4. Relation between the Einstein coeffecients. (lasers)

To find out the relation, we assume that:

i. The atoms and the radiation are thermal equilibrium.

ii. The radiation is identical with black body radiation and consistent with Planck's radiation law for any value of T.

iii. The population densities N_1 and N_2 at the lower and upper energy levels respectively, are constant in time and are distributed according to Boltzmann law in the energy levels.

The above conditions indicate that ~~rate of change of atoms at the level E_2 must be equal to zero~~. the number of transitions from E_2 to E_1 must be equal to the transitions from E_1 to E_2.

so, we have, $N_{12} = N_{21}$

$$\therefore \quad N_1 P_{12} = N_2 P_{21}$$

$$\therefore \quad N_1 B_{12} Q = N_2 [A_{21} + B_{21} Q]$$

$$\therefore \quad N_1 B_{12} Q = N_2 A_{21} + N_2 B_{21} Q$$

$$\therefore \quad N_2 A_{21} = Q (N_2 B_{21} - N_1 B_{12})$$

$$\therefore \quad Q = \frac{N_2 A_{21}}{N_2 B_{21} - N_1 B_{12}}$$

$$\therefore \quad Q = \frac{A_{21}/B_{21}}{\left[\frac{N_1 B_{12}}{N_2 B_{21}} - 1\right]}$$

According to Boltzmann distribution law, no. of ~~items~~ atoms N_1 and N_2 in energy states E_1 and E_2 in thermal equilibrium at temperature T are given,

$$N_1 = N_0 e^{-(E_1/K_B T)} \quad \text{and} \quad N_2 = N_0 e^{-(E_2/K_B T)}$$

where, N_0 is the total no. of atoms and ~~K~~ K_B is boltzmann's constant.

$$\frac{N_1}{N_2} = e^{(E_2/k_BT) - (E_1/k_BT)} = e^{(E_2 - E_1/k_BT)}$$

$$\therefore \frac{N_1}{N_2} = e^{(h\nu/k_BT)} \quad \ldots\ E_2 - E_1 = h\nu$$

- energy of each photon of radiation

from (1),

$$Q = \frac{(A_{21}/B_{21})}{\left[\frac{B_{12}}{B_{21}} e^{(h\nu/k_BT)} - 1\right]} \quad \text{— (2)}$$

according to planck's radiation formula,

$$Q = \frac{8\pi h\nu^3}{c^3} \frac{1}{\left[e^{(h\nu/k_BT)} - 1\right]} \quad \text{— (3)}$$

comparing equation (2) and (3),

$$\frac{B_{12}}{B_{21}} = 1 \Rightarrow B_{12} = B_{21}$$ - answer

also, $$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$$ - answer

These equations present the relation between Einstein's coeffecients.