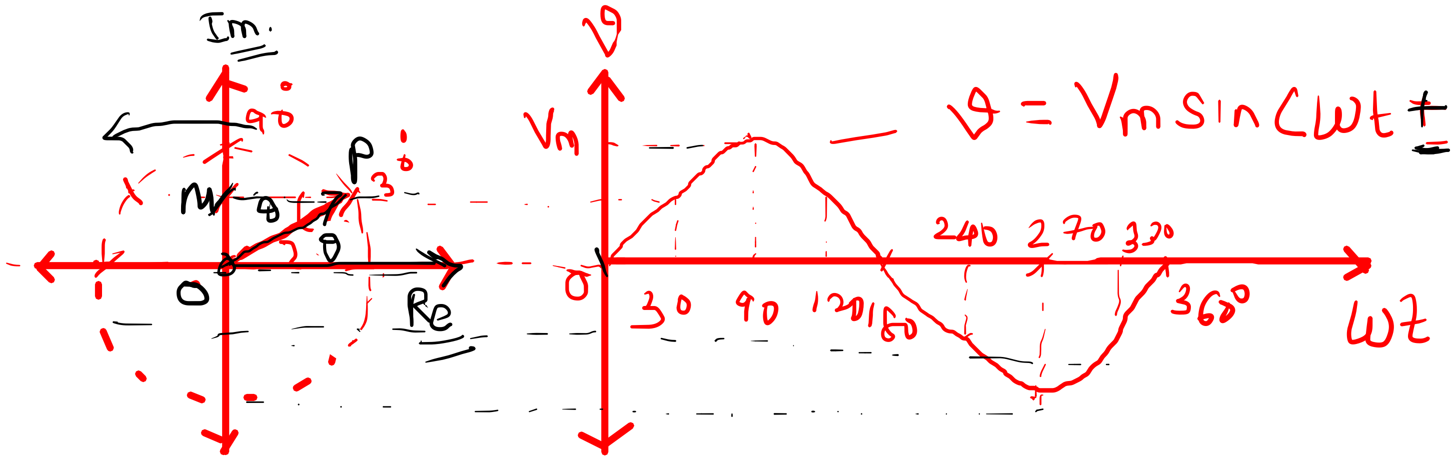
Phasor Representation of alternating quantities

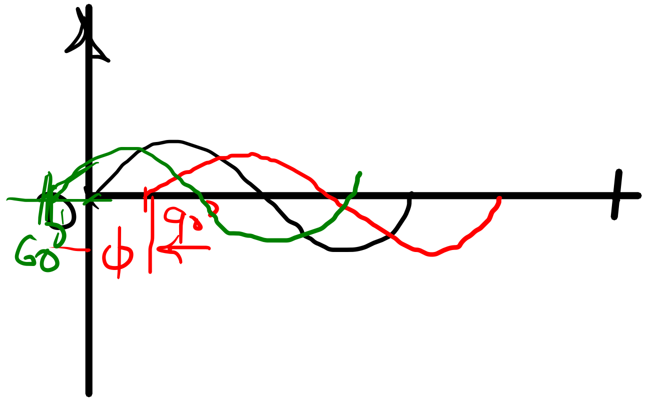


$$v = V_m \sin(\omega t \pm \phi)$$

$$v = V_m \sin\theta = V_m \sin \omega t$$

$$\sin\theta = \frac{OM}{OP} \qquad OM = OP \sin\theta$$

$$v = V_m \sin(\omega t \pm \phi)$$



Lagging ← $v_2 = V_m \sin(\omega t - 90^\circ)$

Leading $v_3 = V_m \sin(\omega t + 60^\circ)$

Mathematical representation of phasor

① Rectangular form

$$\bar{v} = \underline{x} \underline{\pm} j \underline{y} \qquad |v| = \sqrt{x^2+y^2}$$

Phase angle $\phi = \tan^{-1}\left(\frac{y}{x}\right)$

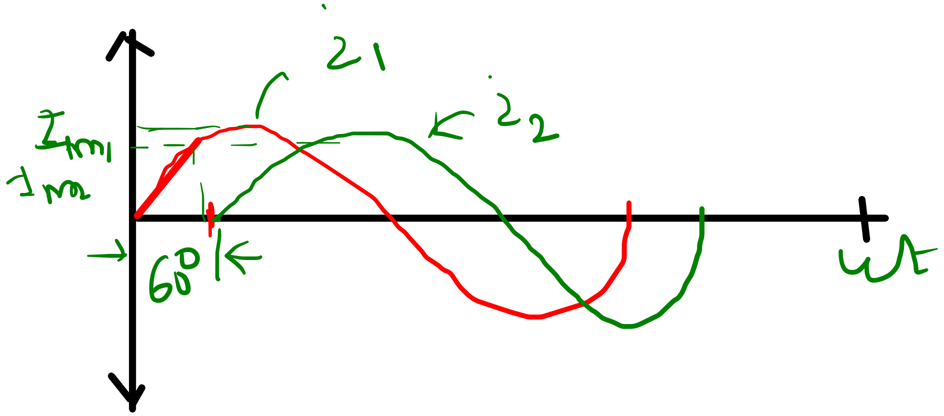
② Triangular

$$v = V_m[\cos\phi \pm j\sin\phi]$$

③ Exponential $\quad v = V e^{\pm j\phi}$

④ polar form $\quad v = |v| \angle \phi$

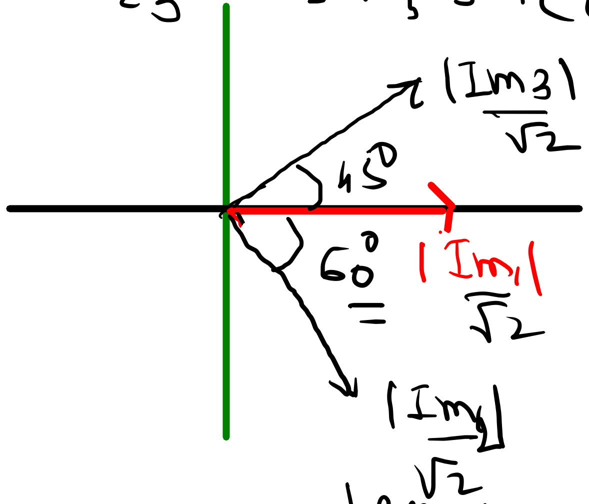
(1)



$i_1 = I_{m1}\sin(\omega t) = |I_{m1}| \angle 0^\circ$

$i_2 = I_{m2}\sin(\omega t - 60^\circ) = |I_{m2}| \angle -60^\circ$

$i_3 = I_{m3}\sin(\omega t + 45^\circ) = |I_{m1}| \angle 45^\circ$



✓ I_{m2} is lagging I_{m1} by 60°

✓ I_{m1} is leading I_{m2} by 60°

Length of phasor is Rms value of alternating quantity

1. Two sinusiodal currents are given as

$$i_1 = 10\sqrt{2}\sin\omega t \quad \& \quad i_2 = 20\sqrt{2}\sin(\omega t + 90^\circ)$$

Find sum of the currents

$$\Rightarrow \quad \bar{i} = \bar{i}_1 + \bar{i}_2$$

$$= \left(\frac{10\sqrt{2}}{\sqrt{2}}\right) \angle 0^\circ + \frac{20\sqrt{2}}{\sqrt{2}} \angle 90^\circ$$

$$= 10\angle 0^\circ + 20\angle 90^\circ$$

$$= 10[\cos 0 + j\sin 0) + 20[\cos 90 + j\sin 90^\circ]$$

$$= 10[1 + 0] + 20[0 + j\sin 90]$$

$$\bar{i} = (10 + j20\sin 90^\circ)$$

1. Two sinusiodal currents are given as

$$i_1 = 10\sqrt{2}\sin\omega t \quad \& \quad i_2 = 20\sqrt{2}\sin(\omega t + 90^\circ)$$

