Electromagnetism

Concept of Field


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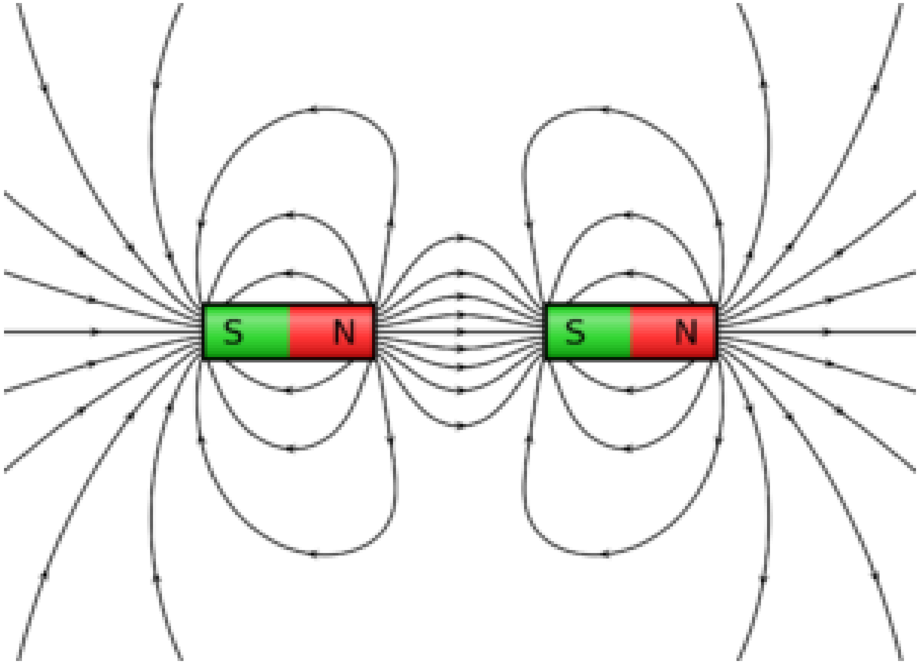
