

Let the initial intensity of the pumping radiation be I_0 . Intensity of radiation after traversing distance x from the active medium is given by

$$I(x) = I_0 e^{(\gamma-\alpha)x}$$

On reflection at mirror 1, intensity of radiation becomes $I(x) = R_1 I_0 e^{(\gamma-\alpha)x}$ and after reflection at mirror 2 the intensity gets further modified to $I(x) = R_1 R_2 I_0 e^{(\gamma-\alpha)x}$. Thus, after one round trip, the intensity will become

$$I = I(x = 2L) = R_1 R_2 I_0 e^{(\gamma-\alpha)2L}$$

$$\therefore \frac{I}{I_0} = R_1 R_2 e^{(\gamma-\alpha)2L}$$

$\frac{I}{I_0} < 1 \Rightarrow$ net attenuation, laser radiation dies out.

$\frac{I}{I_0} > 1 \Rightarrow$ net amplification

$\frac{I}{I_0} = 1 \Rightarrow$ sustained oscillations

The last two conditions together correspond to light amplification, which is an essential step for laser emission. In particular, the last condition tells about the condition on overall gain factor for onset of stimulated emission dominance over spontaneous emission.

By taking $\frac{I}{I_0} = 1$, we get $R_1 R_2 e^{(\gamma-\alpha)2L} = 1$ or $e^{(\gamma-\alpha)2L} = \frac{1}{R_1 R_2}$

Taking log on both sides,

$$(\gamma - \alpha)2L = \ln\left(\frac{1}{R_1 R_2}\right). \text{ Rearranging, we get}$$

$$\gamma = \alpha + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right) \quad \dots (5)$$

Equation (5) above is called as "threshold condition for lasing action".