Computer Vision

Lecture 6 – Applications of Graphical Models

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Agenda

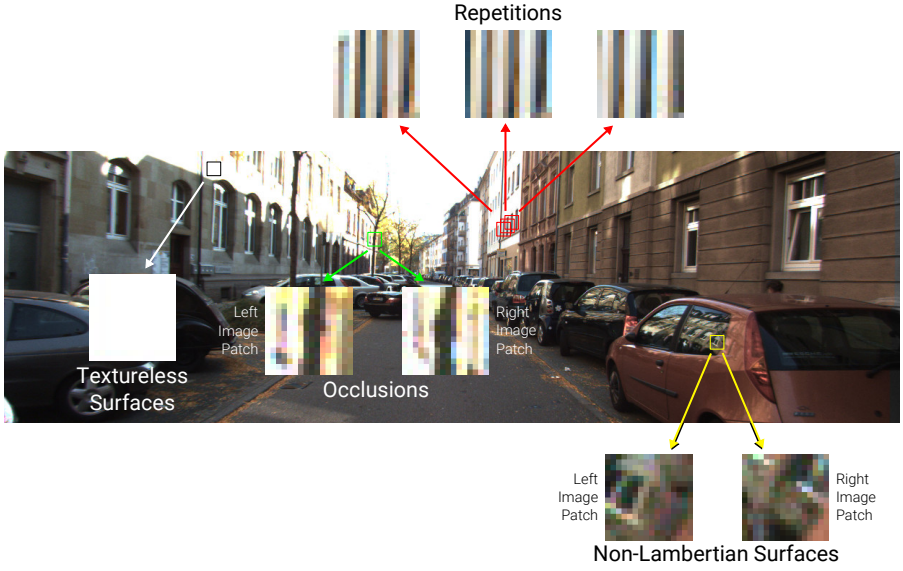
6.1 Stereo Reconstruction

6.2 Multi-View Reconstruction

6.3 Optical Flow

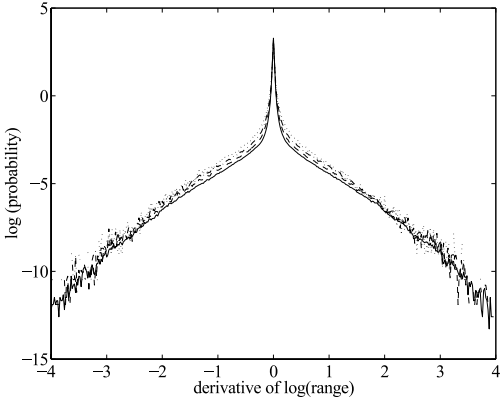
6.1
Stereo Reconstruction

Stereo Matching Ambiguities



How does the real world look like?

- Analyze **real-world statistics** (e.g., Brown range image database)
- Conclusion: Depth varies slowly except at object discontinuities which are sparse

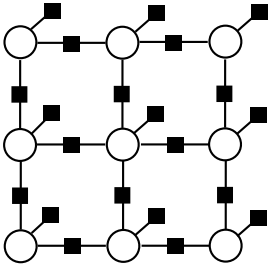


Huang, Lee and Mumford: Statistics of Range Images. CVPR, 2000.

Stereo MRF

Specify a loopy **Markov Random Field (MRF)** on a grid and solve for the whole disparity map $\mathbf{D}$ at once (MAP solution = minimum of energy):

$$p(\mathbf{D}) \propto \prod_i f_{data}(d_i) \times \prod_{i \sim j} f_{smooth}(d_i, d_j)$$



Or equivalently:

$$p(\mathbf{D}) \propto \exp\left\{ \underbrace{-\left(\sum_i \psi_{data}(d_i) + \lambda \sum_{i \sim j} \psi_{smooth}(d_i, d_j) \right)}_{\text{Energy E}} \right\}$$

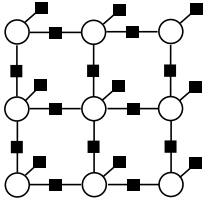
- $\psi(\cdot) := -\log f(\cdot), \quad i \sim j$: neighboring pixels on a 4-connected grid
- $\psi_{data}(d)$: unary terms, and $\psi_{smooth}(d, d')$ pairwise terms

Boykov, Veksler & Zabih: Fast Approximate Energy Minimization via Graph Cuts. PAMI, 1999.

Stereo MRF

Markov Random Field (MRF) from last slide:

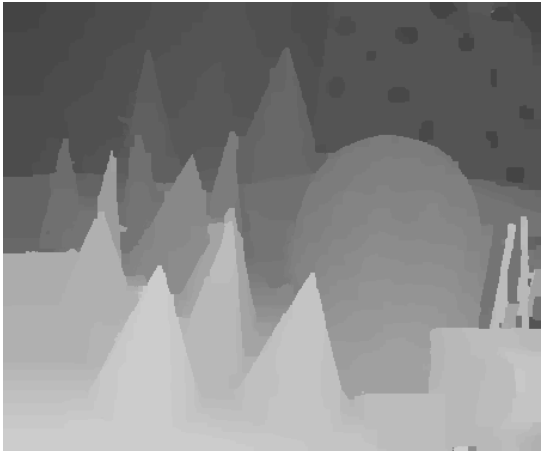
$$p(\mathbf{D}) \propto \exp\left\{ \underbrace{-\left(\sum_i \psi_{data}(d_i) + \lambda \sum_{i \sim j} \psi_{smooth}(d_i, d_j) \right)}_{\text{Energy E}} \right\}$$



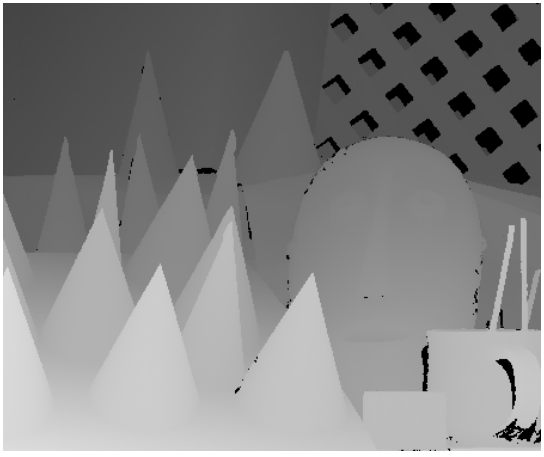
- $i \sim j$: neighboring pixels on a 4-connected grid
- **Unary terms:** Matching cost $\psi_{data}(d)$
- **Pairwise terms:** Encourages smoothness between adjacent pixels, e.g.:
  - Potts model: $\psi_{smooth}(d, d') = [d \neq d']$
  - Truncated l_1 penalty: $\psi_{smooth}(d, d') = \min(|d - d'|, \tau)$
- The parameter λ is added to control the strength of the smoothness prior
- Solve MRF approximately for disparity map $\mathbf{D}$ using BP, graph cuts, etc.

Boykov, Veksler & Zabih: Fast Approximate Energy Minimization via Graph Cuts. PAMI, 1999.

Stereo MRF – Results



Inference Results



Ground Truth

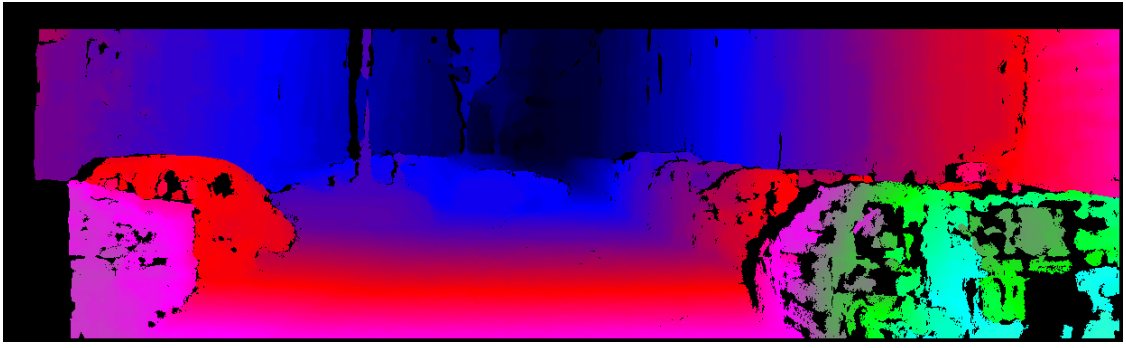
Non-local Priors

Non-local Priors

$$E(\mathbf{D}) = \underbrace{\sum_i \psi_i^{\mathcal{A}}(d_i)}_{\text{Appearance}} + \lambda \underbrace{\sum_{i \sim j} \psi_{ij}^{\mathcal{S}}(d_i, d_j)}_{\text{Smoothness}}$$

Güney & Geiger: Displets: Resolving Stereo Ambiguities using Object Knowledge. CVPR, 2015.

