* Def 1). V: real vector space. } inner product space.
$\langle \underline{u}, \underline{v} \rangle$: inner product.

4가지 조건 만족 시 연산자 사용가능. (= Axiom).
1. $\langle \underline{u}, \underline{v} \rangle = \langle \underline{v}, \underline{u} \rangle$
2. $\langle \underline{u}+\underline{v}, \underline{w} \rangle = \langle \underline{u}, \underline{w} \rangle + \langle \underline{v}, \underline{w} \rangle$.
3. $\langle k\underline{u}, \underline{v} \rangle = k\langle \underline{u}, \underline{v} \rangle$
4. $\langle \underline{v}, \underline{v} \rangle \geq 0$. Equality $\Leftrightarrow$ $\underline{v} = 0$.

• $\mathbb{R}^n$ $\langle \underline{u}, \underline{v} \rangle = \underline{u} \cdot \underline{v}$
$= u_1v_1 + u_2v_2 + \cdots + u_nv_n$.

* Dot Product is Inner Product in Euclidean Space.
= Standard inner product.

V가 $\langle \underline{u}, \underline{v} \rangle$에 의해 정의 or $\langle \underline{u}, \underline{v} \rangle$의 inner product space = V.

*Def 2). V: real inner product space.

• $\|\underline{v}\| = \sqrt{\langle \underline{v}, \underline{v} \rangle}$
• $d(\underline{u}, \underline{v}) = \|\underline{u} - \underline{v}\| = \sqrt{\langle \underline{u}-\underline{v}, \underline{u}-\underline{v} \rangle}$
• $\|\underline{u}\| = 1 \Rightarrow u$: unit vector.

*Thm 6.1.1). (a) $\|\underline{v}\| \geq 0$. Equalitey $\Leftrightarrow$ $\underline{v} = \underline{0}$.
(b) $\|k\underline{v}\| = k\|\underline{v}\|$.
(c) $d(\underline{u}, \underline{v}) = d(\underline{v}, \underline{u})$.
(d) $d(\underline{u}, \underline{v}) \geq 0$. Equality $\Leftrightarrow$ $\underline{u} = \underline{v}$

• $\mathbb{R}^n$ $\langle \underline{u}, \underline{v} \rangle = w_1u_1v_1 + \cdots + w_nu_nv_n$.
→ Weighted Euclidean inner product.

* ex1). $\mathbb{R}^2$ $\langle \underline{u}, \underline{v} \rangle = 3u_1v_1 + 2u_2v_2$.

Axiom 1. $\langle \underline{v}, \underline{u} \rangle = 3v_1u_1 + 2v_2u_2$.

Axiom 2. $\langle \underline{u}+\underline{v}, \underline{w} \rangle = 3(u_1+v_1)w_1 + 2(u_2+v_2)w_2$.
$= 3u_1w_1 + 2u_2w_2 + 3v_1w_1 + 2u_2w_2$.
$= \langle \underline{u}, \underline{w} \rangle + \langle \underline{v}, \underline{w} \rangle$.

Axiom 3. $\langle k\underline{u}, \underline{v} \rangle = 3(ku_1)v_1 + 2(ku_2)v_2$.
$= k(3u_1v_1 + 2u_2v_2)$.

Axiom 4. $\langle \underline{v}, \underline{v} \rangle = 3v_1v_1 + 2v_2v_2$
$= 3v_1^2 + 2v_2^2 \geq 0$.

NO.

* Def 3. $U = \{ \underline{u} \mid \|\underline{u}\| = 1 \}$.

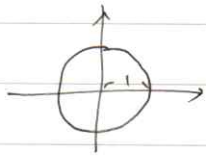
U : unit sphere.

* ex3). $\langle \underline{u}, \underline{v} \rangle = u_1 v_1 + u_2 v_2$.

$\| \underline{u} \| = \sqrt{\langle \underline{u}, \underline{u} \rangle}$

$= \sqrt{u_1^2 + u_2^2} = 1$.

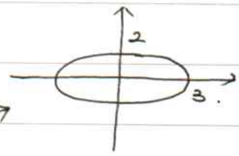
$u_1^2 + u_2^2 = 1$.



$\langle \underline{u}, \underline{v} \rangle = 1/9 \; u_1 v_1 + 1/4 \; u_2 v_2$.

$\| \underline{u} \| = \sqrt{1/9 \; u_1^2 + 1/4 \; u_2^2} = 1$.

$1/9 \, u_1^2 + 1/4 \, u_2^2 = 1$.



Inner Products Generated by Matrices.

*. dot product.
↓.
inner product.
↓.
weight inner product.
↓.
inner product gen by mat.

· matrix inner product $\langle \underline{u}, \underline{v} \rangle = A\underline{u} \cdot A\underline{v}$.

$= (A\underline{v})^T A\underline{u}$

$= \underline{v}^T A^T A \underline{u}$

* ex4). $\langle \underline{u}, \underline{v} \rangle = \underline{u} \cdot \underline{v}$

$= I\,\underline{u} \cdot I\,\underline{v}$

let $A = \begin{bmatrix} \sqrt{w_1} & & & 0 \\ & \sqrt{w_2} & & \\ & & \ddots & \\ 0 & & & \sqrt{w_n} \end{bmatrix}$

$(\sqrt{w_1} u_1, \sqrt{w_2} u_2, \cdots \sqrt{w_n} u_n)$ ·

$(\sqrt{w_1} v_1, \sqrt{w_2} v_2, \cdots, \sqrt{w_n} v_n)$

$\Rightarrow$ w_1^2 ..? $\rightarrow$ w. ...

* ex6). M_{nn}.

