Institut für Mechatronische Systeme
imes Leibniz Universität Hannover

Leibniz Universität Hannover

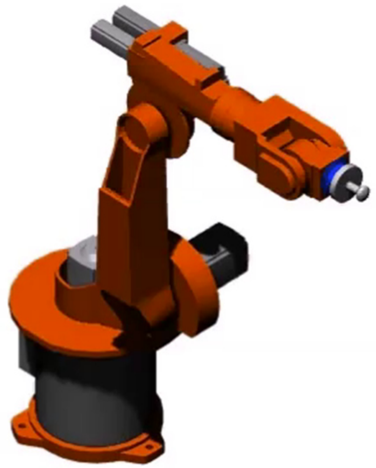
Robotics I

05. Kinematically Redundant Robots (Introduction)

Review

How do I relate end effector and joint angular velocities and how do I determine important properties of my robot?

- Differential kinematics
- Jacobian matrix
- Manipulability
- Static force and torque model
- Singularities



Literature

B. Heimann et al. – Mechatronik (Komponenten – Methoden – Beispiele), S. 190–199
J. J. Craig – Introduction to Robotics Mechanics and Control, S. 152–182
L. Sciavicco et al. – Modeling and Control of Robot Manipulators, S. 69–117

Total content

Control

- Sensors
- Advanced Control Methods
- Multi-Axis Control
- Single-Axis Control
- Dynamics: Newton-Euler and Lagrange
- Path Planning
- **Kinematically Redundant Robots**
- Jacobian Matrix – Velocities and Forces
- Forward and Inverse Kinematics
- Coordinate Transformations
- Introduction

Kinematics and Dynamics

KUKA LBR (kinematically redundant)
(dim($\boldsymbol{q}$) = 7, dim($\boldsymbol{x}_\mathrm{E}$) = 6)

Kinematically Redundant Robots

What (constructive) measures can be taken to avoid collisions and singularities and improve other performance characteristics?

- Introduction
- Redundancy and (differential) kinematics
- Null space as a homogeneous solution of differential kinematics
- Redundancy and inverse kinematics

KUKA LBR

Literature:
G. Hirzinger et al. – DLR's torque-controlled light weight robot III – are we reaching the technological limits now?
A. Albu-Schäffer et al. – The DLR lightweight robot: design and control concepts for robots in human environments
N. N. – Eigenwerte und Eigenvektoren von Matrizen

Introduction

Definition Redundancy
Number of minimum coordinates $\boldsymbol{q}$ is larger than the number of independent end effector degrees of freedom $\boldsymbol{x}_\mathrm{E}$ (dimension task space): $\dim(\boldsymbol{q}) > \dim(\boldsymbol{x}_\mathrm{E})$

LBR (DLR) – catching a ball

Application:
- Obstacle avoidance
- Singularity avoidance
- Local performance increase through reconfiguration

KUKA LBR
$(\dim(\boldsymbol{q}) = 7, \dim(\boldsymbol{x}_\mathrm{E}) = 6)$

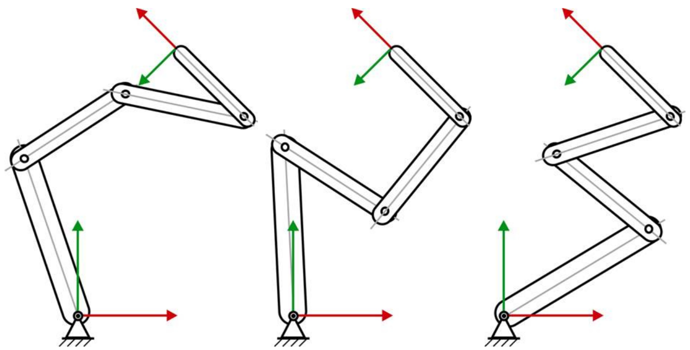
Robotics I

05. Kinematically Redundant Robots (Kinematics)

Redundancy and Kinematics

Effects
- No effect on the solvability of forward kinematics
- Inverse kinematics not uniquely solvable, infinitely many solutions (null space motion possible)

KineMedic (DLR)
(Nullspace motion)



Example: Planar RRRR-robot
$(\dim(\boldsymbol{q}) = 4, \dim(\boldsymbol{x}_\mathrm{E}) = 3)$

