LASER

(Light Amplification through Stimulated Emission of Radiation) *

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1.1 Absorption of Radiation

An atom has a numbers of quantized energy states. Initially an atom is in ground state i.e. all of its electrons possess lowest possible energy states. When energy is given to an atom, it goes to excited state, i.e., its electron jump to higher energy state by absorbing a quantum of radiation or photon. This process is called *absorption of radiation.* If E_1 and E_2 are energies of electron in initial and final states and ν is the frequency of absorbed radiation, then

$$E_2 - E_1 = h\nu,$$

$$\Rightarrow \quad \nu = \frac{E_2 - E_1}{h} \tag{1}$$

where h is Plank's constant.

*[*] **Syllabus of FET-MITS**: **LASER** Einstein's coefficients (expression for energy density). Requisites of a Laser system. Condition for laser action. Principle, Construction and working of CO2 laser and semiconductor Laser. Applications of Laser - Laser welding, cutting and drilling. Measurement of atmospheric pollutants. Holography-Principle of Recording and reconstruction of images, applications of holography.*

[†] This lecture note is made for B. Tech first semester students of MITS-FET.

The absorption is an induced (or stimulated) process. The probable rate of transition $1 \rightarrow 2$ depends on the properties of state 1 and state 2 and is proportional to the energy density $u(\nu)$ of the radiation of frequency ν incident on atom. Thus

$$P_{12} = B_{12} \,.\, u(\nu), \tag{2}$$

where B_{12} is Einstein's coefficient for absorption of radiation and depends on the properties of state 1 and state 2.

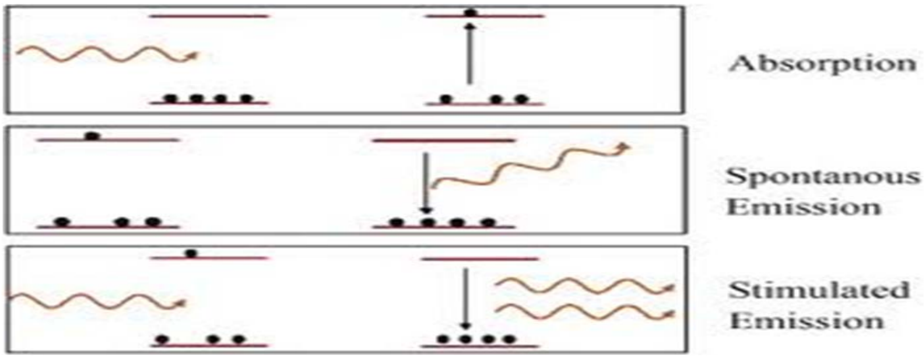

Figure 1: Absorption, Spontaneous and Stimulated Emission of Radiation

1.2 Spontaneous Emission of Radiation

An atom in exited state remains for only about $10^{-8} sec.$ It then, of own accord, jumps to lower energy state, emitting a radiation. If initially the atom is in exited state 2, then it spontaneously jump to sate 1, emitting a photon of frequency ν given by,

$$\nu = \frac{E_2 - E_1}{h}.$$

If there are large number of atoms in exited state, the photons emitted by different atoms have a random phase and hence they are incoherent.

The probability of spontaneous emission $2 \rightarrow 1$ depends only on the properties of state 1 and state 2. According to Einstein, the probable rate of spontaneous emission is denoted by

$$(P_{21})_{spontanous} = A_{21}, \tag{3}$$

where A_{21} is Einstein's coefficient for spontaneous emission of radiation and depends on the properties of state 1 and state 2.

1.3 Stimulated Emission of Radiation

If an atom is in its exited state, then an incident photon of correct energy may cause the atom to jump to lower state, emitting an additional photon of same frequency. Thus now two photons of same frequency and same phase are present. This phenomenon is called *Stimulated Emission of Radiation*. These two photons are coherent and travel in the same direction. For example, suppose we have two energy states E_1 and E_2; E_1 is ground and E_2 is exited. If the atom is in exited state and a photon of frequency ν is made incident on it, the atom jumps to lower energy state, emitting an additional photon of frequency ν.

The probability of stimulated emission from energy sate $2 \rightarrow 1$ depends on the properties of state 1 and state 2 and is proportional to the energy density $u(\nu)$ of the radiation of frequency ν incident on atom. Thus

$$(P_{21})_{Stimulated} = B_{21} \,.\, u(\nu), \tag{4}$$

where B_{21} is Einstein's coefficient for stimulated emission of radiation and depends on the properties of state 1 and state 2.

