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Examination 3 Math 4100 Closed Book Prof. Chjan Lim

All electronic devices must be turned off for the duration of this examination. You must write legibly and provide full justification of each step in your solution to receive full credit for a problem. Only solutions / proofs on the blank sheets stapled here will be graded.

(1) *Prove or disprove:* If $B = \{v_1, \dots, v_m\}$ is a basis for U , and $C = \{w_1, \dots, w_m\}$ is linearly independent in U , then $\{v_1 + w_1, \dots, v_m + w_m\}$ is linearly independent in U .

(2) *Prove or disprove:* *There* is a basis for $P_3(R) = \{\text{all real polynomials with degree less than or equal to 3}\}$ such that none of the elements of this basis has degree 2.

(3) Suppose that U and W are subspaces of R^8 , $\dim(U) = 3$, $\dim(W) = 5$, and $U + W = R^8$. Prove that $R^8 = U \oplus W$.

(4) Let $T : V \rightarrow W$ be a linear map, $\{v_1, \dots, v_m\}$ is a set of vectors in V , and $\{Tv_1, \dots, Tv_m\}$ is linearly independent in W . Prove or disprove that

$\{v_1, \dots, v_m\}$ is linearly independent in V .

(5) Give an example of a linear map $T : R^4 \rightarrow R^4$ such that $\text{Range } T = \text{Null } T < R^4$.

W

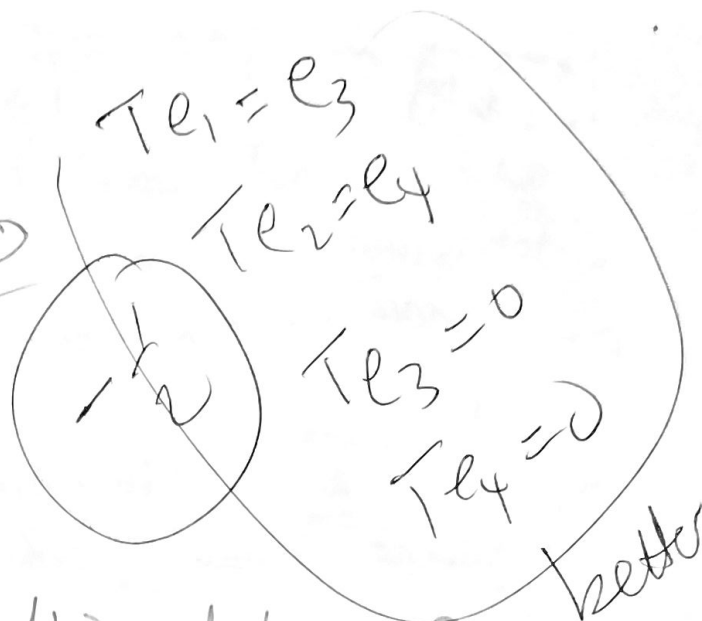
5. $e_1, e_2, e_3, e_4 \in \mathbb{R}^4$

$T e_1 = e_2$ $T e_2 = e_1$, $T e_3 = T e_4 = 0$

$\text{Range} = 2$

$\text{Nullity} = 2$

$R + N = \mathbb{R}^4$



this maps 2 vectors to 0, giving $N=2$, and also maps 2 vectors to each other giving $R=2$, $N+R = \mathbb{R}^4$, $2+2 = \mathbb{R}^4$ ✓

1. Both $V_1 \rightarrow V_m$ and $W_1 \rightarrow W_m$ are ~~se~~ separately Linearly Independent, but that doesn't promise that the addition of them are L.I. For example for $1 \leq i \leq m$, $V_i = -W_i$ then you get $V_1 + W_1 = 0, V_2 + W_2 = 0, \dots$ which thus ~~implies~~ implies that they are linearly Dependent.

polynomial

2. $1, X, X^2, X^3 \in P_3(F)$, a basis of $P_3(F)$ s.t. ~~no~~ P_i has a ~~degree of 2~~ is: $1+X^3, X+X^3, X^2+X^3, X^3$. Using Linear Combinations, all ^{via X} can be gotten, but no ~~single~~ single polynomial has a degree of 2.

3. $\dim U = 3, \dim W = 5$, if $U+W = \mathbb{R}^8$, then $U \cap W = \mathbb{R}^8$
 this is because $\dim(U+W) = \dim U + \dim W - \dim(U \cap W) = 8$
 thus simplifying this gives $\dim(U+W) = 3 + 5 - \dim(U \cap W) = 8$
 thus $\dim(U \cap W) = 3 + 5 - 8 = 0$, which forces the intersection of U and W to be \emptyset , which implies a direct sum.

4. let $a_1, \dots, a_m \in F$, suppose $a_1 V_1 + a_2 V_2 + \dots + a_m V_m = 0$. This means that $T(a_1 V_1 + \dots + a_m V_m) = T a_1 V_1 + T a_2 V_2 + \dots + T a_m V_m$, and since we know $T V_1, \dots, T V_m$ is Linearly Independent, this implies that $T a_1 V_1, \dots, T a_m V_m$ is equal to \emptyset , meaning that all $a_1, \dots, a_m = 0$, thus implying that $\{V_1, \dots, V_m\}$ is L.I.