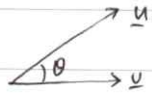
직교성.

선형대수학 - Chap 3.3 Orthogonality NO. 24.04.18.

orthogonality.



$\underline{u} \cdot \underline{v} = \|\underline{u}\| \cdot \|\underline{v}\| \cos\theta$

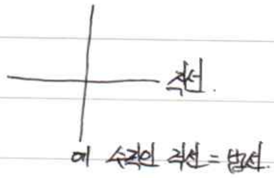
$\hookrightarrow 90°??$

Def 1) $\underline{u} \cdot \underline{v} = 0 \iff \underline{u}$ and $\underline{v}$ orthogonal (perpendicular).

Ex 1) $\underline{u} \cdot \underline{v} = (-2)(1) + (3)(2) + (1)(0) + (4)(-1) = 0.$

Lines and Planes Determined by Points and Normals.

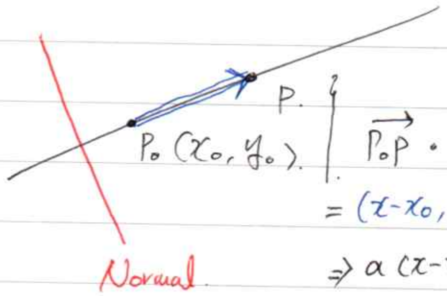
법선.



* 점, 법선 제시 ⇒ 방정식 결정.

- $P_0(x_0, y_0) \Rightarrow \underline{n} = (a, b)$

$P(x, y)$.



$\overrightarrow{P_0P} \cdot \underline{n}$

$= (x - x_0, y - y_0)\,(a, b)$

$\Rightarrow a(x - x_0) + b(y - y_0) = 0.$

$P_0(x_0, y_0, z_0)$. $\underline{n} = (a, b, c)$.

$\Rightarrow a(x - x_0) + b(y - y_0) + c(z - z_0) = 0.$

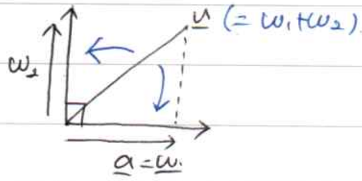
Thm 3.3.1. $ax + by + c = 0.$ $ax + by + cz + d = 0.$

$\underline{n} = (a, b)$ $\underline{n} = (a, b, c)$.

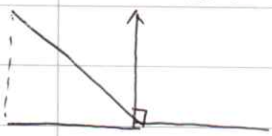
* homogeneous $\cdots$ $n \cdot x = 0$.

(x = 상수항 0일 때).

orthogonal Projections.

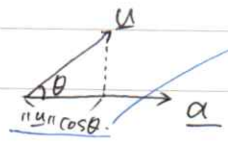


* Vector는 orthogonal Projection으로 표현할 수 있다.



*Thm 3.3.2) $w_1 = \text{proj}_{a} u$

u의 a에 대한 projection.



$$= \frac{u \cdot a}{\|a\|^2} \cdot a$$

* $u \cdot a = \|u\| \|a\| \cos\theta$.

$\|u\| \cos\theta = (u \cdot a) / \|a\|$

$$= \left(\frac{u \cdot a}{\|a\|^2}\right) a$$

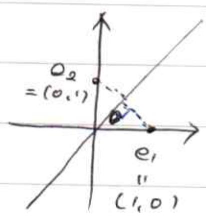
"C"

Ex4). $u = (2, -1, 3)$, $a = (4, -1, 2)$

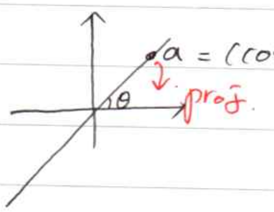
$$\text{proj}_a u = \frac{8+1+6}{4^2+1^2+2} (4, -1, 2).$$

$$= \frac{15}{21} (4, -1, 2).$$

Ex5).



*직선은 무수히 많은 벡터의 set.



$$\text{* } \text{proj}_a e_1 = \frac{\cos\theta}{\cancel{\cos^2\theta + \sin^2\theta}_{\,1}} (\cos\theta, \sin\theta).$$

$$= (\cos^2\theta, \sin\theta\cos\theta).$$

$$\text{proj}_a e_2 = \frac{\sin\theta}{\sin^2\theta + \cos^2\theta} (\cos\theta, \sin\theta)$$

$$= (\sin\theta\cos\theta, \sin^2\theta).$$

* Generalization

$$P_\theta = \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

NO.

Ex6). $x = (1, 5)$, $\theta = \pi/6$. orthogonal projection.

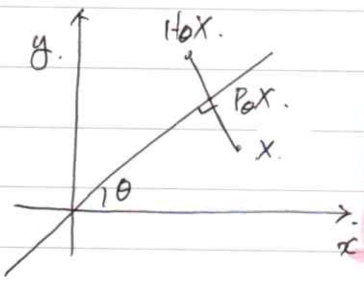
$$P_{\pi/6} = \begin{bmatrix} \cos^2(\pi/6) & \sin(\pi/6)\cos(\pi/6) \\ \sin(\pi/6)\cos(\pi/6) & \sin^2(\pi/6) \end{bmatrix}$$

$$= \begin{bmatrix} 3/4 & \sqrt{3}/4 \\ \sqrt{3}/4 & 1/4 \end{bmatrix}$$

$$P_{\pi/6} \cdot X = \begin{bmatrix} 3/4 & \sqrt{3}/4 \\ \sqrt{3}/4 & 1/4 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3+5\sqrt{3}}{4} \\ \frac{3+5}{4} \end{bmatrix}.$$