Find sum of the currents

$$\Rightarrow \quad \bar{i} = \bar{i}_1 + \bar{i}_2$$

$$= \left(\frac{10\sqrt{2}}{\sqrt{2}}\right)\angle 0^\circ + \frac{20\sqrt{2}}{\sqrt{2}}\angle 90^\circ$$

$$= 10\angle 0^\circ + 20\angle 90^\circ$$

$$= 10[\cos 0 + j\sin 0) + 20[\cos 90 + j\sin 90^\circ]$$

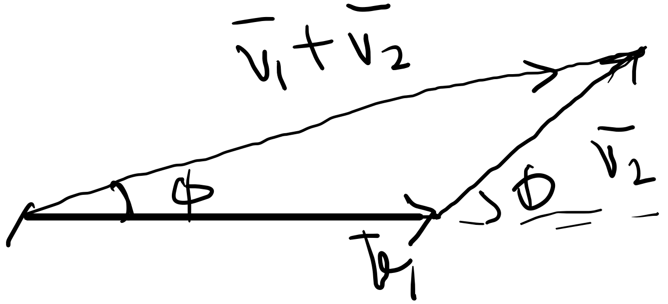
$$= 10[1 + 0] + 20[0 + j\sin 90]$$

$$\bar{i} = (10 + j20\sin 90^\circ)$$

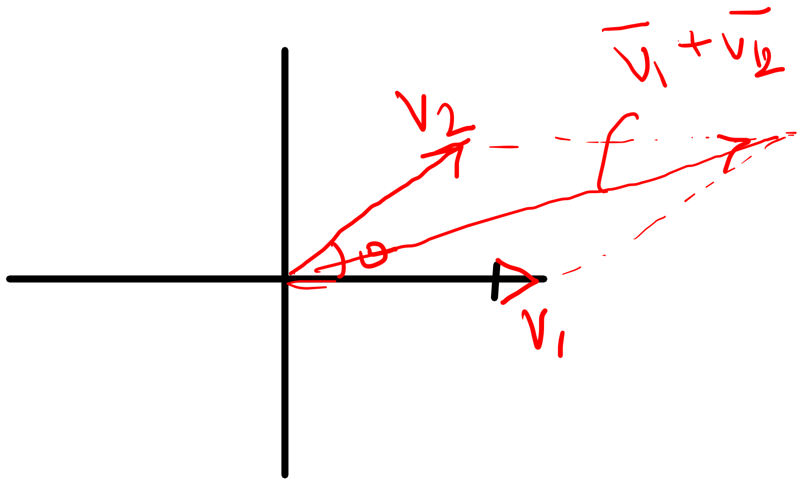
⇒ Resultant of phasors:

⇒ Law of triangle; If two vectors acting at a point are represented in magnitude and direction by two adjacent sides of the triangle taken in order then closing side of triangle taken in reverse order represents the resultant

$v_1 = V_m \sin\omega t$ & $v_2 = V_m \sin(\omega t + \theta)$

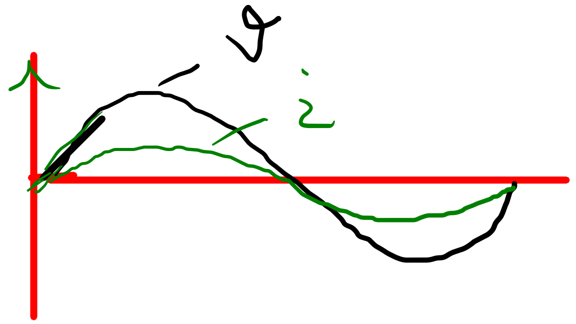


⇒ Law of parallelogram

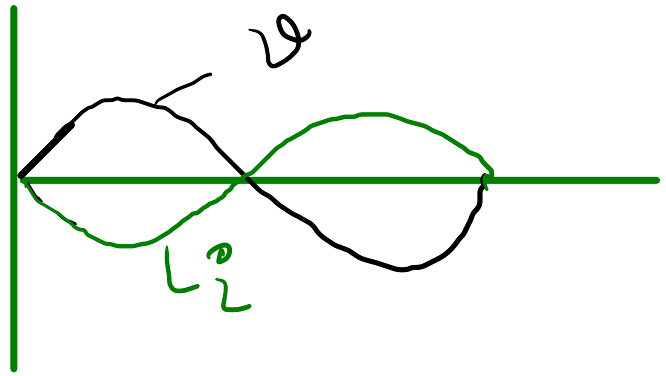


✓ Inphase

✓ Outof phase

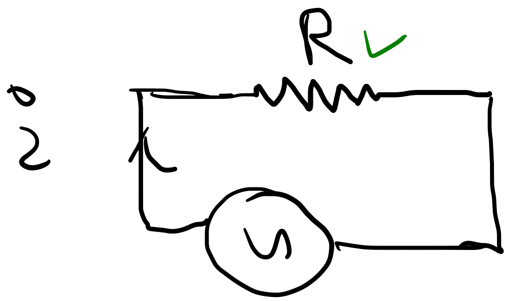


v & i are in phase



v & i are out of phase
phase diff of 180°.

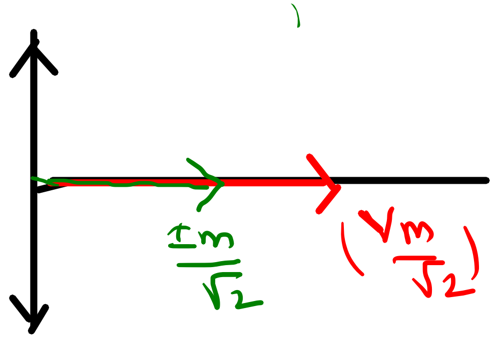
Pure resistor with AC input



$v = V_m \sin \omega t$

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

Phasor Diagram



Voltage & Current are inphase

<u>Impedance</u> $= \frac{v}{i} = \frac{V_m \sin \omega t}{I_m \sin \omega t} = \frac{V_m}{I_m} = R$

Power factor $= \cos \phi = \cos(0) = 1$

Instanteneous Power

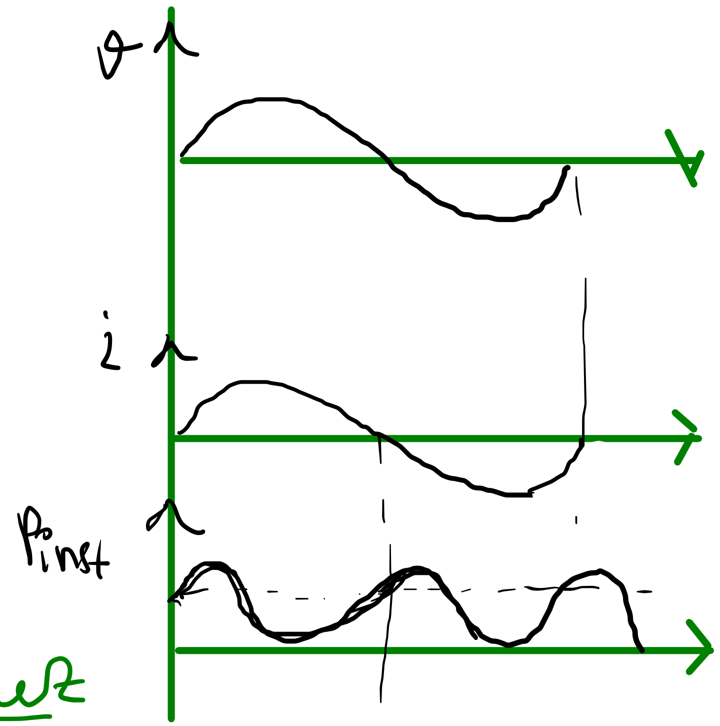
Instantaneous power $= \underline{v} \times \underline{i}$

