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Exercise Set 4.6.

#2.

~~3 1 1 1~~
~~5 -1 1 -1~~
~~8 4 2 0~~
~~4 0 1 0~~

$$\left[\begin{array}{cccc|c} +1 & -1 & 0 & -1 & 0 \\ 0 & 4 & 1 & 4 & 0 \end{array}\right]$$

$x_1 - x_2 \quad - x_4 = 0.$

$4x_2 + x_3 + 4x_4 = 0$

$$\begin{cases} x_1 = t + s \\ x_2 = t \\ x_3 = -4t - 4s \\ x_4 = s \end{cases}$$

$S.S = [\, t+s,\ t,\ -4t-4s,\ s \,]$

$\dim(S.S) = 2.$

#6.

~~1 1 1~~
~~3 2 -2~~
~~4 3 1~~
~~6 5 1~~
0 1 5
~~0 -1 -5~~
0 0 0
~~0 -1 -5~~
0 0 0
1 0 -4

$$\Rightarrow \begin{matrix} 1 & 0 & -4 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0. \end{matrix}$$

$x_1 \quad -4x_3 = 0.$

$x_2 + 5x_3 = 0.$

$$\begin{cases} x_1 = 4t. \\ x_2 = -5t. \\ x_3 = t. \end{cases}$$

S.S = [undefined...

$\dim(S.S) = 0.$

#8.

a. basis: {(1, 0, 0, 0), (0, 1, 0, 0), (0, 0, 1, 0)}.

dim: 3.

b. (a, b, a-b, a+b)

basis: {(1, 0, 1, 1), (0, 1, -1, 1)}.

dim: 2.

#8. c. basis: $\{(1,1,1,1)\}$.

dim $= 1$.

#12. a. ~~$-1, 2, 3$~~

~~$1, 2, -2$~~

$0 \ 0 \ 1$

$1 \ -2 \ 0$

basis: $\{(0,0,1), (1,-2,0)\}$.

dim: 2.

$\therefore$ Cannot be added.

b. $1 \ -1 \ 0$

~~$3 \ 1 \ 2$~~

~~$4 \ 0 \ 2$~~

$2 \ 0 \ -1$

basis: $\{(1,-1,0), (2,0,-1)$.

dim: 2.

$\therefore$ Cannot be added.

#14

$\{\underline{u}_1, \underline{u}_2, \underline{u}_3\} = \{\underline{v}_1, \underline{v}_1 + \underline{v}_2, \underline{v}_1 + \underline{v}_2 + \underline{v}_3\}$. $\subset V$.

$\{\underline{u}_1, \underline{u}_2, \underline{u}_3\} + \{\underline{u}_4, \underline{u}_5, \underline{u}_6\} = \{\underline{v}_1 + \underline{v}_4, \underline{v}_1 + \underline{v}_2 + \underline{v}_4 + \underline{v}_5, \underline{v}_1 + \underline{v}_2 + \underline{v}_3 + \underline{v}_4 + \underline{v}_5 + \underline{v}_6\}$ ... v.a.

$k\{\underline{u}_1, \underline{u}_2, \underline{u}_3\} = \{k\underline{v}_1, k\underline{v}_1 + k\underline{v}_2, k\underline{v}_1 + k\underline{v}_2 + k\underline{v}_3\}$. ... s.m.

$\subset V$.

$\therefore \{\underline{u}_1, \underline{u}_2, \underline{u}_3\}$ is basis.

Exercise Set 4.7.

#4

$\cancel{-6 \quad -2 \quad -2 \mid -3 \quad -3 \quad 1}$

$\cancel{-6 \quad -6 \quad -3 \mid 0 \quad 2 \quad 6}$

$\cancel{0 \quad 4 \quad 7 \mid -3 \quad -1 \quad -1}$

$\cancel{0 \quad 4 \quad 1 \quad -3 \quad 5 \quad 5}$

$\cancel{0 \quad 0 \quad 6 \quad 0 \quad 4 \quad 4}$

$0 \quad 0 \quad 1 \quad 0 \quad 2/3 \quad 2/3$

$\cancel{0 \quad 4 \quad 0 \quad -3 \quad 17/3 \quad -17/3}$

$0 \quad 1 \quad 0 \quad -3/4 \quad -17/12 \quad -17/12$

$\cancel{-6 \quad 0 \quad 0 \quad 3-6/2, \; -3-17/6+4/3, \; 1-17/6+4/3}$

$-6 \quad \cancel{3-9/6}\,3/2. \quad 1-9/6\,3/2$

$-9/2 \quad -1/2.$

$1 \quad 0 \quad 0 \quad 1 \quad 1/4 \quad 1/12$

$$\therefore \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1/4 & 1/12 \\ 0 & 1 & 0 & -3/4 & -17/12 & -17/12 \\ 0 & 0 & 1 & 0 & 2/3 & 2/3 \end{array}\right]$$

$$\therefore P_{W' \to W} = \begin{bmatrix} 1 & 1/4 & 1/12 \\ -3/4 & -17/12 & -17/12 \\ 0 & 2/3 & 2/3 \end{bmatrix}.$$

