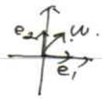
- $(3,2) = 3e_1 + 2e_2$ ⋯ (3,2)를 표현할 수 있는 유일한 방법.

  e를 이용해

  (1,0)과 (1,1)을 이용해

  ⋮

(3,2)를 e_1, e_2, w로 표현하는 방법 { 해 두개 이상. / 없거나. ().

$(w = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}))$

왜??

$$\begin{bmatrix} 1 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ w \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$ 를 푼것.

$e_1, e_2, w \rightarrow w = \frac{1}{\sqrt{2}} e_1 + \frac{1}{\sqrt{2}} e_2.$

* Def.1 $S = \{\underline{v_1}, \underline{v_2}, \cdots, \underline{v_r}\} \subset V$. : linearly independent set.

  if no vector in S can be expressed as a linear combination of the others.

* Thm 4.4.1. Linearly independent. $\Leftrightarrow k_1\underline{v_1} + \cdots + k_r\underline{v_r} = \underline{0}$. for only $k_1 = k_2 = \cdots = k_r = 0$.

  ∴ 계수 모두가 0일 때 0벡터가 되는 경우 를 제외하면 0벡터가 되는 경우, 없어야 함.

$k_1\underline{v_1} + k_2\underline{v_2} + \cdots + k_r\underline{v_r}$

$$\left.\begin{aligned} &\Rightarrow \underline{v_1} + \tfrac{k_2}{k_1}\underline{v_2} + \cdots + \tfrac{k_r}{k_1}\underline{v_r} = 0 \\ &v_1 = \tfrac{k_2}{-k_1} v_2 + \cdots + \tfrac{k_r}{-k_1}\underline{v_r} \end{aligned}\right\}.$$

* ex1 $\underline{e_1}, \underline{e_2}, \cdots, e_n$ is linearly independent?

* ex2 $\underline{v_1} = (1,-2,3)$, $\underline{v_2} = (5,6,-1)$, $\underline{v_3} = (3,2,1)$.

  $k_1(1,-2,3) + k_2(5,6,-1) + k_3(3,2,1) = (0,0,0)$. 이 trivial solution only 여야 선형독립

  $$\det\begin{bmatrix} 1 & 5 & 3 \\ -2 & 6 & 2 \\ 3 & -1 & 1 \end{bmatrix} ??$$ ∴ $\det = 0$. ⋯ linearly dependent.

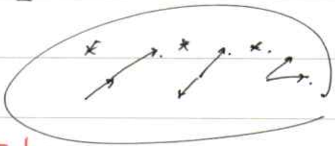
* ex3　$v_1 = (1, 2, 2, -1)$.
$v_2 = (4, 9, 9, -4)$.
$v_3 = (5, 8, 9, -5)$.　$k_1\underline{v_1} + k_2\underline{v_2} + k_3\underline{v_3} = 0$.

$$\begin{array}{c}k_1 \quad k_2 \quad k_3\\ \begin{pmatrix}1 & 4 & 5\\ 2 & 9 & 8\\ 2 & 9 & 9\\ -1 & -4 & -5\end{pmatrix}\end{array} \xrightarrow{\text{RREF}} \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{pmatrix}$$

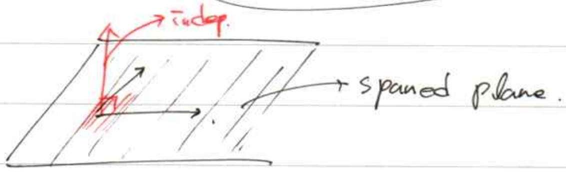
* ex4.　$1, x, x^2, \cdots, x^n \rightarrow$ linearly independent?
x^n cannot be expressed as linear combination of $1, x, \cdots x^{n-1}$.

* thm 4.4.2.　(a) $\underline{0} \in S$: linearly dep.
(b) $S = \{\underline{v_1}, \underline{v_2}\}$　$\underline{v_1} = k\underline{v_2}$: linearly dep.

*) $k\underline{v_1} + k\underline{v_2} + \cdots + k_r\underline{v_r} = \underline{0}$
$\underline{0}$
$\cancel{0}$
$1\underline{v_1} - k\underline{v_2} = 0$



+)



⇓
세 개 벡터, 같은 평면 : dep.

* ex6　$F(-\infty, \infty)$　$f_1 = x$
$f_2 = \sin x$.　} dep?　$x = k\sin x$. 가능하면 dep.
But 표현 × ⇒ linearly indep.

$g_1 = \sin 2x$
$g_2 = \sin x \cos x$　}　$\underline{g_1} = 2\underline{g_2}$ ⇒ linearly dep.

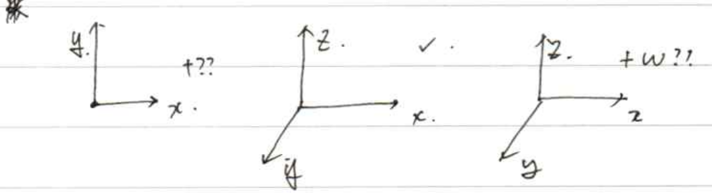
* thm 4.4.3.　$S = \{ \underline{v_1}, \underline{v_2}, \cdots, \underline{v_r} \} \subset \mathbb{R}^n$.

$r > n \Rightarrow S$: linearly dep.

그보다 크면

"차원 당 하나 … 차원 상에서 linearly dep 한 것이 존재함."

$n=2,\ r=3$.　　$n=3,\ r=3$.　　$n=\cancel{4},\ r=3$.



* ex7　Nonzero rows in RREF or REF are linearly indep.

$$\begin{bmatrix} 1 & \cdots & & & \cdots \\ 0 & 1 & \cdots & & \cdots \\ 0 & 0 & 1 & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

→ 나머지 row들로 어떤 row 만들 수 있나?
~~외~~ all zero row만 있다면 가능!!

Linear Independence of Functions.

$\begin{cases} \underline{f_1} = \sin x \\ \underline{f_2} = \cos^2 x. \\ \underline{f_3} = 5. \end{cases}$ → 하나가 나머지로 표현되는가?　$f_1 + f_2 = 1. \Rightarrow \underline{5(f_1 + f_2) = f_3}$

linearly depend.

* Def 2.

$\left.\begin{array}{l} \underline{f_1} = f_1(x). \\ \underline{f_2} = f_2(x). \\ \vdots \\ \underline{f_n} = f_n(x). \end{array}\right\}$　Wronskian of $f_1, \ldots, f_n$.

$$W(x) = \begin{vmatrix} f_1(x) & f_2(x) & \cdots & f_n(x). \\ f_1'(x) & f_2'(x) & \cdots & f_n'(x) \\ \vdots & \vdots & \ddots & \vdots \\ f_n^{(n-1)}(x) & f_n^{(n-2)}(x) & \cdots & f_n^{(n-n+1)}(x) \end{vmatrix}$$

* $W(x) = 0 \Rightarrow$ linearly dep.

* ex8

$\left.\begin{array}{l} \underline{f_1} = x. \\ f_2 = \sin x. \end{array}\right\}$ indep?　$W(x) = \begin{vmatrix} \overset{x}{\cancel{\sin x}} & \sin x. \\ 1 & \cos x \end{vmatrix} = x\cos x - \sin x. \neq 0.$

$\Rightarrow$ indep.