Thus the total probability of emission transition $2 \rightarrow 1$ is the sum of spontaneous and stimulated emission probabilities, i.e.,

$$P_{21} = (P_{21})_{Spon} + (P_{21})_{Stim} = A_{21} + B_{21} \,.\, u(\nu). \tag{5}$$

2 Relationship amongst Einstein's A and B coefficients:

Consider an assumable of atoms in the thermal equilibrium at temperature T with radiation of frequency ν and the energy density $u(\nu)$. Let N_1 and N_2 be the number of atoms in energy states 1 and 2, respectively, at any instant.
The number of atoms in state 1 that can absorb a photon and give rise to absorption per unit time

$$= N_1 P_{12} = N_1 B_{12} \,.\, u(\nu) \;. \tag{6}$$

Conversely, the number of photons in its state 2 that can cause emission process (spontaneous + stimulates) per unit time,

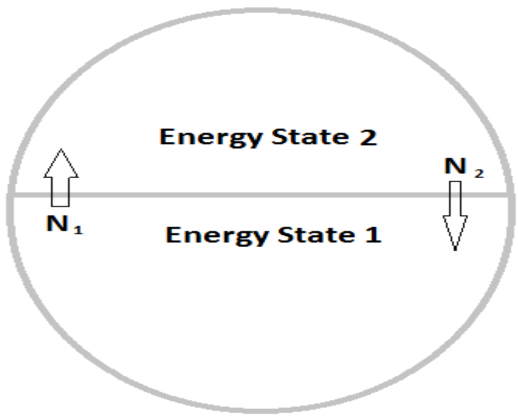

Figure 2: Equilibrium of ground and excited states.

$$= N_2 P_{21} = N_2[A_{21} + B_{12} \,.\, u(\nu)]. \tag{7}$$

For equilibrium, the absorption and emission rates must be equal, i.e.,

$$
\begin{aligned}
N_1 P_{12} &= N_2 P_{21} \\
\Rightarrow N_1 B_{12} u(\nu) &= N_2 [A_{21} + B_{21} \,.\, u(\nu)] \\
\Rightarrow u(\nu) \,.\, [N_1 B_{12} - N_2 B_{21}] &= N_2 A_{21} \\
\Rightarrow u(\nu) &= \frac{N_2 A_{21}}{[N_1 B_{12} - N_2 B_{21}]} = \frac{A_{21}}{B_{21}} \left(\frac{1}{\frac{N_1}{N_2} \frac{B_{12}}{B_{21}} - 1} \right) \qquad (8)
\end{aligned}
$$

According to Boltzmann distribution law, number of atoms N_1 and N_2 in energy state E_1 and E_2 in thermal equilibrium at temperature T are given by,

$$
N_1 = N_0 e^{-E_1/kT} \quad \text{and} \quad N_2 = N_0 e^{-E_2/kT}
$$

where $N_0 \rightarrow$ total number of atoms and $k \rightarrow$ Boltzman's constant.

$$
\frac{N_1}{N_2} = e^{(E_2 - E_1)/kT} = e^{h\nu/kT} \qquad (9)
$$

putting (9) into (8),

$$
u(\nu) = \frac{A_{21}}{B_{21}} \left(\frac{1}{e^{h\nu/kT} \frac{B_{12}}{B_{21}} - 1} \right) \qquad (10)
$$

Comparing (10) with Plank's radiation formula,

$$
u(\nu) = \frac{8\pi h \nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1} \qquad (11)
$$

we get,

$$
\frac{A_{21}}{B_{21}} = \frac{8\pi h \nu^3}{c^3} \quad \text{and} \quad \frac{B_{12}}{B_{21}} = 1. \qquad (12)
$$

Conclusion:

1. $B_{21} = B_{12}$ i.e., probability of stimulated emission is same as that of absorption.

2. $\frac{A_{21}}{B_{21}} \propto \nu^3$, i.e., ratio of spontaneous emission to stimulated emission is proportional to ν^3. It means that the probability of spontaneous emission dominates over induced emission more and more as the energy difference between two states increases.

3 Population Inversion

According to Boltzmann's distribution law, number of atoms in energy state E per unit volume is given by

$$
N_i = N_0 . e^{-E_i/kT},
$$

where N_0 is total number of atoms and T is absolute temperature.

As $E_1 < E_2 < E_3$, hence in normal state $N_1 > N_2 > N_3$.

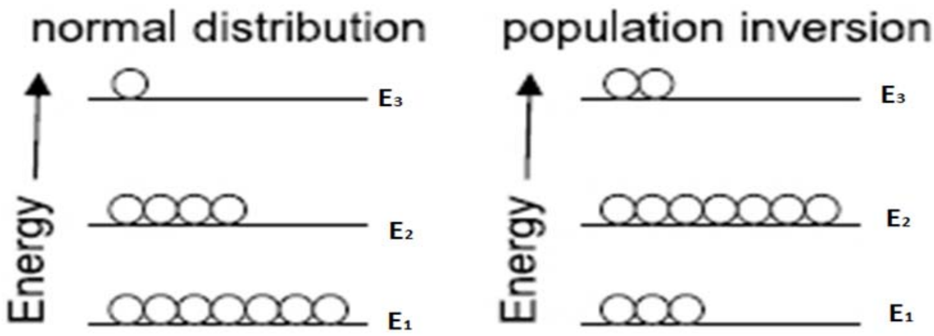

Figure 3: Explanation of Population Inversion

We know that the rate of absorption and stimulated emission processes are proportional to the number of atoms in the ground and excited states, N_1 and N_2, respectively. If the ground state has a higher population than the excited state (i.e., $N_1 > N_2$), the process of absorption dominates and there is a net attenuation of photons. If the populations of the two states are the same (i.e., $N_1 = N_2$), the rate of absorption of light exactly balances the rate of emission; the medium is then said to be optically transparent.

