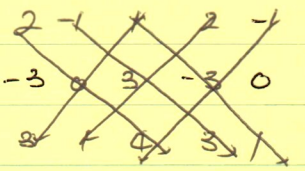
Exercise Set 2.1.

#4.

a. $M_{32} = \begin{vmatrix} 2 & -1 & 1 \\ -3 & 0 & 3 \\ 3 & 1 & 4 \end{vmatrix}$



$(-9-3)-(6+12)$

$= -30$

$= -30$

$a_{32} = 30.$

b. $M_{44} = \begin{vmatrix} 2 & 3 & -1 \\ -3 & 2 & 0 \\ 3 & -2 & 1 \end{vmatrix}$

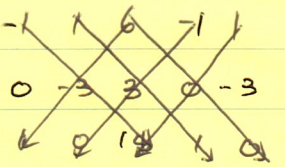
$(4-6)-(-6-9)$

$= 13$

$= 13$

$a_{44} = 13.$

C. $M_{41} = \begin{vmatrix} -1 & 1 & 6 \\ 0 & -3 & 3 \\ 1 & 0 & 14 \end{vmatrix}$



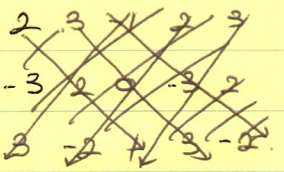
$-18-(42+3)$

$= -63$

$= -63.$

$a_{41} = 63.$

d. $M_{24} = \begin{vmatrix} 2 & 3 & -1 \\ 3 & -2 & 1 \\ 3 & -2 & 1 \end{vmatrix}$



$= 18.$

$a_{24} = 18.$

$2\ 3\ -1\ 2\ 3$

$3\ -2\ 1\ 3\ -2$

$3\ -2\ 1\ 3\ -2.$

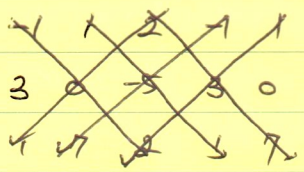
$(-4+9+6)-(-12-4+9)$

$= 11+7$

#6. Not invertible.

#8
$$\frac{1}{\sqrt{2}\sqrt{3}-4\sqrt{6}}\begin{bmatrix}\sqrt{3} & -\sqrt{6}\\ -4 & \sqrt{2}\end{bmatrix}$$
$$= -\frac{1}{3\sqrt{6}}\begin{bmatrix}\sqrt{3} & -\sqrt{6}\\ -4 & \sqrt{2}\end{bmatrix}$$
$$= -\frac{\sqrt{6}}{3\cdot 6}\begin{bmatrix}\sqrt{3} & -\sqrt{6}\\ -4 & \sqrt{2}\end{bmatrix}$$
$$= \begin{bmatrix}-\sqrt{2}/6 & 1/3\\ 2\sqrt{6}/9 & -\sqrt{3}/3\end{bmatrix}$$

#12.



$(-5+21)-(35+6)$

$= -4.$

$\therefore -4.$

#16

$$\begin{vmatrix}\lambda-4 & 0 & 0 & \lambda-4 & 0\\ 0 & \lambda & 2 & 0 & \lambda\\ 0 & 3 & \lambda-1 & 0 & 3\end{vmatrix}$$

$(\lambda-4)\lambda(\lambda-1)-6(\lambda-4).$

$=(\lambda-4)(\lambda^2-\lambda-6)$

$=(\lambda-4)(\lambda-3)(\lambda+2).$

$\lambda=4,\ \lambda=3,\ \lambda=-2$ then $\det(A)=0.$

#22.
$$\det(A) = -\begin{vmatrix}3 & 1\\ 1 & 5\end{vmatrix} - 4\begin{vmatrix}3 & 1\\ 1 & 5\end{vmatrix}$$
$$= -14 - 42$$
$$= -56.$$

#32. 18.

Exercise set 2.2.

