Exercise Set 1.1.

#2.

a. linear.

b. non-linear.

c. linear

d. non-linear

e. non-linear

f. linear.

#12.

$\frac{2}{4} = \frac{-3}{-6} \quad \cdots \quad \frac{a}{b}.$

- no solutions. : $2a \neq b$.
- One solution : (null)
- infinitely many : $2a = b$.

#16.

a. let $x_2 = t$. $\therefore (x_1, x_2) = (-\frac{1}{3}(4+t), t)$

then $3x_1 + t = -4$.

$x_1 = (-4-t)\frac{1}{3}$

b. let $y = t$, $z = k$. $\therefore (x, y, z) = (\frac{1}{2}(t-2k-4), t, k)$.

then $2x - t + 2k = -4$.

$\cancel{2}x = (t - 2k - 4)/2$

$x = \frac{1}{2}(t - 2k - 4)$

#18.

a. $r_1 = 3r_1 + r_3$

$$\begin{bmatrix} 1 & 16 & -16 & 33 \\ 7 & 1 & 4 & 3 \\ -5 & 4 & 2 & 7 \end{bmatrix}$$

b. $r_1 = r_1 + r_3$

$$\begin{bmatrix} 1 & -1 & -3 & 6 \\ 3 & -1 & 8 & 1 \\ -6 & 3 & -1 & 4 \end{bmatrix}$$

#20. a. $k \neq -\frac{5}{2}$

b. $k \neq -4$

Exercise Set 1.2.

#2.

a. Row echelon form

b. Row echelon form

c. Both reduced row echelon form and row echelon form.

d. Row echelon form.

e. Neither reduced row echelon form nor row echelon form.

f. Row echelon form.

g. Both reduced row echelon form and row echelon form.

#4.

(problem)	pivot rows	pivot columns	solution.
a.	1, 2, 3.	1, 2, 3.	$(x_1, x_2, x_3) = (-3, 0, 7)$.
b.	1, 2, 3.	1, 2, 3.	$(x_1, x_2, x_3, x_4) = (8+7k, 2-3k, -5-k, k)$.
c.	1, 2, 3.	1, 2, 4.	$(x_1, x_2, x_3, x_4, x_5) = (-2+6k-3t, k, 7-4t, 8-5t, t)$.
d.	1, 2.	1, 3.	no solution.

#8.

$$\begin{bmatrix} 0 & -2 & 3 & 1 \\ 3 & 6 & -3 & -2 \\ 6 & 6 & 3 & 5 \end{bmatrix} \Rightarrow \begin{array}{cccc} \cancel{0} & \cancel{-2} & \cancel{3} & \cancel{1} \\ \cancel{3} & \cancel{6} & \cancel{-3} & \cancel{-2} \\ \cancel{6} & \cancel{6} & \cancel{3} & \cancel{5} \\ \cancel{0} & \cancel{6} & \cancel{9} & \cancel{9} \\ \cancel{3} & \cancel{0} & \cancel{6} & \cancel{5} \\ 1 & 0 & 2 & \frac{5}{3} \\ 0 & -2 & 3 & 3 \\ 0 & 0 & 0 & 2 \end{array} \Rightarrow \begin{bmatrix} 1 & 0 & 2 & \frac{5}{3} \\ 0 & -2 & 3 & 3 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$\therefore$ no solution.

#12.

$$\begin{bmatrix} 1 & -3 & 7 & 2 & 5 \\ 0 & 1 & 2 & -4 & 1 \\ 0 & 0 & 1 & 6 & 9 \\ 0 & 0 & 0 & a & b \end{bmatrix} \Rightarrow \begin{matrix} \cancel{1} & \cancel{-3} & \cancel{7} & \cancel{2} & \cancel{5} \\ \cancel{0} & \cancel{1} & \cancel{2} & \cancel{-4} & \cancel{1} \\ 0 & 0 & 1 & 6 & 9 \\ 0 & 0 & 0 & a & b \\ \cancel{1} & \cancel{0} & \cancel{13} & \cancel{-10} & \cancel{8} \\ 0 & 1 & 0 & -16 & -17. \\ 1 & 0 & 0 & -88 & -109. \end{matrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & -88 & -109 \\ 0 & 1 & 0 & -16 & -17 \\ 0 & 0 & 1 & 6 & 9 \\ 0 & 0 & 0 & a & b \end{bmatrix}$$

a. $a=0$, $b=1$. no solution.

b. $a=0$, $b=0$. infinitely many solutions.

