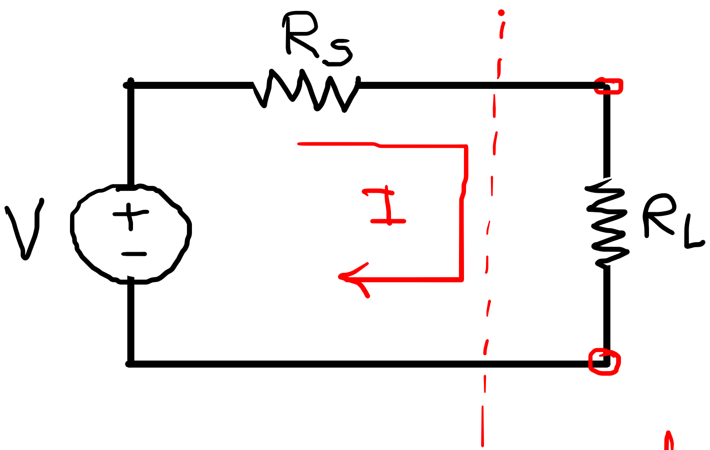
Maximum Power transfer Theorem

Statement:

The maximum power is delivered from a source to a load when load resistance is equal to source resistance.



$$I = \frac{V}{R_S + R_L} \quad \text{-----} \; (1)$$

$R_L = R_S$

Power delivered to load

$$P_L = I^2 R_L = \frac{V^2}{(R_S+R_L)^2} \times R_L$$

to determine value of R_L for which maximum power is delivered

$$\frac{dP_L}{dR_L} = 0$$

$$\frac{dP_L}{dR_L} = \frac{d}{dR_L}\left(\frac{V^2 R_L}{(R_S+R_L)^2}\right) = \frac{V^2(R_S+R_L)^2 - V^2 \cdot R_L \cdot 2(R_S+R_L)}{(R_S+R_L)^4} = 0$$

