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Examination 3 Math 4100 Closed Book Prof. Chjan Lim

All electronic devices must be turned off for the duration of this examination. You must write legibly and provide full justification of each step in your solution to receive full credit for a problem. Only solutions / proofs on the blank sheets stapled here will be graded.

(1) Prove or disprove: If $B = \{v_1, ..., v_m\}$ is a basis for U, and $C = \{w_1, ..., w_m\}$ is linearly independent in U, then
$\{v_1 + w_1, ..., v_m + w_m\}$ is linearly independent in U.

(2) Prove or disprove: *There* is a basis for $P_3(R) = \{$*all* real polynomials with degree less than or equal to 3$\}$ such that none of the elements of this basis has degree 2.

(3) Suppose that U and W *are* subspaces of R^8, $\dim(U) = 3$, $\dim(W) = 5$, and $U + W = R^8$. Prove that $R^8 = U \oplus W$.

(4) Let $T : V \to W$ be a linear map, $\{v_1, ..., v_m\}$ is a set of vectors in V, and $\{Tv_1, ..., Tv_m\}$ is linearly independent in W. Prove or disprove that

$\{v_1, ..., v_m\}$ is linearly independent in V.

(5) Give an example of a linear map $T : R^4 \to R^4$ such that *Range* $T =$ *Null* $T < R^4$.

5. $e_1, e_2, e_3, e_4 \in \mathbb{R}^4$

$Te_1 = e_2 \quad Te_2 = e_1, \quad Te_3 = Te_4 = 0$

Range $= 2$ Nullity $= 2$

$R_g + N < \mathbb{R}^4$

1/2

$Te_1 = e_3$

$Te_2 = e_4$

$Te_3 = 0$

$Te_4 = 0$

better

this maps 2 vectors to $\emptyset$, giving $N=2$, and also maps 2 vectors to each other giving $R_g = 2$, $N + R = \mathbb{R}^4$, $2+2 = \mathbb{R}^4$ ✓

1. Both $V_1 \to V_m$ and $W_1 \to W_m$ are seperately Linearly Independant, but that doesn't promise that the addition of them are L.I. for example for $1 \leq i \leq m$, $V_i = -W_i$ then you get $V_1 + W_1 = 0, V_2 + W_2 = 0 \ldots$, which thus implies that they are linearly Dependant

2. $1, X, X^2, X^3 \in P_3(F)$, a basis of $P_3(F)$ s.t no Polynomial P_i has a degree of 2 is: $1+X^3, X+X^3, X^2+X^3, X^3$. Using Linear Combinations, all via LA can be gotten, but no single Polynomial has a degree of 2.

3. $\dim U = 3$, $\dim W = 5$, if $U + W = R^8$, then $U \oplus W = R^8$ this is because $\dim(U+W) = \dim U + \dim W - \dim(U \cap W) = R^8(8)$, thus simplifying this gives $\dim(U+W) = 3 + 8 - \dim(U \cap W) = R^8(8)$ thus $\dim(U+W) = 8 + \dim(U \cap W) = 8$, which forces the intersection of U and W to be $\emptyset$, which implies a direct Sum.

4. let $a_1 \to a_m \in F$, suppose $a_1 V_1 + a_1 V_1 \cdots a_m V_m = 0$, this means that $T(a_1 V_1 \ldots + a_m V_m) = Ta_1 V_1 + Ta_2 V_2 \cdots + Ta_m V_m$, and since we know $TV_1 \ldots TV_m$ is Linearly Independant, this implies that $Ta_1 V_1 \ldots Ta_m V_m$ is equal to $\emptyset$, meaning that all $a_1 \ldots a_m = 0$, thus implying that $\{V_1 \ldots V_n\}$ is L.I.