Non-local Priors



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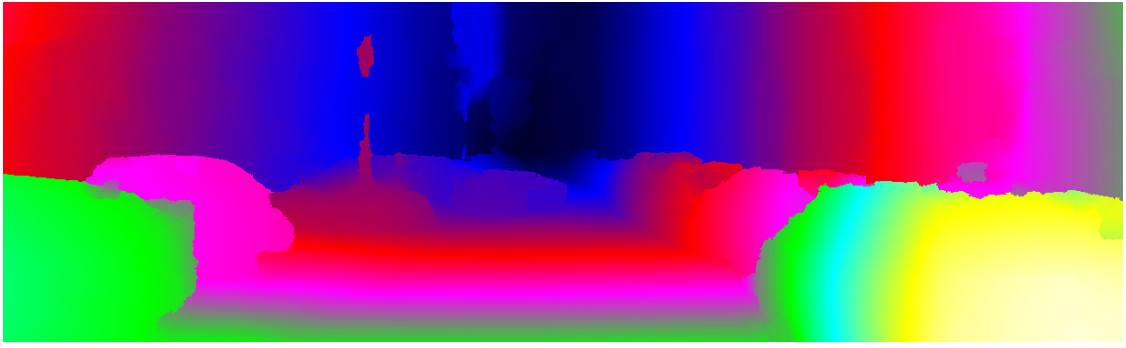
Non-local Priors

$$E(\mathbf{D},\mathbf{O}) = \underbrace{\sum_i \psi_i^{\mathcal{A}}(d_i)}_{\text{Appearance}} + \lambda_{\mathcal{S}} \underbrace{\sum_{i\sim j} \psi_{ij}^{\mathcal{S}}(d_i,d_j)}_{\text{Smoothness}} + \lambda_{\mathcal{O}} \underbrace{\sum_k \psi_k^{\mathcal{O}}(o_k)}_{\text{Object Semantics}} + \lambda_{\mathcal{C}} \underbrace{\sum_k \sum_i \psi_{ki}^{\mathcal{C}}(o_k,d_i)}_{\text{3D Consistency}}$$

Güney & Geiger: Displets: Resolving Stereo Ambiguities using Object Knowledge. CVPR, 2015.

Non-local Priors



$$E(\mathbf{D},\mathbf{O}) = \underbrace{\sum_i \psi_i^{\mathcal{A}}(d_i)}_{\text{Appearance}} + \lambda_{\mathcal{S}} \underbrace{\sum_{i\sim j} \psi_{ij}^{\mathcal{S}}(d_i,d_j)}_{\text{Smoothness}} + \lambda_{\mathcal{O}} \underbrace{\sum_k \psi_k^{\mathcal{O}}(o_k)}_{\text{Object Semantics}} + \lambda_{\mathcal{C}} \underbrace{\sum_k \sum_i \psi_{ki}^{\mathcal{C}}(o_k,d_i)}_{\text{3D Consistency}}$$

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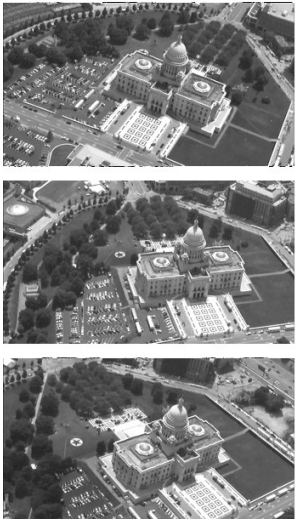
Stereo Reconstruction Summary

- Block matching suffers from **ambiguities**
- Choosing window size is problematic (tradeoff)
- Incorporating **smoothness constraints** can resolve some of the ambiguities and allows for choosing small windows (no bleeding artifacts)
- Can be formulated as **MAP inference in a discrete MRF**
- MAP solution can be obtained using belief propagation, graph cuts, etc.
- Integrating recognition cues can further regularize the problem

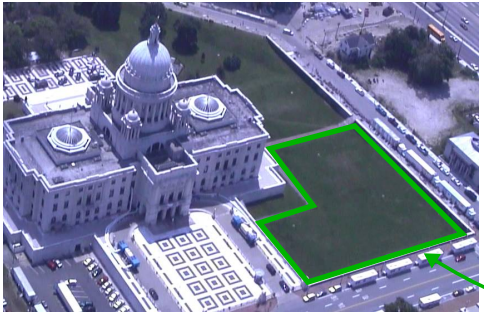
6.2
Multi-View Reconstruction

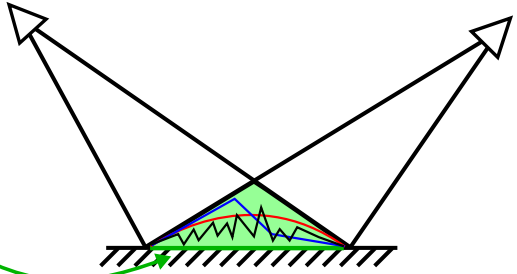
Probabilistic Dense Multi-View 3D Reconstruction



Ulusoy, Geiger & Black: Towards Probabilistic Reconstruction. 3DV, 2015.

Reconstruction Ambiguities



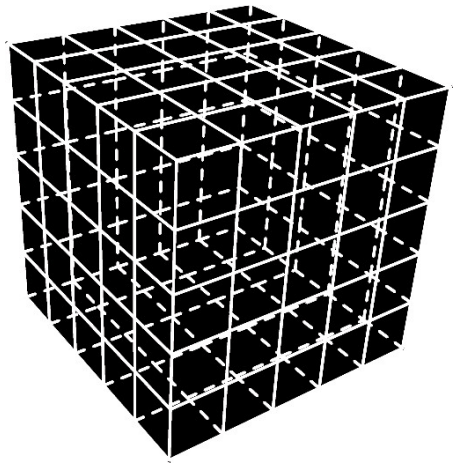


- Image-based 3D reconstruction is a highly **ill-posed** problem
  ⇒ Coping with and exposing **uncertainty** is essential

Can we formulate dense 3D reconstruction in a probabilistic way?

Ulusoy, Geiger & Black: Towards Probabilistic Reconstruction. 3DV, 2015.

Representation



- **Voxel occupancy:**

$$o_i = \begin{cases} 1 & \text{if voxel } i \text{ is occupied} \\ 0 & \text{if voxel } i \text{ is empty} \end{cases}$$

- **Voxel appearance:**

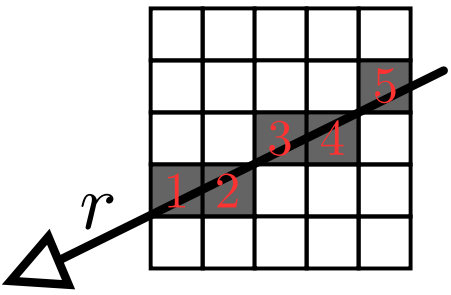
$$a_i \in \mathbb{R}$$

- **Shorthand notation:**

$$\mathbf{o}_r = \{o_1^r, \ldots, o_{N_r}^r\}$$
$$\mathbf{a}_r = \{a_1^r, \ldots, a_{N_r}^r\}$$

Ulusoy, Geiger & Black: Towards Probabilistic Reconstruction. 3DV, 2015.

Representation



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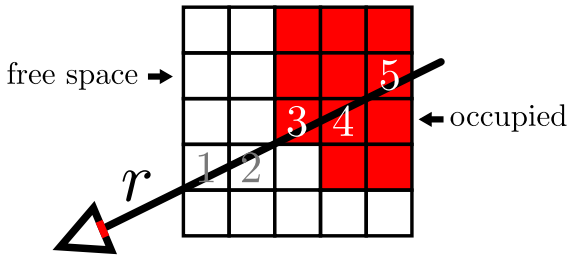
Ulusoy, Geiger & Black: Towards Probabilistic Reconstruction. 3DV, 2015.

Image Formation Process



$$I_r = \sum_{i=1}^{N} o_i \prod_{j<i} (1 - o_j)\, a_i$$

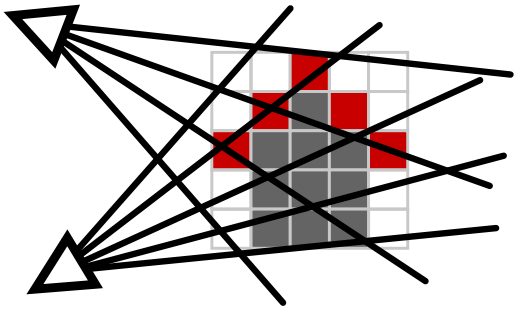
- I_r: intensity at pixel r o_i: occupancy of voxel i a_i: appearance of voxel i

Ulusoy, Geiger & Black: Towards Probabilistic Reconstruction. 3DV, 2015.

Probabilistic Model

Joint Distribution:

$$p(\mathbf{O}, \mathbf{A}) = \frac{1}{Z} \prod_{v \in \mathcal{V}} \underbrace{\varphi_v(o_v)}_{\text{unary}} \prod_{r \in \mathcal{R}} \underbrace{\psi_r(\mathbf{o}_r, \mathbf{a}_r)}_{\text{ray}}$$



Ulusoy, Geiger & Black: Towards Probabilistic Reconstruction. 3DV, 2015.

