_1 \end{bmatrix}$

b. $\begin{bmatrix} 1/k & 0 & 0 & 0 \\ -1/k^2 & 1/k & 0 & 0 \\ 1/k^3 & -1/k^2 & 1/k & 0 \\ -1/k^4 & 1/k^3 & -1/k^2 & 1/k \end{bmatrix}$

Chap 1-6.

#2.

$$\begin{bmatrix} 4 & -3 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -3 \\ 2 & -5 \end{bmatrix}^{-1}\begin{bmatrix} 4 & -3 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 2 & -5 \end{bmatrix}^{-1}\begin{bmatrix} 3 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{-20+6}\begin{bmatrix} 21 \\ 63 \end{bmatrix}$$

$$= \frac{1}{-14}\begin{bmatrix} 21 \\ 63 \end{bmatrix}$$

$$= \begin{bmatrix} -3/2 \\ -9/2 \end{bmatrix}$$

$$\therefore (x_1, x_2) = (-3/2, -9/2).$$

#10.

$$\left[\begin{array}{ccc|ccc} 1 & 4 & 1 & 1 & 0 & 0 \\ 1 & 9 & -2 & 0 & 1 & 0 \\ 6 & 4 & -8 & 0 & 0 & 1 \end{array}\right] \cdot \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 32 & -18 & 8 \\ 0 & 1 & 0 & 2 & -1 & 1/2 \\ 0 & 0 & 1 & 25 & -14 & 6 \end{array}\right]$$

$$0\ 13\ -1\ |\ 1\ 1\ 0$$

$$0\ 28\ 2\ |\ 6\ 0\ 1$$

$$1\ 4\ 1\ |\ 1\ 0\ 0$$

$$0\ 2\ 0\ |\ 4\ -2\ 1$$

$$0\ 1\ 0\ |\ 2\ -1\ \tfrac{1}{2}$$

$$1\ 0\ 1\ |\ 7\ -4\ 2$$

$$0\ 0\ -1\ |\ -26\ 14\ -\tfrac{13}{2}$$

$$0\ 0\ 1\ |\ 25\ -14\ 6$$

$$1\ 0\ 0\ |\ 32\ -18\ 8$$

$$A^{-1} = \begin{bmatrix} 32 & -18 & 8 \\ 2 & -1 & 1/2 \\ 25 & -14 & 6 \end{bmatrix}$$

$$\text{i.} \begin{bmatrix} 32 & -18 & 8 \\ 2 & -1 & 1/2 \\ 25 & -14 & 6 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -18 \\ -1 \\ -14 \end{bmatrix}$$

ii. $\begin{bmatrix} 32 & -18 & 8 \\ 2 & -1 & 1/2 \\ 25 & -14 & 6 \end{bmatrix}\begin{bmatrix} -3 \\ 4 \\ -5 \end{bmatrix} = \begin{bmatrix} -3(32)+4(-18)-5(8) \\ -3(2)+4(-1)-5(1/2) \\ -3(25)+4(-14)-5(6) \end{bmatrix} = \begin{bmatrix} -208 \\ -25/2 \\ -161 \end{bmatrix}$

#16.

~~1 -2 -1 $b_1$~~

~~-4 5 2 $b_2$~~

~~-4 7 4 $b_3$~~

~~0 -2 2 $b_2-b_2$~~

~~1 0 1 $b_1-b_2+b_3$~~

~~0 7 8 $4b_1-4b_2+5b_3$~~

~~0 1 1 $-\frac{1}{2}b_2+\frac{1}{2}b_3$~~

0 0 1 $4b_1-\frac{1}{2}b_2-\frac{3}{2}b_3$

1 0 0 $5b_1-\frac{3}{2}b_2-\frac{1}{2}b_3$

0 1 0 $-4b_1+2b_3$

$\begin{cases} 5b_1-\frac{3}{2}b_2-\frac{1}{2}b_3 \neq 0 \\ 4b_1-\frac{1}{2}b_2-\frac{3}{2}b_3 \neq 0 \\ -4b_1+2b_3 \neq 0 \end{cases}$