#6.

$$\left[\begin{array}{cc|cc} 6 & 10 & 2 & 3 \\ 3 & 2 & 0 & 2 \end{array}\right].$$

$\cancel{6 \quad 10 \mid 2 \quad 3}$

$\cancel{3 \quad 2 \mid 0 \quad 2}$

$\cancel{0 \quad 6 \quad 2 \quad -1}$

$\cancel{0 \quad 2 \quad 2/3 \quad -1/3}$

$\cancel{3 \quad 0 \quad -2/3 \quad 7/3}$

$0 \quad 1 \quad 1/3 \quad -1/6$

$1 \quad 0 \quad -2/9 \quad 7/9.$

$$\left[\begin{array}{cc|cc} 1 & 0 & -2/9 & 7/9 \\ 0 & 1 & 1/3 & -1/6 \end{array}\right]$$

$B \to B'$

$\cancel{6 \quad 10 \mid 2 \quad 3}$

$\cancel{3 \quad 2 \mid 0 \quad 2}$

$\cancel{9/2 \quad 3 \quad 0 \quad 3}$

$\cancel{3/2 \quad 7 \quad 2 \quad 0}$

$3/2 \quad 1 \quad 0 \quad 1$

$3/4 \quad 7/2. \quad 1 \quad 0$

$$\left[\begin{array}{cc|cc} 1 & 0 & 3/4 & 7/2 \\ 0 & 1 & 3/2 & 1 \end{array}\right]$$

$B \leftarrow B'$

a. $\begin{bmatrix} 3/4 & 7/2 \\ 3/2 & 1 \end{bmatrix}$

b. $\begin{bmatrix} -2/9 & 7/9 \\ 1/3 & -1/6 \end{bmatrix}$

#6

c. $\begin{bmatrix} -2/9 & 7/9 \\ 1/3 & -1/6 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \end{bmatrix} = \begin{bmatrix} -1/9 \\ -9/6 \end{bmatrix}.$

$\therefore \begin{bmatrix} -1/9 \\ -3/2 \end{bmatrix}.$

#8.

a. $\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}^{-1} = \frac{1}{8+3} \begin{bmatrix} 4 & 3 \\ -1 & 2 \end{bmatrix}$

$\therefore \begin{bmatrix} 4/11 & 3/11 \\ -1/11 & 2/11 \end{bmatrix}$

b. $\left[\begin{array}{cc|cc} 1 & 0 & 2 & -3 \\ 0 & 1 & 1 & 4 \end{array}\right] \quad \therefore \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}.$

c. $\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} 4/11 & 3/11 \\ -1/11 & 2/11 \end{bmatrix}.$

d. $[w]_B = \begin{bmatrix} 19 \\ -7 \end{bmatrix}.$

$[w]_S = P_{B \to S} [w]_B$

$= \begin{bmatrix} 4/11 & 3/11 \\ -1/11 & 2/11 \end{bmatrix} \begin{bmatrix} 19 \\ -7 \end{bmatrix}$

$= \begin{bmatrix} (76-21)/11 \\ (-19-14)/11 \end{bmatrix}$

$= \begin{bmatrix} 5 \\ -3 \end{bmatrix}. \quad \therefore (5, -3).$

$\therefore (5, -3)$

e. $[w]_S = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$

$[w]_B = P_{S \to B} [w]_S = \begin{bmatrix} 4 & 3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ -5 \end{bmatrix}$

$= \begin{bmatrix} 12-15 \\ -3-10 \end{bmatrix} = \begin{bmatrix} -3 \\ -13 \end{bmatrix}.$

$\therefore (-3, -13).$

#12.

$$\left[\begin{array}{cc|cc} 3 & 1 & 7 & 2 \\ 5 & 2 & 4 & -1 \end{array}\right].$$

$\cancel{3 \quad 1 \quad 7 \quad 2}$. $\qquad \cancel{3 \quad 1 \quad 7 \quad 2}$.

$\cancel{5 \quad 2 \quad 4 \quad -1}$. $\qquad \cancel{5 \quad 2 \quad 4 \quad -1}$

$+1 \quad 0 \quad +10 \quad +5.$ $\qquad \cancel{+3\ 5 \quad 15 \quad 0}$.