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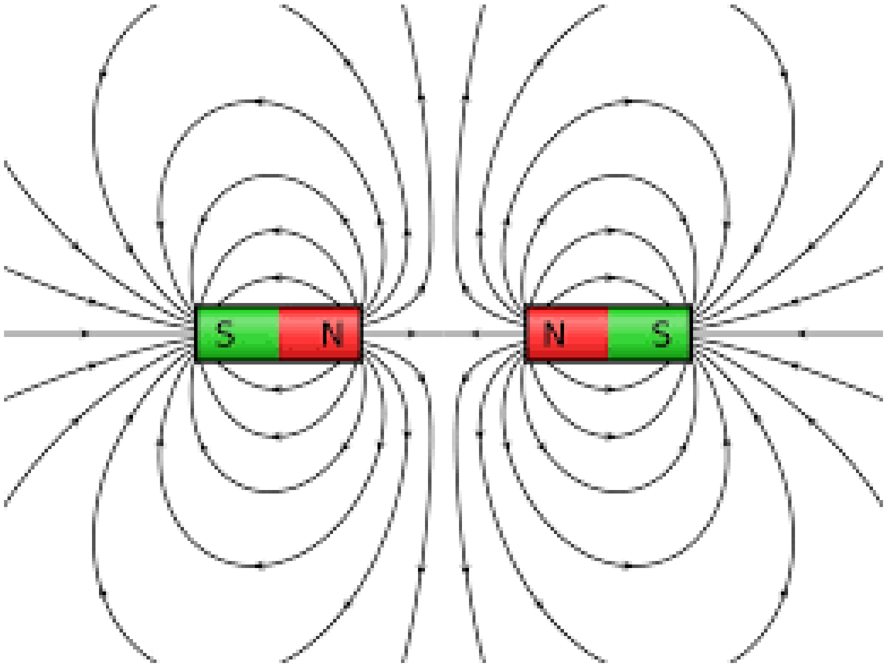
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