Interferencea nd Diffraction of Light

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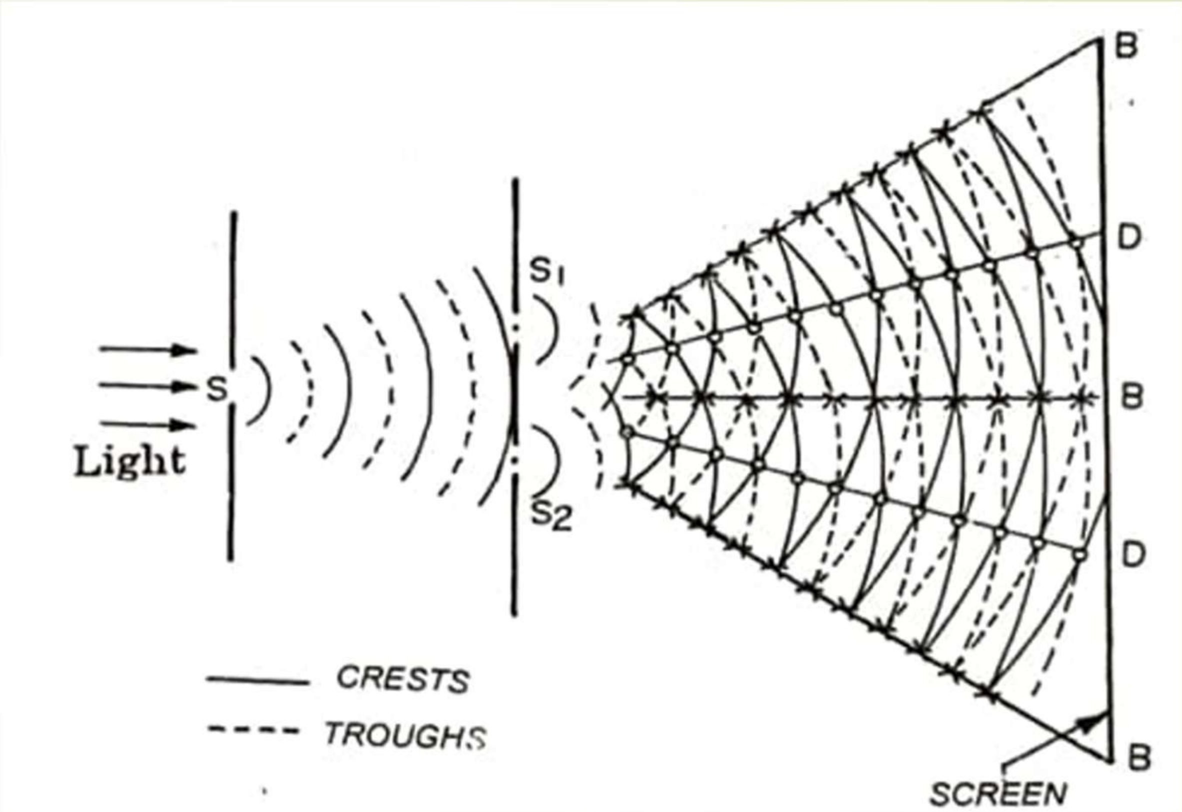
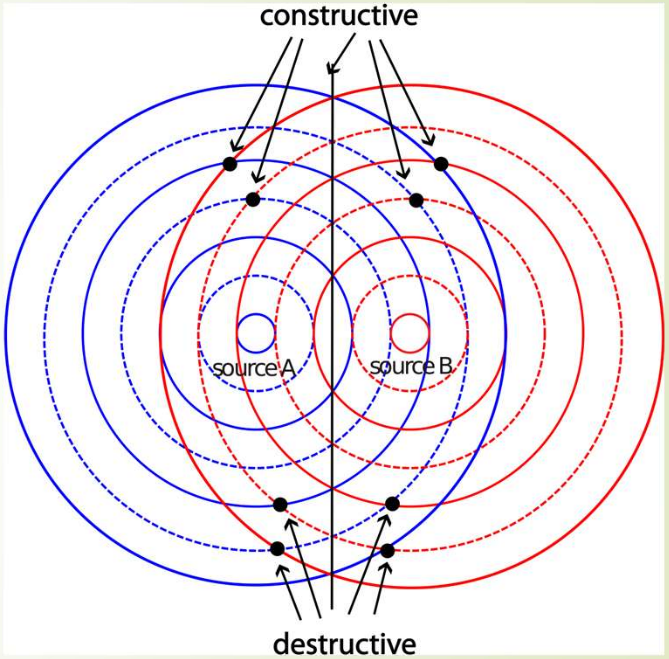


Principle of Superposition

- **Definition:** The **principle of superposition** states that when two or more waves overlap in space, the resultant displacement at any point is the **algebraic sum of the displacements** due to each individual wave.

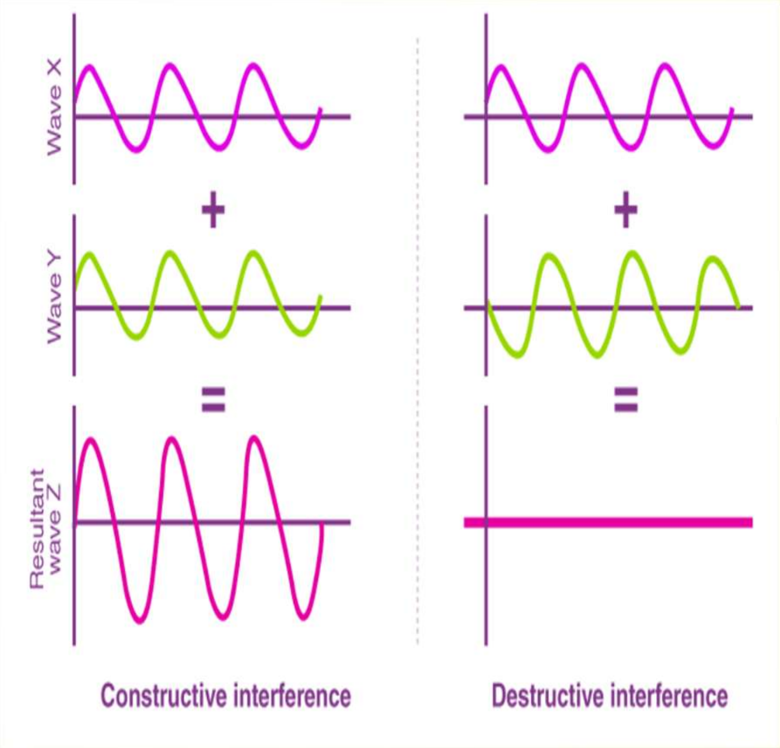
$$Y = Y_1 + Y_2 + \cdots$$

- When two light waves superpose, they **interfere constructively** (bright fringe) or **destructively** (dark fringe) depending on the **phase difference** between them.