$0 \quad 1 \quad -23 \quad -13$ $\qquad \frac{13}{15} \quad \frac{1}{3} \quad 1 \quad 0.$

$B_1 \to B_3$. $\qquad \cancel{\frac{75}{15} \quad \frac{52}{15} \quad \frac{4}{3} \quad 0 \quad -1}$.

$-\frac{23}{15} \quad -\frac{2}{3} \quad 0 \quad 1$

$$P_{B_3 \to B_1} = \begin{bmatrix} \frac{13}{15} & \frac{1}{3} \\ -\frac{23}{15} & -\frac{2}{3} \end{bmatrix}.$$

#16.

$$P_{? \to w} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 2 \\ 0 & 1 & 1 \end{bmatrix}.$$

$$w = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

$$\left.\begin{array}{ccc} a & b & c \\ d & e & f \\ \cancel{g} & \cancel{h} & \cancel{i} \end{array}\right| \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0. \end{array}$$

$1 \quad 0 \quad 0.$

$a-1 \quad b \quad c \qquad 0 \quad 1 \quad 1.$

$\overset{\shortparallel}{0} \quad \overset{\shortparallel}{0} \quad \overset{\shortparallel}{1}$

$d-1 \quad e \quad f \qquad 0 \quad 1 \quad 0.$

$d-1 \quad e \quad f+2 \qquad 0 \quad 3 \quad 2.$

$\overset{\shortparallel}{0} \quad \overset{\shortparallel}{1} \quad \overset{\shortparallel}{0}.$

$a=1. \quad b=0 \quad c=1.$
$d=1 \quad e=1 \quad f=-2.$
$g=1 \quad h=0 \quad i=0.$

$$\therefore \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & -2 \\ 1 & 0 & 0 \end{bmatrix}.$$

Exercise Set 4.8.

#4.

a. $\left\{ \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \right\}$

1 -1 1
~~-1 1 -1~~ 0.
~~-1 -1 1~~.
0 0 0.
2 0 0.

=>

1 -1 1
2 0 0
0 0 0.

∴ b is in the column space.

b. $\begin{bmatrix} \cancel{1 \quad 2 \quad 0 \quad 1} \\ \cancel{0 \quad 1 \quad 2 \quad 1} \\ \cancel{1 \quad 2 \quad 1 \quad 3} \\ \cancel{0 \quad 1 \quad 2 \quad 2} \end{bmatrix}$

0 0 0 1
~~0 0 0 2~~
0 0 0 0
~~1 2 0 0~~.
0 1 2 0.
1 0 -4 0

1 0 -4 0
0 1 2 0
0 0 0 1
0 0 0 0.
s t ~~-4s+2t~~ k.
4 3 ~~-4+6~~ -10 7.

∴ b is not in the column space.

#8.

a.

$$\begin{array}{cccc|c|c} 1 & -2 & 1 & 2 & -1 & 0 \\ \cancel{2} & \cancel{-4} & \cancel{2} & \cancel{4} & \cancel{-2} & \cancel{0} \\ \cancel{-1} & \cancel{2} & \cancel{-1} & \cancel{-2} & \cancel{1} & 0 \\ \cancel{3} & \cancel{-6} & \cancel{3} & \cancel{6} & \cancel{-3} & \cancel{0} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$x_1 - 2x_2 + x_3 + 2x_4 = -1 \quad \cdots Ax = b.$

$x_1 - 2x_2 + x_3 + 2x_4 = 0 \quad \cdots Ax = \underline{0}$

$\therefore$ General Solution of $Ax = \underline{b}$. $:= \left\{ \begin{array}{l} x_1 = 2s - t - 2k - 1. \\ x_2 = s \\ x_3 = t. \\ x_4 = k. \end{array} \right\}$.

General Solution of $Ax = \underline{0}$ $: \left\{ \begin{array}{l} x_1 = 2s - t - 2k \\ x_2 = s \\ x_3 = t \\ x_4 = k \end{array} \right\}$.

b.

$$\begin{array}{cccc|c|c} 1 & 2 & -3 & 1 & 4 & 0 \\ \cancel{-2} & \cancel{1} & \cancel{2} & \cancel{1} & \cancel{-1} & \cancel{0} \\ \cancel{-1} & \cancel{3} & \cancel{-1} & \cancel{2} & \cancel{3} & \cancel{0} \\ \cancel{4} & \cancel{-7} & \cancel{0} & \cancel{-5.} & \cancel{-5} & \cancel{0} \end{array}$$

$$\begin{array}{cccccc} 0 & -5 & 4 & -3 & -7 & 0 \\ \cancel{0} & \cancel{5} & \cancel{-4} & \cancel{3} & \cancel{7} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \cancel{-1} & \cancel{3} & \cancel{-1} & \cancel{2} & \cancel{3} & \cancel{0} \\ \cancel{0} & \cancel{5} & \cancel{-4} & \cancel{3} & \cancel{7} & \cancel{0} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$x_1 + 2x_2 - 3x_3 + x_4 = 4$