Reflection about a Line through the Origin.



$$\boxed{\begin{aligned} &*. P_\theta x - x = 1/2\,(H_\theta x - x). \\ &H_\theta X = (2P_\theta - I)X. \\ &H_\theta = 2P_\theta - I \end{aligned}}$$

$$= \begin{bmatrix} 2\cos^2\theta - 1 & \sin\theta\cos\theta \\ \sin\theta\cos\theta & 2\sin^2\theta - 1 \end{bmatrix}$$

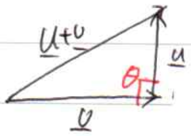
$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$$

Ex7). $H_\theta X = H_{\pi/6} X = \begin{bmatrix} 1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

* θ가 둔각일 때, cosθ는 음수… 지금까지 식은 잘못됨 처리해야함

NO.

The theorem of Pythagoras.



$\rightsquigarrow \|u\|^2 + \|v\|^2 = \|u+v\|^2.$

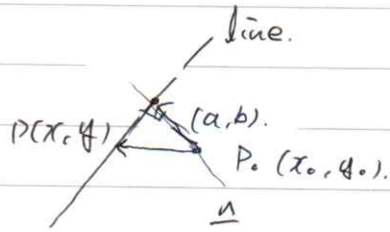
$= (u+v)\cdot(u+v).$

$= \|u\|^2 + 2u\cdot v + \|v\|^2.$

$= \|u\|^2 + \|v\|^2 \rightarrow \theta = \frac{\pi}{2} \cdots \cos\theta = 0.$

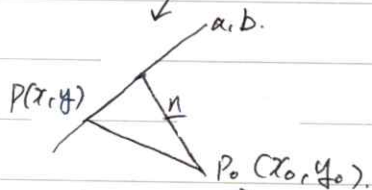
* Distance between line and point.

$$\text{"} \quad D = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2+b^2}} \quad \text{"}$$



$ax+by+c=0.$

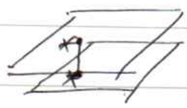
$\Rightarrow$ normal vector $=(a,b).$



$D = \|\text{proj}_n \overrightarrow{PP_0}\|.$

$= \|\text{proj}_n (x-x_0, y-y_0)\|.$

$= \dfrac{\sqrt{a(x-x_0) + b(y-y_0)}}{|a^2+b^2|}.$

Ex 10). $x+2y-2z=3.$, $2x+4y-4z=7.$ normal $=(1,2,-2)$, $(2,4,-4)$.  parallel.

let. $(3,0,0)$.

distance between $(3,0,0)$. and $2x+4y-4z=7.$

The Geometry of Linear Systems. (연립방정식의 기하학).

Dot Product Form of a Linear Systems.

- A linear system.

$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b.$

$\Rightarrow \underline{a} = (a_1, a_2, \cdots, a_n)$

$\underline{x} = (x_1, x_2, \cdots, x_n)$

$\underline{a} \cdot \underline{x} = b \longleftarrow$ 두 벡터의 dot 곱은 상수다.

- The Corresponding Homogeneous Equation (동차방정식)

$a_1x_1 + a_2x_2 + \cdots + a_nx_n = 0 \quad \cdots b$

$\Downarrow$

$\underline{a} \cdot \underline{x} = 0.$ (Orthogonal, 직교)

- Linear System

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots &+ a_{1n}x_1 = b_1 \\ a_{21}x_1 + \cdots &+ a_{2n}x_n = b_2 \\ &\vdots \\ a_{m1}x_1 + \cdots &+ a_{mn}x_n = b_n \end{aligned}$$

The Corresponding Homogeneous System

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$A\underline{x} = \underline{0}$

$\sim$

$$\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_m \end{bmatrix} \underline{x} = 0. \Rightarrow \left.\begin{aligned} \underline{r_1} \cdot \underline{x} &= 0 \\ \underline{r_2} \cdot \underline{x} &= 0 \\ &\vdots \\ \underline{r_m} \cdot \underline{x} &= 0. \end{aligned}\right\}$$

NO.

*Thm 3.4.3. $A: m\times n$.

The solution of $\boxed{A\underline{x}=\underline{0}}$ (homogeneous) consists of all vectors in R^n that are orthogonal to every row vector of A.

직교

*Ex 6

(=Example 6 of section 1.2).

$$\begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 \\ 2 & 6 & -5 & -2 & -4 & -3 \\ 0 & 0 & 5 & 10 & 0 & 15 \\ 2 & 6 & 0 & 8 & 4 & 18 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 = -3r - 4s - 2t$

$x_2 = r$.

$x_3 = -2s$

$x_4 = s$.

$x_5 = t$.

$x_6 = 0$

$\begin{cases} r_1 = (1, 3, -2, 0, 2, 0) \\ r_2 = (2, 6, -5, -2, -4, -3) \\ \vdots \\ \underline{x} = (-3r-4s-2t, r, -2s, s, t, 0) \end{cases}$

$\downarrow$

$\underline{r_1} \cdot x = 0$.

$\underline{r_2} \cdot x = 0$

$(-\cancel{3r} - \cancel{4s} - \cancel{2t}) + \cancel{3r} + \cancel{4s} + 0 + \cancel{2t} + 0$

$= 0$.