$\underline{u} = U$, $\underline{v} = V$.

$\langle \underline{u}, \underline{v} \rangle = tr(U^T V)$. $\rightarrow$ standard inner product.

NO.

*ex6) M_{nn}.

$\underline{u} = U$, $\underline{v} = V$. → standard inner product.

$\langle \underline{u}, \underline{v} \rangle = tr(U^T V)$. → $u_1v_1 + u_2v_2 + u_3v_3 + u_4v_4$.

$U = \begin{bmatrix} u_1 & u_2 \\ u_3 & u_4 \end{bmatrix}$, $V = \begin{bmatrix} v_1 & v_2 \\ v_3 & v_4 \end{bmatrix}$, $U^T V = \begin{bmatrix} u_1 & u_3 \\ u_2 & u_4 \end{bmatrix} \begin{bmatrix} v_1 & v_2 \\ v_3 & v_4 \end{bmatrix}$

$= \begin{bmatrix} u_1v_1 + u_2v_2 & \\ & u_3v_3 + u_4v_4 \end{bmatrix}$

$\|\underline{u}\| = \sqrt{\langle \underline{u}, \underline{u} \rangle}$

$= \sqrt{tr(U^T U)}$

$= \sqrt{u_1^2 + u_2^2 + u_3^2 + u_4^2}$

*ex7). P_n

$\underline{p} = a_0 + a_1 x + \cdots + a_n x^n$

$\underline{q} = b_0 + b_1 x + \cdots + b_n x^n$

$\langle \underline{p}, \underline{q} \rangle = a_0 b_0 + a_1 b_1 + \cdots + a_n b_n$. → standard inner product.

$\|\underline{p}\| = \sqrt{\langle \underline{p}, \underline{p} \rangle} = \sqrt{a_0^2 + a_1^2 + \cdots + a_n^2}$

*ex8). P_n

$\underbrace{x_0, x_1, \cdots, x_n}_{n+1 \text{개}}$ (sample points).

← distinct real number.

evaluation inner product.

$\langle \underline{p}, \underline{q} \rangle = p(x_0)q(x_0) + p(x_1)q(x_1) + \cdots + p(x_n)q(x_n)$.

$\|\underline{p}\| = \sqrt{\langle \underline{p}, \underline{p} \rangle} = \sqrt{p(x_0)^2 + p(x_1)^2 + \cdots + p(x_n)^2}$

NO.

*ex9.) P_2 $x_0 = -2,\ x_1 = 0.\ x_2 = 2.$

$$\left.\begin{array}{l} \underline{p} = x^2. \\ \underline{q} = 1 + x. \end{array}\right)$$

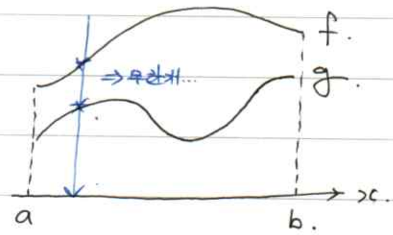
$$\langle p, q \rangle = p(-2)q(-2) + p(0)q(0) + p(2)q(2).$$
$$= 4(-1) + 0\cdot 1 + 4\cdot 3 = 8.$$

$$\|p\| = \sqrt{p(x_0)^2 + p(x_1)^2 + p(x_2)^2}$$
$$= \sqrt{4^2 + 0^2 + 4^2} = 4\sqrt{2}$$

*ex10). $C[a, b]$. $f = f(x)$, $g = g(x)$.

$$\langle f, g \rangle = \int_a^b f(x)g(x)\,dx.$$

→ standard Inner Product.



*f *g 다곱해라.

→ 함수를 inner product 하면...

무한한 친구들을 Sum. = 적분..

*Thm 6.1.2). Dot곱 Axiom의 Inner Product3의 확장.

(a) $\langle \underline{0}, \underline{v} \rangle = \langle \underline{v}, \underline{0} \rangle = 0.$

(b) $\langle \underline{u}, \underline{v} + \underline{w} \rangle = \langle \underline{u} + \underline{v} \rangle + \langle \underline{u}, \underline{w} \rangle.$

(c) $\langle \underline{u}, \underline{v} - \underline{w} \rangle = \langle \underline{u}, \underline{v} \rangle - \langle \underline{u}, \underline{w} \rangle.$

(d) $\langle \underline{u} - \underline{v}, \underline{w} \rangle = \langle \underline{u}, \underline{w} \rangle - \langle \underline{v}, \underline{w} \rangle.$

(e) $k\langle \underline{u}, \underline{v} \rangle = \langle \underline{u}, k\underline{v} \rangle.$

*ex12)

$$\langle \underline{u} - 2\underline{v}, 3\underline{u} + 4\underline{v} \rangle = \langle \underline{u}, 3\underline{u} + 4\underline{v} \rangle - \langle 2\underline{v}, 3\underline{u} + 4\underline{v} \rangle.$$
$$= \langle \underline{u}, 3\underline{u} \rangle + \langle \underline{u}, 4\underline{v} \rangle - \langle 2\underline{v}, 3\underline{u} \rangle - \langle 2\underline{v}, 4\underline{v} \rangle.$$
$$= 3\langle \underline{u}, \underline{u} \rangle + 4\langle \underline{u}, \underline{v} \rangle - 6\langle \underline{u}, \underline{v} \rangle - 8\langle \underline{v}, \underline{v} \rangle.$$
$$= 3\|\underline{u}\|^2 - 2\langle \underline{u}, \underline{v} \rangle - 8\|\underline{v}\|^2.$$

* Chap 6.2 is same as 3.2.

(inner product.) (dot product.)