Definition of Interference

- **Interference** is the phenomenon that occurs when **two or more coherent light waves** superpose in space, resulting in a **variation in intensity** due to constructive or destructive combination of the wave amplitudes.
  - **Constructive Interference** → Bright fringes (waves in phase)
  - **Destructive Interference** → Dark fringes (waves out of phase)
- **Key Requirement:** Coherent sources (same frequency and constant phase difference)

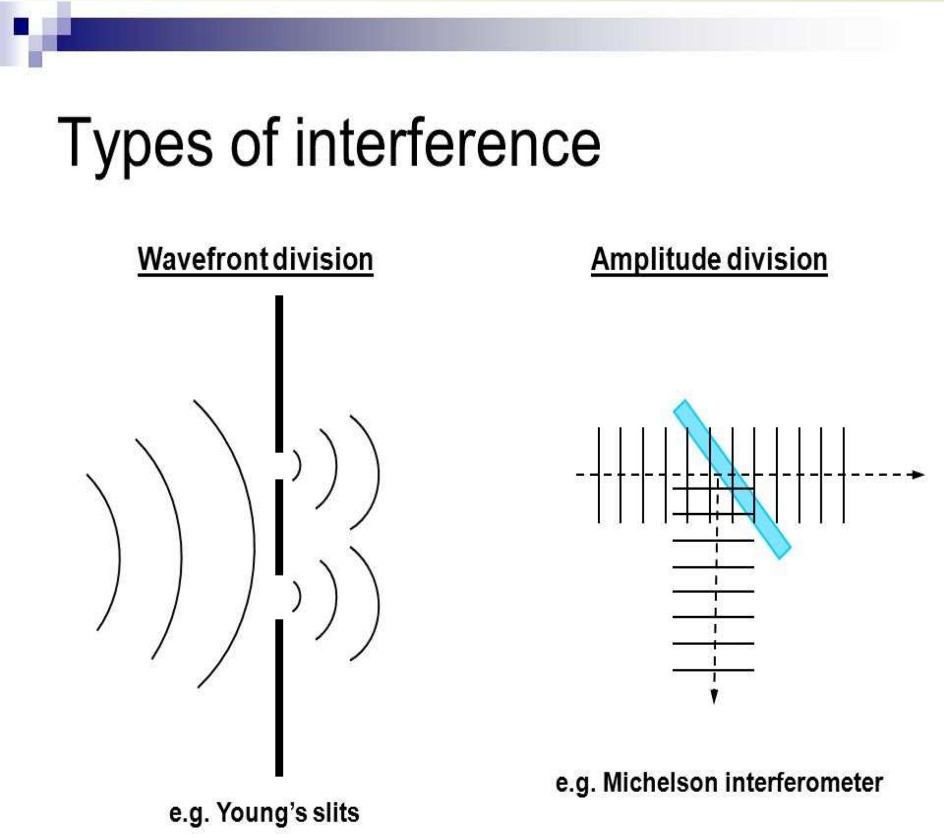


Conditions for Observing Interference

1. The two waves must be **coherent** (i.e., maintain a constant phase difference).
2. They must have the **same frequency**.
3. The waves should have **nearly equal amplitudes** for clear fringe visibility.
4. The **phase difference** between the waves must remain constant.
5. The light source should be **monochromatic** (single wavelength).
6. The waves should **travel close together** and **in the same direction**.
7. For **polarized light**, the waves must have the **same plane of polarization**.

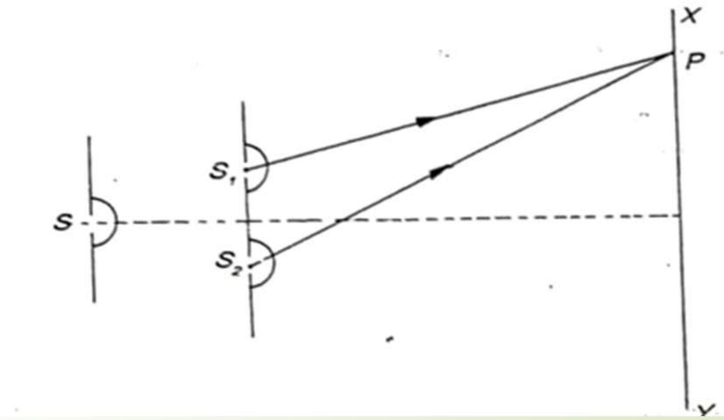
Methods for getting coherent Sources

1. **Division of wavefront**: A single wavefront is split into two parts using slits or mirrors. The parts travel different paths and then recombine. **Examples**: Young's Double-Slit Experiment, Fresnel's Biprism, Lloyd's Mirror
2. **Division of amplitude**: A single light beam is partially reflected and transmitted using beam splitters or thin films. The resulting beams follow different paths and then recombine. **Examples**: Newton's Rings, Michelson Interferometer, Fabry–Pérot Interferometer



Superposition of two waves

$$\text{Phase difference } \delta = \frac{2\pi x}{\lambda} = \frac{2\pi}{\lambda} \times (\text{path difference})$$



$$\begin{aligned} y &= y_1 + y_2 \\ &= a_1 \sin \omega t + a_2 \sin(\omega t + \delta) \\ &= a_1 \sin \omega t + a_2 \sin \omega t \cos \delta + a_2 \cos \omega t \sin \delta \\ &= \sin \omega t\,(a_1 + a_2 \cos \delta) + \cos \omega t\,(a_2 \sin \delta) \end{aligned}$$

Let $a_1 + a_2 \cos \delta = R \cos \theta$ ...(*i*)

$a_2 \sin \delta = R \sin \theta$ ...(*ii*)

where R and θ are new constants. This gives

$$y = \sin \omega t\, R \cos \theta + \cos \omega t\, R \sin \theta$$

$$y = R \sin(\omega t + \theta)$$

Hence the resultant displacement at P is simple harmonic and of amplitude R. Squaring and adding eqn. (*i*) and eqn. (*ii*), we get

$$R^2 \cos^2 \theta + R^2 \sin^2 \theta = (a_1 + a_2 \cos \delta)^2 + (a_2 \sin \delta)^2$$

$$R^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \delta$$

The resultant intensity I at P, which is proportional to the square of the resultant amplitude, is given by

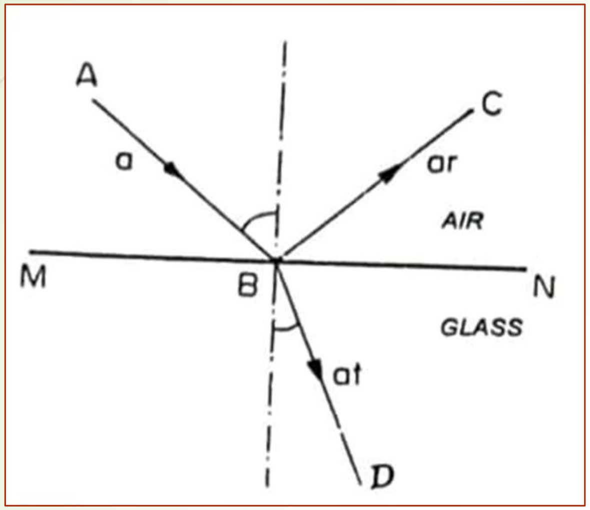
$$I = R^2$$

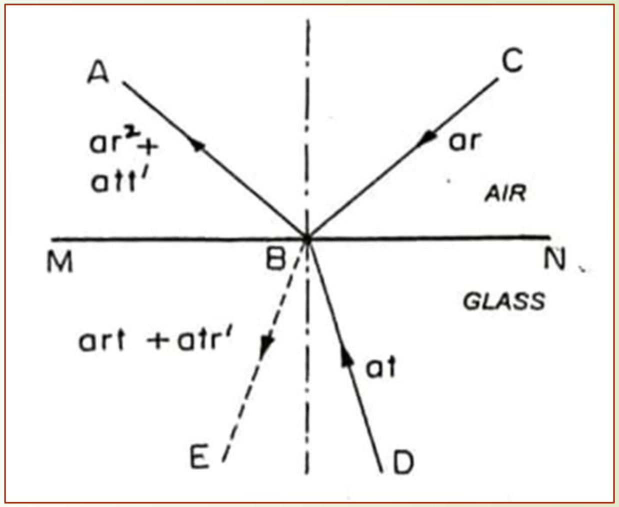
$$\boxed{I = a_1^2 + a_2^2 + 2a_1 a_2 \cos \delta} \quad \boxed{I_{max} = (a_1 + a_2)^2} \quad \boxed{I_{min} = (a_1 - a_2)^2}$$

