*Thm 4.6.1. All bases for a f.d.v.s. have the same number of vectors.

*Thm 4.6.2. V: finite dimensional vector space

$\{\underline{v_1}, \underline{v_2}, \cdots, v_n\}$: basis of V.

(a) A set in V more than n vectors. ⇒ linearly dep.

(b) less than n vectors ⇒ does not span V.

*Def 1 Dimension of a finite-dimensional vector space.

denoted by dim(V) = Number of vectors in a basis.

*ex1).

$\dim(\mathbb{R}^n) = n.$

$\dim(P_n) = n+1.$

$\dim(M_{mn}) = mn.$

M_{22} $\begin{bmatrix}1&0\\0&0\end{bmatrix}\begin{bmatrix}0&1\\0&0\end{bmatrix}\begin{bmatrix}0&0\\1&0\end{bmatrix}\begin{bmatrix}0&0\\0&1\end{bmatrix}$.

*ex3)

$$\left.\begin{aligned}
x_1 + 3x_2 - 2x_3 \quad + 2x_5 \quad &= 0\\
2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 &= 0.\\
5x_3 + 10x_4 \quad + 15x_6 &= 0.\\
2x_1 + 6x_2 \quad + 8x_4 + 4x_5 + 18x_6 &= 0.
\end{aligned}\right)$$

☆

$$\rightarrow \left.\begin{aligned}
x_1 &= -3r - 4s - 2t\\
x_2 &= r\\
x_3 &= -2s\\
x_4 &= s\\
x_5 &= t.\\
x_6 &= 0.
\end{aligned}\right\} \rightarrow \text{Solution Space.}$$

*dim(SS)?

$$\begin{array}{rcccccccl}
 & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & \\
r(& -3, & 1, & 0, & 0, & 0, & 0 &)\\
+s(& -4, & 0, & -2, & 1, & 0, & 0 &)\\
+t(& -2, & 0, & 0, & 0, & 1, & 0 &).
\end{array}$$

*independent

Span → indep. ∴ dim = 3.

* Some Fundamental Theorems.

* Thm 4.6.3. $S \subset V$.

(a) S is linearly indep.

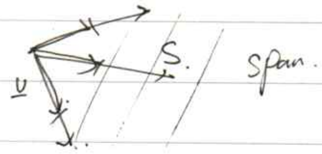
$\underline{v} \notin span(S)$.

$\Rightarrow S \cup \{\underline{v}\}$ linearly indep.

(b) $\underline{v} \in S$. $\underline{v} \in span(S - \{\underline{v}\})$.

$\Rightarrow span(S) = span(S - \{\underline{v}\})$.

*dep.



* ex 4

$P_1 = 1 - x^2$.

$P_2 = 2 - x^2$. $\longrightarrow$ 3×3 det $\neq 0$. … 계산함.

$P_3 = x^3$.

빨리 찾기.

$P_1 \cdots P_2$ 를 먼저 확인 $\Rightarrow$ indep.

$span\{\underbrace{P_1, P_2}_{\text{indep}}\} \not\ni \underbrace{P_3}_{\text{indep.}}$ $\quad \therefore$ indep.

using thm 4.6.3.

* Thm 4.6.4. V: n-dim v.s. $|S| = n$. $S \subset V$. (S is subset of V).

S: basis $\Longleftrightarrow$ ① S span V. ② S: linearly indep.

둘중 하나 체크 $\Rightarrow$ other 보장.

* ex 5.

$\mathbb{R}^2$ $V_1 = (-3, 7)$.

$V_2 = (5, 5)$.

indep. $V_1 \neq k\, \underline{V_2}$

$\mathbb{R}^3$ $\underline{V_1} = (2, 0, -1)$.

$\underline{V_2} = (4, 0, 7)$. 정수배인가? 아님.

$\underline{V_3} = (-1, 1, 4)$. $\longrightarrow$ span하면 표현되나?

$\times \Rightarrow$ indep.

* thm 4.6.5. S: finite set in a f.d.v.s. V.

(a) S spans V but is not a basis.

$\Rightarrow S$ can be reduced to a basis by removing some vectors from S.

(b) S is linearly indep. but does not span V.

$\Rightarrow S$ can be enlarged to a basis by adding some indep vectors.