C. $a=1$, $b=0$.

$$\begin{bmatrix} 1 & 0 & 0 & -88 & -109. \\ 0 & 1 & 0 & -16 & -17 \\ 0 & 0 & 1 & 6 & 9 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -109 \\ 0 & 1 & 0 & 0 & -17 \\ 0 & 0 & 1 & 0 & 9 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$\therefore (x_1, x_2, x_3, x_4) = (-109, -17, 9, 0)$.

#26.

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & -2 & 3 & 1 \\ 1 & 2 & -(a^2-3) & a \end{bmatrix} \Rightarrow \begin{array}{cccc} \cancel{1} & \cancel{2} & \cancel{1} & \cancel{2} \\ \cancel{2} & \cancel{-2} & \cancel{3} & \cancel{1} \\ \cancel{1} & \cancel{2} & \cancel{(a^2-3)} & \cancel{a} \\ \cancel{0} & \cancel{-6} & \cancel{2} & \cancel{-1} \\ 0 & 0 & -a^2+2 & a-2 \\ \cancel{0} & \cancel{-2} & \cancel{\frac{2}{3}} & \cancel{-\frac{1}{3}} \\ 1 & 0 & \frac{5}{3} & \frac{5}{3} \\ 0 & 1 & -\frac{1}{3} & +\frac{1}{6} \end{array}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & \frac{5}{3} & \frac{5}{3} \\ 0 & 1 & -\frac{1}{3} & \frac{1}{6} \\ 0 & 0 & -a^2+2 & a-2 \end{bmatrix}$$

if $\left\{ \begin{array}{l} -a^2+2 \neq 0 \\ a-2=0 \end{array} \right\} \Rightarrow a=2$ then system has no solutions.

if $\left\{ \begin{array}{l} -a^2+2 \neq 0 \\ a-2 \neq 0 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} a \neq \pm\sqrt{2} \\ a \neq 2 \end{array} \right\}$ then system has one solution.

if $\left\{ \begin{array}{l} -a^2+2 = 0 \\ a-2 = 0 \end{array} \right\}$ then system has infinitely many solutions.
but $a^2=2$ and $a=2$ cannot be satisfied at the same time.

$\therefore$ No cases of infinitely many solutions.

Exercise Set 1.3.

#2.

a. 5×4

b. Not defined.

c. 4×2.

d. 2×4.

e. 5×2.

f. Not defined.

#6.

a.
$$\left(2\begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix} - \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix} \right) \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -3 & 3 \\ 11 & -1 & 2 \\ 0 & 1 & 5 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -6 & -3 \\ 36 & 0 \\ 4 & 7 \end{bmatrix}$$

b. Not defined.

c.
$$\left(- \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} \right)^T + 5 \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & -12 & -6 \\ -5 & 2 & -8 \\ -4 & -5 & -7 \end{bmatrix} + \begin{bmatrix} 5 & -5 & 15 \\ 25 & 0 & 10 \\ 10 & 5 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -17 & 9 \\ 20 & 2 & 2 \\ 6 & 0 & 13 \end{bmatrix}$$

#6

d. $$\left(\begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix} - 2 \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} \right)^T$$

$$= \begin{bmatrix} 4 & -6 & 3 \\ 0 & 4 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 8 & 4 \\ 6 & 2 & 10 \end{bmatrix} = \begin{bmatrix} 2 & -14 & -1 \\ -6 & 2 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -6 \\ -14 & 2 \\ -1 & -8 \end{bmatrix}$$

e. Not defined.

f. $D^T E^T - (ED)^T$

$= (ED)^T - (ED)^T$

$= \underline{0}$

$$\therefore \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

#10

a. $$6 \begin{bmatrix} 3 \\ -2 \\ 7 \end{bmatrix} + 0 \begin{bmatrix} 3 \\ -2 \\ 7 \end{bmatrix} + 7 \begin{bmatrix} 3 \\ -2 \\ 7 \end{bmatrix}$$

$$-2 \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix} + 1 \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix} + 7 \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}$$

$$4 \begin{bmatrix} 0 \\ 4 \\ 9 \end{bmatrix} + 3 \begin{bmatrix} 6 \\ 4 \\ 9 \end{bmatrix} + 5 \begin{bmatrix} 6 \\ 4 \\ 9. \end{bmatrix}$$