$\therefore 2b_1 \neq b_3$

$8b_1 \neq b_2+3b_3$

$10b_1 \neq 3b_2+b_3.$

#20

~~-2 0 1 1 0 0~~

~~0 1 1 0 1 0~~

~~1 1 4 0 0 1~~

~~0 2 7 1 0 2~~

~~1 0 -5 0 1 1~~

~~0 0 9 0 3 1~~

0 0 1 0 $-\frac{1}{3}$ $-\frac{1}{9}$

~~0 1 0 0 $\frac{2}{3}$ $-\frac{1}{9}$~~

0 1 0 0 $-\frac{2}{3}$ $\frac{1}{9}$

1 0 0 0 $-\frac{2}{3}$ $\frac{4}{9}$

$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \; -2/3 \; 4/9 \\ 0 & 1 & 0 & 0 \; -1/3 \; 1/9 \\ 0 & 0 & 1 & 0 \; -1/3 \; -1/9 \end{array}\right]$

$\therefore \begin{bmatrix} 0 & -2/3 & 4/9 \\ 0 & -1/3 & 1/9 \\ 0 & -1/3 & -1/3 \end{bmatrix}\begin{bmatrix} 4 & 3 & 2 & 1 \\ 6 & 7 & 8 & 9 \\ 1 & 3 & 7 & 9 \end{bmatrix}$

$= \begin{bmatrix} -4+4/9 & -14/3+4/3 & -16/3+28/9 & -6+4 \\ -2+1/9 & -7/3+1/3 & -8/3+7/9 & -3+1 \\ -2-1/3 & -7/3-1 & -8/3-7/3 & -3-3 \end{bmatrix}$

$= \begin{bmatrix} -32/9 & -10/3 & -20/9 & -2 \\ -17/9 & -2 & -17/9 & -2 \\ -7/3. & -10/3 & -5 & -6 \end{bmatrix}$

Chap 1.7.

#4

$$\begin{bmatrix} -4(-2) & 0 & 0 \\ 0 & 3(1) & 0 \\ 0 & 0 & 2(-5) \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -10 \end{bmatrix}$$

#10.

$$A^2 = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 16 & 0 & 0 \\ 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}, \quad A^{-1} = \begin{bmatrix} -\frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{4} & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix},$$

$$A^k = \begin{bmatrix} (-2)^k & 0 & 0 & 0 \\ 0 & (-4)^k & 0 & 0 \\ 0 & 0 & (-3)^k & 0 \\ 0 & 0 & 0 & 2^k \end{bmatrix}$$

#20 invertible

#22 not invertible.

#26.

$3 = a - 2b + 2c$ $\quad c = -2 - a.$ $\quad 3 = a - 2(2-a) + 2(-2-a)$

$0 = 2a + b + c$ $\quad 0 = 2a + b - 2 - a$ $\quad 3 = a - 8.$

$-2 = a + c.$ $\quad 2 = a + b.$ $\quad b = 2 - a.$

$\therefore (a, b, c) = (11, -9, -13).$

#32

$$A = \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Chap 1.8.

#12. a. $\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 3 & -2 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. $\therefore \begin{bmatrix} -1 & 1 \\ 3 & -2 \\ 5 & -7 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

#14. a. $Ax = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

b. $Ax = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

c. $Ax = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix} x$

d. $Ax = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & -8 \end{bmatrix} x$.

#18 a. $\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} x = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} -6 \\ 0 \end{bmatrix}$

#18 b. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ 0 \end{bmatrix}$

#28. $\begin{bmatrix} 2 & 1 & 3 \\ -3 & -1 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6+2+3 \\ -9-2+0 \\ 3+0+2 \end{bmatrix} = \begin{bmatrix} 11 \\ -11 \\ 5 \end{bmatrix}$

#46 $T \cdots Ax$

$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$\begin{cases} a + c = 2 & a+b+c = 1 & -3a-b+2c = -5 \\ d + f = -3 & d+e+f = -3 & -3d-e+2f = -11 \\ g + i = 10. & g+h+i = 8. & -3g-h+i = 9. \end{cases}$