Probabilistic Model

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Unary Potentials:

$$\varphi_v(o_v) = \gamma^{o_v} (1 - \gamma)^{1 - o_v}$$

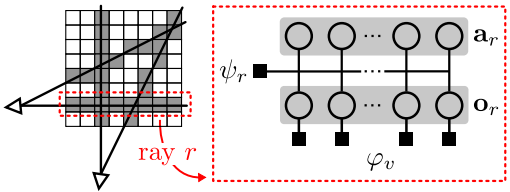
- Most voxels are empty $\Rightarrow \gamma < 0.5$

Probabilistic Model

Joint Distribution:

$$p(\mathbf{O}, \mathbf{A}) = \frac{1}{Z} \prod_{v \in \mathcal{V}} \underbrace{\varphi_v(o_v)}_{\text{unary}} \prod_{r \in \mathcal{R}} \underbrace{\psi_r(\mathbf{o}_r, \mathbf{a}_r)}_{\text{ray}}$$

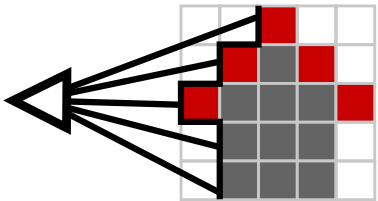
Ray Potentials:



$$\psi_r(\mathbf{o}_r, \mathbf{a}_r) = \sum_{i=1}^{N_r} o_i^r \prod_{j<i} (1 - o_j^r) \underbrace{\mathcal{N}(a_i^r | I_r, \sigma)}_{\text{Gaussian Noise}}$$

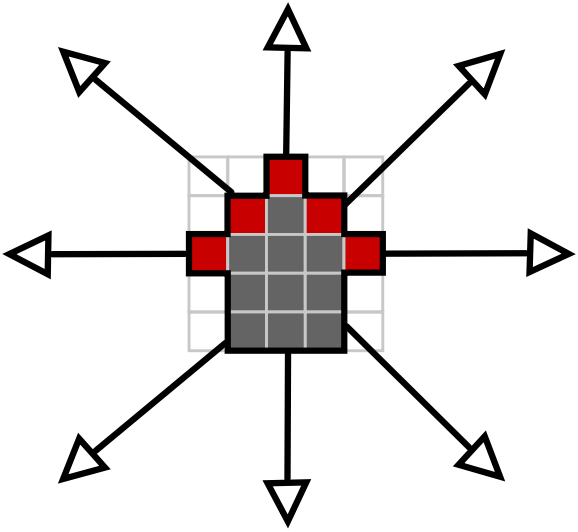
Ulusoy, Geiger & Black: Towards Probabilistic Reconstruction. 3DV, 2015.

3D Reconstruction



Ulusoy, Geiger & Black: Towards Probabilistic Reconstruction. 3DV, 2015.

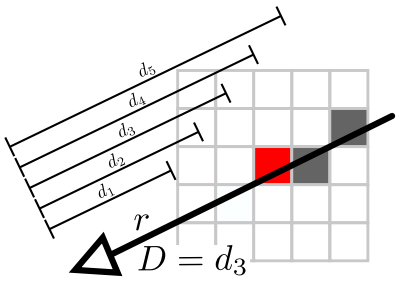
3D Reconstruction



Ulusoy, Geiger & Black: Towards Probabilistic Reconstruction. 3DV, 2015.

Bayes Optimal Depth Estimation

- Consider a single ray r in space
- Let d_k be the distance from the camera to voxel k along ray r
- Depth $D \in \{d_1, \ldots, d_N\}$: distance to closest occupied voxel



Ulusoy, Geiger & Black: Towards Probabilistic Reconstruction. 3DV, 2015.

Bayes Optimal Depth Estimation

- Consider a single ray r in space
- Let d_k be the distance from the camera to voxel k along ray r
- Depth $D \in \{d_1, \ldots, d_N\}$: distance to closest occupied voxel
- **Optimal depth estimate:**

$$
\begin{aligned}
D^* &= \underset{D'}{\operatorname{argmin}} \ \text{Risk}(D') \\
&= \underset{D'}{\operatorname{argmin}} \ \mathbb{E}_{p(D)}[\,\Delta(D, D')\,] \\
&= \begin{cases} \text{mean}\,(\,p(D)\,) & \text{if } \Delta(D, D') = (D - D')^2 \\ \text{median}\,(\,p(D)\,) & \text{if } \Delta(D, D') = |D - D'| \end{cases}
\end{aligned}
$$

- Requires **marginal depth distribution** $p(D)$ along each ray

Depth Distribution for Single Ray

Let:

$$o_1 = 0 \quad \cdots \quad o_{k-1} = 0 \quad o_k = 1$$

Then:

$$p(D = d_k) = \underbrace{\sum_{\mathbf{o}_{>k}} \int_{\mathbf{a}} p(\mathbf{o}, \mathbf{a})}_{= p(o_1, \ldots, o_k)}$$

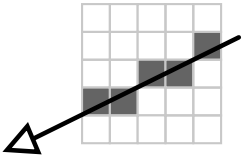
Depth Distribution for Single Ray

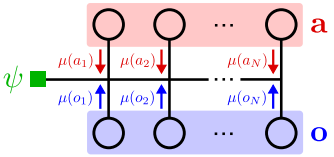
Let:

$$o_1 = 0 \quad \cdots \quad o_{k-1} = 0 \quad o_k = 1$$

Then:

$$p(D = d_k) \propto \sum_{\mathbf{o}_{>k}} \int_{\mathbf{a}} \psi(\mathbf{o}, \mathbf{a}) \prod_i \mu(o_i) \prod_i \mu(a_i)$$





Marginal = Product of Factor & Incoming Messages

Depth Distribution for Single Ray

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$$\psi(\mathbf{o}, \mathbf{a}) = \underbrace{\sum_i o_i \prod_{j<i} (1 - o_j) \mathcal{N}(a_i | I, \sigma)}_{= \mathcal{N}(a_k | I, \sigma)}$$

Depth Distribution for Single Ray

Let:

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Then:

$$p(D = d_k) \propto \sum_{\mathbf{o}_{>k}} \int_{\mathbf{a}} \mathcal{N}(a_k | I, \sigma) \prod_i \mu(o_i) \prod_i \mu(a_i)$$

Ulusoy, Geiger & Black: Towards Probabilistic Reconstruction. 3DV, 2015.

Depth Distribution for Single Ray

Let:

$$o_1 = 0 \quad \cdots \quad o_{k-1} = 0 \quad o_k = 1$$

Then:

$$p(D = d_k) \propto \sum_{\mathbf{o}_{>k}} \int_{\mathbf{a}} \mathcal{N}(a_k | I, \sigma) \prod_{i>k} \mu(o_i) \prod_{i} \mu(a_i) \times \prod_{i \leq k} \mu(o_i)$$

Depth Distribution for Single Ray

Let:

$$o_1 = 0 \quad \cdots \quad o_{k-1} = 0 \quad o_k = 1$$

Then:

$$p(D = d_k) \propto \sum_{\mathbf{o}_{>k}} \int_{\mathbf{a}_{\neq k}} \prod_{i>k} \mu(o_i) \prod_{i \neq k} \mu(a_i)$$
$$\times \prod_{i \leq k} \mu(o_i) \int_{a_k} \mathcal{N}(a_k | I, \sigma) \mu(a_k)$$

Depth Distribution for Single Ray

Let:

$$o_1 = 0 \quad \cdots \quad o_{k-1} = 0 \quad o_k = 1$$

Then:

$$p(D = d_k) \propto \underbrace{\sum_{\mathbf{o}_{>k}} \int_{\mathbf{a}_{\neq k}} \prod_{i>k} \mu(o_i) \prod_{i \neq k} \mu(a_i)}_{=1}$$
$$\times \prod_{i \leq k} \mu(o_i) \int_{a_k} \mathcal{N}(a_k | I, \sigma) \mu(a_k)$$

Depth Distribution for Single Ray

Let:

$$o_1 = 0 \quad \cdots \quad o_{k-1} = 0 \quad o_k = 1$$

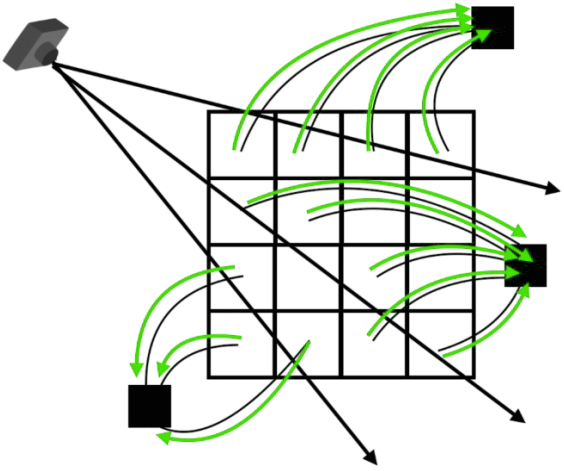
Then:

$$p(D = d_k) \propto \prod_{i \leq k} \mu(o_i) \int_{a_k} \mathcal{N}(a_k | I, \sigma) \mu(a_k)$$

Intuition: Depth $D = d_k \Leftrightarrow$ Voxel k is occupied and visible and explains the observed pixel value.