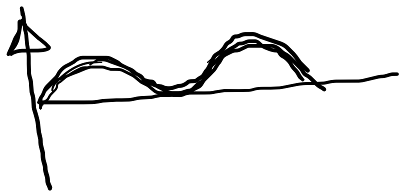
$= V_m \sin\omega t \cdot I_m \sin\omega t$

Instantaneous power $= V_m I_m \sin^2\omega t$

$= V_m I_m \left[\frac{1 - \cos 2\omega t}{2} \right]$

$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$





Instanteneous Power

Instantaneous power $= \underline{v} \times \underline{i}$

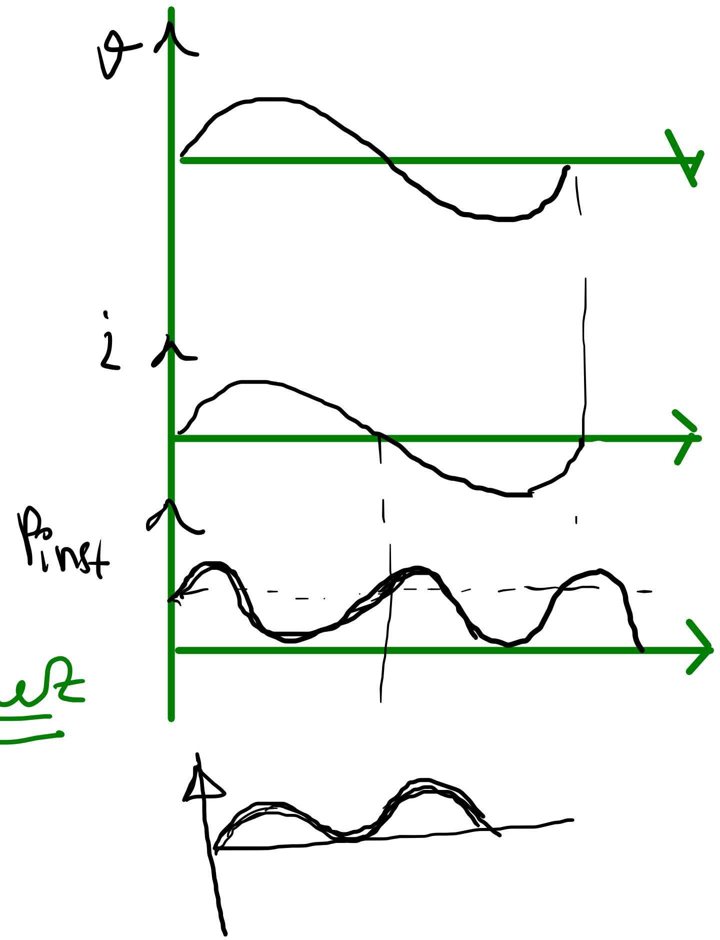
$= V_m \sin \underline{\omega t} \cdot I_m \sin \underline{\omega t}$

Instantaneous power $= \underline{\underline{V_m I_m \sin^2 \omega t}}$

$= V_m I_m \left[\frac{1 - \cos 2\omega t}{2} \right]$

$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\underline{\omega t}$

Waveforms



Average power consumed in resistor

$$\text{Average power} = \frac{1}{2\pi}\int_0^{2\pi} P_{inst}\, d\omega t$$

$$= \frac{1}{2\pi}\int_0^{2\pi} V_m I_m \sin^2\omega t\, d\omega t$$

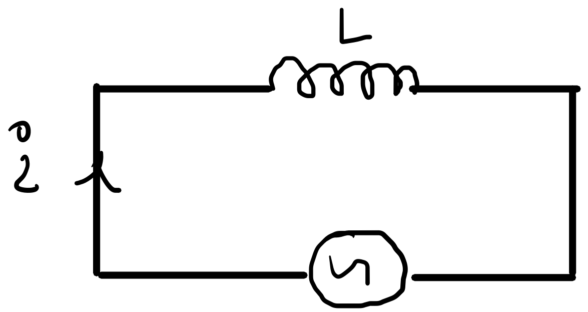
$$= \frac{V_m I_m}{2\pi}\int_0^{2\pi} \frac{1-\cos 2\omega t}{2}\, d\omega t$$

$$= \frac{V_m I_m}{2\pi}\,\frac{1}{2}\left[\underset{=}{\omega t} - \frac{\sin 2\omega t}{2}\right]_0^{2\pi}$$

$$= \frac{V_m I_m}{\cancel{2\pi}\cdot 2}\left[\cancel{2\pi} - 0 - 0 + 0\right]$$

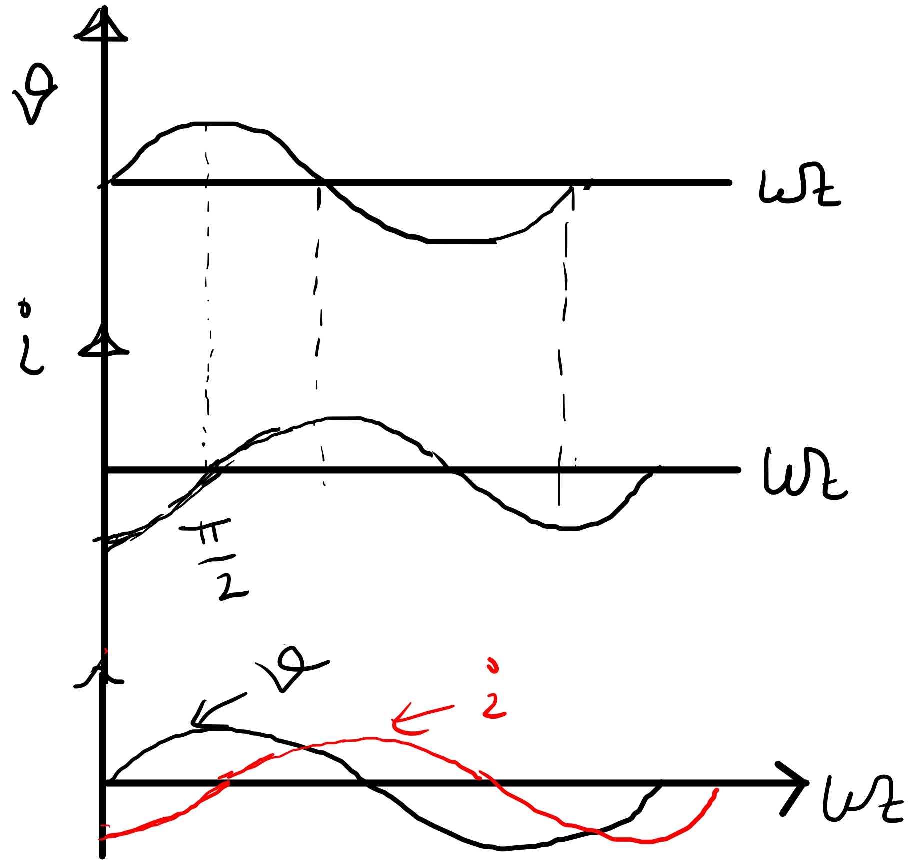
$$= \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}}\cdot\frac{I_m}{\sqrt{2}} = V_{rms}\cdot I_{rms}$$

