*Def 1. $\underline{v_1}, \underline{v_2}, \cdots, \underline{v_r}, \underline{w} \in V.$

$k_1\underline{v_1} + k_2\underline{v_2} + \cdots + k_r\underline{v_r} = \underline{w}$

→ a linear combination of $v_1, \cdots, v_r$

k_i is coefficient.

*Thm 4.3.1. $S = \{\underline{v_1}, \underline{v_2}, \cdots, \underline{v_r}\} \subset V$

$W = \{\underline{k_1 v_1} + \cdots + k_r \underline{v_r} \mid k_1 \cdots k_r \in \mathbb{R}\}$. (→ size: inf.)

↳ s.s. of V.   ∴ V에서 r개의 벡터 뽑아서 선형결합 ⇒ V의 sub space.

Smallest subspace of containing (r개를 포함한 선형결합, 모음으로).

* S spans W

W is spanned by S.

$W = \text{span}\{\underline{v_1}, \underline{v_2}, \cdots, \underline{v_r}\} = \text{span}(S).$

*ex1)

$\underline{e_1} = (1, 0, \cdots, 0)$

$\underline{e_2} = (0, 1, \cdots, 0).$

$\underline{e_r} = (0, 0, \cdots, 1).$

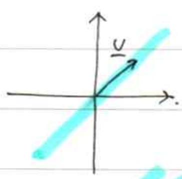
$\underline{v} \in \mathbb{R}^n$ $\underline{v} = (v_1, v_2, \cdots, v_n).$

$\underline{v} = \underbrace{v_1\underline{e_1} + v_2\underline{e_2} + \cdots + v_r\underline{e_r}}_{}$

linear combination of $e_1, e_2, \cdots, e_r$

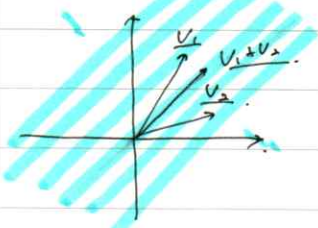
$e_1, e_2, \cdots, e_r$ spans $\mathbb{R}^n$

*ex2)



* span($\underline{v}$)? → $\underline{v}$로 만들 수 있는 모든 선형결합.

⇒ (any) $\underline{v}$ = 직선.



span ($\underline{v_1}$, +$\underline{v_2}$)

⇒(any) v_1 +(any v_2) ⇒ 평면.

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*ex3. $1, x, \cdots x^n \in P_n$.

$\underline{P} = a_0 \cdot 1 + a_1 x_1 + a_2 x^2 + \cdots a_n x^n$.

spans P_n.

*ex4. $\underline{u} = (1, 2, -1)$, $\underline{v} = (6, 4, 2)$.

$\underline{w} = (9, 2, 7)$, $\underline{w}' = (4, -1, 8)$.

linear Combination.

$(9, 2, 7) = k_1 (1, 2, -1) + k_2 (6, 4, 2)$.

$$\left.\begin{aligned} 9 &= k_1 + 6k_2 \\ 2 &= 2k_1 + 4k_2 \\ 7 &= -k_1 + 2k_2 \end{aligned}\right\}$$

*ex5.

$$\left.\begin{aligned} \underline{v_1} &= (1, 1, 2) \\ \underline{v_2} &= (1, 0, 1) \\ v_3 &= (2, 1, 3) \end{aligned}\right\} \text{ spans } \mathbb{R}^3?$$

sol). $\underline{v} \in \mathbb{R}^3$

$\underline{v} = (v_1, v_2, v_3)$.

$(v_1, v_2, v_3) = k_1(1, 1, 2) + k_2(1, 0, 1) + k_3(2, 1, 3)$.

$$\left.\begin{aligned} k_1 + k_2 + 2k_3 &= v_1 \\ k_1 \qquad + k_3 &= v_2 \\ 2k_1 + k_2 + 3k_3 &= v_3 \end{aligned}\right\} \rightarrow \text{Consistent?}$$

$\det \neq 0$.

*ex6. (a) $S = \{1 + x + x^2, -1 - x, 2 + 2x + x^2\}$.

$P = P_0 + P_1 x + P_2 x^2 \in P_2$.

$$\left.\begin{aligned} &= k_1 (1 + x + x^2) \\ &+ k_2 (-1 - x) \\ &+ k_3 (2 + 2x + x^2) \end{aligned}\right\}$$

$$\begin{aligned} P_0 &= k_1 - k_2 + 2k_3 \\ P_1 &= k_1 - k_2 + 2k_3 \\ P_2 &= k_1 \qquad + k_3 \end{aligned} \rightarrow \det \begin{bmatrix} 1 & -1 & 2 \\ 1 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix} = 0.$$

Coefficient.

(b) $S = \{x + x^2, x - x^2, 1 + x, 1 - x\}$

Spans P_2?

식 세개. 응과 두개... 무수히 많음.

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 1 & 1 & P_0 \\ 1 & 1 & 1 & -1 & P_1 \\ 1 & -1 & 0 & 0 & P_2 \end{bmatrix}$$

Compute RREF.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & (-P_0 + P_1 + P_2)/2 \\ 0 & 1 & 0 & 0 & (-P_0 + P_1 - P_2)/2 \\ 0 & 0 & 1 & -1 & P_0 \end{bmatrix}$$

* ex 7. $S = \left\{ \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right\}$.

S spans M_{22}?

$\underline{u} \in M_{22}$. $\underline{u} = \begin{bmatrix} u_1 & u_2 \\ u_3 & u_4 \end{bmatrix}$

$$= k_1 \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + k_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + k_3 \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} + k_4 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 0 & 2 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$$

… $\det \neq 0$?

* Thm 4.3.2. V.

$\mathrm{span}\{v_1, v_2, \cdots, v_r\} = \mathrm{span}\{w_1, w_2, \cdots, w_k\}$.

$\Longleftrightarrow$ v_i is expressed as a linear combination of $w_1, w_2, \cdots, w_k$.

w_i is expressed as a linear combination of $v_1, v_2, \cdots, v_r$.

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