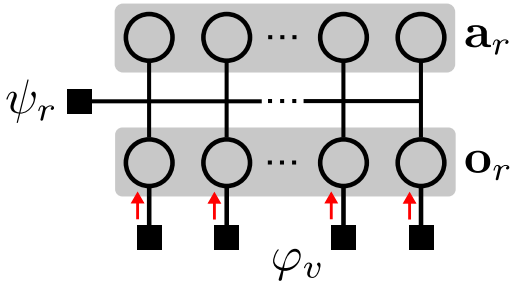
- How can we obtain $\mu(o_i)$ and $\mu(a_k)$?

Message Passing



Ulusoy, Geiger & Black: Towards Probabilistic Reconstruction. 3DV, 2015.

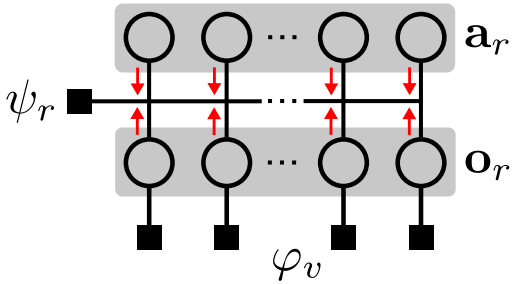
Message Passing



$$\mu_{\varphi_v \to o_v}(o_v) = \varphi_v(o_v)$$

Ulusoy, Geiger & Black: Towards Probabilistic Reconstruction. 3DV, 2015.

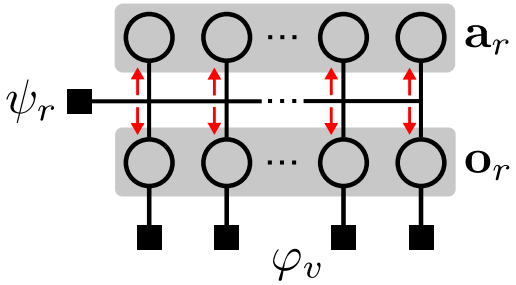
Message Passing



$$\mu_{x \to \psi_r}(x) = \prod_{f \in \mathcal{F} \setminus \psi_r} \mu_{f \to x}(x)$$

Ulusoy, Geiger & Black: Towards Probabilistic Reconstruction. 3DV, 2015.

Message Passing



$$\mu_{\psi_r \to x}(x) = ?$$

- Can be computed in **linear time** (see paper for technical derivation)

Inference

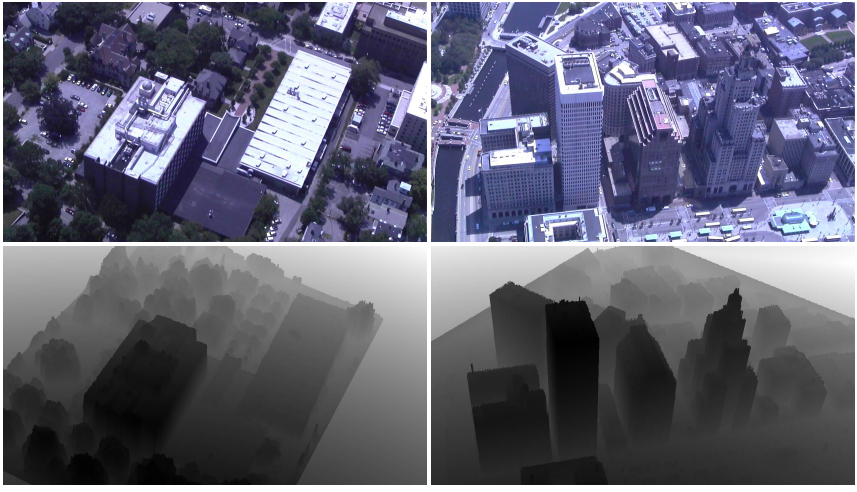
Challenges:

1. MRF comprises **discrete and continuous variables**
2. Ray potentials are **high-order**
3. **Many factors:** each pixel defines a factor

Solution:

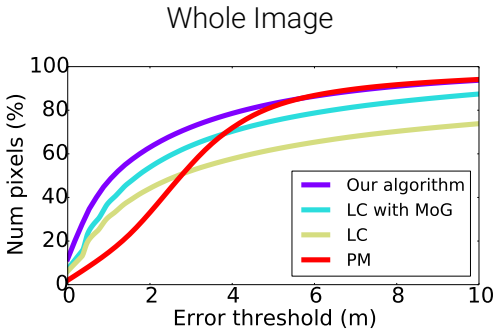
1. Approximate continuous belief propagation
   $\Rightarrow$ Update MoG's via importance sampling
2. Messages can be calculated in linear time
   $\Rightarrow$ Exact (but technical) derivation of messages
3. Octree implementation & GPGPU parallelization

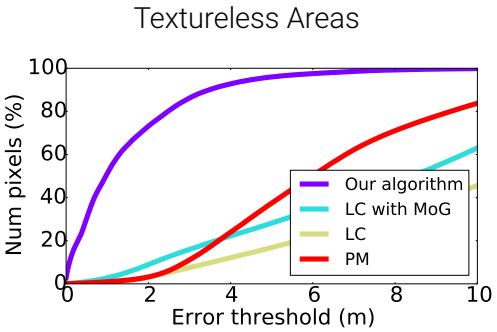
Experimental Results



Ulusoy, Geiger & Black: Towards Probabilistic Reconstruction. 3DV, 2015.

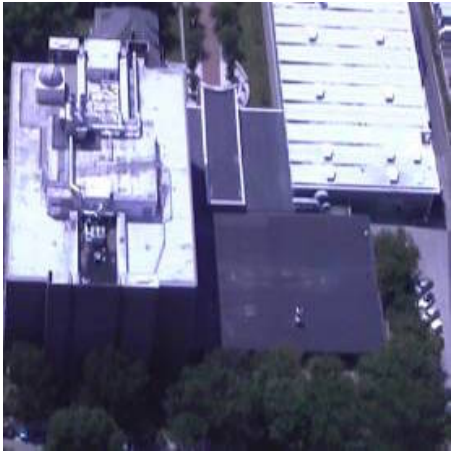
Quantitative Results





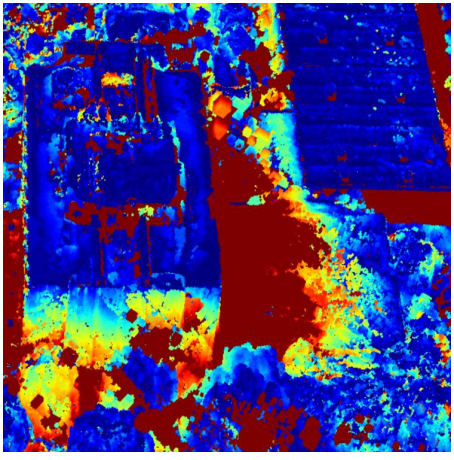
Ulusoy, Geiger & Black: Towards Probabilistic Reconstruction. 3DV, 2015.

Qualitative Results



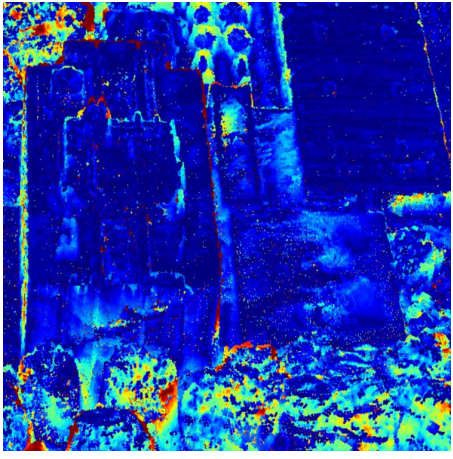
Input Image

Qualitative Results



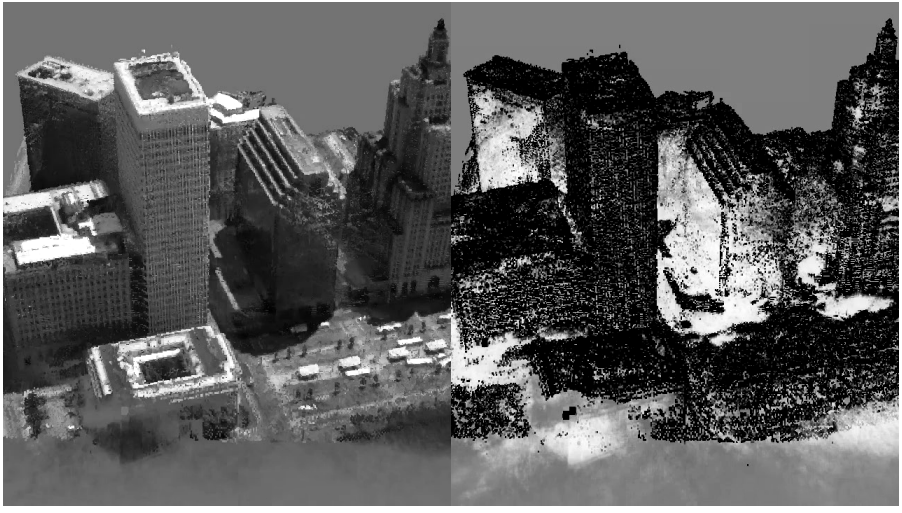
Error Map (Liu & Cooper, 2014)