$a_1 = a_2 = a$, then

$$\boxed{I = 4a^2 \cos^2 \frac{\delta}{2}} \quad \boxed{I_{max} = 4a^2} \quad \boxed{I_{min} = 0}$$

Stokes treament





The component along BE should be zero and that along BA should be equal to a. That is

$$art + atr' = 0 \qquad \ldots(39)$$

and

$$ar^2 + att' = a \qquad \ldots(40)$$

From eqns. (39) and (40), we get

$$r' = -r \qquad \ldots(41)$$

and

$$tt' = 1 - r^2 \qquad \ldots(42)$$

Thin film interference

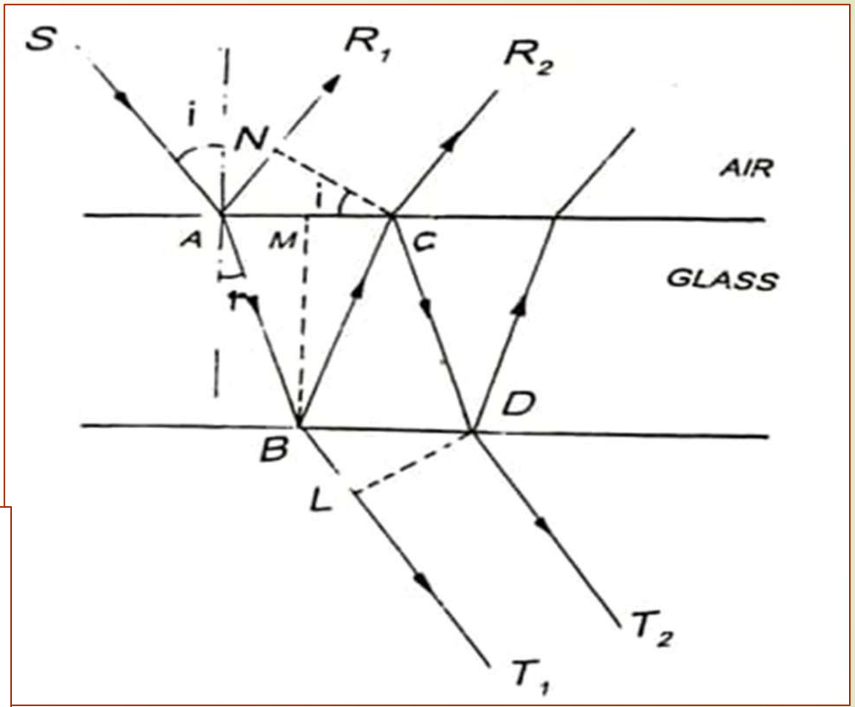
Path difference $= \text{path } ABC \text{ in film} - \text{path } AN \text{ in air}$
$= \mu\,(AB + BC) - AN$

Now, $AB = BC = \dfrac{BM}{\cos r} = \dfrac{t}{\cos r}$,

$AN = AC \sin i$
$= (AM + MC) \sin i = (BM \tan r + BM \tan r) \sin i$
$= 2t \tan r \sin i$

$$= 2t \cdot \frac{\sin r}{\cos r} \sin i = 2t \frac{\sin r}{\cos r} (\mu \sin r) \quad [\therefore \ \sin i = \mu \sin r)$$

$$= 2\mu t \frac{\sin^2 r}{\cos r}$$



Therefore, using these values, path difference becomes :

$$\text{path difference} = \mu \left(\frac{t}{\cos r} + \frac{t}{\cos r} \right) - 2\mu t \frac{\sin^2 r}{\cos r}$$

$$= \frac{2\mu t}{\cos r} - \frac{2\mu t \sin^2 r}{\cos r} = \frac{2\mu t}{\cos r}(1 - \sin^2 r) = \frac{2\mu t}{\cos r} \cdot \cos^2 r$$

$\therefore$ Path difference $= 2\mu t \cos r$. ...(44)

Thin film interference

Since the ray AR_1, suffers reflection at the surface of a denser medium, therefore, it undergoes a phase change of π $\left(\text{or path difference of } \frac{\lambda}{2}\right)$.

Hence the *effective path difference* between rays AR_1 and CR_2 is :

$$= 2\mu t \cos r + \frac{\lambda}{2} \qquad \ldots(45)$$

Conditions of Maxima and Minima in Reflected Light

The two rays will produce *constructive interference* if the path difference between them is an integral multiple of λ

$$2\mu t \cos r + \frac{\lambda}{2} = n\lambda \qquad \text{when } n = \cancel{0}, 1, 2, \ldots\ldots$$

or

$$2\mu t \cos r = (2n-1)\frac{\lambda}{2} \qquad \ldots(46)$$

(condition of maxima)

The two rays will produce *destructive interference* if the path difference between them is an odd multiple of $\frac{\lambda}{2}$

$$2\mu t \cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2} \qquad \text{where } n = 0, 1, 2\ldots\ldots$$

or

$$2\mu t \cos r = n\lambda \qquad \ldots(47)$$

(condition of minima)

Wadge shape film

effective path difference $2\mu t + \frac{\lambda}{2}$.

The condition for *maximum intensity* (bright fringe) is

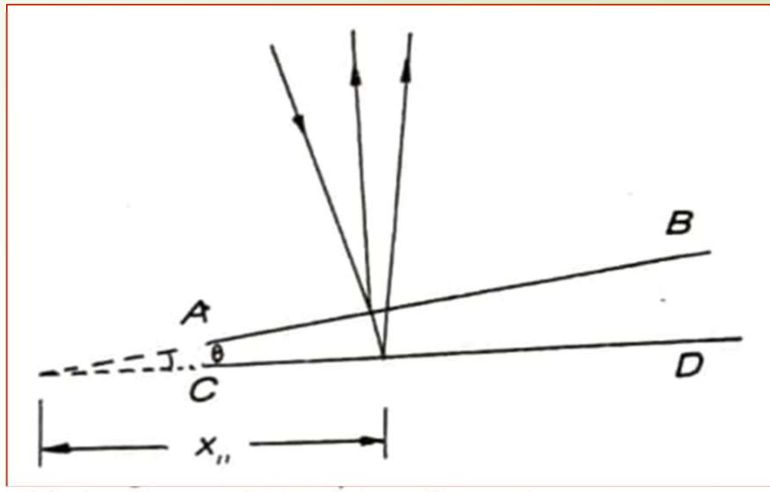
$$2\mu t + \frac{\lambda}{2} = n\lambda$$

$$2\mu t = (2n - 1)\frac{\lambda}{2}$$

The condition for *minimum intensity* (dark fringe) is

$$2\mu t + \frac{\lambda}{2} = (2n + 1)\frac{\lambda}{2}$$

$$\therefore \quad 2\mu t = n\lambda$$



Let this fringe be obtained at a distance x_n from the edge

$$t = x_n \tan\theta$$

as θ is small.