Pure inductor with AC input



$\vartheta = V_m \sin \omega t$

$$\vartheta = L \frac{di}{dt} \quad \text{OR} \quad i = \frac{1}{L}\int \vartheta \, dt$$



$$i = \frac{1}{L}\int V_m \sin \omega t \, dt$$

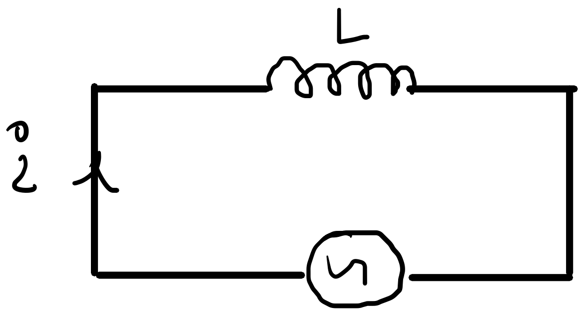
$$= \frac{V_m}{L}\left[-\frac{\cos \omega t}{\omega}\right]$$

$$= \frac{V_m}{\omega L}\left[\sin\left(\omega t - \frac{\pi}{2}\right)\right]$$

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

where $I_m = \dfrac{V_m}{\omega L}$

Pure inductor with AC input



$v = V_m \sin \omega t$

$$v = L\frac{di}{dt} \quad \text{OR} \quad i = \frac{1}{L}\int v\, dt$$

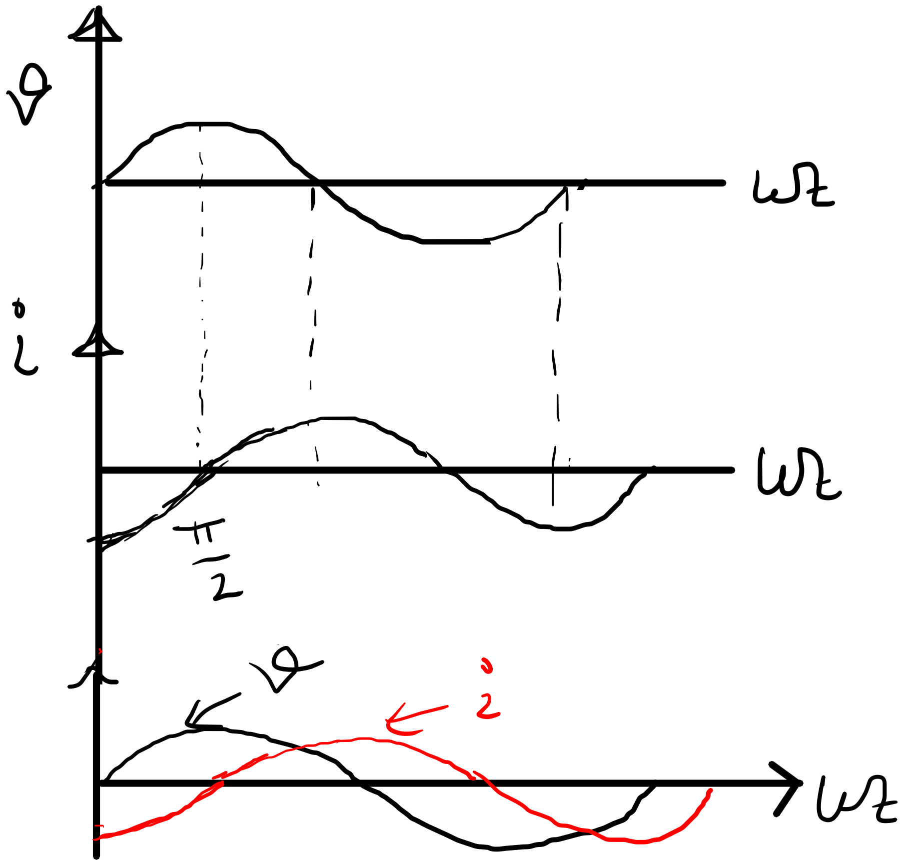
$$i = \frac{1}{L}\int V_m \sin \omega t \, dt$$

$$= \frac{V_m}{L}\left[-\frac{\cos \omega t}{\omega}\right]$$

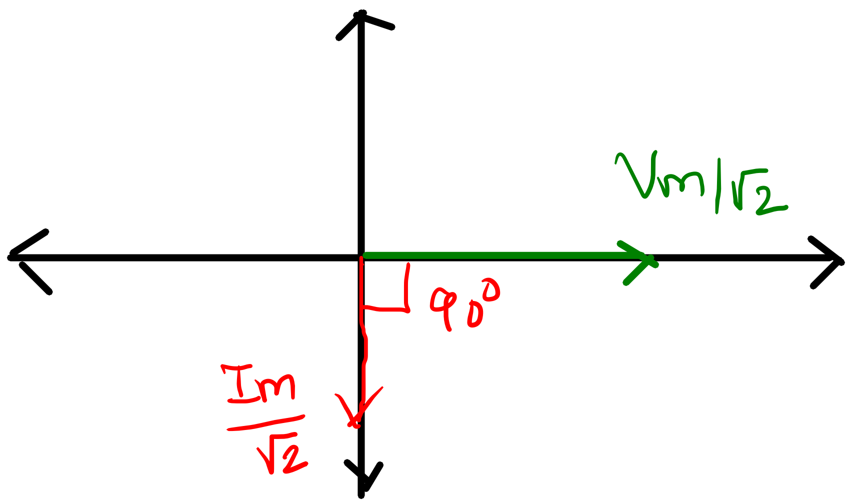
$$= \frac{V_m}{\omega L}\left[\sin\left(\omega t - \frac{\pi}{2}\right)\right]$$

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

where $I_m = \dfrac{V_m}{\omega L}$



Phasor diagram



Current lags the voltage by 90° or $\frac{\pi}{2}$

Voltage leads the current by 90° or $\frac{\pi}{2}$

Impedance

$$Z = \frac{v}{i} = \frac{V_m}{I_m} = \frac{V_m}{V_m/\omega L} = \omega L$$

$\omega L = X_L \Rightarrow$ Inductive Reactance $= 2\pi f \cdot L$

For DC input $f = 0$ so $X_L = 2\pi f L = 0\ \Omega$ so inductor acts as short circuit

Power factor: It is cosine of angle between the voltage and current phasors.

$$PF = \cos\phi = \cos(90^\circ) = 0$$

Instanteneous Power

$$P_{inst} = v \cdot i$$

$$= V_m \sin\omega t \times (-I_m \cos\omega t)$$

$$= -V_m I_m \sin\omega t \cdot \cos\omega t$$

$$\boxed{P_{inst} = -\frac{V_m I_m}{2} \sin 2\omega t}$$

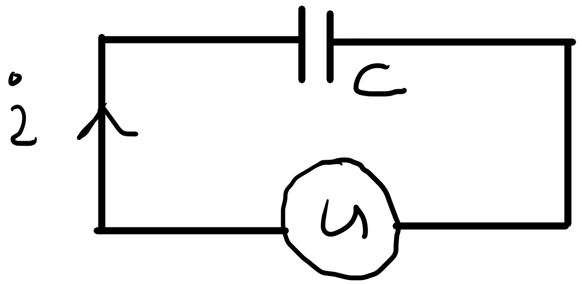
Average power

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} P_{inst}\, d\omega t = \frac{1}{2\pi}\left[\int_0^{2\pi} -\frac{V_m I_m}{2} \sin 2\omega t\, d\omega t\right.$$

$$= \frac{1}{2\pi}\left[-\frac{V_m I_m}{2}\left[-\frac{\cos 2\omega t}{2}\right]_0^{2\pi}\right. = \frac{V_m I_m}{8\pi}\left[\cos 4\pi - \cos 0\right] = 0$$

$P_{av} = 0$ Hence power consumed by pure inductor is zero.