Qualitative Results



Error Map (Bayes Optimal)

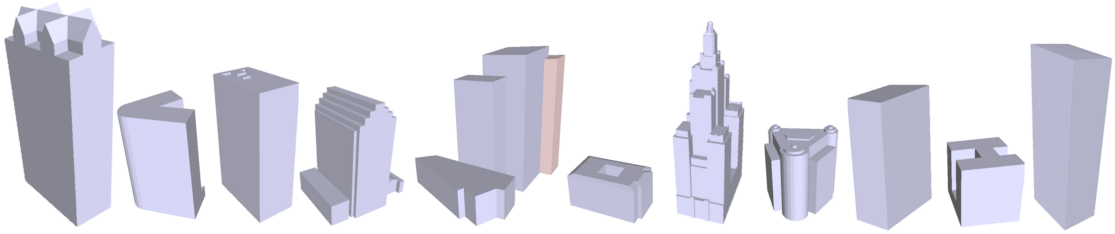
Qualitative Results: Our Results



Ulusoy, Geiger & Black: Towards Probabilistic Reconstruction. 3DV, 2015.

Integrating 3D Shape Priors

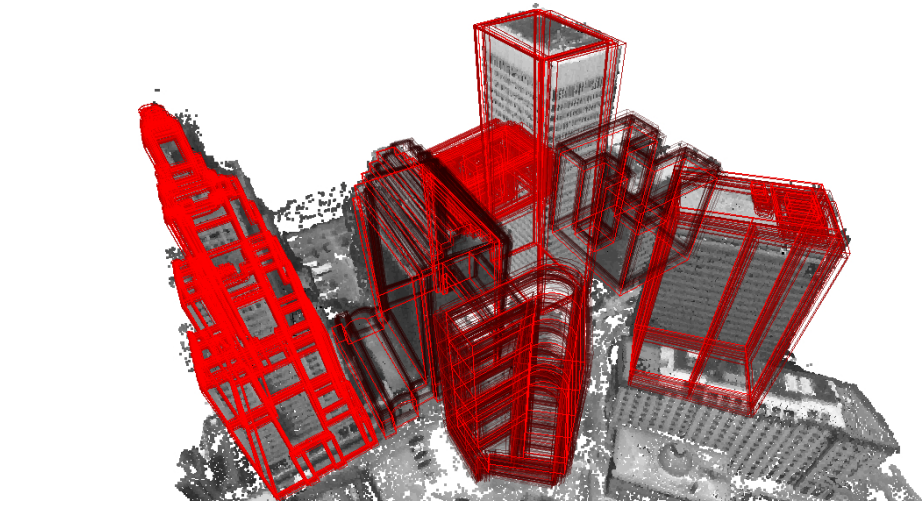
3D Shape Priors



- For many scenes **exclusive prior knowledge** is available
  - GPS tags can be used to retrieve 3D Warehouse models
  - 3D models of IKEA furniture for indoor scenes
- Challenges:
  - Often only coarse and inaccurate models
  - Unknown orientation and approximate location
  - Occlusions and object size

Ulusoy, Black & Geiger: Semantic Multi-view Stereo: Jointly Estimating Objects and Voxels. CVPR, 2017.

Probabilistic Model Fitting and 3D Reconstruction



Ulusoy, Black & Geiger: Semantic Multi-view Stereo: Jointly Estimating Objects and Voxels. CVPR, 2017.

Probabilistic Multi-View Reconstruction Summary

Pros:

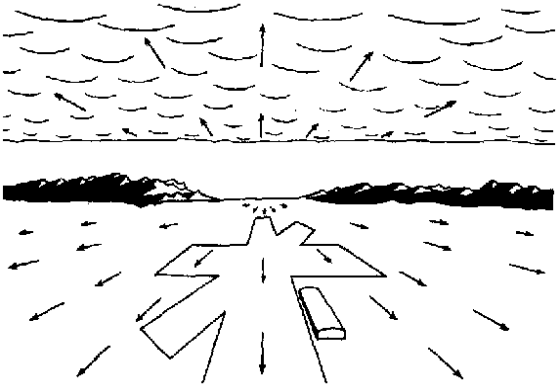
- Probabilistic formulation is tractable as ray factors decompose
- Non-local constraints via joint inference in 2D and 3D
- CAD priors can help disambiguate textureless regions
- Using octrees, reconstructions up to 1024^3 voxels are possible

Cons:

- Only approximate inference possible (highly loopy)
- Relatively slow: several minutes per scene on a GPU
- Appearance term very simplistic and not robust
- Resolution limited by discrete voxels (as opposed to meshes)

6.3
Optical Flow

Optical Flow



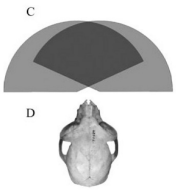
- Optical flow is the **apparent motion** of objects, surfaces, and edges in a visual scene caused by the relative motion between an observer and a scene.

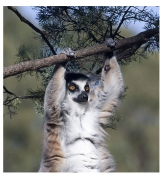
J. J. Gibson, 1950: The Ecological Approach to Visual Perception

Stereo vs. Optical Flow

Stereo

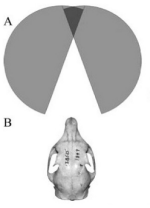
- 2 images at the same time
- Only camera motion
- 1D estimation problem
- Monkeys



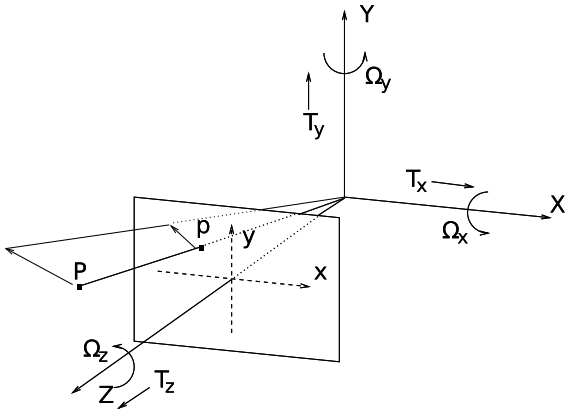


Optical Flow

- 2 images at 2 time steps
- Camera and object motion
- 2D estimation problem
- Squirrels



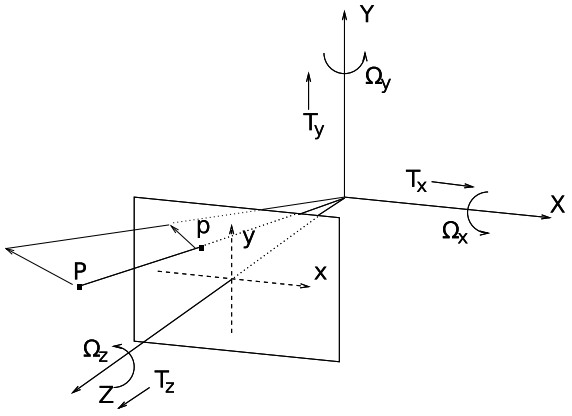
Optical Flow



Motion field:

- 2D motion field representing the **projection of the 3D motion** of points in the scene onto the image plane
- Can be the result of camera motion or object motion (or both)

Optical Flow



Optical flow:

- 2D velocity field describing the **apparent motion** in the image (i.e., the displacement of pixels looking “similar”)
- Optical flow $\neq$ motion field! Why?

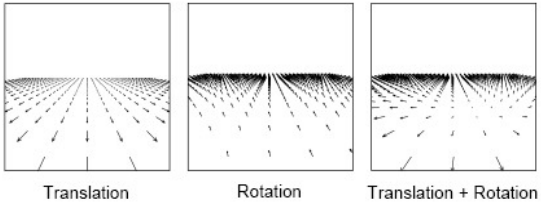
Thought Experiment

- Lambertian ball
  rotating in 3D
- What does the 2D
  motion field look like?
- What does the 2D
  optical flow field look like?

Thought Experiment

- Stationary specular ball
  moving light source
- What does the 2D
  motion field look like?
- What does the 2D
  optical flow field look like?

