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Borderline F.
Consider dropping!

15/15?

Examination 1 Math 4100 Closed Book Prof. Chjan Lim

All electronic devices must be turned off for the duration of this examination. You must write legibly and provide full justification of each step in your solution to receive full credit for a problem. Makeups requires a note from advisor or Dean's office.

You can assume that the vector spaces concerned are over the reals.

(1) Solve the linear system for the vector x :

$$Mx = y$$

$$M = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}, y = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Handwritten solution for (1):

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$x_1 = 0$
 $x_2 = 1$

Final answer circled: -2

(2) Does the following linear system have a solution? If YES find the solution; If NOT explain why:

$$Mx = y$$

$$M = \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}, y = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

No Solution; L.D. (20=10) rows

(3) Find the eigenvalues and eigenvectors :

Subst last

$$\begin{bmatrix} 1-\lambda & 1 \\ 0 & -\lambda \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$Mv = \lambda v$$

$$M = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Handwritten calculation:

$$(1-\lambda)(1-\lambda) = 0$$

$$\lambda = 1$$

eigenvector -1

(4) Calculate the inverse of the following matrix if it exists or explain why there is no inverse:

O.K.

$$M = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

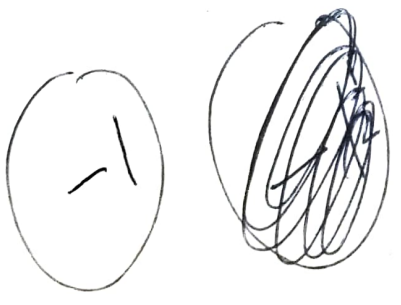
No inverse; with elementary row operations we can get $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$, which can't be inverted by anything to get $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ due to the second row being 0, all L.D. rows

(5) What is the span of the following vectors:

$$x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, y = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

Handwritten solution for (5):

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \xrightarrow{L.I. \quad L.I.} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \rightarrow \text{Span}(x,y) = 2$$



x, y are L.I. thus span is 2
span is V -space $\neq 2$