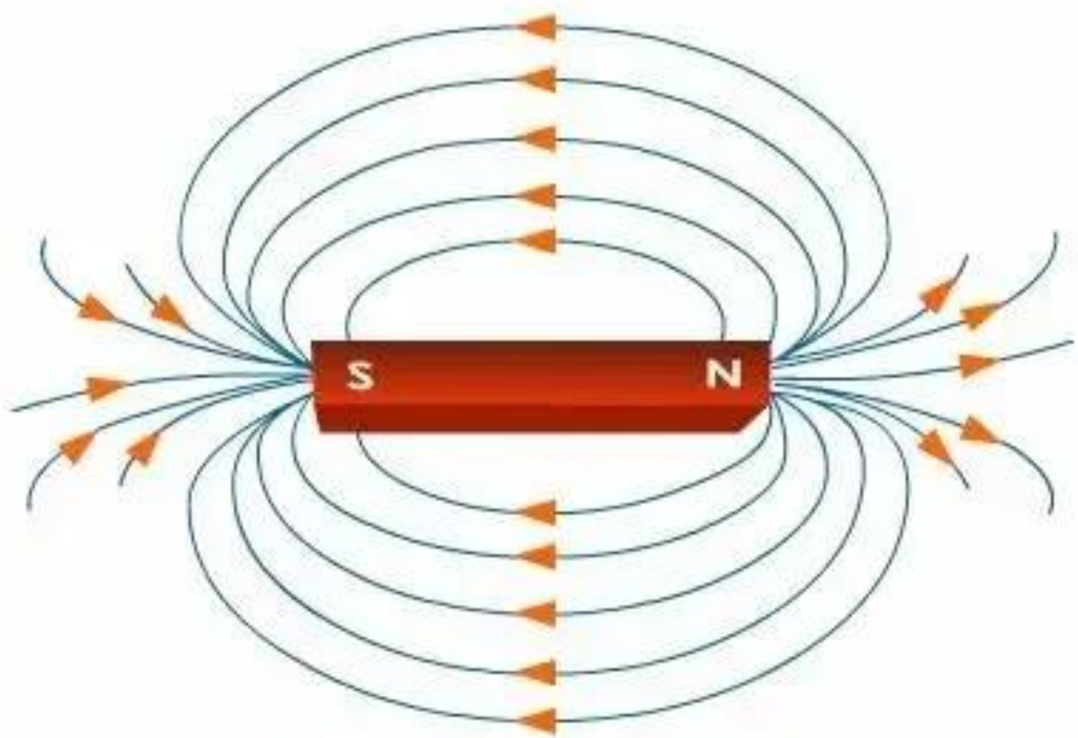
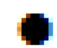
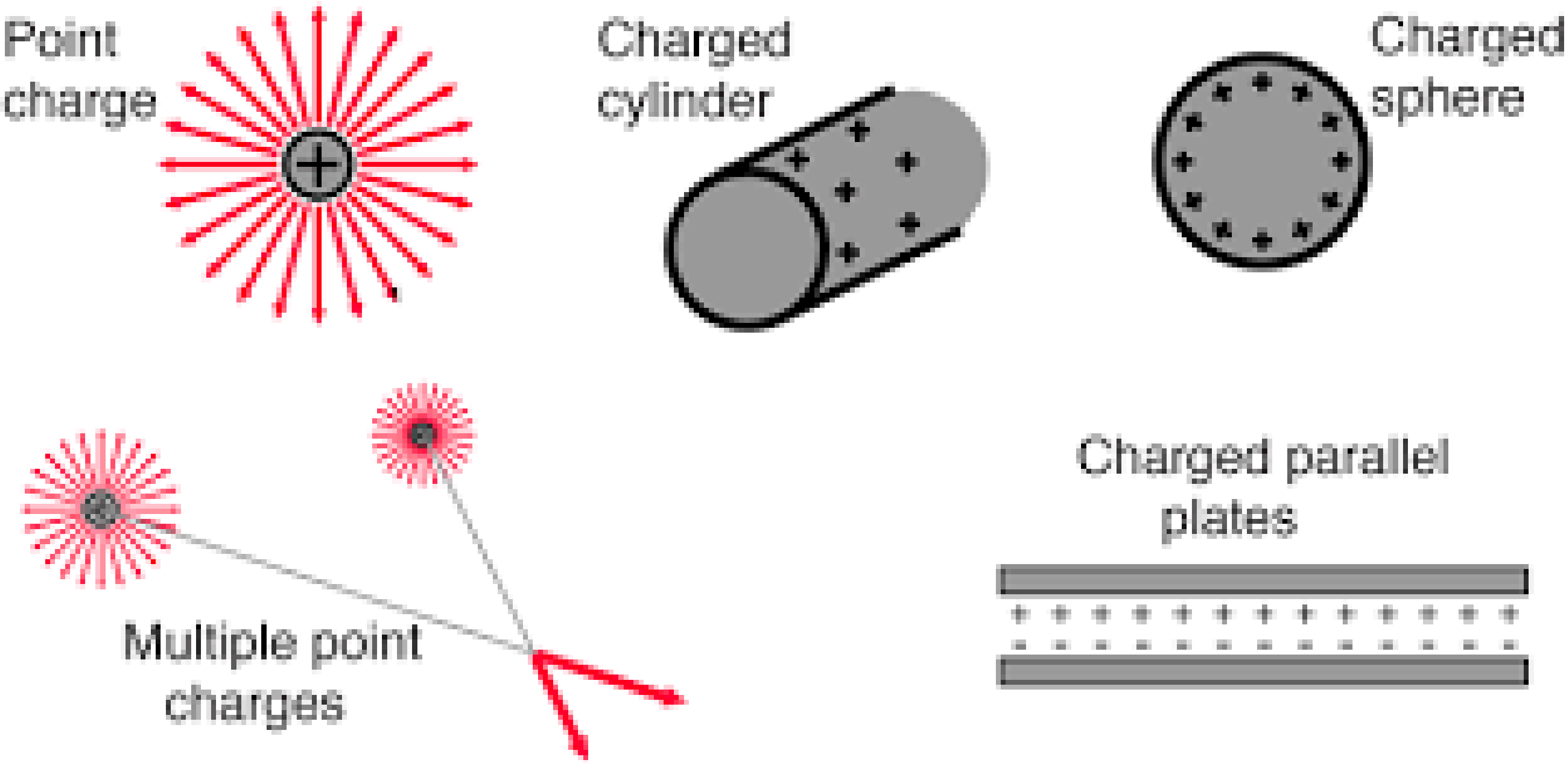
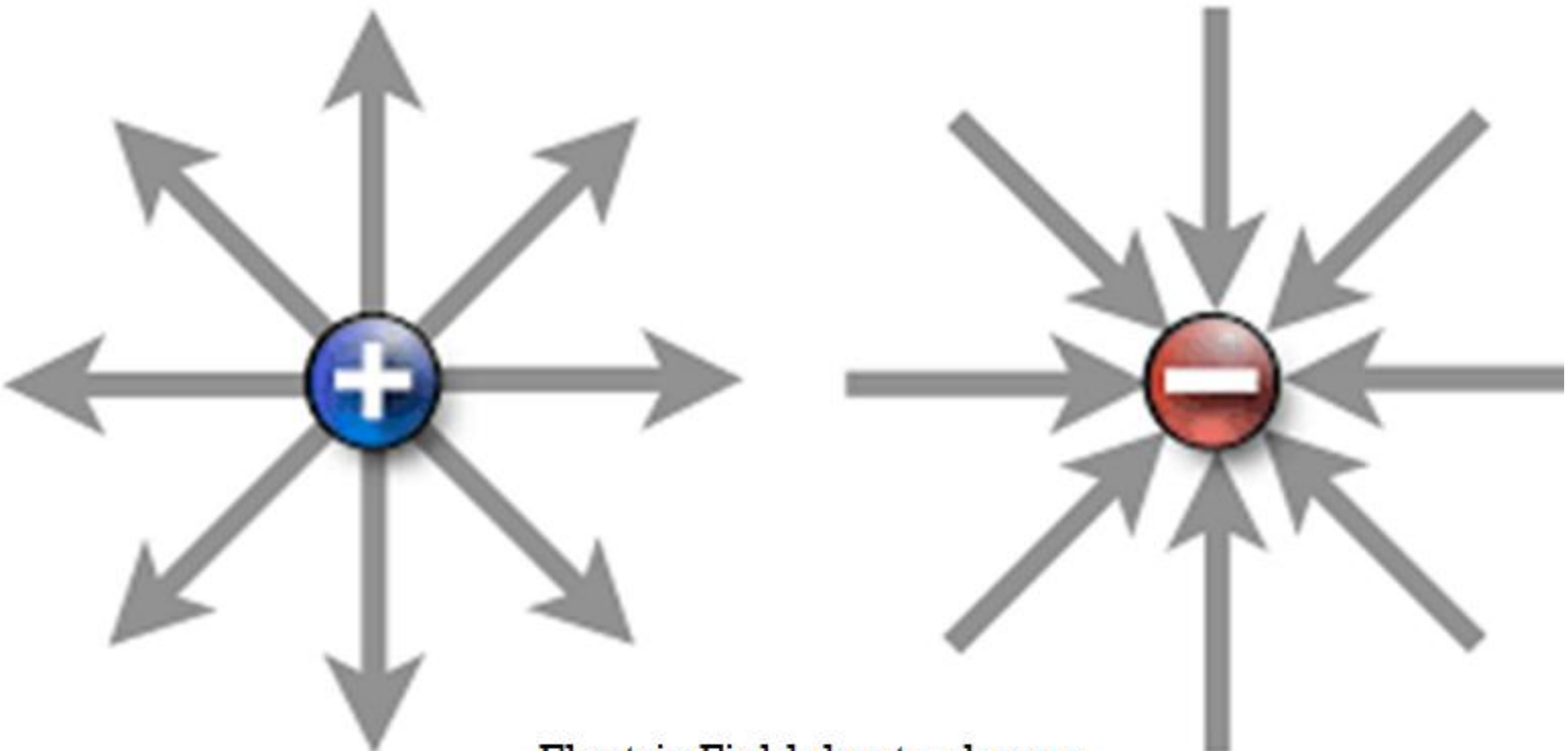