Pure Capacitor with AC input



$v = V_m \sin \omega t$

$$i = C\frac{dv}{dt}$$

$$i = C \cdot \frac{d}{dt}[V_m \sin \omega t]$$

$$= V_m \cdot C\,[\cos \omega t \times \omega]$$

$$i = V_m \cdot \omega C\,[\cos \omega t]$$

$$i = I_m \cos \omega t$$

$$i = I_m \sin(\omega t + \frac{\pi}{2})$$

where $I_m = \dfrac{V_m}{1/\omega C}$

So $\dfrac{V_m}{I_m} = \dfrac{1}{\omega C}$

Impedance $Z = \dfrac{V_m}{I_m} = \dfrac{1}{\omega C} = X_C$

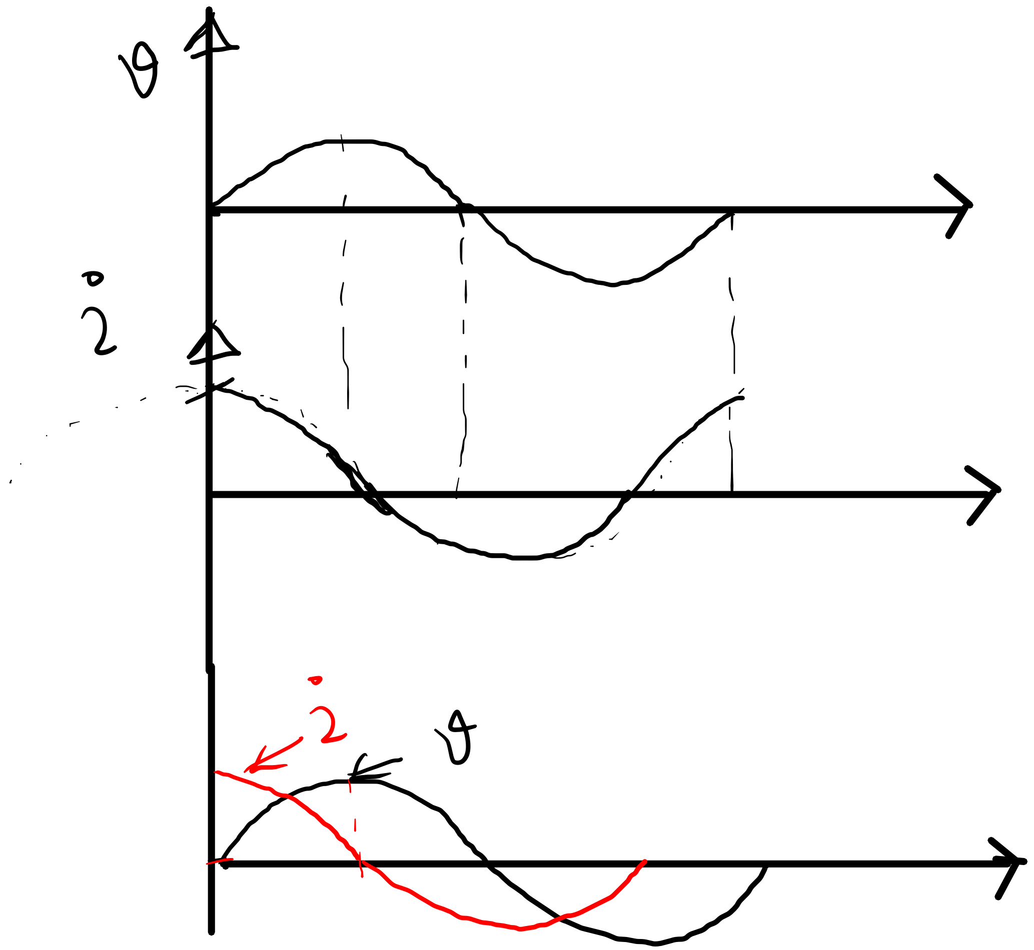
where $X_C = \dfrac{1}{\omega C}$ is Capacitive Reactance

$$X_C = \frac{1}{2\pi f C}$$

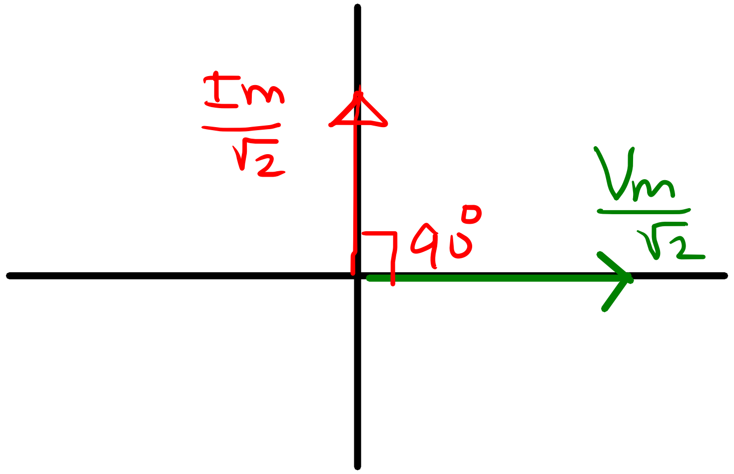
for DC input $f = 0$ so $X_C = \frac{1}{0} = \infty$

So for DC input capacitor acts Open Circuit.

Waveforms:



Phasor diagram



Power factor= $\cos\phi = \cos(90^\circ) = 0$

Instanteneous power

$$P_{inst} = v \cdot i$$

$$= V_m \sin\omega t \cdot I_m \cos\omega t$$

$$= V_m I_m \sin\omega t \cdot \cos\omega t$$

$$P_{inst} = \frac{V_m I_m}{2} \sin 2\omega t$$

$$\text{Average power} = P_{av} = \frac{1}{2\pi}\int_0^{2\pi} P_{inst} \cdot d\omega t$$

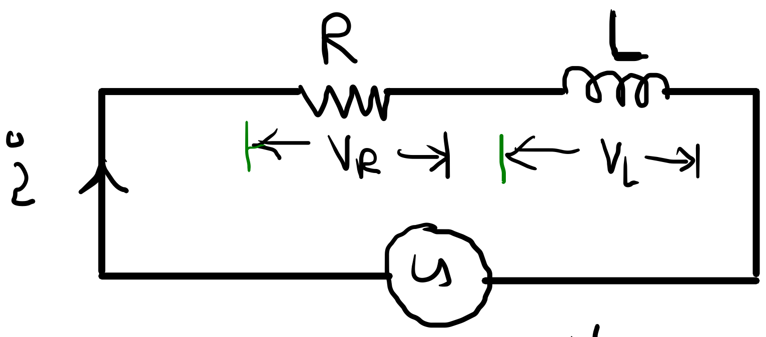
$$= \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\omega t \, d\omega t$$

$$= \frac{V_m I_m}{8\pi}\Big[-\cos 2\omega t\Big]_0^{2\pi}$$

$$= \frac{V_m I_m}{8\pi}\Big[-\cos 4\pi + \cos 0\Big] = 0$$

So power consumed by pure capacitor is zero.