#12:

$$\begin{vmatrix} 1 & -3 & 0 \\ -2 & 4 & 1 \\ 5 & -2 & 2 \end{vmatrix} = \begin{vmatrix} 1 & -3 & 0 \\ -2 & 4 & 1 \\ 9 & -10 & 0 \end{vmatrix}$$

$$= 1 \begin{vmatrix} 1 & -3 \\ 9 & -10 \end{vmatrix}$$

$$= -10 + 27$$

$$= 17$$

#14

$$\begin{vmatrix} 1 & -2 & 3 & 1 \\ 5 & -9 & 6 & 3 \\ -1 & 2 & -6 & -2 \\ 2 & 8 & 6 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -2 & 3 & 1 \\ 0 & -1 & -9 & -2 \\ 0 & 0 & -3 & -1 \\ 0 & 12 & 0 & 1 \end{vmatrix}$$

$$= 1 \begin{vmatrix} -1 & -9 & -2 \\ 0 & -3 & -1 \\ 12 & 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & 0 & 1 \\ 0 & -3 & -1 \\ 13 & 0 & 0 \end{vmatrix}$$

$$= -13 \begin{vmatrix} 0 & 1 \\ -3 & -1 \end{vmatrix}$$

$$= -13(-(-3))$$

$$= -39.$$

#20 $-12$.

#29.

$$\begin{vmatrix} -5 & -4 & 3 \\ -2 & 8^{\circ} & 4 \\ 3 & 2^{\circ} & 5 & 1 \\ 1 & 16^{\circ} & 6 & 5 \\ 4 & -6^{\circ} & 4 & -8 \\ -1 & & 0 & 0 \end{vmatrix} = \begin{vmatrix} -5^{0} & 0 & -4^{1} & 3^{4} \\ 8^{0} & 0 & 5 & 1 \\ 1^{0} & 0 & 6_{1} & 5_{4} \\ -1 & 0 & 0 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 1 & 4 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 1 & 4 \\ -1 & 0 & 0 & 0 \end{vmatrix}$$

$\therefore \det(A) = 0.$

Exercise Set 2.3.

#4

$\det(kA) = k^n \det(A)$

$$\det(kA) = \begin{bmatrix} 3 & 3 & 3 \\ 0 & 6 & 9 \\ 0 & 3 & -6 \end{bmatrix} = 3 \begin{vmatrix} 6 & 9 \\ 3 & -6 \end{vmatrix} = 3(-36-27) = 3 \cdot (-63).$$

$$k^n \det(A) = 3^3 \cdot 1 \begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = 3^3(-7)$$

#12.

$$\det(A) = \begin{vmatrix} 1 & 0 & -1 \\ 9 & -1 & 4 \\ 8 & 9 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & -1 \\ 13 & -1 & 0 \\ 7 & 9 & 0 \end{vmatrix}$$

$$= -1 \begin{vmatrix} 13 & -1 \\ 7 & 9 \end{vmatrix}$$

$\therefore 117$

#22

$\det(A) = 2(24+15)$ ... A is invertible.

$= 2 \cdot 39 > 0$

$\det(A) A^{-1} = \text{adj}(A)$.

$$\begin{array}{cccc|cccc} 1 & 0 & 0 & & 1/2 & 0 & 0 & \\ 8 & 1 & 0 & & 0 & 1 & 0 & \\ 5 & 3 & 6 & & 0 & 0 & 1 & \\ 0 & 1 & 0 & & -4 & 1 & 0 & \\ 0 & 3 & 6 & & \frac{5}{2} & 0 & 1 & \\ 0 & 0 & 1 & & \frac{29}{12} & -3/6 & 1/6 & \end{array}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -4 & 1 & 0 \\ \frac{29}{12} & -1/2 & 1/6 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 39 & 0 & 0 \\ -3/2 & 78 & 0 \\ 39/3 & -39 & 13 \end{bmatrix}$$

#27

$$\det(A) = 4\begin{vmatrix} -3 & 1 \\ -1 & 0 \end{vmatrix} - 3\begin{vmatrix} 1 & -3 \\ 2 & -1 \end{vmatrix}$$

$$= 4 - 3(5)$$

$$= -11$$

$\Rightarrow A$ is invertible.

$$\det(A_1) = 3\begin{vmatrix} 4 & -3 \\ -2 & -1 \end{vmatrix}$$

$$= 3(-10)$$

$$= -30.$$

$$\det(A_2) = 4\begin{vmatrix} 4 & 1 \\ -2 & 0 \end{vmatrix} + 3\begin{vmatrix} 1 & 4 \\ 2 & -2 \end{vmatrix}$$

$$= 4 \cdot 2 + 3(-10)$$

$$= -22.$$

$$\det(A_3) = 4\begin{vmatrix} -3 & 4 \\ 1 & -2 \end{vmatrix}$$

$$= 4(6-4)$$

$$= -8.$$

$$\therefore (x_1, x_2, x_3) = (\det(A_1)/\det(A), \det(A_2)/\det(A), \det(A_3)/\det(A))$$

$$= (30/11, 22/11, 8/11)$$

$$= (30/11, 2, 8/11).$$

#30

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A)$$

$$= 1\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Exercise Set 3.1.

