[redacted]

6

Borderline F. Consider dropping!

1515?

Examination 1 Math 4100 Closed Book Prof. Chjan Lim

All electronic devices must be turned off for the duration of this examination. You must write legibly and provide full justification of each step in your solution to receive full credit for a problem. Makeups requires a note from advisor or Dean's office.

You can assume that the vector spaces concerned are over the reals.

(1) Solve the linear system for the vector x :

$$\mathbf{M}x = y$$
$$\mathbf{M} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}, y = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = $

$x_1 = 0$
$x_2 = 1$

X −2

(2) Does the following linear system have a solution? If YES find the solution; If NOT explain why:

$$\mathbf{M}x = y$$
$$\mathbf{M} = \begin{array}{c} ① \\ ② \end{array}\begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}, y = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

No Solution; L.D. (2① = 1②) rows ✓

(3) Find the eigenvalues and eigenvectors :

$$\mathbf{M}v = kv$$
$$\mathbf{M} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

Sub in last λ

$\begin{bmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{bmatrix} \to \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$(1-\lambda)(1-\lambda) = 0$

$\lambda = 1$

eigenvectors −1

(4) Calculate the inverse of the following matrix if it exists or explain why there is no inverse:

$$\mathbf{M} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}.$$

O.K.

No inverse; with elementary Row operations, we can get $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$, which easily can't be row reduced by anything to get $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ due to the second row being 0, null ✓

L.D. rows

(5) What is the span of the following vectors:

$$x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, y = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}.$$

L.I L.I

$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \xrightarrow{1} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \to$ span(x,y) = 2

x,y are L.I thus span is 2

span is V-space ≠ 2

−1