Redundancy and Differential Kinematics

board notes

Effects

Jacobian matrix has more columns than rows $(m < n)$:

$$\boldsymbol{J}(\boldsymbol{q}) = \frac{\partial \boldsymbol{f}(\boldsymbol{q})}{\partial \boldsymbol{q}} \in \mathbb{R}^{\overset{\dim(\boldsymbol{x}_\mathrm{E})}{m} \times \underset{\dim(\boldsymbol{q})}{n}}$$

Infinitely many solutions for inverse differential kinematics

➡ Underdetermined, linear system of equations with constraints for solving the inverse problem:

$\dot{\boldsymbol{q}} = \boldsymbol{J}^{+}(\boldsymbol{q})\, \dot{\boldsymbol{x}}_\mathrm{E}$

Moore-Penrose pseudoinverse:

$\boldsymbol{J}^{+}(\boldsymbol{q}) = \boldsymbol{J}^\mathrm{T}(\boldsymbol{q}) \left(\boldsymbol{J}(\boldsymbol{q})\, \boldsymbol{J}^\mathrm{T}(\boldsymbol{q})\right)^{-1}$

is determined by solving the minimization problem:

$g(\dot{\boldsymbol{q}}) = \frac{1}{2} \dot{\boldsymbol{q}}^\mathrm{T} \dot{\boldsymbol{q}}$

Redundancy and Differential Kinematics

Application

We are looking for the EE motion with minimum joint motion

$$g(\dot{\boldsymbol{q}}) = \frac{1}{2}\,\dot{\boldsymbol{q}}^{\mathrm{T}}\,\boldsymbol{W}\,\dot{\boldsymbol{q}} \quad \rightarrow \quad \min$$

$(n \text{ x } n)$ weighting matrix $\boldsymbol{W} = \mathrm{diag}(\boldsymbol{w})$,
with $w_i > 0$

with the constraint $\dot{\boldsymbol{x}}_{\mathrm{E}} = \boldsymbol{J}(\boldsymbol{q})\,\dot{\boldsymbol{q}}$.

results from specification
of the target motion.

KUKA LBR
$(\dim(\boldsymbol{q}) = 7, \;\; \dim(\boldsymbol{x}_{\mathrm{E}}) = 6)$

Lagrange multiplier technique provides solution:

irrelevant for
minimization

$$g(\dot{\boldsymbol{q}}) = \cancel{\frac{1}{2}}\,\dot{\boldsymbol{q}}^{\mathrm{T}}\,\boldsymbol{W}\,\dot{\boldsymbol{q}} \quad \rightarrow \quad \min$$

$$\dot{\boldsymbol{x}}_{\mathrm{E}} = \boldsymbol{J}(\boldsymbol{q})\,\dot{\boldsymbol{q}}$$

$$\dot{\boldsymbol{q}} = \overbrace{\boldsymbol{W}^{-1}\,\boldsymbol{J}^{\mathrm{T}}(\boldsymbol{q})\left(\boldsymbol{J}(\boldsymbol{q})\,\boldsymbol{W}^{-1}\,\boldsymbol{J}^{\mathrm{T}}(\boldsymbol{q})\right)^{-1}}^{\text{Moore-Penrose pseudoinverse } \boldsymbol{J}^{+}(\boldsymbol{q}) \text{ for } \boldsymbol{W} = \boldsymbol{E}}\,\dot{\boldsymbol{x}}_{\mathrm{E}}$$

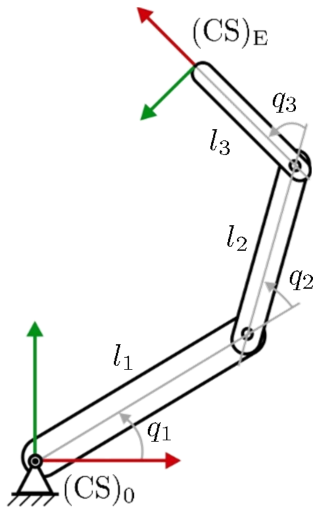
Example: Redundant, Planar RRR-robot

$$\boldsymbol{x}_{\mathrm{E}} = \left({}_{(0)}x_{\mathrm{E}},\ {}_{(0)}y_{\mathrm{E}}\right)^{\mathrm{T}}$$
$$\boldsymbol{q} = (q_1,\ q_2,\ q_3)^{\mathrm{T}}$$