$$(R_S+R_L)^2 - 2R_L(R_S+R_L) = 0$$

$$R_S^2 + R_L^2 + \cancel{2R_L R_S} - \cancel{2R_L R_S} - 2R_L^2 = 0$$

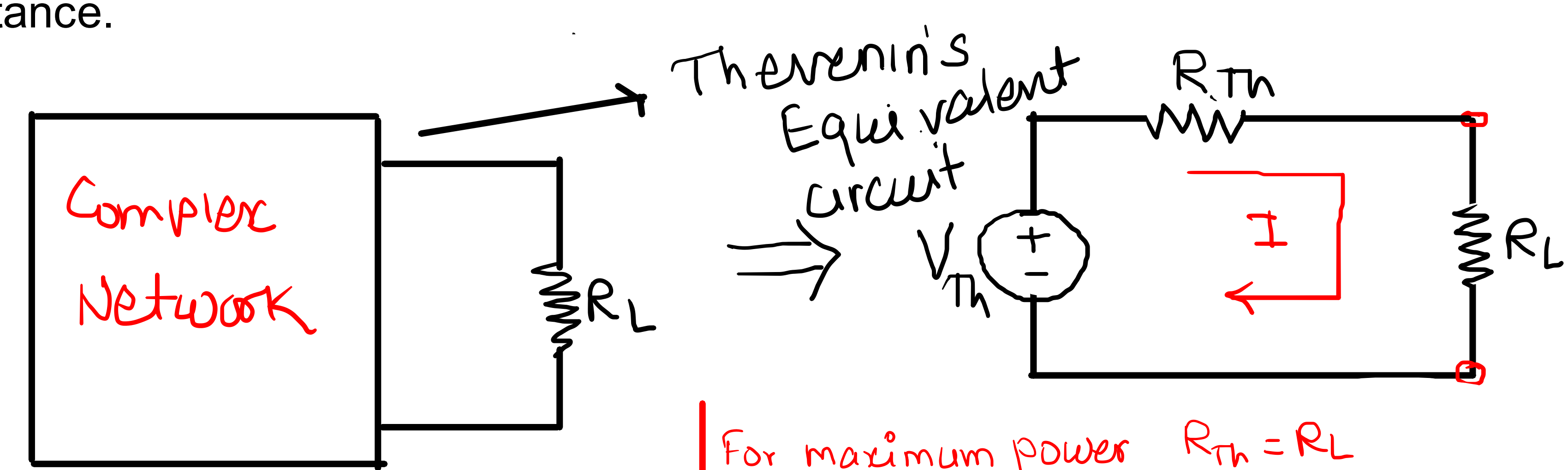
$$R_S^2 - R_L^2 = 0$$

$R_S = R_L$ is condition for maximum power transfer

Maximum Power transfer Theorem

Statement:

The maximum power is delivered from a source to a load when load resistance is equal to source resistance.



Steps:

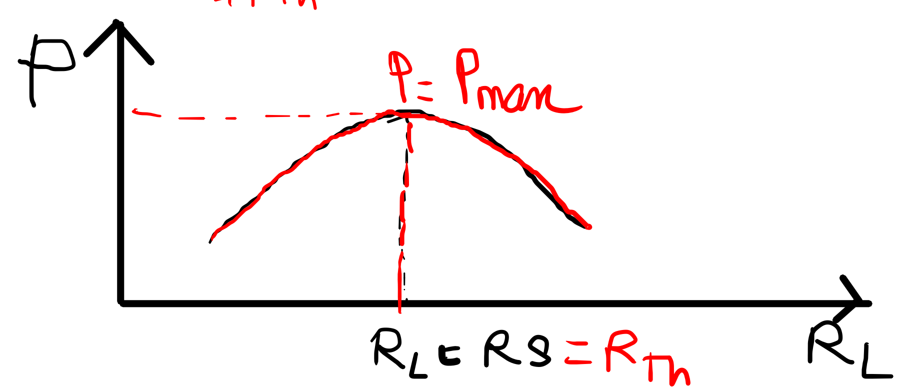
1. Remove Load ✓
2. Find Open circuit Voltage Vth ✓
3. Find Rth ✓
4. Find RL for Maximum power transfer (RL=Rth) ✓
5. Find Maximum Power

$$P_{max} = \frac{(V_{th})^2}{4R_{th}}$$ ✓

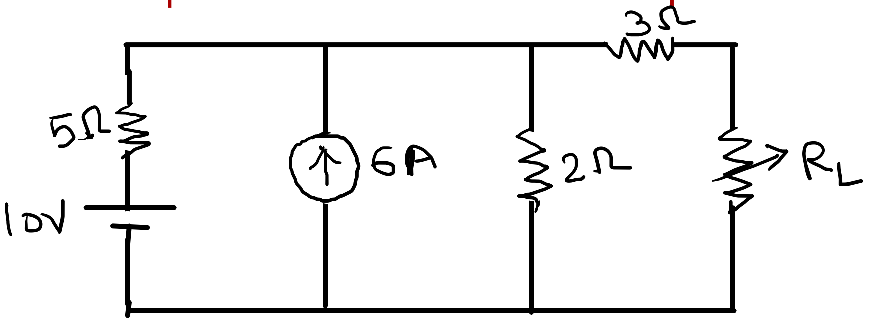
For maximum power $R_{Th} = R_L$

$$P_L = \frac{(V_{Th})^2}{(R_L+R_{Th})^2} \times R_L \quad \text{OR} \quad P_{max} = \frac{(V_{Th})^2}{(R_{Th}+R_{Th})^2} \times R_{Th}$$

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}^{\cancel{2}}} \times \cancel{R_{Th}} \quad \therefore P_{max} = \frac{(V_{Th})^2}{4R_{Th}}$$