In process of *population inversion*, the higher energy state has a greater population than the lower energy state (i.e., $N_1 < N_2$). Due to which the emission process dominates, and light in the system undergoes a net increase in intensity. A population inversion is required for laser operation which is possible only in atoms with meta-stable states.

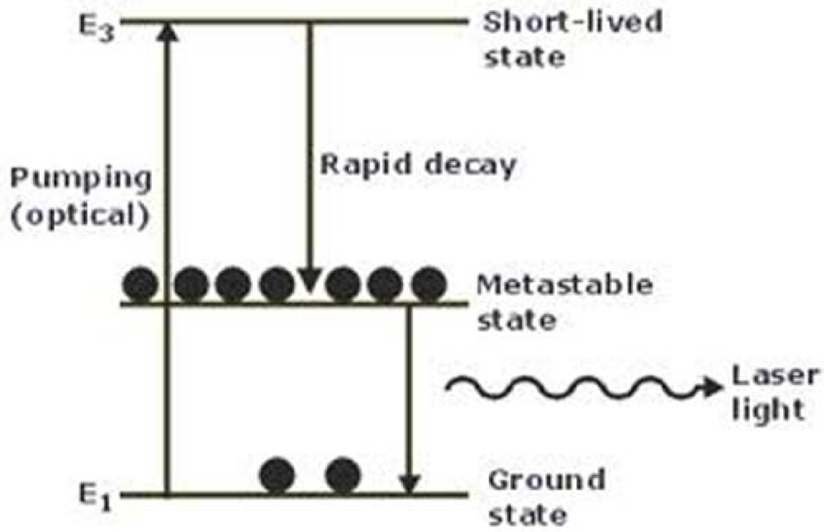

Figure 4: Explanation of Population Inversion

The energy levels, which has the spontaneous lifetime of the order of microseconds to a few milliseconds are called *meta-stable states*. The probability of transitions involving meta-stable levels is relatively low. If an atom is excited into a meta-stable state it can stay there long enough for a photon of the correct frequency to arrive.

4 Pumping Methods

Atoms are energized from the ground state to the excited state by a process called *pumping*,

1. Optical Pumping (Ruby Laser)
2. Electric Discharge (He-Ne Laser)
3. Inelastic-atom-atom-collision (He-Ne Laser)
4. Direct Conversion (Semiconductor Laser)
5. Chemical reactions (CO_2 Laser)

5 Main components of Laser

1. Energy source to raise atoms in excited state.
2. Active medium: materials (solid, liquid or gas) whose atoms go into excited state.
3. Optical resonator or Laser cavity:

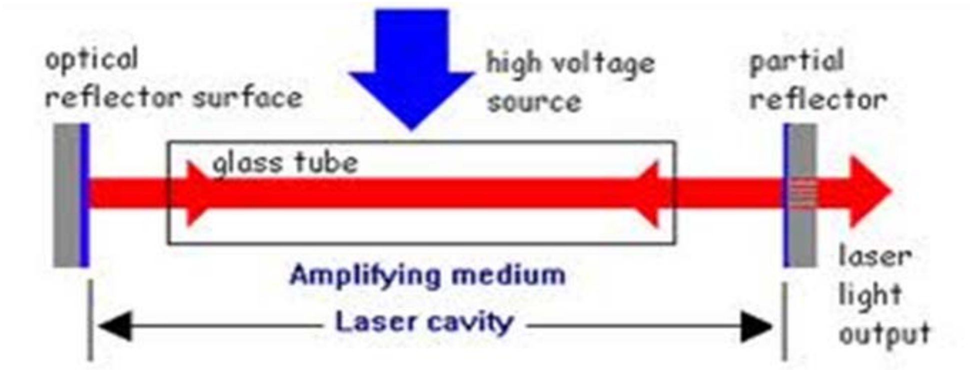

Figure 5: Laser cavity or optical resonator.

   (a) An optical cavity or optical resonator is an arrangement of mirrors that forms a standing wave cavity resonator for light waves.
   (b) Optical cavities are surrounds the gain medium and providing feedback of the laser light.
   (c) Light confined in the cavity reflect multiple times producing standing waves for certain resonance frequencies.
   (d) The standing wave patterns produced are called modes; longitudinal modes differ only in frequency while transverse modes differ for different frequencies and have different intensity patterns across the cross section of the beam.
   (e) Different resonator types are distinguished by the focal lengths of the two mirrors and the distance between them.

(f) The geometry (resonator type)must be chosen so that the beam remains stable (that the size of the beam does not continually grow with multiple reflections).

(g) Resonator types are also designed to meet other criteria such as minimum beam waist or having no focal point inside the cavity.

Books Used:

1. A Text Book of Engineering Physics by Gupta and Kumar.
2. Laser Fundamentals by W. T. Silfvast.
3. http://en.wikipedia.org