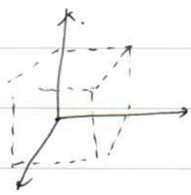
Euclidean $\mathbb{R}^n$.



실수
Real Vector Spaces

Def 1) Vector Space Axioms (공리).
(V.S.)

V : Set of objects. (vector...) { vector addition. (v.a.) / scalar multiplication (s.m.) } given. ⇙

$u, v, w \in V$, $\underline{u}+\underline{v} \in V$.

1. closed under v.a.
2. Commutating under v.a.
3. associativity under v.a.
4. There exists an identity for va → $\underline{u}+\underline{0} = \underline{0}+\underline{u} = \underline{u}$
5. There exists an inverse of $\underline{u}$ for v.a. → $\underline{u} + \underset{\text{→ negative}}{(-\underline{u})} = (-\underline{u}) + \underline{u} = 0$.
6. closed under s.m.
7. $k(\underline{u}+\underline{v}) = k\cdot\underline{u} + k\underline{v}$
8. $(k+m)\underline{u} = k\underline{u} + m\underline{u}$
9. associativity for s.m. $k(m\underline{u}) = (km)\underline{u}$
10. $1\underline{u} = \underline{u}$ (identity for s.m).

distributive laws.

ex1) $V = \{\underline{0}\}$. v.a. $= \underline{0}+\underline{0} = \underline{0}$.
s.m. $= k\underline{0} = \underline{0}$

NO.

* Ex 2). $\mathbb{R}^n$: VS.

$\underline{u}+\underline{v} = (\underline{u}_1+\underline{v}_1, \underline{u}_2+\underline{v}_2, \cdots, \underline{u}_n+\underline{v}_n)$.

$k\underline{u} = (k\underline{u}_1, k\underline{u}_2, \cdots, k\underline{u}_n)$.

* Ex 3) $\mathbb{R}^\infty$

$\underline{u} = (\underline{u}_1, \underline{u}_2, \cdots \cdots)$.

$\begin{cases} \underline{u}+\underline{v} = (\underline{u}_1+\underline{v}_1, \cdots) \\ k\underline{u} = (k\underline{u}_1, \cdots). \end{cases}$

* Ex 4) $V = \left\{ \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} \middle| \ u_{11}, u_{12}, u_{21}, u_{22} \in \mathbb{R} \right\}$.

$\underline{u}, \underline{v} \in V$

$\underline{u} = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}$, $\underline{v} = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}$.

* v.a. : $\underline{u}+\underline{v} = \begin{bmatrix} u_{11}+v_{11} & u_{12}+v_{12} \\ u_{21}+v_{21} & u_{22}+v_{22} \end{bmatrix} \in V$.

s.m. : $k\underline{u} = \begin{bmatrix} ku_{11} & ku_{12} \\ ku_{21} & ku_{22} \end{bmatrix} \in V$.

$\underline{u} + (-\underline{u}) = \underline{0}$.

M_{22}. $\cdots$ M_{mn} : V.S.

* Ex 6). V : set of real-valued fuctions defined on $(-\infty, \infty)$

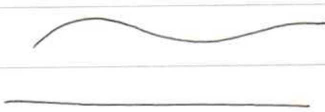
$\underline{f}, \underline{g} \in V$, $\underline{f} = f(x)$, $\underline{g} = g(x)$.

$\Rightarrow$ v.a. $\underline{f}+\underline{g} = f(x)+g(x)$. $\in V$.

s.m. $k\underline{f} = kf(x)$. $\in V$.

$\boxed{\text{V.S. } F(-\infty, \infty).}$

* V.S. of function f.



f.

$(\cdots, 3.1, 3.2, \cdots)$.

NO.

*ex7). $V = \mathbb{R}^2$. … Vector Space아! ⇒ 틀림. Axiom 확인해야.

$\Rightarrow$ v.a. $\underline{u}+\underline{v} = (u_1+v_1, u_2+v_2) \in \mathbb{R}^2$.

s.m. $k\underline{u} = (ku_1, 0) \in \mathbb{R}^2$.

(Specific Defined) Scalar Multiplication.

*ex8)

$$\boxed{\begin{aligned} &V = \{u \in \mathbb{R} \mid u > 0\} \text{ (define)} \\ &\Rightarrow \text{v.a. } \underline{u}+\underline{v} = \underline{u}\,\underline{v} \in V. \\ &\quad\ \text{s.m. } k\underline{u} = \underline{u}^k \in V. \end{aligned}}$$

$\in V$ … ①

$\in V$. … ⑥.

$\underline{u}+\underline{v} = \underline{u}\underline{v}$
$\underline{v}+\underline{u} = \underline{v}\underline{u}$ } same … ②.

$\underline{u}+(\underline{v}+\underline{w}) = \underline{u}(\underline{v}\underline{w})$.
$(\underline{u}+\underline{v})+\underline{w} = (\underline{u}\underline{v})\underline{w}$ } same … ③.

$\underline{u}+\underline{0} = \underline{u}$.
$u \cdot 1 = u$ $\Rightarrow \underline{0} = 1$ … ④.

$\underline{u}+(-\underline{u}) = \underline{0}$
$-\underline{u} = \frac{1}{\underline{u}}$
($*\ \underline{0} = 1$). } … ⑤.

$k(\underline{u}+\underline{v}) = (\underline{u}\underline{v})^k$
$k\underline{u}+k\underline{v} = u^k v^k$ } … ⑦.

$1 \cdot \underline{u} = \underline{u}^1 = u$ … ⑩.

$(k+m)\underline{u} = \underline{u}^{k+m}$
$k\underline{u}+m\underline{u} = \underline{u}^k\underline{u}^m$ } … ⑧.

$(km)\underline{u} = \underline{u}^{km}$
$k(m\underline{u}) = (u^m)^k$ } … ⑨.

NO.

* Thm 4.1.1. V: Vector space.

$\underline{u} \in V$.

* using v.s. of ex 8.

(a) $0\,\underline{u} = \underline{0}$.

- (a). $u^0 = 1$. ∴ $(\underline{0} = 1.)$ … 성립.

(b) $k\,\underline{0} = \underline{0}$.

(c) $(-1)\underline{u} = -\underline{u}$

(d) $k\underline{u} = \underline{0} \Rightarrow k = 0$ or $\underline{u} = \underline{0}$.