$$l_1 = 6\,\mathrm{m},\ l_2 = 5\,\mathrm{m},\ l_3 = 4\,\mathrm{m}$$

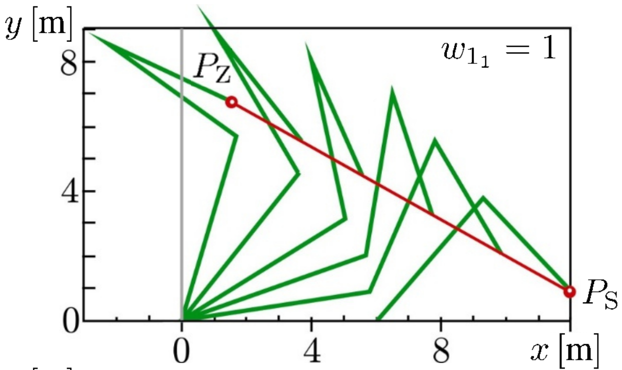
Specification: Movement from a starting point P_{S} to a target point P_{Z} at a constant velocity $\boldsymbol{v}_0$:

$$\boldsymbol{x}_{\mathrm{E}} = P_{\mathrm{S}} + v_0\, t\, (P_{\mathrm{Z}} - P_{\mathrm{S}})$$
$$\dot{\boldsymbol{x}}_{\mathrm{E}} = \mathrm{const}$$



Planar RRR-robot

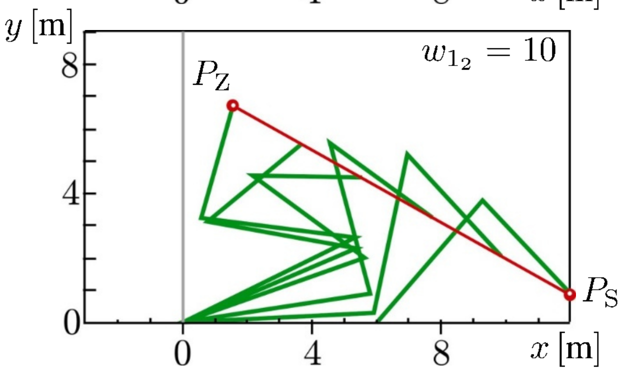
Example Redundant, Planar RRR-robot

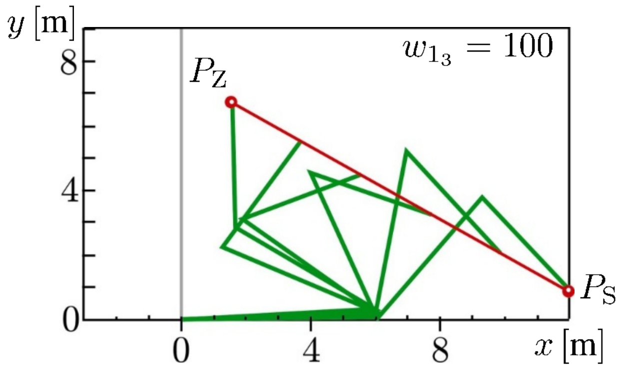


$$\dot{\boldsymbol{q}} = \boldsymbol{W}^{-1}\,\boldsymbol{J}^\mathrm{T}(\boldsymbol{q})\left(\boldsymbol{J}(\boldsymbol{q})\,\boldsymbol{W}^{-1}\,\boldsymbol{J}^\mathrm{T}(\boldsymbol{q})\right)^{-1}\dot{\boldsymbol{x}}_\mathrm{E}$$

Joint motion depending on the weighting matrix $\boldsymbol{W}_i$:

$$\boldsymbol{W}_i = \begin{pmatrix} w_{1_i} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$





Robotics I

05. Kinematically Redundant Robots (Null Space/Inverse Kinematics)

Null Space as a Homogeneous Solution

Null space as a homogeneous solution of
$\dot{\boldsymbol{x}}_\mathrm{E} = \boldsymbol{J}(\boldsymbol{q})\,\dot{\boldsymbol{q}}$