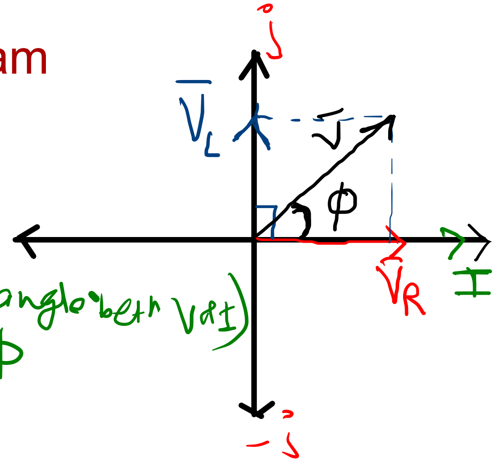
Series R-L circuit with AC input



$v = V_m \sin \omega t$

$$\bar{V} = \bar{V}_R + \bar{V}_L$$

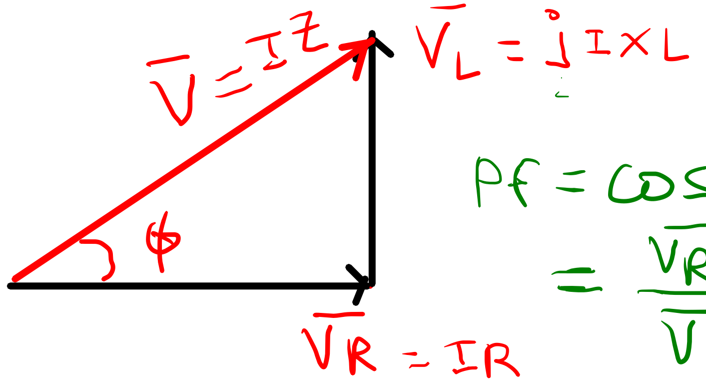
Phasor diagram



$PF = \cos\phi$

$= \cos$ (angle betn V & I)

Voltage triangle



$$PF = \cos\phi = \frac{\bar{V}_R}{\bar{V}}$$

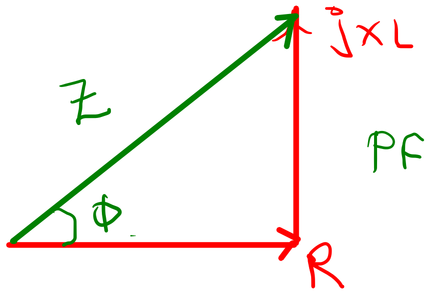
Impedance triangle

$$Z = R + jX_L$$

$$|Z| = \sqrt{R^2 + X_L^2}$$

$$\angle Z = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$\boxed{\phi = \angle Z = \tan^{-1}\left(\frac{\omega L}{R}\right)}$$

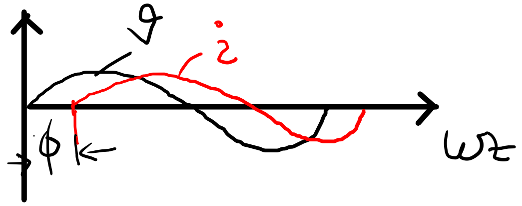


$PF = \cos\phi = \frac{R}{Z}$ (lagging PF)

Waveforms

$$v = V_m \sin \omega t$$

$$i = I_m \sin(\omega t - \phi)$$



Instanteneous Power

$$P_{inst} = v \times i = V_m \sin \omega t \times I_m \sin(\omega t - \phi)$$

$$P_{inst} = \frac{V_m I_m}{2}\left[\cos(\omega t - \omega t + \phi) - \cos(\omega t + \omega t - \phi)\right]$$

$$P_{inst} = \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t - \phi)\right]$$

Average power

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t - \phi)\right] d\omega t$$

$$P_{av} = \frac{1}{2\pi}\left[\frac{V_m I_m}{2}\cos\phi\left[\omega t\right]_0^{2\pi} - \frac{V_m I_m}{2}\left[\frac{\sin(2\omega t - \phi)}{2}\right]_0^{2\pi}\right]$$

$$P_{av} = \frac{1}{2\pi}\left[\frac{V_m I_m}{2}\cos\phi\,(2\pi - 0) - \frac{V_m I_m}{4}\left[\sin(4\pi - \phi) - \sin(0 - \phi)\right]\right]$$

$$P_{av} = \frac{1}{\cancel{2\pi}}\left[\frac{V_m I_m}{2}\cos\phi \times \cancel{2\pi} - \cancel{\frac{V_m I_m}{4}\left(-\sin\phi + \sin\phi\right)}_{0}\right]$$

$$P_{av} = \frac{V_m I_m}{2}\cos\phi$$

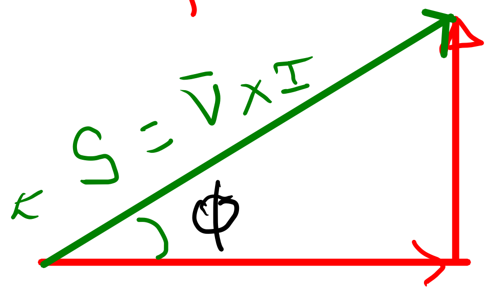
$$\text{So} \Rightarrow P_{av} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}\cos\phi = V_{rms}\cdot I_{rms}\cos\phi$$

Active power:

$P_{av} = P_{active} = VI\cos\phi$ & unit is watts

Reactive power : circulating power

$Q = VI\sin\phi$ & unit VAR (Volt.Ampere Reactive)



$\bar{V}_L \times I = Q = VI\sin\phi$

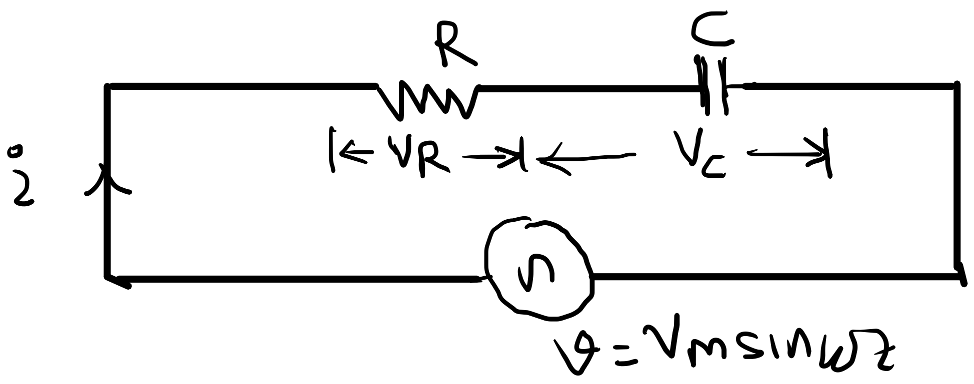
← (power triangle)

$\bar{V}_R \times I = P = VI\cos\phi$

Apparent Power (S) $S^2 = P^2 + Q^2$

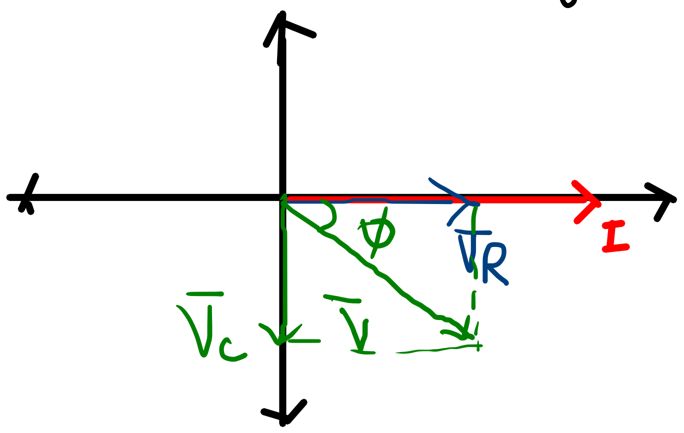
$S = V \times I$ & unit is VA (Volt-Ampere)

Series RC circuit with AC input



$$\bar{V} = \bar{V}_R + \bar{V}_C$$

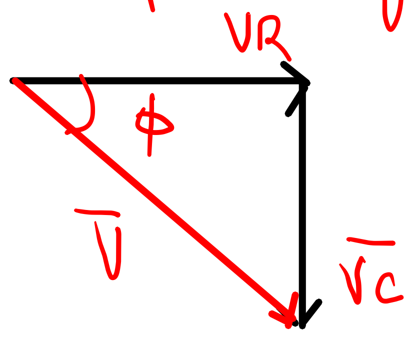
⇒ phasor Diagram



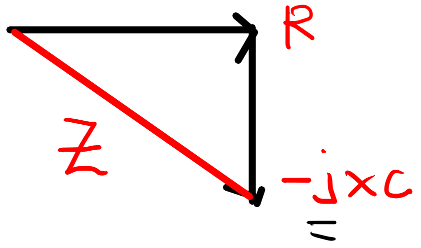
$PF = \cos \phi$ (leading PF)

leading pF because current leads the voltage.