Optical Flow Field



The optical flow fields tell us something (maybe ambiguous) about:

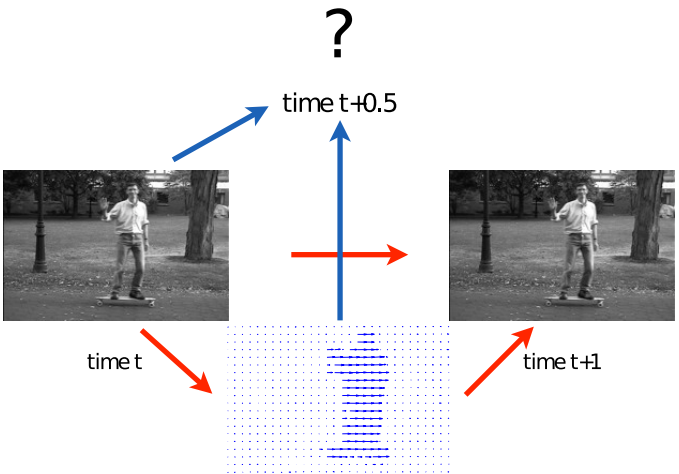
- The **3D structure** of the world
- The **motion of objects** in the viewing area
- The **motion of the observer** (if any)

In contrast to stereo:

- No epipolar geometry $\Rightarrow$ **2D estimation problem!**

The Northern Gannet

Applications: Video Interpolation / Frame Rate Adaption

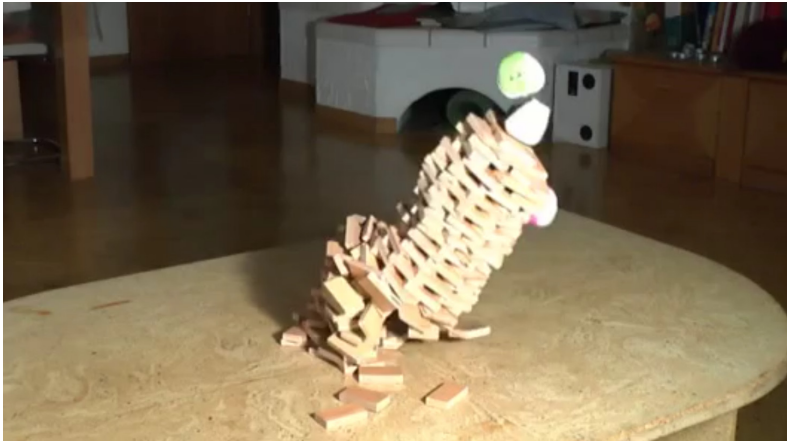


- If we know the image motion we can compute images at intermediate frames

Applications: Video Interpolation / Frame Rate Adaption

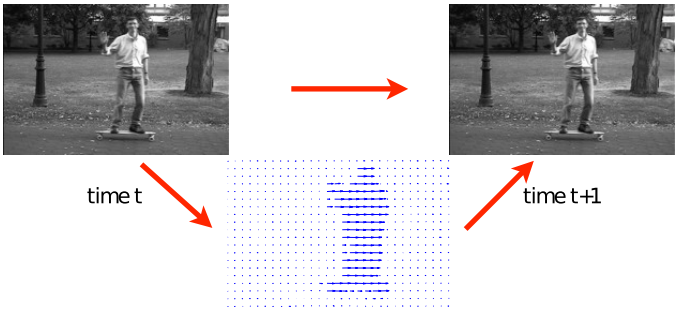
- If we know the image motion we can compute images at intermediate frames

Applications: Video Interpolation / Frame Rate Adaption



- If we know the image motion we can compute images at intermediate frames

Applications: Video Compression

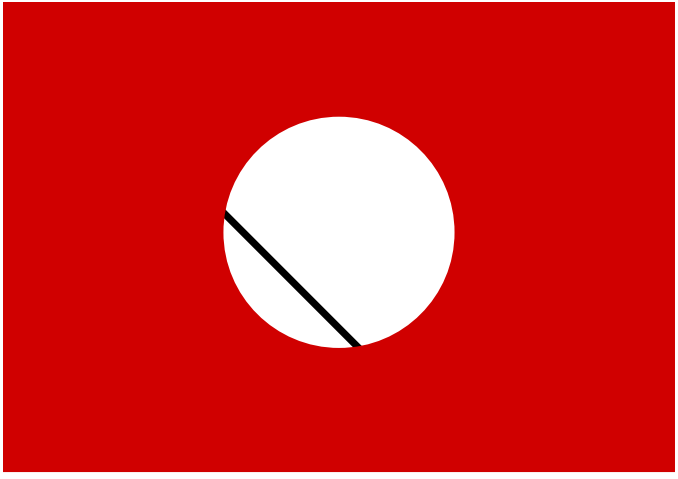


- To compress an image sequence, we can predict new frames using the optical flow field and only store how to “fix” the prediction
- Flow fields are smooth, thus easier to compress/store than images!

Applications: Autonomous Driving

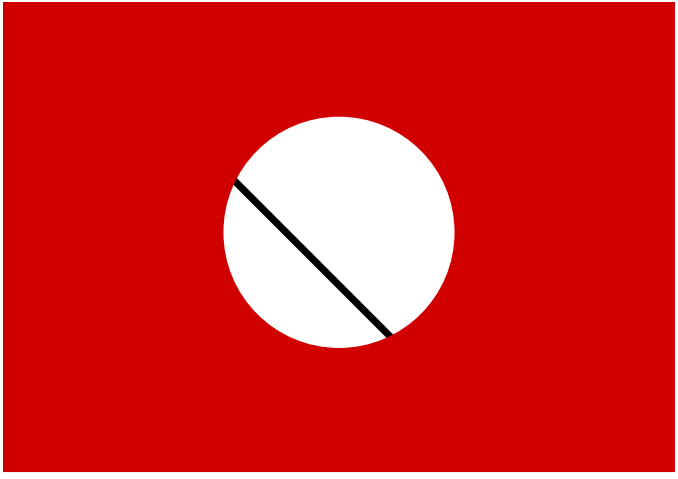
Aperture Problem

In which direction does the line move?



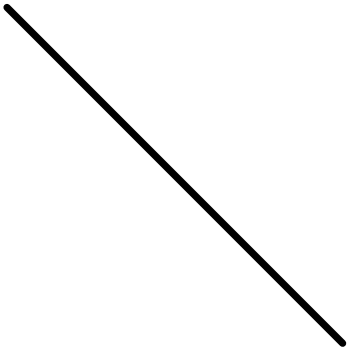
Aperture Problem

In which direction does the line move?



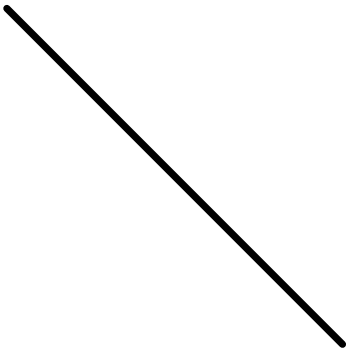
Aperture Problem

Now the full picture …



Aperture Problem

Now the full picture …



Aperture Problem

- Aperture problem: A single observation is not enough to determine flow
- Barber Pole: What is the motion field? What is the optic flow field?

Determining Optical Flow

Horn-Schunck Optical Flow

- Consider the image I as a function of continuous variables x, y, t
- Consider $u(x, y)$ and $v(x, y)$ as continuous flow fields (functions)
- Minimize the following **energy functional**

$$
\begin{aligned}
E(u, v) &= \iint \underbrace{\left(I(x + u(x, y), y + v(x, y), t + 1) - I(x, y, t)\right)^2}_{\text{quadratic penalty for brightness change}} \\
&+ \lambda \cdot \underbrace{\left(\|\nabla u(x, y)\|^2 + \|\nabla v(x, y)\|^2\right)}_{\text{quadratic penalty for flow change}} \, dx \, dy
\end{aligned}
$$

with regularization parameter λ and gradient $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$.

Horn & Schunck: Determining Optical Flow. Artificial Intelligence, 1981.

Horn-Schunck Optical Flow

$$
\begin{aligned}
E(u,v) &= \iint \left(I(x+u(x,y), y+v(x,y), t+1) - I(x,y,t)\right)^2 \\
&+ \lambda \cdot \left(\|\nabla u(x,y)\|^2 + \|\nabla v(x,y)\|^2 \right) dx\, dy
\end{aligned}
$$

- Minimizing this directly is a hard problem because the energy is highly **non-convex** and has many local optima
- Solution: **linearize** the brightness constancy assumption

Horn & Schunck: Determining Optical Flow. Artificial Intelligence, 1981.