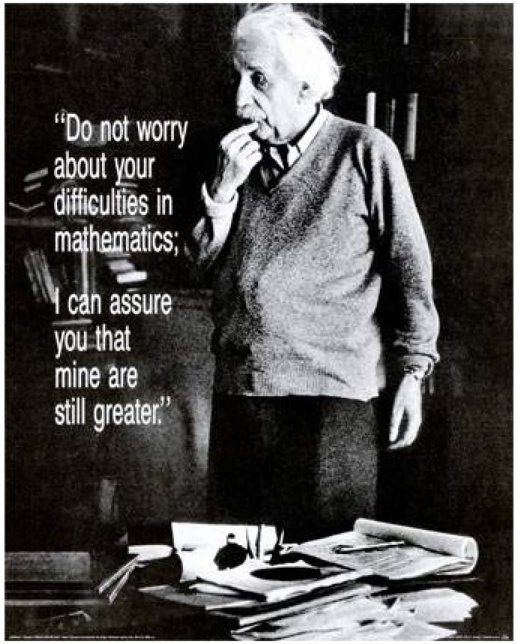
Review:
Equation for systems of linear equations $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$
is solvable if $\mathrm{Rank}(\boldsymbol{A}|\boldsymbol{b}) = \mathrm{Rank}(\boldsymbol{A})$.

Solution has the form $\boldsymbol{x} = \boldsymbol{x}_\mathrm{p} + \boldsymbol{x}_\mathrm{h}$
where $\boldsymbol{x}_\mathrm{p}$ is the particular solution and
$\boldsymbol{x}_\mathrm{h}$ is the homogeneous solution ($\boldsymbol{0} = \boldsymbol{A}\boldsymbol{x}_\mathrm{h}$).

Selection of free variables:
n - $\mathrm{Rank}(\boldsymbol{A})$, with $n = \dim(\boldsymbol{x})$

➡ Transferred to

$\dot{\boldsymbol{x}}_\mathrm{E} = \boldsymbol{J}(\boldsymbol{q})\,\dot{\boldsymbol{q}}$



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Null Space as a Homogeneous Solution

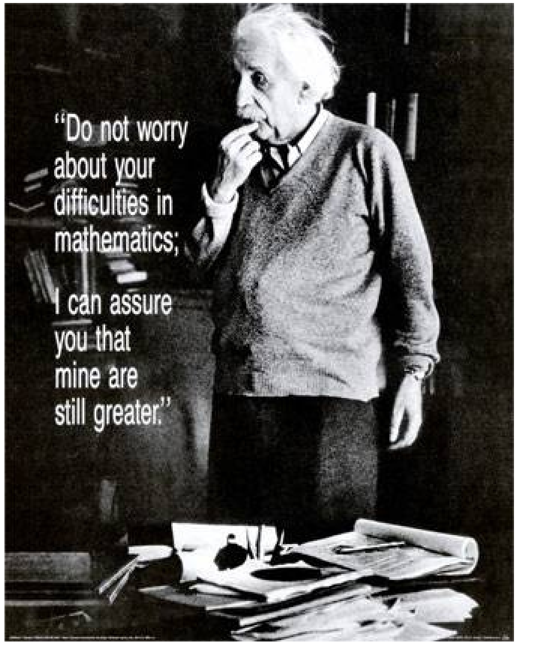
board notes

Null space as a homogeneous solution of
$\dot{\boldsymbol{x}}_\mathrm{E} = \boldsymbol{J}(\boldsymbol{q})\,\dot{\boldsymbol{q}}$, with $\boldsymbol{J}(\boldsymbol{q}) \in \mathbb{R}^{m\times n} \quad (m < n)$
and $\mathrm{Rank}(\boldsymbol{J}(\boldsymbol{q})) = m$

Full rank, no singularity

Transferred to the system of equations above:
System is solvable because
$\mathrm{Rank}(\boldsymbol{J}(\boldsymbol{q})\,|\,\dot{\boldsymbol{x}}_\mathrm{E}) = \mathrm{Rank}\,(\boldsymbol{J}(\boldsymbol{q})) = m$

There is a solution
$\dot{\boldsymbol{q}} = \dot{\boldsymbol{q}}_\mathrm{p} + \dot{\boldsymbol{q}}_\mathrm{h}$
with the following number of free variables
$\dim(\dot{\boldsymbol{q}}) - m = n - m$



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Redundancy and Inverse Kinematics