$$\therefore \quad \tan\theta = \theta$$

$$\therefore \quad t = x_n\theta$$

$$\therefore \quad 2\mu x_n\theta = n\lambda$$

Similarly, if $(n + 1)$ dark fringe is obtained at a distance, then

$$2\mu x_{n+1}\theta = (n + 1)\lambda$$

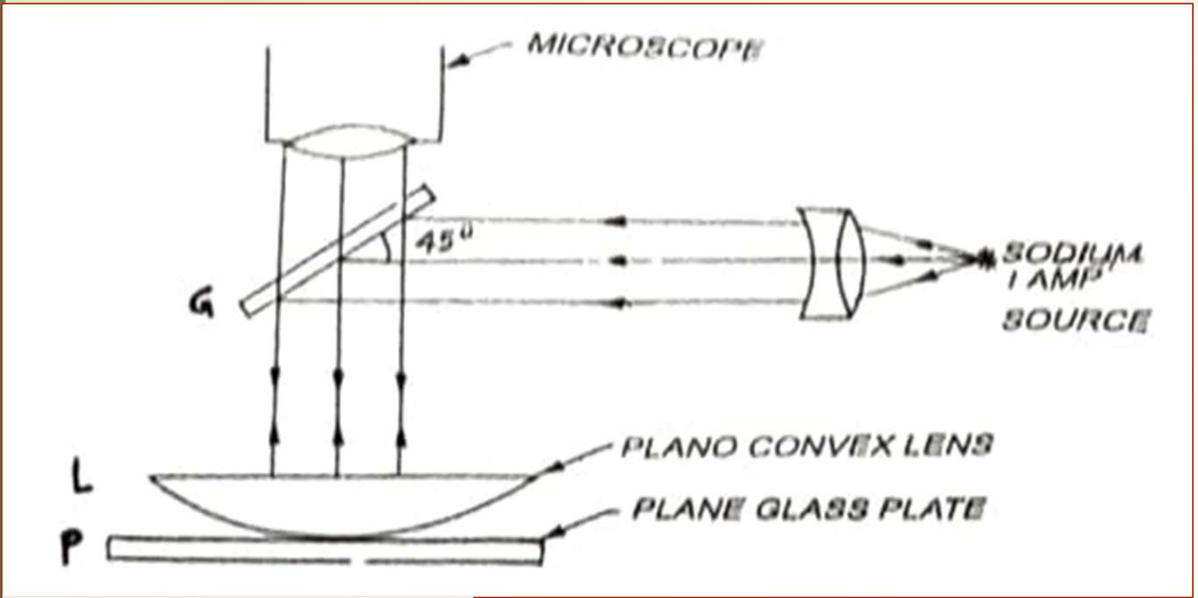
Subtracting eqn. (53) from eqn. (54), we get

$$2\mu\theta\,(x_{n+1} - x_n) = \lambda$$

$$\therefore \quad \text{Fringe width } \overline{X} = x_{n+1} - x_n = \frac{\lambda}{2\mu\theta}$$

$$\boxed{\overline{X} = \frac{\lambda}{2\mu\theta}}$$

Newton's Ring



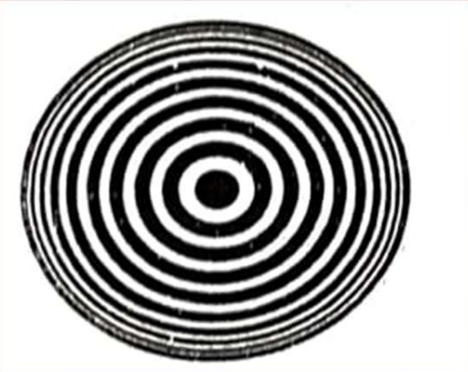
Path difference $2\mu t \cos r + \frac{\lambda}{2}$

For r=0 and air film

$\therefore$ Path difference $= 2t + \frac{\lambda}{2}$

At the point of contact ; $t = 0$

$\therefore$ Path difference $= \frac{\lambda}{2}$



For maxima (bright-fringe), the path-difference $= n\lambda$

$$\therefore \quad 2t + \frac{\lambda}{2} = n\lambda$$

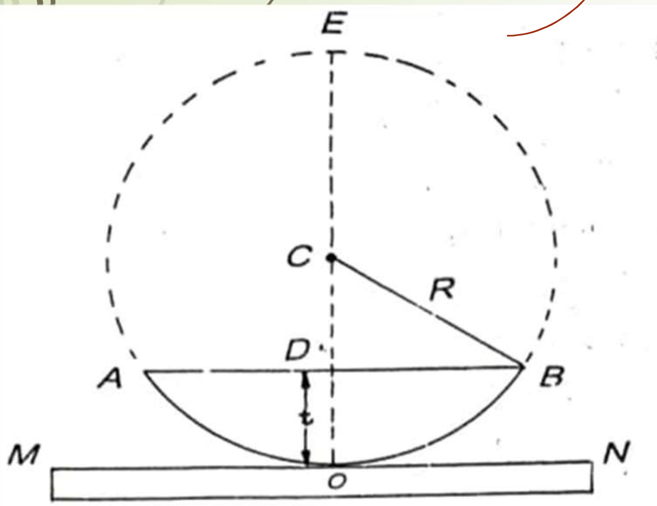
or

$$2t = (2n - 1)\frac{\lambda}{2}$$

where

$$n = 1, 2, 3, \ldots\ldots$$

Newton's Ring



For minima (dark fringe), the path difference $= (2n+1)\frac{\lambda}{2}$

or $$2t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

or $$2t = n\lambda$$

where $$n = 0, 1, 2, 3, \ldots\ldots$$

From the geometrical property of a circle,

$$AD \times DB = OD \times DE$$

$$r \times r = OD\,(2R - OD)$$

$$r^2 = t\,(2R - t)$$

$$r^2 = 2Rt - t^2 \qquad \ldots(59)$$

Since t is very small compared to R, hence t^2 can be neglected. Hence

$$r^2 = 2Rt$$

or $$2t = \frac{r^2}{R} \qquad \ldots(60)$$

Substituting the value of $2t$ in eqn. (57) for ***bright rings***, we have

$$\frac{r^2}{R} = (2n-1)\frac{\lambda}{2}$$

$\therefore$ Radius of nth bright ring

$$r_n = \sqrt{\frac{(2n-1)\lambda R}{2}} \qquad \ldots(61)$$

The diameter of nth bright ring $D_n = 2r_n$

$\therefore$ $$D_n^2 = 2\,(2n-1)\,\lambda R$$

$$D_n = \sqrt{2\lambda R}\sqrt{(2n-1)} \qquad \ldots(62)$$

or $$D_n \propto \sqrt{(2n-1)}$$

Thus, ***the diameters of bright rings are proportional to the square-roots of the odd natural numbers.***