Horn-Schunck Optical Flow

First-Order Multivariable Taylor Series:

$$f(x,y) \overset{a,b}{\approx} f(a,b) + \frac{\partial f(a,b)}{\partial x}(x-a) + \frac{\partial f(a,b)}{\partial y}(y-b)$$

Therefore, we have:

$$\begin{aligned} & I(x+u(x,y), y+v(x,y), t+1) \\ \overset{x,y,t}{\approx}\ & I(x,y,t) + I_x(x,y,t)(x+u(x,y)-x) \\ & + I_y(x,y,t)(y+v(x,y)-y) + I_t(x,y,t)(t+1-t) \\ =\ & I(x,y,t) + I_x(x,y,t)\,u(x,y) + I_y(x,y,t)\,v(x,y) + I_t(x,y,t) \end{aligned}$$

Horn & Schunck: Determining Optical Flow. Artificial Intelligence, 1981.

Horn-Schunck Optical Flow

Thus, the optical flow energy functional

$$\begin{aligned} E(u,v) &= \iint \left(I(x+u(x,y), y+v(x,y), t+1) - I(x,y,t)\right)^2 \\ &+ \lambda \cdot \left(\|\nabla u(x,y)\|^2 + \|\nabla v(x,y)\|^2\right) dx\, dy \end{aligned}$$

is approximated by the following **linearized equation:**

$$\begin{aligned} E(u,v) &= \iint \left(I_x(x,y,t)u(x,y) + I_y(x,y,t)v(x,y) + I_t(x,y,t)\right)^2 \\ &+ \lambda \cdot \left(\|\nabla u(x,y)\|^2 + \|\nabla v(x,y)\|^2\right) dx\, dy \end{aligned}$$

Horn & Schunck: Determining Optical Flow. Artificial Intelligence, 1981.

Horn-Schunck Optical Flow

Spatial discretization of the equation

$$\begin{aligned} E(u,v) &= \iint \left(I_x(x,y,t)u(x,y) + I_y(x,y,t)v(x,y) + I_t(x,y,t)\right)^2 \\ &+ \lambda \cdot \left(\|\nabla u(x,y)\|^2 + \|\nabla v(x,y)\|^2\right) dx\, dy \end{aligned}$$

leads to the following **discretized objective:**

$$\begin{aligned} E(\mathbf{U},\mathbf{V}) &= \sum_{x,y} \left(I_x(x,y)\, u_{x,y} + I_y(x,y)\, v_{x,y} + I_t(x,y)\right)^2 \\ &+ \lambda \cdot \left((u_{x,y} - u_{x+1,y})^2 + (u_{x,y} - u_{x,y+1})^2 + (v_{x,y} - v_{x+1,y})^2 + (v_{x,y} - v_{x,y+1})^2\right) \end{aligned}$$

This objective is quadratic in the flow maps $\mathbf{U}$, $\mathbf{V}$ and thus has a unique optimum.

Horn & Schunck: Determining Optical Flow. Artificial Intelligence, 1981.

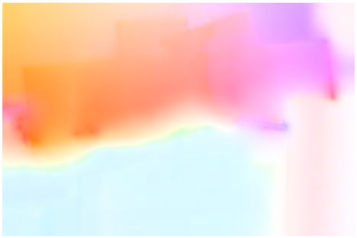
Horn-Schunck Optical Flow

$$
\begin{aligned}
E(\mathbf{U},\mathbf{V}) &= \sum_{x,y} \left(I_x(x,y)\,u_{x,y} + I_y(x,y)\,v_{x,y} + I_t(x,y)\right)^2 \\
&+ \lambda \cdot \big((u_{x,y} - u_{x+1,y})^2 + (u_{x,y} - u_{x,y+1})^2 + \\
&\qquad (v_{x,y} - v_{x+1,y})^2 + (v_{x,y} - v_{x,y+1})^2\big)
\end{aligned}
$$

- Differentiate wrt. $\mathbf{U}, \mathbf{V}$ and set the gradient to 0
- Results in a huge but **sparse linear system**
- Can be solved using **standard techniques** (e.g., Gauss-Seidel, SOR)
- However, linearization works only for **small motions**
- Solution: Iterative estimation & warping, coarse-to-fine estimation

Horn & Schunck: Determining Optical Flow. Artificial Intelligence, 1981.

Results of Horn & Schunck



Horn & Schunck: Determining Optical Flow. Artificial Intelligence, 1981.

Horn & Schunck Optical Flow

- The HS results are quite plausible already
- However, the flow is very smooth, i.e., to overcome ambituities we need to set λ to a high value which **oversmooths** flow discontinuities. Why?
- We use a **quadratic penalty** for penalizing changes in the flow
- This does not allow for **discontinuities** in the flow field
- In other words, it penalizes large changes too much and causes oversmoothing

Horn & Schunck: Determining Optical Flow. Artificial Intelligence, 1981.

Robust Estimation of Optical Flow

Probabilistic Interpretation

The HS optimization problem can be interpreted as **MAP inference in a MRF:**

$$p(\mathbf{U}, \mathbf{V}) = \frac{1}{Z} \exp\{-E(\mathbf{U}, \mathbf{V})\}$$

with the following **Gibbs energy:**

$$\begin{aligned} E(\mathbf{U}, \mathbf{V}) &= \sum_{x,y} \left(I_x(x,y)\, u_{x,y} + I_y(x,y)\, v_{x,y} + I_t(x,y)\right)^2 \\ &+ \lambda \cdot \big((u_{x,y} - u_{x+1,y})^2 + (u_{x,y} - u_{x,y+1})^2 + \\ &\qquad (v_{x,y} - v_{x+1,y})^2 + (v_{x,y} - v_{x,y+1})^2\big) \end{aligned}$$

Remark: As $\mathbf{U}, \mathbf{V}$ are continuous, we solve inference with gradient descent (not BP)

Black & Anandan: A Framework for Robust Estimation of Optical Flow. ICCV, 1993.

Probabilistic Interpretation

The HS optimization problem can be interpreted as **MAP inference in a MRF:**

$$p(\mathbf{U}, \mathbf{V}) = \frac{1}{Z} \exp\{-E(\mathbf{U}, \mathbf{V})\}$$

The corresponding **Gibbs distribution** is Gaussian:

$$\begin{aligned} p(\mathbf{U}, \mathbf{V}) \quad &\propto \quad \prod_{x,y} \exp\left\{-\left(I_x(x,y)\, u_{x,y} + I_y(x,y)\, v_{x,y} + I_t(x,y)\right)^2\right\} \\ &\times \quad \exp\left\{-\lambda(u_{x,y} - u_{x+1,y})^2\right\} \times \exp\left\{-\lambda(u_{x,y} - u_{x,y+1})^2\right\} \\ &\times \quad \exp\left\{-\lambda(v_{x,y} - v_{x+1,y})^2\right\} \times \exp\left\{-\lambda(v_{x,y} - v_{x,y+1})^2\right\} \end{aligned}$$

Remark: As $\mathbf{U}, \mathbf{V}$ are continuous, we solve inference with gradient descent (not BP)

Black & Anandan: A Framework for Robust Estimation of Optical Flow. ICCV, 1993.

Robust Regularization

Quadratic penalties translate to Gaussian distributions:

$$
\begin{aligned}
p(\mathbf{U},\mathbf{V}) \quad \propto \quad & \prod_{x,y} \exp\left\{-\left(I_x(x,y)\,u_{x,y} + I_y(x,y)\,v_{x,y} + I_t(x,y)\right)^2\right\} \\
\times \quad & \exp\left\{-\lambda(u_{x,y} - u_{x+1,y})^2\right\} \times \exp\left\{-\lambda(u_{x,y} - u_{x,y+1})^2\right\} \\
\times \quad & \exp\left\{-\lambda(v_{x,y} - v_{x+1,y})^2\right\} \times \exp\left\{-\lambda(v_{x,y} - v_{x,y+1})^2\right\}
\end{aligned}
$$

- Both assumptions are invalid (e.g., discontinuities at object boundaries). Why?
- Gaussian distributions correspond to squared loss functions
- Squared loss functions are not robust to outliers!
- Outliers occur at object boundaries (violation of smoothness/regularizer)
- Outliers occur at specular highlights (violation of photoconsistency/data term)

Black & Anandan: A Framework for Robust Estimation of Optical Flow. ICCV, 1993.