⇒ Voltage triangle



⇒ Impedance triangle



$$\bar{V} = \bar{V}_R + \bar{V}_C$$
$$= \bar{I} \cdot R - jX_C \cdot \bar{I}$$
$$\frac{\bar{V}}{\bar{I}} = R - jX_C = Z$$

Instanteneous Power :

$$P_{inst} = v \times i$$

$$= V_m \sin\omega t \times I_m \sin(\omega t + \phi)$$

$$P_{inst} = V_m I_m \sin\omega t \cdot \sin(\omega t + \phi)$$

$$P_{inst} = \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t + \phi)\right]$$

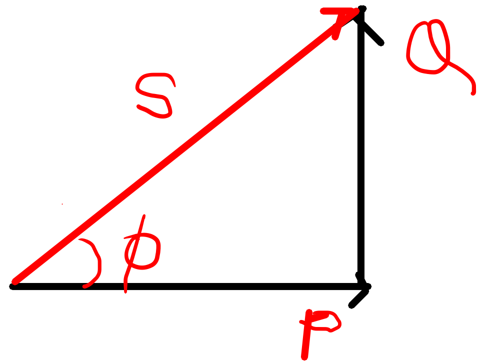
Average Power

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} P_{inst}\, d\omega t = \frac{V_m I_m}{2}\cos\phi = V_{rms} I_{rms} \cos\phi$$

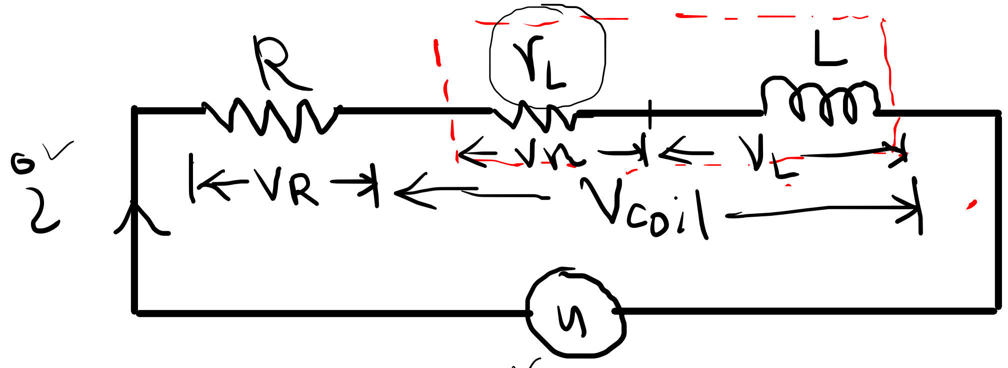
$$P_{av} = P_{active} = VI\cos\phi \text{ (watts)}$$

Reactive power $\Rightarrow$ $Q = VI\sin\phi$ (VAR)

Apparent Power $\Rightarrow$ $S = VI = \sqrt{P^2 + Q^2}$ (VA)



R-L series circuit with AC input where inductor is Impure

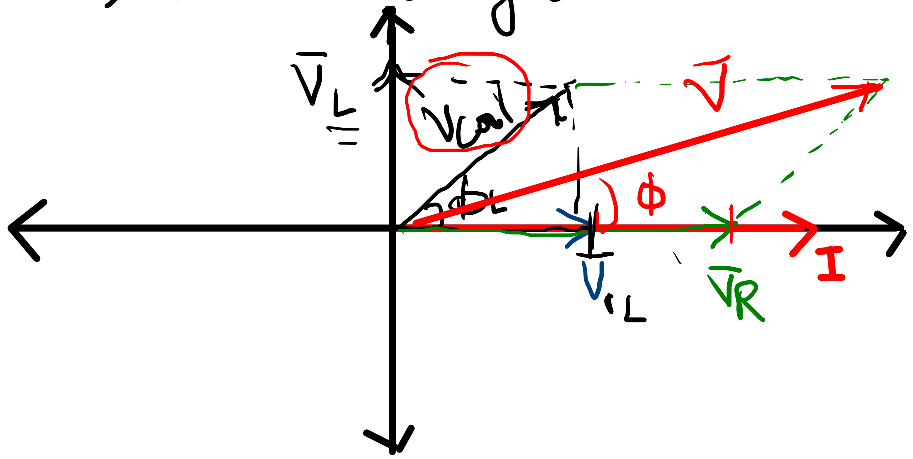


$v = V_m \sin \omega t$

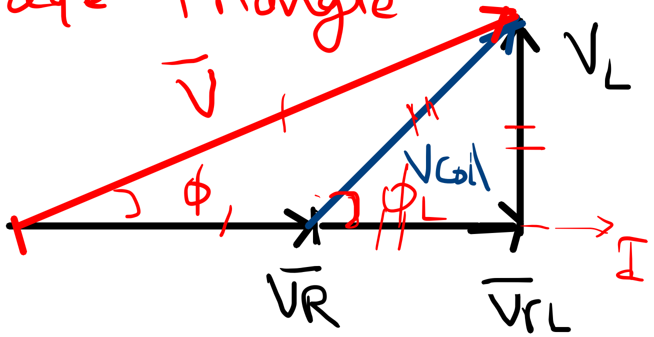
$\bar{V} = \bar{V}_R + \bar{V}_{coil}$ ---- (1)

$\bar{V}_{coil} = \bar{V}_{rL} + \bar{V}_L$

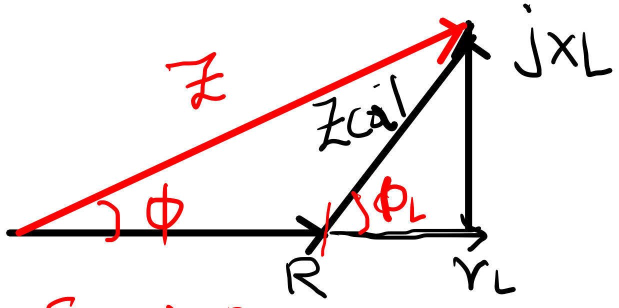
⇒ phasor diagram.



⇒ Voltage triangle



⇒ Impedance triangle



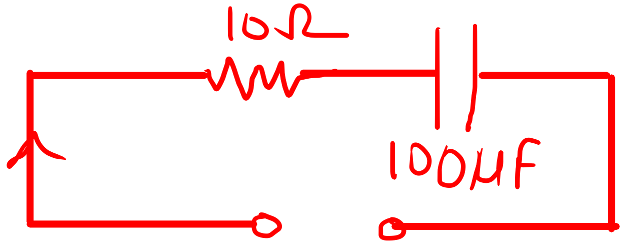
$Z = R + Z_{coil}$

$Z = (R + r_L) + jX_L$

$P = VI \cos\phi$ $\quad Q = VI \sin\phi$ $\quad S = VI$

Example ① ⇒ A capacitor which has an internal resistance of 10Ω & capacitance value of $100\mu F$ is connected to ac voltage $v(t) = 100\sin(314t)$. Calculate current flowing through the circuit & construct voltage triangle.