$-5x_2 + 4x_3 - 3x_4 = -7.$

$$\cancel{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 2 \\ -5 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -3 \\ 4 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} 1 \\ -3 \\ 0 \\ 0 \end{bmatrix} x_4 + \begin{bmatrix} -4 \\ 7 \\ 0 \\ 0 \end{bmatrix} =}$$

#8.

b.

$$\begin{matrix} 1 & 2 & -3 & 1 \\ 0 & -5 & 4 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0. \end{matrix}$$

$x_2 = (4t - 3k)/(-5) = -4/5\, t + 3/5\, k.$

$x_3 = t.$

$x_4 = k.$

$x_1 = -8/5\, t + 6/5\, k - 3t + k$

$= -23/5\, t + 11/5\, k.$

$$Ax = \underline{b} : \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -23/5\, t + 11/5\, k + -4/5. \\ -4/5\, t + 3/5\, k - 1 \\ t + 3 \\ k + 5 \end{bmatrix}$$

$$Ax = \underline{0} : \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -23/5\, t + 11/5\, k \\ -4/5\, t + 3/5\, k \\ t \\ k \end{bmatrix}.$$

#10.

a.

$$\begin{matrix} \cancel{1} & \cancel{4} & \cancel{5} & \cancel{2} \\ \cancel{2} & \cancel{1} & \cancel{3} & \cancel{0}. \\ \cancel{-1} & \cancel{3} & \cancel{2} & \cancel{2}. \\ \cancel{0} & \cancel{4} & \cancel{4} & \cancel{6/7}. \\ \cancel{0} & \cancel{-7} & \cancel{-7} & \cancel{-4}. \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & -2/7 \\ 0 & 1 & 1 & 4/7. \end{matrix}$$

$$\begin{array}{c|cccc|c} & x_1 & x_2 & x_3 & x_4 & \\ k_1 & 1 & 0 & 1 & -2/7 & x_1 \\ k_2 & 0 & 1. & 1 & 4/7 & x_2 \\ k_3 & 0 & 0 & 0 & 0. & x_3 \\ & & & & & x_4 \end{array}$$

$k_1(1, 0, 0).$

$k_2(0, 1, 0).$

$k_3(1, 1, 0).$

$k_4(-2/7, 4/7, 0).$

$$\therefore \begin{bmatrix} 1 & 0 & 1 & -2/7 \\ 0 & 1 & 1 & 4/7 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 1 & -2/7 \\ 0 & 1 & 1 & 4/7 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

#10.

b. ~~1 4 5 6 9~~ P_1

~~3 -2 1 4 -1~~ P_2

~~1 0 1 2 1~~ P_3

~~2 3 5 7 8~~ P_4

~~0 4 4 4 8~~ P_1+P_3

~~0 -5 -5 -5 -10~~ P_4-2P_1

~~0 -14 -14 -4 -28~~ P_2-3P_1

0 1 1 1 2 $(P_1+P_3)/4 = (P_4-2P_1)/(-5) = (P_2-3P_1)/(-14)$.

1 0 1 2 7 $\frac{3}{4}P_1+\frac{1}{4}P_3 = \frac{3}{5}P_1 - \frac{1}{5}P_4 = \frac{11}{14}P_1 - \frac{1}{14}P_2$.

0 0 0 0 0

0 0 0 0 0.

1 0 1 2 7

0 1 1 1 2

0 0 0 0 0

0 0 0 0 0

$$\therefore \begin{bmatrix} 1 & 0 & 1 & 2 & 7 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

#14 is not spans $\mathbb{R}^4$.

#16.

~~1 3 1 5~~

~~0 3 3 3~~

~~1 7 9 5~~

~~1 1 3 1~~

~~1 6 6 4~~

~~0 -4 -4 -6~~

~~0 4 3 3~~

~~1 -2 0 -2~~

~~0 0 11 13~~

~~0 4 2 0~~

~~0 1 1 1~~

1 0 2 0.

~~0 0 -2 -4~~

0 1 0 -2.

~~0 0 1 2~~

0 0 0 1

0 0 1 0.

$\therefore (e_1, e_2, e_3, e_4)$.