Robust Regularization

Solution: Use **robust data term** and smoothness penalties $\rho(\cdot)$:

$$\begin{aligned} p(\mathbf{U},\mathbf{V}) \quad &\propto \quad \prod_{x,y} \exp\left\{-\rho_D\left(I_x(x,y)\,u_{x,y} + I_y(x,y)\,v_{x,y} + I_t(x,y)\right)\right\} \\ &\times \quad \exp\left\{-\lambda\,\rho_S(u_{x,y} - u_{x+1,y})\right\} \times \exp\left\{-\lambda\,\rho_S(u_{x,y} - u_{x,y+1})\right\} \\ &\times \quad \exp\left\{-\lambda\,\rho_S(v_{x,y} - v_{x+1,y})\right\} \times \exp\left\{-\lambda\,\rho_S(v_{x,y} - v_{x,y+1})\right\} \end{aligned}$$

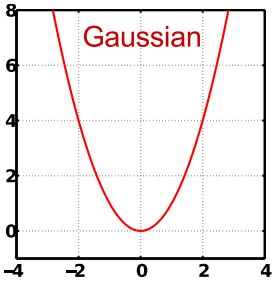
- How to choose $\rho_D(\cdot)$ and $\rho_S(\cdot)$? We want a prior that allows for discontinuities in the optical flow and a likelihood that allows for outliers and occlusions
- Replace Gaussian with (heavy-tailed) **Student-t distribution** (=Lorentzian penalty):

$$p(x) \propto \left(1 + \frac{x^2}{2\sigma^2}\right)^{-\alpha} \quad \Rightarrow \quad \rho(x) = \quad -\log\left(p(x)\right) = \alpha \log\left(1 + \frac{x^2}{2\sigma^2}\right)$$

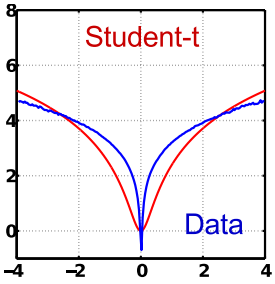
Black & Anandan: A Framework for Robust Estimation of Optical Flow. ICCV, 1993.

Robust Regularization

Gaussian penalty (squared loss) vs. robust Student-t penalty:



negative
log-density
(i.e. energy)



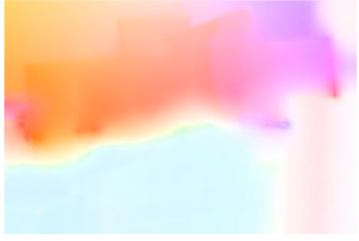
$$p(x) \propto \left(1 + \frac{x^2}{2\sigma^2}\right)^{-\alpha}$$

$$\rho(x) = \alpha \log\left(1 + \frac{x^2}{2\sigma^2}\right)$$

- Proposed by [Black-Anandan, ICCV 1993], [Sun et al., CVPR 2010] and others

Black & Anandan: A Framework for Robust Estimation of Optical Flow. ICCV, 1993.

Result of Horn & Schunck



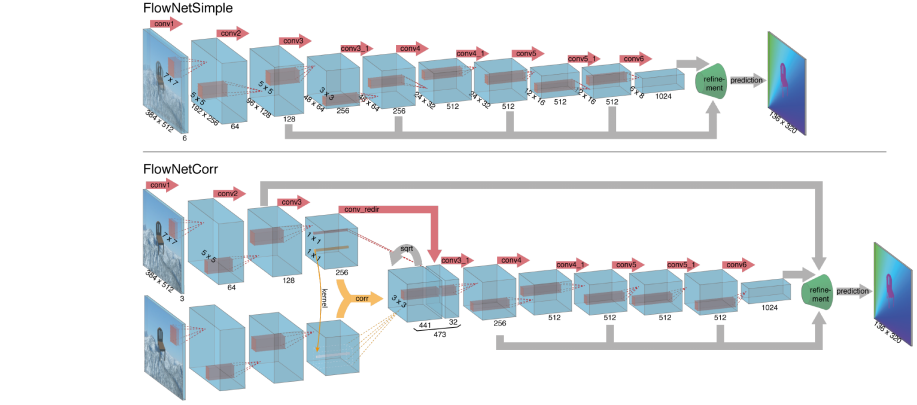
Black & Anandan: A Framework for Robust Estimation of Optical Flow. ICCV, 1993.

Results of Sun / Black & Anandan

Black & Anandan: A Framework for Robust Estimation of Optical Flow. ICCV, 1993.

End-to-End Deep Learning

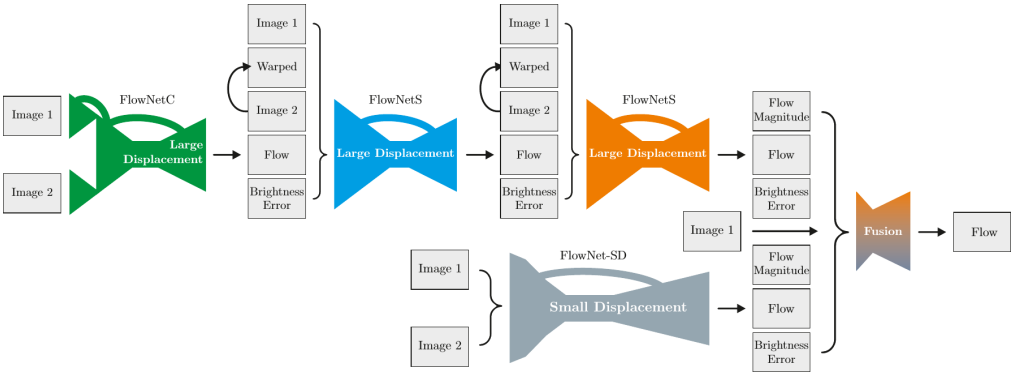
FlowNet



- Encoder: Convolution with stride, decoder: Upconvolution, skip-connections
- Multi-scale loss (EPE in pixels), curriculum learning, synth. training data

Dosovitskiy et al.: FlowNet: Learning Optical Flow,with Convolutional Networks. ICCV, 2015.

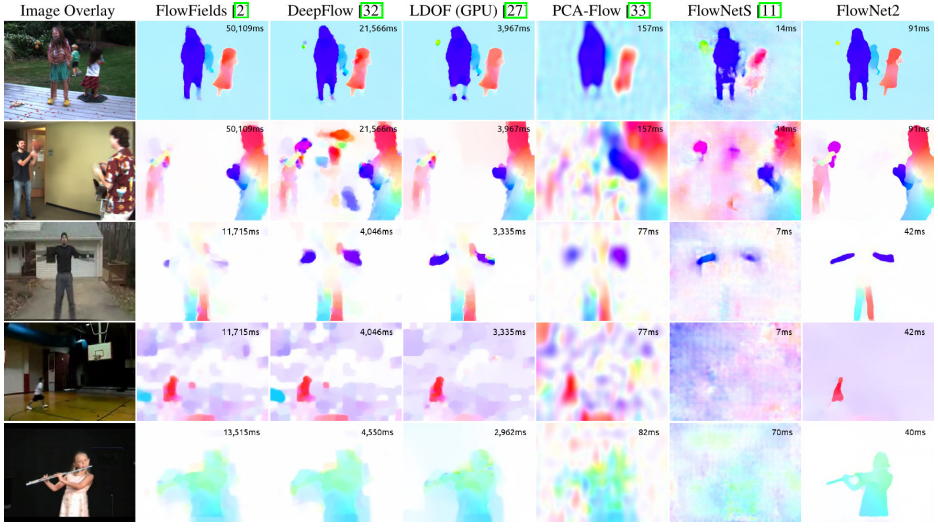
FlowNet2



[Ilg et al., CVPR 2017]

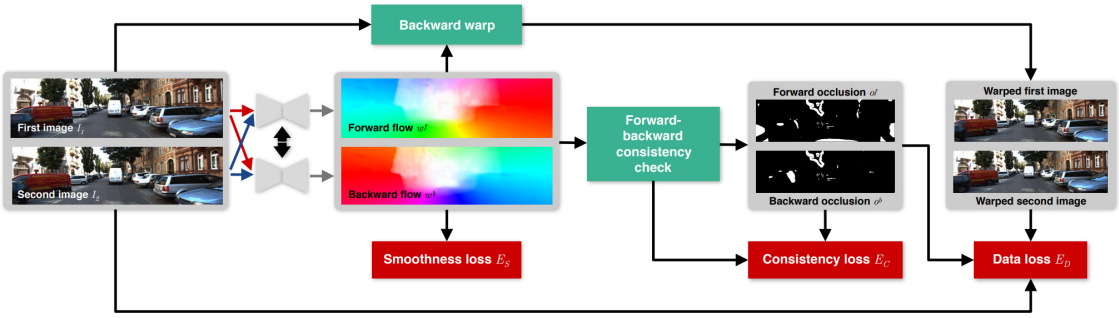
- Combination (stacking) of multiple FlowNets with warping

FlowNet2 – Results



Ilg, Mayer, Saikia, Keuper, Dosovitskiy and Brox: FlowNet 2.0: Evolution of Optical Flow Estimation with Deep Networks. CVPR, 2017.

UnFlow: Unsupervised Optical Flow



- Learn optical flow without supervision by warping images into the other frame
- Uses photometric/smoothness terms as loss functions for training the neural net
- More about self-supervised learning in Lecture 12

Meister, Hur & Roth: UnFlow: Unsupervised Learning of Optical Flow with a Bidirectional Census Loss. AAAI, 2018.

Optical Flow Summary

- Classical OF approaches state-of-the-art until 2016
- DL based methods on par or better since 2017
- But require
  - Big models
  - Enormous amount of (synthetic) training data
  - GPU compute time
  - Sophisticated curriculum learning schedules
- Top performing DL methods borrow many elements from classical methods (e.g., warping, cost volume, coarse-to-fine estimation, loss functions)