#2. a. $(-5, 0)$.

b. $(-3, 4, 0)$.

#6. a. $(1, -2)$.

b $(-1, 1, -3)$.

#8 a. @ $(5, 10, -8)$

b. @ $(-7, -4, -2)$

#12. a. $(7, 5, -5, -1, 5)$.

b. $(6, 3, -12, 27, -6)$.

C. $(-33, -6, 14, 12, -11)$.

d. $(a/2, a/2, -5, 11/2, -3/2)$.

# 16. a. $t = 2$.

b. Cannot determine $t$.

C. $t = 2\bar{2}$.

#20.

$$\begin{bmatrix} -1 & 2 & 1 \\ 0 & 2 & -2 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} -6 \\ 12 \\ 4 \end{bmatrix}$$

~~$-1 \quad 2 \quad 1 \quad -6$~~

~~$0 \quad 2 \quad -2 \quad 12$~~

~~$2 \quad -2 \quad 1 \quad 4$~~

~~$1 \quad 0 \quad 2 \quad -2$~~

~~$0 \quad 2 \quad 3 \quad 8$~~

~~$0 \quad 0 \quad 5 \quad 20$~~

$0 \quad 0 \quad 1 \quad -4$

~~$0 \quad 2 \quad 0 \quad 4$~~

$0 \quad 1 \quad 0 \quad 2$

$1 \quad 0 \quad 0 \quad -10.$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & & & -10 \\ & 1 & & 2 \\ & & 1 & -4 \end{array}\right]$$

$\therefore (-10, 2, -4)$.

Exercise Set 3.2.

#2. a. $v = (1, -1, 2)$

$\therefore \sqrt{6}$.

b. $\sqrt{29}$.

#6. a. $\sqrt{46} + \sqrt{2}\sqrt{84} + \sqrt{3}\sqrt{42}$.

$= (2\sqrt{2} + \sqrt{6})\sqrt{21} + \sqrt{46}$.

b. $(2-1, -2+3, 3-4)$.

$\sqrt{3}\sqrt{36+4+1+1}$.

$= 3\sqrt{14}$.

#12. a. $\sqrt{16+1+25+4}$

$= \sqrt{46}$.

b. $\sqrt{4+1+4+1}$

$= \sqrt{10}$.

#19. a. The circle that has center as $r_0$ and radius as 1.

b. The circle that has center as $r_0$ and radius ~~lesser~~ than 1.

c. The circle that has center as $r_0$ and radius greater than 1.

#26.


$\arg\min \|U-W\| = 1$

$\arg\max \|U-W\| = 5$.

$\therefore \min(\text{smallest}) = 1$.

$\max(\text{largest}) = 5$.

Exercise Set 3.3

#4. $\underline{P} \cdot \underline{n} = 0$.

$\therefore n$ is orthogonal, $n$ is normal.

#8. $(1, -4, -3)$ $(3, -12, -9)$ Let $P_0 \left(\frac{7}{3}, 0, 0\right)$ that on $3x - 12y - 9z - 7 = 0$.

$$D = \frac{7/3}{\sqrt{1+16+9}} > 0$$

$\therefore$ given planes are parallel.

#14. a. $\frac{u \cdot a}{\|a\|^2} a = \frac{10-6}{5} (2, -1)$

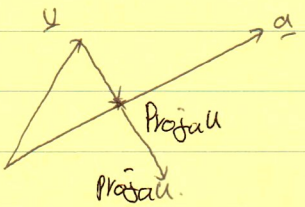
$= 4/5 (2, -1)$.

$\therefore (8/5, -4/5)$.

b. $\frac{3 - 4 - 42}{1 + 4 + 49} (1, 2, -7) = -\frac{43}{54} (1, 2, -7)$.

$\therefore (-43/54, -43/27, 301/54)$

#20




$\text{Proj}_a u = \frac{10+1+3-7}{4+1+1+1} (2, 1, -1, -1)$.

$2\text{Proj}_a u = (4, 2, -2, -2)$.

$\therefore (4, 2, -2, -2)$.

#30

a. $v \cdot w = (a(-b)) + ab = 0$.

b. $(3, 2)$

$(-3, -2)$.

c. $(4, 3) \Rightarrow \left(\frac{4\sqrt{5}}{5}, \frac{3\sqrt{5}}{5}\right)$.

$(-4, -3) \Rightarrow \left(-\frac{4\sqrt{5}}{5}, -\frac{3\sqrt{5}}{5}\right)$.