$\downarrow$

$a + c = 2$ $\quad d = -3 - f$ $\quad -3g + i = 5.$

$-3a + 2c = -5.$ $\quad -3d - 12 - 2d = -11.$ $\quad -3g + (10 - g) = 5.$

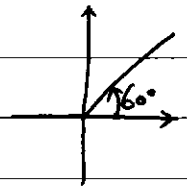
$a = 9/5,\ c = 1/5$ $\quad d = -1/5,\ f = -14/5.$ $\quad g = 4/5,\ i = 46/5.$

$b = -1$ $\quad e = 6$ $\quad h = -2.$

$\therefore \begin{bmatrix} 9/5 & -1 & 1/5 \\ -1/5 & 6 & -14/5 \\ 4/5 & -2 & 46/5 \end{bmatrix}$

Chap 1.9.

#8. a.




reflection $-\cos 60°$

$$\begin{bmatrix} \cos 60° & 0 \\ 0 & \sin 60° \end{bmatrix} = \begin{bmatrix} -1/2 & 0 \\ 0 & 0 \end{bmatrix}$$

projection.

b. $\begin{bmatrix} -\sqrt{2}/2 & 0 \\ 0 & 0 \end{bmatrix}$

c. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

#10. a. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

b. $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

c. $\begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

#12 a. $T_1: \begin{bmatrix} 4 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & -3 & 0 \end{bmatrix}$, $T_2: \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & -1 \\ 4 & 0 & -1 \end{bmatrix}$

b. $T_1T_2: \begin{bmatrix} 4 & 8 & 0 \\ -2 & -4 & -1 \\ -1 & -2 & 3 \end{bmatrix}$, $T_2T_1: \begin{bmatrix} 0 & 2 & 0 \\ 1 & 3 & 0 \\ 17 & 3 & 0 \end{bmatrix}$

#12.

c. $T_1(T_2(x_1, x_2, x_3))$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & -1 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & -3 & 0 \end{bmatrix} \times$$

$$= \begin{bmatrix} 0 & 2 & 0 \\ 1 & 3 & 0 \\ 17 & 3 & 0 \end{bmatrix}$$

$T_2(T_1(x_1, x_2, x_3))$

$$\Rightarrow \begin{bmatrix} 4 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & -3 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & -1 \\ 4 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 8 & 0 \\ -2 & -4 & -1 \\ -1 & -2 & 3 \end{bmatrix}$$

#20.

a. ~~1 2 2 1 0 0~~

~~2 1 1 0 1 0~~

~~1 1 0 0 0 1~~

~~0 1 2 1 0 -1~~

~~1 0 1 0 1 -1~~

~~0 1 -1 0 -1 2~~

~~0 0 3 1 1 -3~~

0 0 1 $1/3$ $1/3$ $-1$

0 1 0 $1/3$ $-2/3$ 1

1 0 0 $-1/3$ $5/3$ $-2$.

$$T^{-1}: \begin{bmatrix} 1/3 & 1/3 & -1 \\ 1/3 & -2/3 & 1 \\ -1/3 & 5/3 & -2 \end{bmatrix} \times$$

b. 1 $-3$ 4 1 0 0

~~1 1 1 0 1 0~~

6 $-2$ 5 0 0 1

0 $-2$ 5 1 1 0

$\therefore$ Not invertible.

#24.

a. 1 2 0 | 1 0 0

~~1 1 1 — 0 1 0~~

~~2 3 1 — 0 0 1~~

0 1 -1 | 1 -1 0

0 -1 1 | -2 0 1

Not invertible.

b. ~~1 1 0 | 1 0 0~~

~~0 1 1 | 0 1 0~~

~~1 0 1 | 0 0 1~~

~~0 1 1 | -1 0 1~~

~~0 0 2 | -1 1 1~~

0 0 1 | $-\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

~~0 -1 0 | $-\frac{1}{2}$ $-\frac{1}{2}$ $\frac{1}{2}$~~

0 1 0 | $\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$

1 0 0 | $\frac{1}{2}$ $-\frac{1}{2}$ $\frac{1}{2}$

$$\therefore \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$