선형대수 - 4.8. Row Space, Column Space, and Null Space.

Matrix Spaces.

* Def 1)

$$A = \begin{bmatrix} \underline{r_1} \\ \underline{r_2} \\ \vdots \\ \underline{r_m} \end{bmatrix} = [\,\underline{c_1}\ \underline{c_2} \cdots \underline{c_n}\,]$$

A : $m \times n$.

$\underline{r_i}$: row vectors.

$\underline{c_i}$: column vectors.

* Def 2) Row space of A : row(A).

$= \text{span}\{\underline{r_1}, \underline{r_2}, \cdots, \underline{r_m}\}$.

Column space of A : col(A)

$= \text{span}\{\underline{c_1}, \underline{c_2}, \cdots, \underline{c_n}\}$.

Null space of A : null$(A) \rightarrow$ Solution space.

$= \{x \mid A\underline{x} = 0\}$.

- Relations between col(A), row(A), null(A).

$A\underline{x} = \underline{0}$, $A\underline{x} = \underline{b}$

* Thm 4.8.1) $A\underline{x} = \underline{b}$: consistent $\Leftrightarrow \underline{b} \in$ col(A).

$$\rightarrow A\underline{x} = [\underline{c_1}, \underline{c_2}, \cdots, \underline{c_n}] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$= \underbrace{x_1 \underline{c_1} + x_2 \underline{c_2} + \cdots + x_n \underline{c_n}}_{\in\ \text{col}(A) = \text{span}\{\underline{c_1}, \cdots, \underline{c_n}\}} = \underline{b}.$$

↑ section 1.2 ex 5·6).

4×6 size A.

$$A \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \longrightarrow \begin{bmatrix} x_1 \\ \vdots \\ x_6 \end{bmatrix} = \underbrace{r[\ ] + s[\ ] + t[\ ]}_{\substack{\text{general solution} \\ \text{of} \\ \text{homogeneous system.}}} + \underbrace{[\ ]}_{\substack{\text{constant.} \\ \text{particular solution} \\ \text{of} \\ \text{Non-homogeneous System.}}}$$

* Bases for Row(A), Col(A), Null(A).

*Thm 4.8.3) a. Row equivalent matrices have the same row space.

↳ 행 동치..

b. null space.

* Elementary Row Operation을 해도 null space (= Solution space)는 변하지 X.

해가 유지..

*Thm 4.8.4 and ex 3).

$$R = \begin{bmatrix} 1 & -2 & 5 & 0 & 3 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

↓. REF

4.

all zero.. Row space에 영향 X.

⇒ leading 1 row 만 Row space에 영향. 서로는 independent. 만 Basis가 됨.

이걸 보았을 때, 이는 leading 1.

모든 0으로 표현 가능.

⇒ leading 1 col는 column space의 Basis. 서로는 independent.

*ex 4)

$$A = \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}$$

row space

||

row space.

GE. → 하면 Basis 찾기 쉬움.

$$R = \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 0 & 0 & 1 & 3 & -2 & -6 \\ 0 & 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

REF

row spaces' basis.

* The basis obtained for row(R) $\boxed{\text{is a basis}}$ for row(A).

Col(R) $\boxed{\text{is not a basis}}$ for column(A).

* Thm 4.8.5) ERO do not change dependency relationships between column vectors.

* Elementary Row Operations가 Column operation는 '아니' 다름.

* ex 6).

$$A = \begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 2 & -5 & -3 & -2 & 6 \\ 0 & 5 & 15 & 10 & 0 \\ 2 & 6 & 18 & 8 & 6 \end{bmatrix} \begin{matrix} \text{independent?} \\ \text{independent?} \\ \text{independent?} \\ \text{independent?} \end{matrix}$$

Choose rows of A to form a basis for row(A).

⟷ ex3 … GE, ERO: row 수정 … 원본 row 복구

· sol) $A^T = \begin{bmatrix} 1 & 2 & 0 & 2 \\ -2 & -5 & 5 & 6 \\ 0 & -3 & 15 & 18 \\ 0 & -2 & 10 & 8 \\ 3 & 6 & 0 & 6 \end{bmatrix}$

$\downarrow R$

$R = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & 1 & -5 & -10 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

$\uparrow \quad \uparrow \qquad \uparrow$

* ex 7).

$\underline{v}_1 = (1, 2, 2, -1)$

$v_2 = (-3, -6, -6, 3)$.

$v_3 = (4, 9, 9, -4)$.

$v_4 = (-2, -1, -1, 2)$.

$v_5 = (5, 8, 9, -5)$.

직접 basis 골라보세요 : Column → ERO

→ Check indep col.

→ transpose.

Why null space is null space?

$$A\underline{x} = \underline{0}. \longrightarrow [\underline{c}_1 \ \underline{c}_2 \ \cdots \ \underline{c}_n] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}.$$

$$= \begin{bmatrix} \underline{r}_1 \\ \underline{r}_2 \\ \vdots \\ \underline{r}_n \end{bmatrix} [\underline{x}]. = \begin{bmatrix} \underline{r}_1 \cdot \underline{x} \\ \underline{r}_2 \cdot \underline{x} \\ \vdots \\ \underline{r}_n \cdot \underline{x} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

→ 두 벡터의 도트곱 0 ⇒ 수직.

$\Downarrow$

$c_1\underline{r}_1 + \cdots + c_n\underline{r}_n$ row(A).

$\Downarrow$

row space의 수직인 space.

⇒ Row space의 모든 도트곱 결과를 nulling 해줌.