#10

b.

$$3\begin{bmatrix}6\\-2\\4\end{bmatrix}+6\begin{bmatrix}6\\-2\\4\end{bmatrix}+0\begin{bmatrix}6\\-2\\4\end{bmatrix}$$

$$-2\begin{bmatrix}6\\1\\3\end{bmatrix}+5\begin{bmatrix}0\\1\\3\end{bmatrix}+4\begin{bmatrix}6\\1\\3\end{bmatrix}$$

$$7\begin{bmatrix}7\\7\\5\end{bmatrix}+4\begin{bmatrix}7\\7\\5\end{bmatrix}+9\begin{bmatrix}7\\7\\5\end{bmatrix}$$

#16

$$\begin{bmatrix}2 & 2 & k\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\2 & 0 & 3\\0 & 3 & 1\end{bmatrix}\begin{bmatrix}2\\2\\k\end{bmatrix}$$

$$=\begin{bmatrix}6 & 4+3k & 6+k\end{bmatrix}\begin{bmatrix}2\\2\\k\end{bmatrix}$$

$$=\begin{bmatrix}(12)+(8+6k)+k(6+k)\end{bmatrix}=\begin{bmatrix}(k+10)(k+2)\end{bmatrix}=\begin{bmatrix}0\end{bmatrix}$$

$\therefore k=-10,\ k=-2$

#24

$$\begin{aligned}a-b&=8\\b+a&=1\end{aligned}\quad\therefore\quad\begin{aligned}a&=9/2\\b&=-7/2\end{aligned}\qquad\begin{aligned}3d+c&=7\\2d-c&=6\end{aligned}\quad\therefore\quad\begin{aligned}c&=-4/5\\d&=13/5\end{aligned}$$

$\therefore (a,b,c,d)=(9/2,\ -7/2,\ -4/5,\ 13/5)$

Exercise Set 1.4.

#8.

$$D^{-1} = \frac{1}{-6+8}\begin{bmatrix} -1 & -4 \\ 2 & 6 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} -1 & -4 \\ 2 & 6 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & -2 \\ 1 & 3 \end{bmatrix}$$

#10.

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}^{-1} = \frac{1}{\cos^2\theta+\sin^2\theta}\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

#16.

$$(5A^T)^{-1} = \begin{bmatrix} -3 & -1 \\ 5 & 2 \end{bmatrix}$$

$$5A^T = \frac{1}{-6+5}\begin{bmatrix} 2 & 1 \\ -5 & -3 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 5 & 3 \end{bmatrix}$$

$$A^T = \begin{bmatrix} -\frac{2}{5} & -\frac{1}{5} \\ 1 & \frac{3}{5} \end{bmatrix}$$

$$A = \begin{bmatrix} -\frac{2}{5} & 1 \\ -\frac{1}{5} & \frac{3}{5} \end{bmatrix}$$

#20

a. $$\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ 12 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 28 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 8 & 0 \\ 28 & 1 \end{bmatrix}$$

b. $$\begin{bmatrix} 8 & 0 \\ 28 & 1 \end{bmatrix}^{-1} = \frac{1}{8}\begin{bmatrix} 1 & 0 \\ -28 & 8 \end{bmatrix} = \begin{bmatrix} \frac{1}{8} & 0 \\ -\frac{2}{7} & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} \frac{1}{8} & 0 \\ -\frac{2}{7} & 1 \end{bmatrix}$$

c. A^2-2A+I
$= (A-I)^2$

$$= \left(\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)^2 = \begin{bmatrix} 1 & 0 \\ 4 & 0 \end{bmatrix}^2 = \begin{bmatrix} 1 & 0 \\ 4 & 0 \end{bmatrix}.$$

#28.

$$\begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{8+2} \begin{bmatrix} 4 & 2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{24}{10} \\ \frac{4}{10} \end{bmatrix} \qquad \therefore \begin{bmatrix} \frac{12}{5} \\ \frac{2}{5} \end{bmatrix}$$