Exercise Set 4.9.

#2

a. $\begin{bmatrix} 1 & 0 & -2 & 1 & 0 \\ 0 & 1 & -3 & 1 & 3 \\ -2 & -1 & 1 & -1 & 3 \\ 0 & 1 & 3 & 0 & -4 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -4 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$

~~-2 0 -4 -2 0~~

~~+1 0 +2 +1 0~~

0 0 0 1 -1

0 0 1 0 0

0 1 0 0 -4

1 0 0 1 0

$\therefore$ Rank = 4

Nullity = 1.

b. $\begin{bmatrix} 1 & 3 & 1 & 3 \\ 0 & 1 & 1 & 0 \\ -3 & 0 & 6 & -1 \\ 3 & 4 & -2 & 1 \\ 2 & 0 & -4 & -2 \end{bmatrix}$ $\begin{matrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$

1 0 -2 -~~1~~ 0

0 0 0 -~~4~~ 0

~~1 2 0 8 0~~

~~3 3 -3 1~~

0 1 1 0

~~0 3 3 4~~

0 0 0 1

~~0 1 1 0~~

0 0 0 0

$\therefore$ Rank = 4

Nullity = 1.

#4. Number of leading variables: 2.

Number of parameters in the $A\underline{x} = \underline{0}$: 1.

#14.

$$\begin{bmatrix} \cancel{3} & \cancel{4} & \cancel{0} & \cancel{7} \\ \cancel{1} & \cancel{-5} & \cancel{2} & \cancel{-2} \\ \cancel{-1} & \cancel{4} & \cancel{0} & \cancel{-3} \\ \cancel{1} & \cancel{-1} & \cancel{2} & \cancel{2} \end{bmatrix}$$

RREF

$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

~~0 -4 0 -4~~

~~1 3 2 -1~~

~~0 1 0 1~~

~~1 4 2 0~~

~~1 0 2 3~~

~~3 0 0 5~~

1 0 0 ~~1~~ 0

~~0 0 2 2~~

0 0 1 ~~1~~ 0

~~0 4 2 -1~~

~~0 5 2 0~~

0 1 0 ~~1~~ 0

~~0 4 2 -1~~

~~0 0 2 -5~~

~~0 0 0 -7~~

0 0 0 1

$\therefore$ Dim = 4.

fundamental spaces = $\{e_1, e_2, e_3, e_4\}$.

#20.

~~1 2 3 1 1 | 1 0 0 0~~

~~2 8 0 1 2 | 0 1 0 0~~

~~0 4 -6 0 1 | 0 0 1 0~~

1 0 0 0 0 | 0 0 0 1

~~0 8 0 1 2 0 1 0 -2~~

~~0 2 3 1 1 1 0 0 -1~~

~~0 0 +1/3 +2 1/2 +1 1/4 +2 1/2 0 -1/4 -2/2~~

~~0 0 -12 -7 -2 4 1 0 2~~

~~0 0 0 +5 +1 +2 -1 +1 0~~

~~0 1 3/2 1/2 1/2 1/2 0 0 -1/2~~

~~0 0 3 1/2 1/4 1/2 0 -1/4 -1/2~~

~~0 1 0 1/4 3/8 1/4 0 1/8 -1/4~~

0 0 1 1/6 1/12 1/6 0 -1/12 -1/6

0 0 0 1 1/5 2/5 -1/5 1/5 0

~~0 1 0 0 3/8 - 1/20 1/4 - 1/10, 1/20, 1/8 - 1/20~~ -1/4

0 1 0 0 13/40 3/20 1/20 3/20 -1/4

~~0 0 1 0 1/12 - 1/30, 1/6 - 2/30, 0 + 1/30, -1/12 - 1/30, -1/6~~

0 0 1 0 ~~3/60~~ 1/20 1/10, 1/30 -7/60 -1/6

1 0 0 0 0 0 0 0 1

0 1 0 0 13/40 3/20 1/20 3/20 -1/4

0 0 1 0 1/20 1/10 1/30 -7/60 -1/6

0 0 0 1 1/5 2/5 -1/5 1/5 0

$$\therefore \begin{bmatrix} 0 & 0 & 0 & 1 \\ 3/20 & 1/20 & 3/20 & -1/4 \\ 1/10 & 1/30 & -7/60 & -1/6 \\ 2/5 & -1/5 & 1/5 & 0 \end{bmatrix}$$

#22. $Tx = (x_1 + 3x_2, x_1 - x_2, x_1)$.

$$T = \begin{bmatrix} 1 & 3 \\ 1 & -1 \\ 1 & 0 \end{bmatrix}.$$

a. 2.

b. 1.