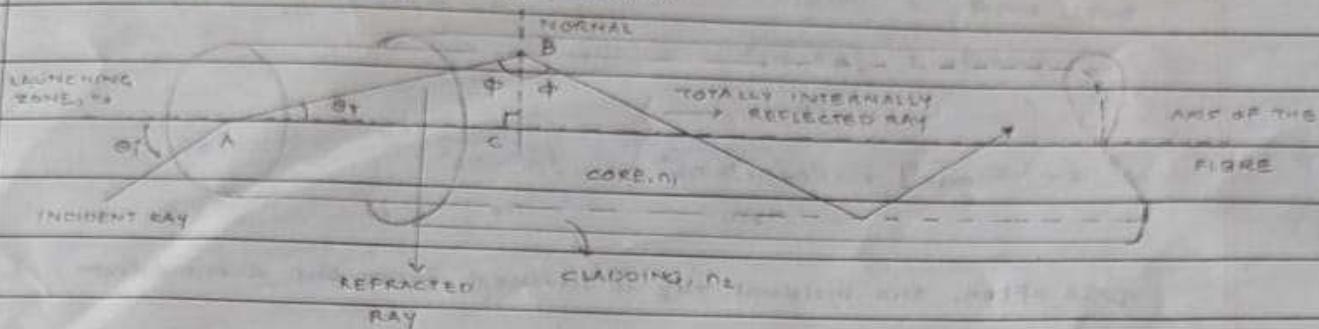


6. Relation for numerical aperture / acceptance angle. (optical fibres)
- consider a step index optical fibre into which light is launched at one end, as shown in figure. Let the refractive index of the core be n_1 , and of the cladding be n_2 ($n_1 > n_2$).



Let n_0 be the refractive index of the medium from which the light is launched into the fibre. Assume that a light ray enters the fibre at an angle θ_i to the axis of the fibre. The ray refracts at an angle θ_r and strikes the core-cladding interface at an angle ϕ . If ϕ is greater than critical angle ϕ_c , the ray undergoes total internal reflection at the interface, since $n_1 > n_2$. As long as the angle ϕ is greater than ϕ_c , the light will stay within the fibre.

Applying Snell's law to the launching face of the fibre, we get,

$$\frac{\sin \theta_i}{\sin \theta_r} = n_1$$

$$\frac{\sin \theta_r}{n_0} = n_0$$

If θ_i is increased beyond a limit, ϕ will drop below the critical value ϕ_c and the ray escapes from the sidewalls of the fibre.

The largest value of θ_i occurs when $\theta_i \times \phi = \phi_c$.

From $\triangle ABC$, it is seen that,

$$\sin \theta_r = \sin(90^\circ - \phi) = \cos \phi$$

$$\therefore \frac{\sin \theta_i}{n_0} = \frac{n_1 \cos \phi}{n_0}$$

when $\phi = \phi_c$.

$$\sin[\theta_{\text{imax}}] = \frac{n_1 \cos \phi_c}{n_0}$$

$$\text{But, } \sin \phi_c = \frac{n_2}{n_1}$$

$$\cos \phi_c = \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$$

$$\therefore \sin[\theta_{\text{imax}}] = \frac{n_1 \sqrt{n_1^2 - n_2^2}}{n_0 n_1} = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

quite often, the incident ray is launched from the air medium, for which, $n_0 = 1$.

designating, $\theta_{\text{imax}} = \theta_o$.

$$\therefore \sin \theta_o = \sqrt{n_1^2 - n_2^2} = NA$$

This is the expression to find numerical aperture, defined as the sine of the acceptance angle.

$$\therefore \theta_o = \sin^{-1} [\sqrt{n_1^2 - n_2^2}] \quad \text{— acceptance angle}$$

defined as, maximum angle that a light ray can have relative to the axis of the fibre and propagate down the fibre.