Newton's Ring

Diameters of Dark Rings

As $$2t = n\lambda \text{ (for dark ring)}$$

and $$2t = \frac{r^2}{R} \quad \ldots(63)$$

$\therefore$ Comparing eqn. (63) and eqn. (58)

$$\frac{r^2}{R} = n\lambda \quad \ldots(64)$$

If D_n be the diameter of the nth dark ring

$$\Rightarrow \quad D_n = 2r_n$$

$$\therefore \quad D_n = \sqrt{4nR\lambda}$$

or $$D_n = \sqrt{4R\lambda}\sqrt{n} \quad \ldots(65)$$

or $$D_n \propto \sqrt{n}.$$

Thus, ***the diameters of dark rings are proportional to the square roots of natural numbers.***

Calculation of wavelength

$$D_n^2 = 4n\lambda R$$

Similarly, the diameter of the $(n + p)$th ring is given by

$$D_{n+p}^2 = 4(n + p)\lambda R$$

Difference of eqns. (67) and (66) gives

$$D_{n+p}^2 - D_n^2 = 4\lambda R\,[(n + p) - n] = 4p\lambda R$$

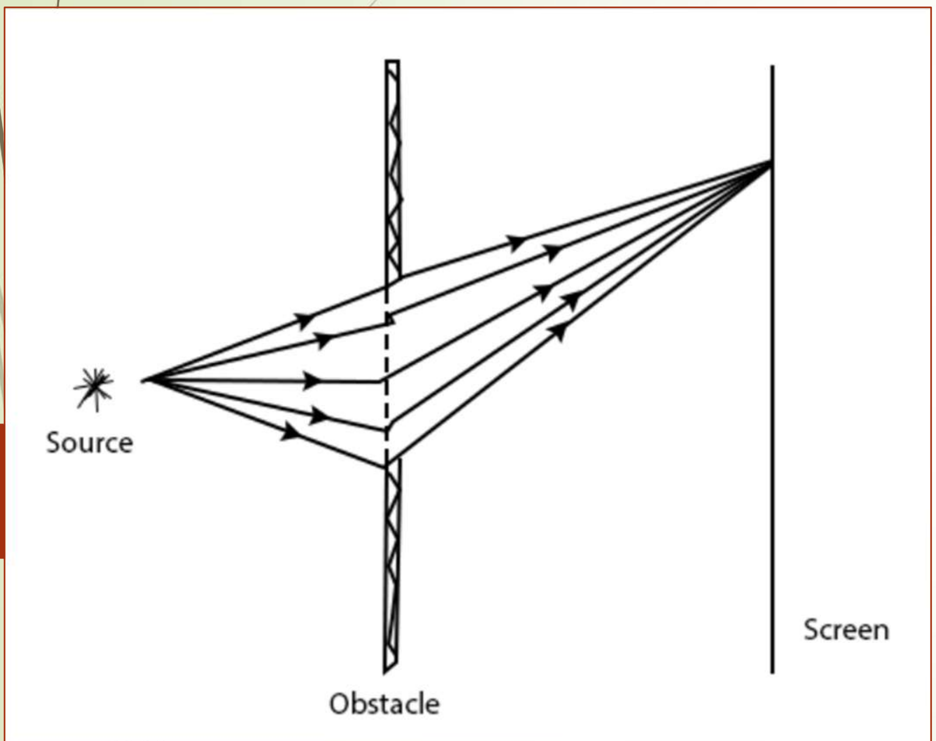
$$\therefore \quad \boxed{\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}}$$

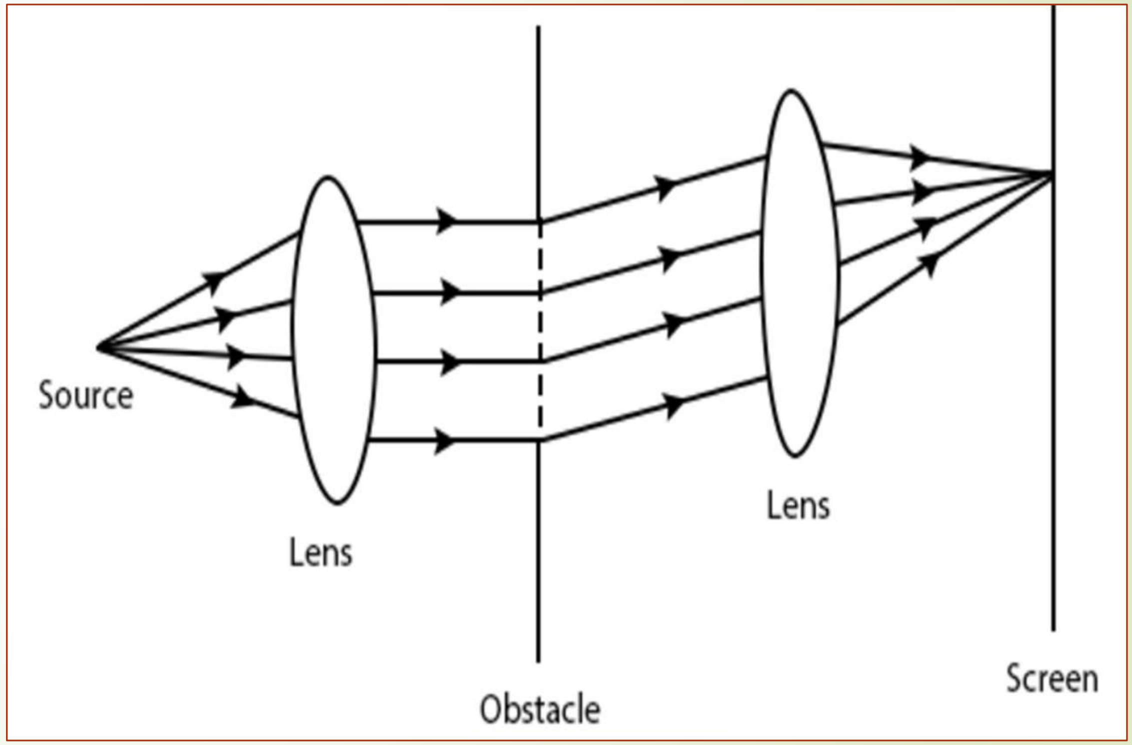
Diffraction

Optical diffraction is the phenomenon in which light waves bend, spread, or interfere when they encounter an obstacle, edge, or aperture whose size is comparable to or smaller than the wavelength of light. It arises from the wave nature of light and results in characteristic patterns of intensity and phase, such as fringes or rings, that cannot be explained by geometric optics alone.

Fresnel Diffraction	*Fraunhofer Diffraction*
(*i*) The source and the screen are at *finite* distance from the diffracting aperture.	(*i*) The source and the screen are at *infinite* distance from the diffracting aperature.
(*ii*) The wave fronts are *divergent* either spherical or cylindrical.	(*ii*) The wave fronts are *plane* which is realized by using convex lens.
(*iii*) *No mirrors* or *lenses* are used for observation.	(*iii*) Diffracted light is collected by a *lens* as in a telescope.
(*iv*) For obtaining Fresnel diffraction *zone plates* are used.	(*iv*) For this, single, double slits or *plane diffraction gratings* are used.
(*v*) The centre of diffraction pattern may be *bright* or *dark* depending upon the number of Fresnel zones.	(*v*) The centre of the diffraction pattern is *always bright* for all paths parallel to the axis of the lens.