Example:1 Find RL for maximum power transfer. Also find maximum power



$$\frac{2V-20+5V}{10} = 6$$

$$7V = 60+20$$

$$V = \frac{80}{7}$$

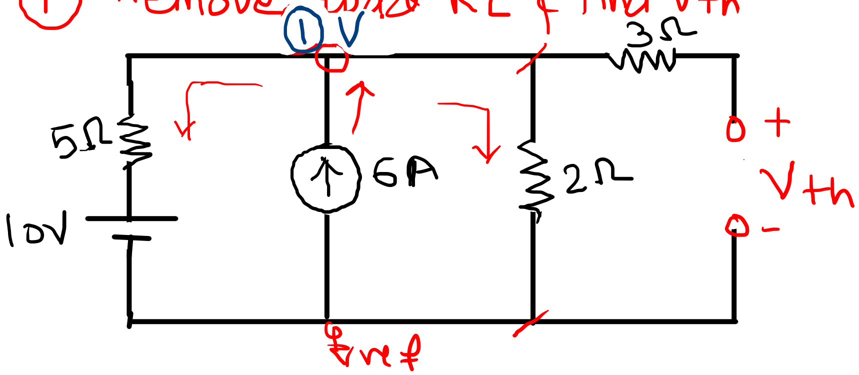
$$V_{th} = V_{2\Omega} = V = 11.43\,V$$

$V_{th} = 11.42V$

$R_{th} = 4.43\Omega$

$P = 7.33w$

① Remove load RL & find Vth

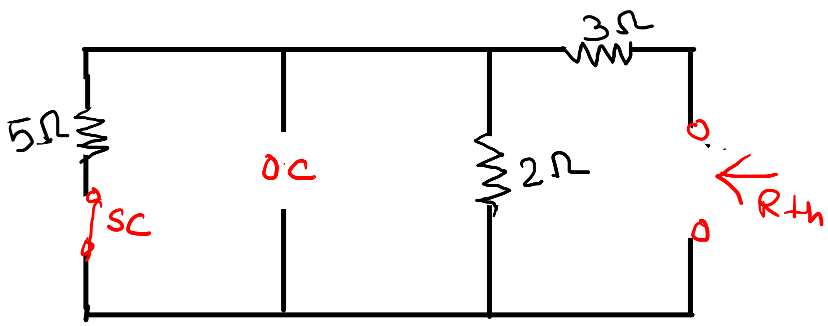


Here $V_{th} = V_{2\Omega}$

using Nodal Analysis so KCL at node

$$\frac{V-10}{5} + \frac{V}{2} = 6$$

② Find Rth.

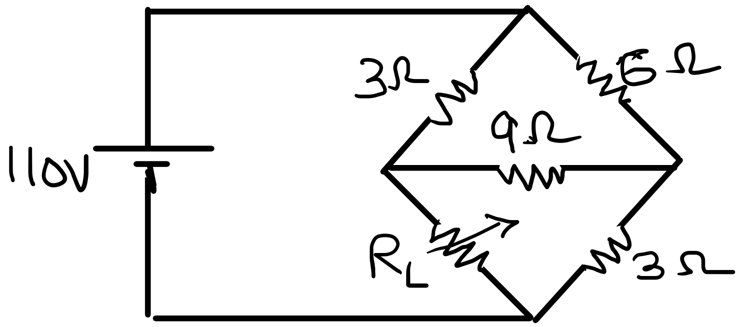


$$R_{th} = (5\|2)+3 = 4.43\Omega$$

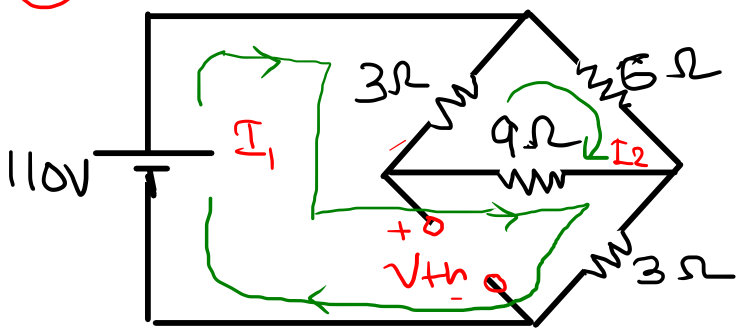
$$R_L = R_{Th} = 4.43\Omega$$

$$P_{max} = \frac{V_{th}^2}{4R_{Th}} = \frac{(11.43)^2}{4\times 4.43} = 7.37\text{ watts.}$$

