Simon Gibson, gibsos3@rpi.edu **Homework 1: Foundations of Computer Science**

Problem 1. Consider the compound proposition: $(p \lor q \lor \neg r) \to (p \land q)$.

(a) Give a truth table for this proposition

p	q	r	$\neg r$	$p \lor q \lor \neg r$	$p \land q$	$(p \lor q \lor \neg r) \to (p \land q)$
T	T	T	F	T	T	F
T	T	F	T	T	T	T
T	F	T	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	T	F	T	T	F	F
F	F	T	F	F	F	T
F	F	F	T	T	F	F

(b) Give the DNF

$(p \lor q \lor \neg r) \to (p \land q)$	
$\neg(p \lor q \lor \neg r) \lor (p \land q)$	Table 7
$(\neg p \land \neg q \land \neg\neg r) \lor (p \land q)$	DeMorgan's Law
$(\neg p \land \neg q \land r) \lor (p \land q)$	Double Negation
DNF Form	

Problem 2.

(a) $p \to (q \lor s)$

(b) $\neg r \to (\neg q \land s)$

(c) $q \to (\neg s \lor r)$

(d) $(q \land p) \to \neg s$

Problem 3.

(a) Let j_1, s_1, k_1 represent the threes attendance, and let j_2, s_2, k_2 represent their happiness. Note that j_1 is independent of any other attendance, while $s_1 \to k_1$ and $k_1 \to j_1$. For happiness, $s_1 \to \neg j_2$, and the other two are independent for happiness.

(b) Thus the combination of friends which allows everyone to be happy is any without Samir's attendance, so either Jasmine and Kanti or just Jasmine.

Problem 4.

$(q \land \neg r) \to (p \to \neg r)$	
$(q \land \neg r) \to (\neg p \lor \neg r)$	Table 7
$\neg(q \land \neg r) \lor (\neg p \lor \neg r)$	Table 7
$(\neg q \lor \neg\neg r) \lor (\neg p \lor \neg r)$	DeMorgan's Law
$(\neg q \lor r) \lor (\neg p \lor \neg r)$	Double Negation
$\neg r \lor r \lor \neg q \lor \neg p$	Commutative/Associative of $\lor$
$T \lor \neg q \lor \neg p$	Negation

$T \vee (\neg q \vee \neg p)$ Associative

$(\neg q \vee \neg p) \vee T$ Commutative

T Domination

Problem 5.

(a) $\exists x(\neg P(x) \vee \neg Q(x))$

(b) $\forall x(Q(x) \rightarrow \exists y(S(x,y)))$

(c) $\exists x_1 \exists x_2(\forall s(\neg S(x_1,s) \wedge \neg S(x_2,s)))$

(d) $\exists x_1 \exists x_2(P(x_1) \wedge P(x_2) \rightarrow \forall s(S(x_1,s) \vee S(x_2,s)))$

(e) $\forall x(\exists s(S(x,s) \rightarrow \neg Q(x)))$

Problem 6.

(a) $\forall x(P(x) \rightarrow Q(x))$

$\forall x(\neg P(x) \vee Q(x))$ Table 7

$\exists x \neg(\neg P(x) \vee Q(x))$ Negated Quantifiers

Can't go further without $\wedge$

(b) $\forall x(\neg Q(x) \iff (P(x) \wedge R(x)))$

$\forall x((\neg Q(x) \rightarrow (P(x) \wedge R(x))) \wedge ((P(x) \wedge R(x)) \rightarrow \neg Q(x)))$ Table 8

$\forall x((\neg\neg Q(x) \vee (P(x) \wedge R(x))) \wedge (\neg(P(x) \wedge R(x)) \vee \neg Q(x)))$ Table 7

$\forall x((Q(x) \vee (P(x) \wedge R(x))) \wedge (\neg Q \vee (\neg P(x) \wedge R(x))))$ Double Negation and Commutative

$\forall x(((Q(x) \vee P(x)) \wedge (Q(x) \vee R(x))) \wedge ((\neg Q(x) \vee \neg P(x)) \wedge (\neg Q(x) \vee R(x))))$

Got Stuck

(c) $\exists x \forall y((S(x,y0 \wedge Q(x)) \rightarrow (R(x) \vee R(y)))$

$\exists x \forall y(\neg(S(x,y) \wedge Q(x)) \vee (R(x) \vee R(y)))$ Table 7

$\exists x \forall y((\neg S(x,y) \vee \neg Q(x)) \vee (R(x) \vee R(y)))$ Distributive

$\exists x \forall y(\neg S(x,y) \vee Q(x) \vee R(x) \vee R(y))$ Commutative

$\exists x \exists y \neg(\neg S(x,y) \vee Q(x) \vee R(x) \vee R(y))$ Quantifier Negation