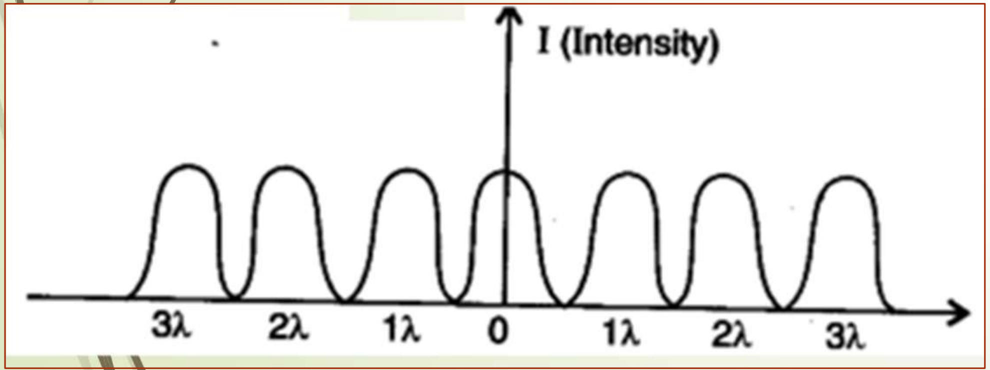
Fresnel vs Fraunhofer Diffraction

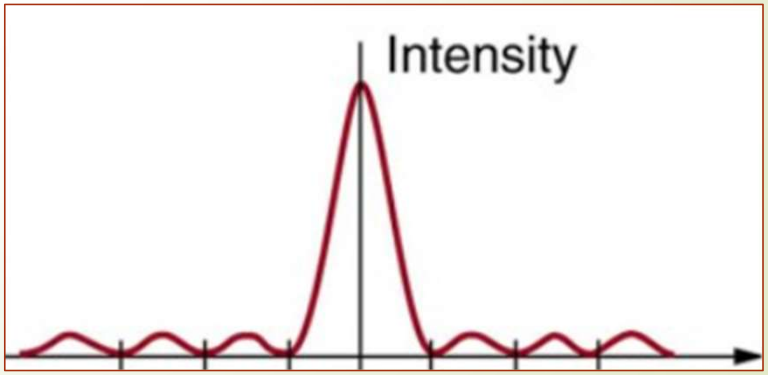




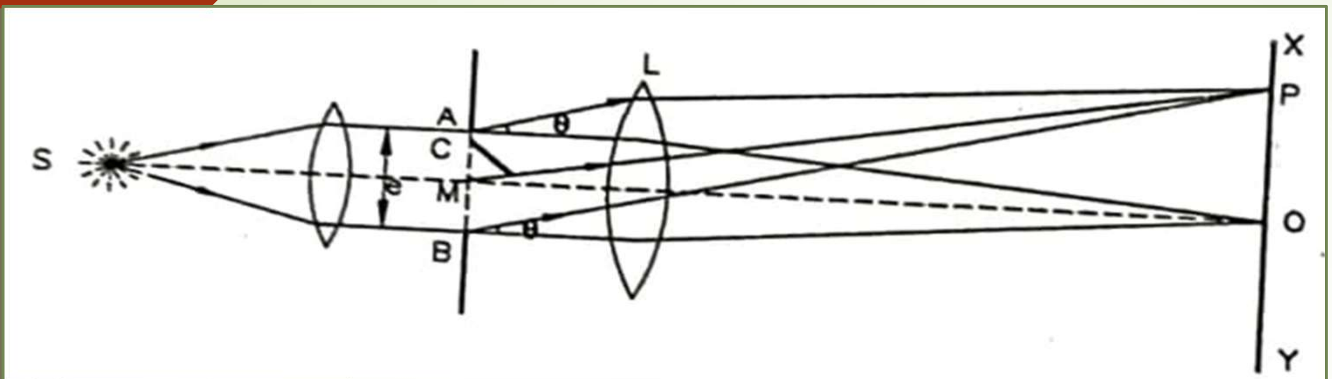
Difference between Diffraction & Interference

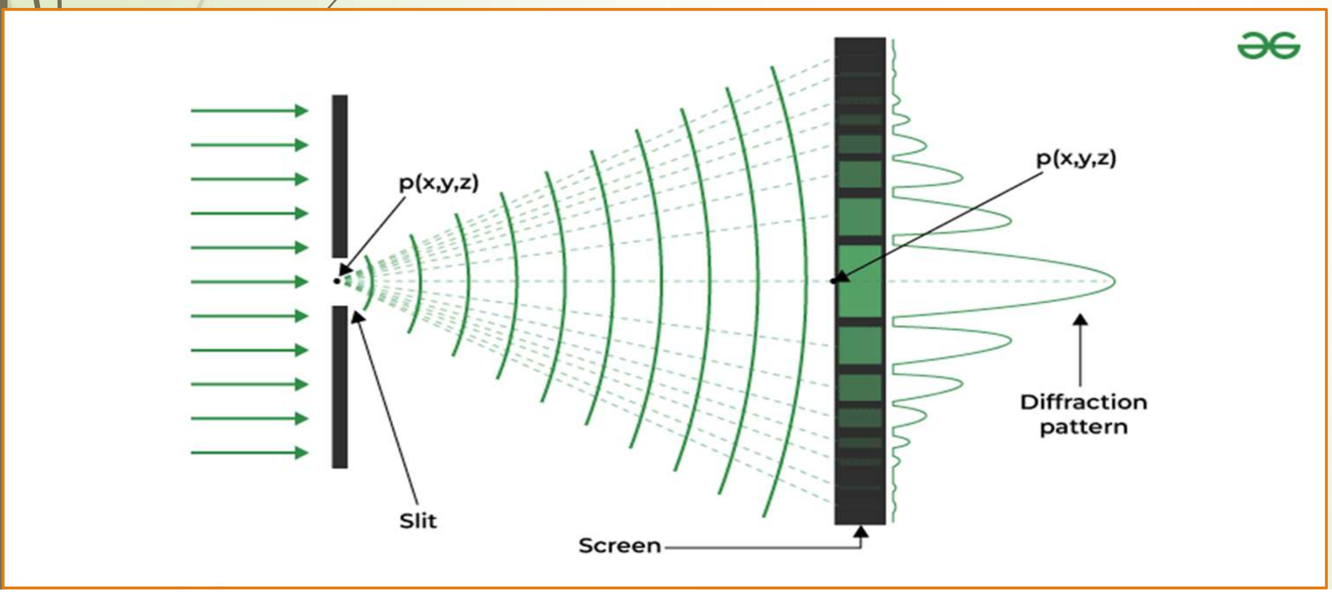
Interference	*Diffraction*
(i) This phenomenon is the result of interaction taking place between two separate wave fronts originating from two coherent sources.	(i) This phenomenon is the result of interaction of light between the secondary wavelets originating from different points of the same wave front.
(ii) The regions of minimum intensity are usually *almost perfectly dark*.	(ii) The regions of minimum intensity are *not perfectly dark*.
(iii) Interference fringes *may* or *may not* be of the same width.	(iii) Diffraction fringes *are not* of the same width.
(iv) All maxima are of same intensity.	(iv) Maxima are of *varying* intensity.