Example:2 Find RL for maximum power transfer. Also find maximum power



1. Remove RL & find Vth



$$V_{th} - V_{9\Omega} - V_{3\Omega} = 0$$

$$V_{th} = V_{9\Omega} + V_{3\Omega}$$

Using mesh Analysis

KVL to mesh (I)

$$110 - 3(I_1 - I_2) - 9(I_1 - I_2) - 3I_1 = 0$$

$$15I_1 - 12I_2 = 110 \quad \text{--- (1)}$$

KVL to mesh (II)

$$-3(I_2 - I_1) - 6I_2 - 9(I_2 - I_1) = 0$$

$$12I_1 - 18I_2 = 0 \quad \text{--- (2)}$$

Solving (1) & (2)

$I_1 = 15.71A$, $I_2 = 10.48A$

$$V_{9\Omega} = 9(I_1 - I_2) = 9(15.71 - 10.48) = 47.07V$$

$$V_{3\Omega} = 3(I_1) = 3 \times 15.71 = 47.13$$

$$V_{th} = 47.07 + 47.13 = 94.2V$$

$R_{Th} = 2.35\Omega$

$V_{Th} = 94.29$

$P = 945.8W$

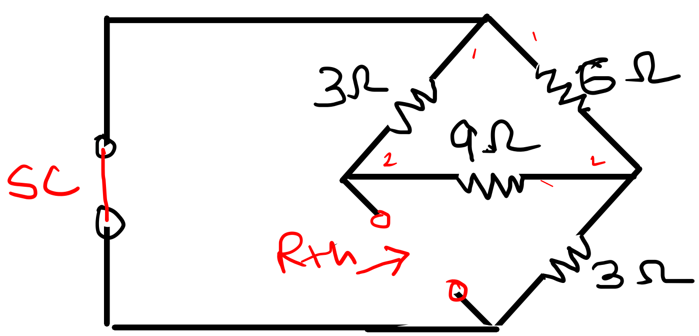
Example:2 Find RL for maximum power transfer. Also find maximum power

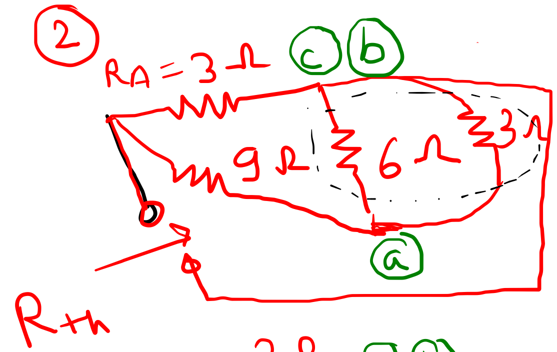
$R_{Th} = 2.35\Omega$

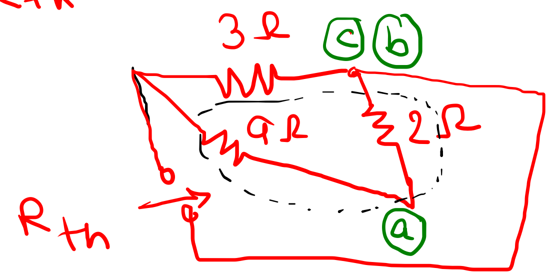
$V_{Th} = 94.29$

$P = 945.8W$

(3) To Find R_{th}.







$R_{th} = 3 \| 11$

$R_{th} = 2.36\,\Omega$

$$R_L = R_{th} = 2.36\Omega$$

$$P_{max} = \frac{V_{th}^2}{4 \cdot R_{th}} = \frac{(94.2)^2}{4 \times 2.36} = 940 \text{ Watts.}$$