Field Lines Around a Bar Magnet

Point charge, Q




Point
charge
Charged
cylinder
Charged
sphere
Multiple point
charges
Charged parallel
plates



Electric Field due to charge

- A region of space where some physical quantity takes different values at different points is called field.
- A **scalar field** is something that has a particular value at every point in space.

  Example: Temperature……
- A **vector field** is having a value and direction at every point in space.
- Example: Electric Field……

Vector Differential Operator

What is the Del Operator?

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

del operator = differential operator

by itself it has no spcific use (like $\frac{d}{dx}$ by itself)

It can be used to find the gradient of a scalar
⇨ vector

" " " " " " divergence of a vector
⇨ scalar

" " " " " " " curl of a vector
⇨ vector

GRADIENT OF A SCALAR FIELD

- The gradient of a scalar function $f(x_1, x_2, x_3, \ldots, x_n)$ is denoted by ∇f or where ∇ (the nabla symbol) denotes the vector differential operator, del. The notation "grad(f)" is also commonly used for the gradient.

- The gradient of f is defined as the unique vector field whose dot product with any vector $\mathbf{v}$ at each point x is the directional derivative of f along $\mathbf{v}$. That is,

$$(\nabla f(x)) \cdot \mathbf{v} = D_{\mathbf{v}} f(x).$$

- In 3-dimensional cartesian coordinate system it is denoted by:

$$\begin{aligned}\nabla f &= \mathbf{i}\frac{\partial f}{\partial x} + \mathbf{j}\frac{\partial f}{\partial y} + \mathbf{k}\frac{\partial f}{\partial z} \\ &= \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}\end{aligned}$$

PHYSICAL INTERPRETATION OF GRADIENT

- Using the language of vector fields, we may restate this as follows: For the given function $f(x, y)$, gravitational force defines a vector field F over the corresponding surface $z = f(x, y)$, and the initial velocity of an object at a point (x, y) is given mathematically by $-\nabla f(x, y)$.

- The gradient also describes directions of maximum change in other contexts. For example, if we think of f as describing the temperature at a point (x, y), then the gradient gives the direction in which the temperature is increasing most rapidly.

Grad Properties

If A and B are two scalars ,then

1) $$\nabla(A \pm B) = \nabla A \pm \nabla B$$

2) $$\nabla(AB) = A(\nabla B) + B(\nabla A)$$

Directional Derivative

Directional derivative of ϕ in the direction of <u>a</u> is

$$\frac{d\phi}{ds} = \hat{a}.grad\,\phi$$

where,

$$\hat{a} = \frac{dr}{|dr|}$$

Which is a unit vector in the direction of dr .

DIVERGENCE

- In vector calculus, divergence is a vector operator that measures the magnitude of a vector field's source or sink at a given point, in terms of a signed scalar.
- More technically, the divergence represents the volume density of the outward flux of a vector field from an infinitesimal volume around a given point.

Divergence of a vector

If $A = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$,the divergence of A is defined as

$$divA = \nabla . A$$

$$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot \left(a_x\hat{i} + a_y\hat{j} + a_z\hat{k}\right)$$

$$\Rightarrow \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}.$$

Curl of a vector

If $A = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$,the curl of A is defined by

$$curlA = \nabla \times A$$

$$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \times \left(a_x\hat{i} + a_y\hat{j} + a_z\hat{k}\right)$$

$$\Rightarrow curlA = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix}$$

CURL

- In vector calculus, the curl is a vector operator that describes the infinitesimal rotation of a 3-dimensional vector field.
- At every point in that field, the curl of that point is represented by a vector.
- The attributes of this vector (length and direction) characterize the rotation at that point.
- The direction of the curl is the axis of rotation, as determined by the right hand rule, and the magnitude of the curl is the magnitude of that rotation.

SOLENOIDAL AND IRROTATIONAL FIELDS

- The with null divergence is called solenoidal, and the field with null-curl is called irrotational field.
- The divergence of the curl of any vector field A must be zero, i.e.

$$\nabla \cdot (\nabla \times A) = 0$$

- Which shows that a solenoidal field can be expressed in terms of the curl of another vector field or that a curly field must be a solenoidal field.

POINTS TO BE NOTED:

- If curl F=0 then F is called an irrotational vector.
- If F is irrotational, then there exists a scalar point function ϕ such that $F=\nabla\phi$ where ϕ is called the scalar potential of F.
- The work done in moving an object from point P to Q in an irrotational field is

  $[\Phi]_P^Q = \phi(Q) - \phi(P)$.
- The curl signifies the angular velocity or rotation of the body.