Single Slit Fraunhofer diffraction





Resultant Intensity

$$I = I_0 \frac{\sin^2 \alpha}{\alpha^2}$$

$$\frac{\pi e \sin \theta}{\lambda} = \alpha$$

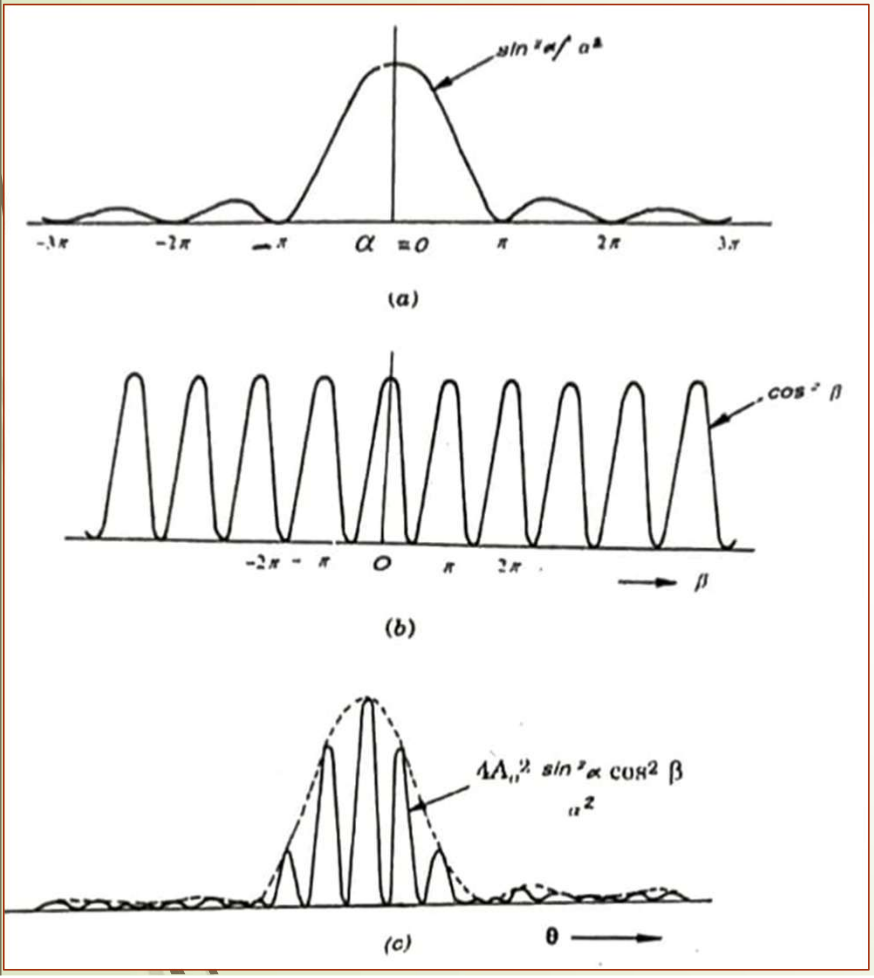
Position of Maxima

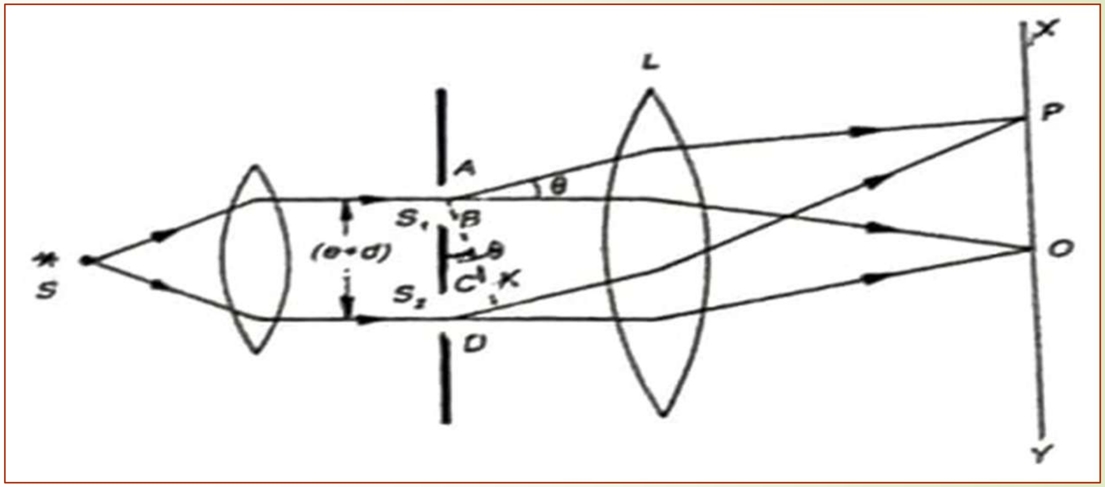
$$\alpha = \tan \alpha$$

Position of Minima

$$e \sin \theta = \pm m\lambda$$

Double Slit Diffraction





Resultant Intensity

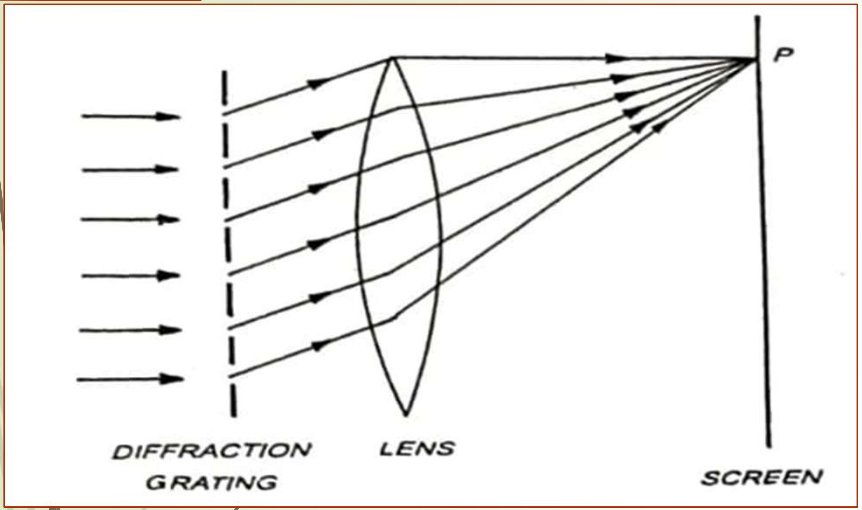
$$I = I_0 \frac{\sin^2 \alpha}{\alpha^2} \cos^2\beta$$

