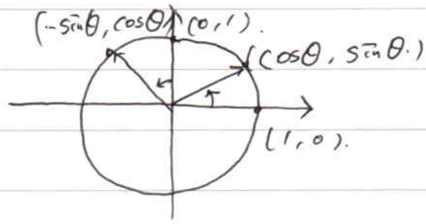
선형대수학 - Ch 1.9. Compositions of Matrix Transformations.

Rotation Operators.

· using radian(θ).



· $T(e_1) = T(1,0) = (\cos\theta, \sin\theta)$

$T(e_2) = T(0,1) = (-\sin\theta, \cos\theta)$

· $A = [T(e_1) \mid T(e_2)]$

$$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

※ Ex 8

$$\cdot\ R_{\pi/6}\, X = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Compositions of Matrix Transformations.

· Composition of Functions.

$f(g(x))$ $(f \circ g)(x)$.

· Compositions of Matrix Transformations.

$$T_A(T_B(x)) = (\underset{\text{2nd}}{T_A} \circ \underset{\text{1st}}{T_B})(x) = \boxed{T_{AB}}$$

$$T_{\underset{2}{A}\underset{1}{B}}$$

$T_A : \mathbb{R}^k \to \mathbb{R}^m.$

$T_B : \mathbb{R}^n \to \mathbb{R}^k.$

$\Rightarrow (T_A \circ T_B) : \mathbb{R}^n \to \mathbb{R}^m.$

※ Ex 1

$T_1(x, y, z) = (x+2y,\ x+2z-y).$

$T_2(x, y) = (3x+y,\ x,\ x-2y).$

$\Downarrow$

$T_1(e_1) = (1,1).\quad T_1(e_2) = (2,-1),\quad T_1(e_3) = (0,2).$

$$\Rightarrow A_1 = \begin{bmatrix} 1 & 2 & 0 \\ 1 & -1 & 2 \end{bmatrix}.$$

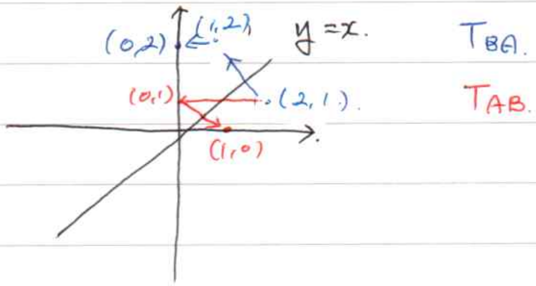
$T_2(e_1) = (3,1,1)$, $T_2(e_2) = (1,0,2).$

$$\Rightarrow A_2 = \begin{bmatrix} 3 & 1 \\ 1 & 0 \\ 1 & 2 \end{bmatrix}.$$

$$A_1 A_2 = \begin{bmatrix} 1 & 2 & 0 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 1 & 0 \\ 1 & 2 \end{bmatrix}.$$

* Ex 2.

$\begin{cases} T_A : \mathbb{R}^2 \to \mathbb{R}^2, \text{ reflection about } y = x. \\ T_B : \mathbb{R}^2 \to \mathbb{R}^2, \text{ orthogonal projection about } y\text{-axis}. \end{cases}$



∴ Composition is not commutative.

(교환 불가).

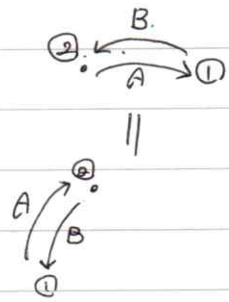
* Ex 3

$$A_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix}.$$

$$A_1 A_2 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix}.$$

$$\parallel$$

$$A_2 A_1 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix}.$$



∴ Composition of Rotations is commutative.

(교환 가능).

* Ex 4.

· Reflection about $-y$.

· $-x$.

$\Big\} \Rightarrow$ about origin.

Invertibility of Matrix Operations.

<Definition>.

- $T_A : \mathbb{R}^n \rightarrow \mathbb{R}^n$.
- $T_A^{-1} = T_{A^{-1}}$ (T_A의 역연산 = A^{-1}이 ~~표준~~ 행렬의 변환).
- $T_A \circ T_A^{-1} = T_A^{-1} \circ T_A = T_I$.

$T_A \circ T_{A^{-1}} = T_{AA^{-1}} = T_I$.

*Ex6

- $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \quad A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \cdots T_A^{-1}, \ R_\theta^{-1}$$

$$= \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix}. \cdots T_{A^{-1}}, \ R_{-\theta}.$$