선형대수학. - 1.3. Matrices and Matrix Operations.

정의 용어.
Matrix Notation and Terminology.

· Matrix : Retangular array of numbers. (↔ Scalar)

· size: (# of rows) $\overset{\text{by}}{\times}$ (# of columns).

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $[4]$ $[1 \;\; 2]$

2×2 1×1 1×2.

· Row matrix (row vector) ... size is $1 \times n$.

(i.e. $[1, 9, -4, 5, 0]$).

Column matrix (column vector) ... size is $n \times 1$.

Generic Def. · $A: m \times n$.

$* \; A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$ → 주대각선 (Main Diagonal)

→ 원소. Entry. Element.

$* \; A = [a_{ij}]_{mn} \quad (1 \le i \le m, \; 1 \le j \le n)$.

$* \; (A)_{ij} = a_{ij}$.

* 책의 약속 : Vector ... 소문자 + Bold.

교재 : 소문자 + Underline.

Square Matrix $n \times n$.

= (of order n).

Equality. A, B is same size.

$(A)_{ij} = (B)_{ij}$

Sum and difference.

ex). $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$.

- A. B : Same Size.
- $(A+B)_{ij} = (A)_{ij} + (B)_{ij}$.

$A+B = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$

- $(A-B)_{ij} = (A)_{ij} - (B)_{ij}$.

$A-B = \begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix}$

Scalar Multiple and Product.

- $(cA)_{ij} = c(A)_{ij}$. (c is scalar Constant).
- AB

$\underset{n \times r}{A} \quad \underset{r \times m}{B} \Rightarrow \underset{n m}{AB}$

r = r
열 = 행

$$(AB)_{ij} = \sum_{k=1}^{r} a_{ik} b_{kj}.$$

*ex)

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix} \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix} = \begin{bmatrix} a & b & c & d \\ e & f & g & h \end{bmatrix}.$$

$b = (AB)_{12}$.

$= \sum_{k=1}^{3} (A)_{1k}(B)_{k2} = \sum_{k=1}^{3} a_{1k} b_{k2}$.

$= a_{11} b_{12} + a_{12} b_{22} + a_{13} b_{32}$.

$= 1 \times 1 + 2 \times (-1) + 4 \times 7$

$= 27$.

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#12.
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* ex 6. $\underset{3\times4}{A}$ $\underset{4\times7}{B}$ $\underset{7\times3}{C}$.   ... 곱이 존재하는 pair? AB. BC. ~~CA~~
~~BA~~ ~~CB~~ ~~AC~~.

* $AB \neq BA$. ; 교환법칙 X.

Partitioned Matrices.

$$A = \left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ \hline a_{31} & a_{32} & a_{33} & a_{34} \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$A = \left[\begin{array}{c|c|c|c} a_{11} & a_{12} & a_{13} & a_{14} \\ \hline a_{21} & a_{22} & a_{23} & a_{24} \\ \hline a_{31} & a_{32} & a_{33} & a_{34} \end{array}\right] = \begin{bmatrix} \underline{r_1} \\ \underline{r_2} \\ \underline{r_3} \end{bmatrix}$$

$$= \begin{bmatrix} \underline{c_1} & \underline{c_2} & \underline{c_3} & \underline{c_4} \end{bmatrix}$$

Matrix multiplication by columns and by rows

$$AB \overset{*}{=} A[\underline{b_1}\ \underline{b_2}\ \cdots\ \underline{b_n}] = [A\underline{b_1}\ A\underline{b_2}\ \cdots\ A\underline{b_n}]$$

column vector.

$$\overset{*}{=} \begin{bmatrix} \underline{a_1} \\ \underline{a_2} \\ \vdots \\ \underline{a_m} \end{bmatrix} B = \begin{bmatrix} \underline{a_1}B \\ \underline{a_2}B \\ \vdots \\ \underline{a_m}B \end{bmatrix}$$

* ex 7).

$$\underbrace{\begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix}}_{A} \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix} = \begin{bmatrix} 12 & 27 & 30 & 13 \\ 8 & -4 & 26 & 12 \end{bmatrix}$$

$\underline{a_1}$; A ; $\underline{b_2}$; $A\underline{b_2}$; $\underline{a_1}B$

Matrix Products as Linear Combinations.

* Def. 6. $A_1, A_2, \cdots A_r$ ⋯ Same Size.
$c_1, c_2, \cdots c_r$ ⋯ scalars.

$$c_1A_1 + c_2A_2 + \cdots + c_rA_r \Rightarrow \text{linear Combination.}$$

coefficients(계수) $= c_1, c_2, \cdots, c_r$.

linear Combination.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$Ax = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{bmatrix} = x_1\begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} + x_2\begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{bmatrix} + \cdots + x_n\begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix}$$

* Thm 1.3.1 A is $m \times n$ matrix, x is $n \times 1$ column vector } Product $Ax \Rightarrow$ linear combination.

* Ex 5.

$$AB = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix} \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix} = \begin{bmatrix} 12 & 27 & 30 & 13 \\ 8 & -4 & 26 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 12 \\ 8 \end{bmatrix} = 4\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 0\begin{bmatrix} 2 \\ 6 \end{bmatrix} + 2\begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 13 \\ 12 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 1\begin{bmatrix} 2 \\ 6 \end{bmatrix} + 2\begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

* Column - Row Expansion.

$$AB = [\underline{c_1}\ \underline{c_2}\ \cdots \underline{c_r}] \begin{bmatrix} \underline{r_1} \\ \underline{r_2} \\ \vdots \\ \underline{r_r} \end{bmatrix} = \underline{c_1}\underline{r_1} + \underline{c_2}\underline{r_2} + \cdots + \underline{c_r}\underline{r_r}$$

* Matrix Form of a Linear System.

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}}_{\text{Coefficient Matrix}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}}_{\text{Unknown Vector (Column Matrix)}} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \rightarrow \underline{b}$$

$Ax = b$

$$[A|b] = \left[\begin{array}{cccc|c} a_{11} & \cdots & & & b_1 \\ \vdots & \ddots & & & b_2 \\ & & \ddots & \vdots & \vdots \\ & & \cdots & a_{mn} & b_n \end{array}\right]$$

전치.

Transpose of a Matrix.

A : m×n matrix.

A^T : n×m matrix.

$(A)_{ij} = (A^T)_{ji}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \qquad A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \\ a_{14} & a_{24} & a_{34} \end{bmatrix}$$

Trace.

"tr(A)" ... when A is square matrix (n×n).

$$tr(A) = \sum_{i=1}^{n} a_{ii}.$$

ex).

$$B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} = 1+6+11+16.$$