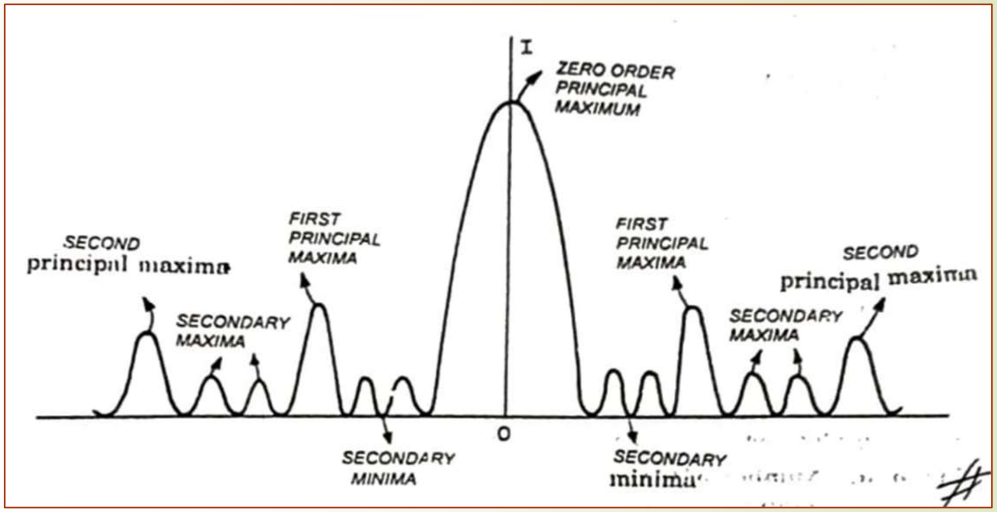
$$\alpha = \frac{\pi e \sin\theta}{\lambda} \qquad \beta = \frac{\pi(e+d)\sin\theta}{\lambda}$$

Condition for missing order

$$\frac{(e+d)}{e} = \frac{n}{m}$$

N-Slit Diffraction





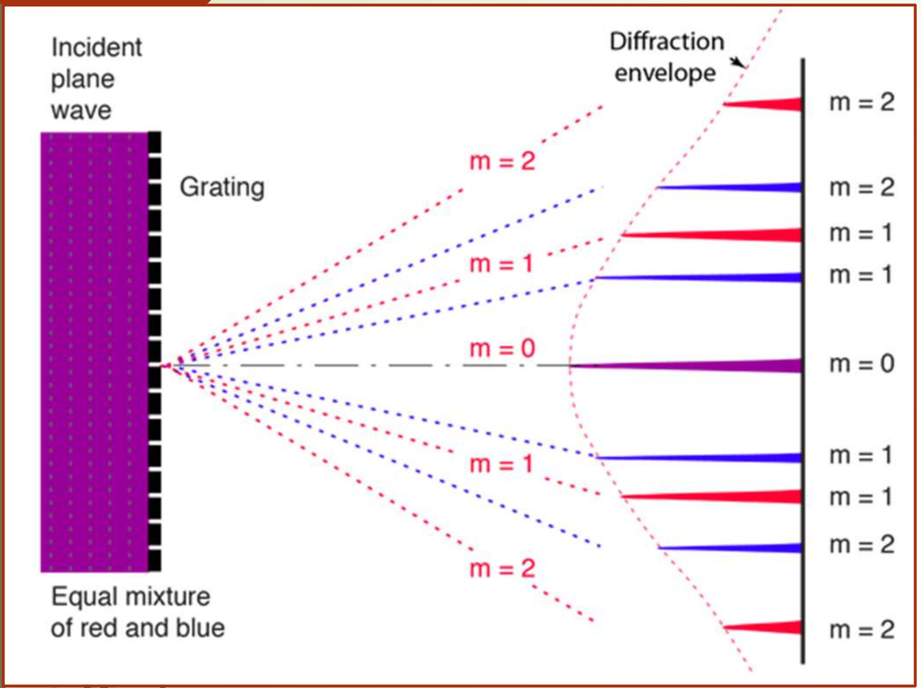
Resultant Intensity

$$I = I_0 \frac{\sin^2 \alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2 \beta}$$

$$\alpha = \frac{\pi e \sin \theta}{\lambda}$$

$$\beta = \frac{\pi (e + d) \sin \theta}{\lambda}$$

Diffraction Grating



A diffraction grating is constructed by scratching a flat piece of transparent material with multiple parallel lines. The material can be scratched with a great number of scratches per cm. The grating to be utilized, for example, contains 6,000 lines per cm. The scratches are opaque, but the spaces between them allow light to pass through. When light falls on a diffraction grating, it forms a multiplicity for the source with a parallel slit.

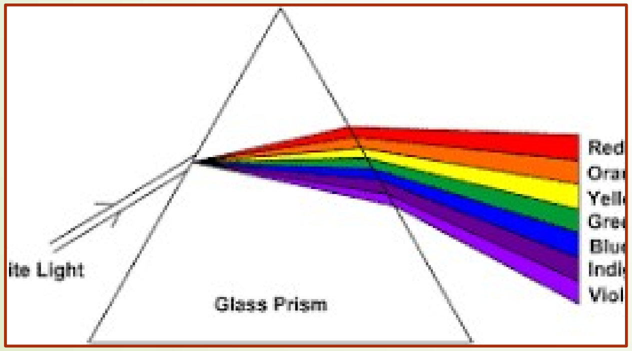
Applications of grating

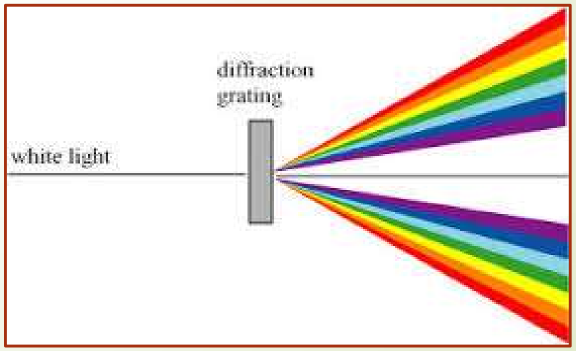
The diffraction grating is an immensely useful tool for the separation of the spectral lines associated with atomic transitions. It acts as a "super prism", separating the different colors of light much more than the dispersion effect in a prism. The illustration shows the hydrogen spectrum. The hydrogen gas in a thin glass tube is excited by an electrical discharge and the spectrum can be viewed through the grating.

The tracks of a compact disc act as a diffraction grating, producing a separation of the colors of white light. The nominal track separation on a CD is 1.6 micrometers, corresponding to about 625 tracks per millimeter. This is in the range of ordinary laboratory diffraction gratings. For red light of wavelength 600 nm, this would give a first order diffraction maximum at about 22° .

Difference between Grating and Prism Spectrum

Prism Spectrum	Grating Spectrum
(i) The prism spectrum is formed by *dispersion*.	(i) The grating spectrum is formed by *diffraction*.
(ii) The prism spectrum is only of one order.	(ii) The grating spectrum forms spectra of different orders.
(iii) The prism spectrum depends on the material of the prism.	(iii) The grating spectrum is independent of the material of the grating.
(iv) The prism spectrum does not show the fine structure of spectral lines.	(iv) The grating spectrum shows the fine structure of spectral lines.
(v) The prism spectral lines are curved.	(v) The grating spectral lines are almost straight.
(vi) In prism spectrum there is greater deviation in the violet region as compared to red region.	(vi) In grating spectrum there is greater deviation in the red region as compared to violet region.





Rayleigh criterion

It states that two point sources are just resolvable when the center of the diffraction pattern (Airy disk) of one source falls on the first minimum of the diffraction pattern of the other. This criterion is crucial for understanding how well an optical system can distinguish between closely spaced objects.