Addendum on Singularities

Loss of at least one degree of freedom in the task space, rank deficiency of $\boldsymbol{J}$:

After elementary operations the following applies for $\mathrm{Rank}(\boldsymbol{J}(\boldsymbol{q})) = n - 1$:

$$\dot{\boldsymbol{x}}_{\mathrm{E}}^{*} = \boldsymbol{J}^{*}\dot{\boldsymbol{q}}, \quad \text{mit} \quad \boldsymbol{J}^{*} = \begin{pmatrix} * & ? & ? & ? \\ 0 & * & ? & ? \\ 0 & 0 & * & ? \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

➡ last row no longer commandable

Deletion of degree of freedom leads to a redundant system:

$$\boldsymbol{J}^{*} = \begin{pmatrix} * & ? & ? & ? \\ 0 & * & ? & ? \\ 0 & 0 & * & ? \end{pmatrix}$$

➡ four joint angle DOF for three (remaining) end effector DOF

Redundancy and Inverse Kinematics

Formulation of Optimization Problem

i.e. manipulability of redundant robots

Determination of the determinant of $\boldsymbol{J}(\boldsymbol{q})\,\boldsymbol{J}^{\mathrm{T}}(\boldsymbol{q})$: $\det\left(\boldsymbol{J}(\boldsymbol{q})\,\boldsymbol{J}^{\mathrm{T}}(\boldsymbol{q})\right)$

Maximization of manipulability:

- Objective function $g(\boldsymbol{q}) = \det\left(\boldsymbol{J}(\boldsymbol{q})\,\boldsymbol{J}^{\mathrm{T}}(\boldsymbol{q})\right) \rightarrow \max$
- Constraints $\boldsymbol{x}_{\mathrm{E}} = \boldsymbol{f}(\boldsymbol{q})$

Solution analogous to the derivation of the Moore-Penrose pseudoinverse:

Coupling constraint by λ

➡ necessary conditions:

$$g(\boldsymbol{q},\,\boldsymbol{\lambda}) = -\det\left(\boldsymbol{J}(\boldsymbol{q})\,\boldsymbol{J}^{\mathrm{T}}(\boldsymbol{q})\right) + \boldsymbol{\lambda}^{\mathrm{T}}\,(\boldsymbol{x}_{\mathrm{E}} - \boldsymbol{f}(\boldsymbol{q}))$$

$$\frac{\partial g(\boldsymbol{q},\,\boldsymbol{\lambda})}{\partial \boldsymbol{q}} \overset{!}{=} \boldsymbol{0}, \qquad \frac{\partial g(\boldsymbol{q},\,\boldsymbol{\lambda})}{\partial \boldsymbol{\lambda}} = \boldsymbol{x}_{\mathrm{E}} - \boldsymbol{f}(\boldsymbol{q}) \overset{!}{=} \boldsymbol{0} \quad \text{solve}$$

Redundancy and Inverse Kinematics

Adding Kinematic Constraints

Definition $k = \dim(\boldsymbol{q}) - \dim(\boldsymbol{x}_\mathrm{E})$ of additional kinematic constraints, for example:

$$q_1 + q_2 = c \quad \Rightarrow \quad h(\boldsymbol{q}) = q_1 + q_2 - c = 0$$

$$\begin{pmatrix} \boldsymbol{x}_\mathrm{E} \\ \boldsymbol{0} \end{pmatrix} = \begin{pmatrix} \boldsymbol{f}(\boldsymbol{q}) \\ \boldsymbol{h}(\boldsymbol{q}) \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} \dot{\boldsymbol{x}}_\mathrm{E} \\ \boldsymbol{0} \end{pmatrix} = \tilde{\boldsymbol{J}}(\boldsymbol{q})\,\dot{\boldsymbol{q}}, \quad \tilde{\boldsymbol{J}}(\boldsymbol{q}) \in \mathbb{R}^{n \times n}$$

Questions for Self Monitoring

Kinematically Redundant Robots

1. What (constructive) measures can be taken to avoid collisions and singularities and improve other performance characteristics?
2. What are the characteristics of a kinematically redundant industrial robot with a serial kinematic structure?
3. What effect do the additional joint degrees of freedom have on the forward and inverse kinematics?
4. Name known approaches or performance characteristics for (optimal) utilization of the additional degrees of freedom!