⇒



$v(t) = 100\sin(314t)$

$X_C = \frac{1}{2\pi f c} = \frac{1}{\omega C}$

$\omega = 314 \quad \therefore X_C = \frac{1}{314 \times 100 \times 10^{-6}}$

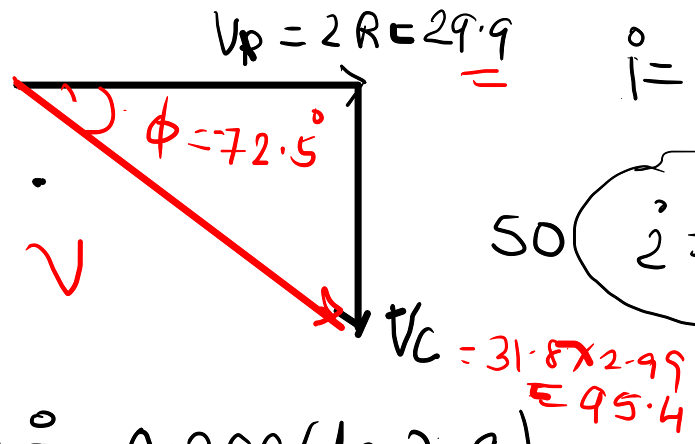
$X_C = 31.8\,\Omega$

$Z = R - jX_C$

$Z = 10 - jX_C$

$Z = (10 - j31.8)$

$\boxed{Z = 33.38\angle -72.5}$



$i = \frac{V}{Z} = \frac{100}{(10 - j31.8)} = \frac{100}{33.38\angle -72.5}$

SO $i = 2.99\angle 72.5^\circ \leftarrow [0.899 + j2.81]$

$|i| = \sqrt{(0.899)^2 + (2.81)^2}$

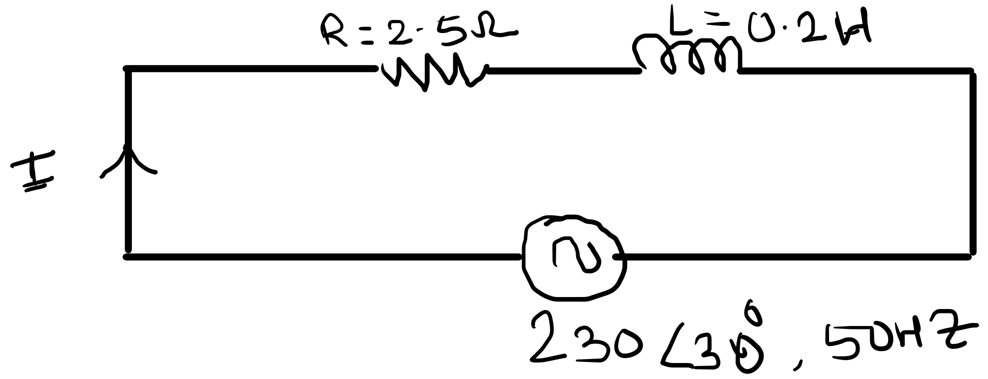
$\angle i = \tan^{-1}\left(\frac{2.81}{0.899}\right)$

$PF = \cos\phi = \cos 72.5^\circ = 0.299$ (leading)

$|\bar{V}| = |\bar{V_R} + \bar{V_C}| = \sqrt{(29.9)^2 + (95.4)^2} \simeq 100V$

3). A R-L series circuit is connected across $230\angle 30^\circ$, 50Hz supply. The value of R is 2.5 ohms and inductor L=0.2 H. Find current flowing through the circuit and power factor of the circuit.

$\Rightarrow$



$$I = \frac{V}{Z} = \frac{230\angle 30^\circ}{Z}$$

$$Z = R + jX_L$$

Here $X_L = \omega L = 2\pi f L$

$$X_L = 2 \times \pi \times 50 \times 0.2$$

$$X_L = 62.8\,\Omega$$

$$Z = (2.5 + j62.8)$$

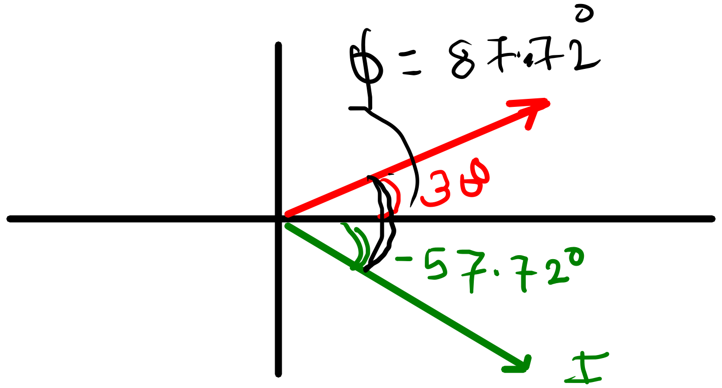
$$Z = 62.87\angle 87.72$$

$$\phi = 87.72$$

$$I = \frac{230\angle 30^\circ}{62.87\angle 87.72}$$

$$I = 3.65\angle -57.72^\circ$$

$$PF = \cos\phi = \cos 87.72 = 0.039 \text{(lagging)}$$

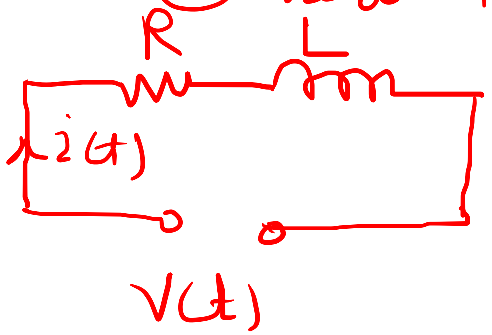


Example (2) In a series circuit containing a pure resistance and a pure inductor. The current & voltage are given as

$$i(t) = 5\sin(314t + \frac{2\pi}{3}) \quad \& \quad V(t) = 15\sin(314t + \frac{5\pi}{6})$$

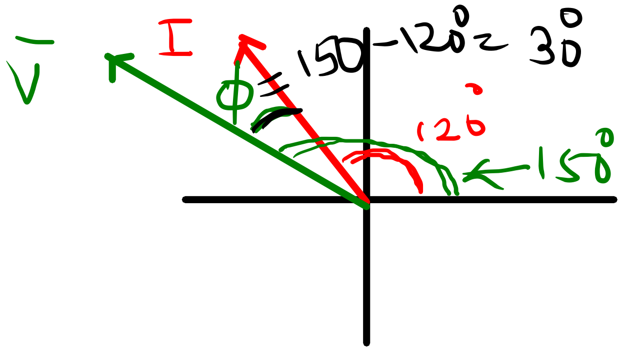
Find (i) Impedance of the circuit (ii) Value of resistance (iii) Value of inductor (iv) Active power (v) Power factor.

$\Rightarrow$



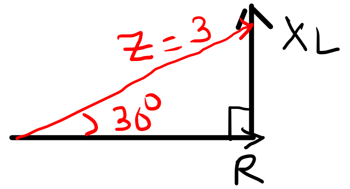
$$\bar{I} = \frac{5}{\sqrt{2}} \angle \frac{2\pi}{3} = \frac{5}{\sqrt{2}} \angle 120^\circ$$

$$\bar{V} = \frac{15}{\sqrt{2}} \angle \frac{5\pi}{6} = \frac{15}{\sqrt{2}} \angle 150$$



So $\phi = 150^\circ - 120^\circ = 30^\circ$

(i) $Z = \frac{V}{I} = \frac{15/\sqrt{2} \angle 150^\circ}{5/\sqrt{2} \angle 120^\circ} = 3\angle 30^\circ$



(ii) $R = Z\cos\phi$ | $X_L = Z\sin\phi$

$R = 3\cos 30^\circ$ | $X_L = 3\sin 30^\circ$

$R = 2.6\Omega$ | $X_L = 1.5\Omega$

$X_L = \omega L \quad \therefore L = \frac{X_L}{\omega} = \frac{1.5}{314} = 4.78\text{mH}$

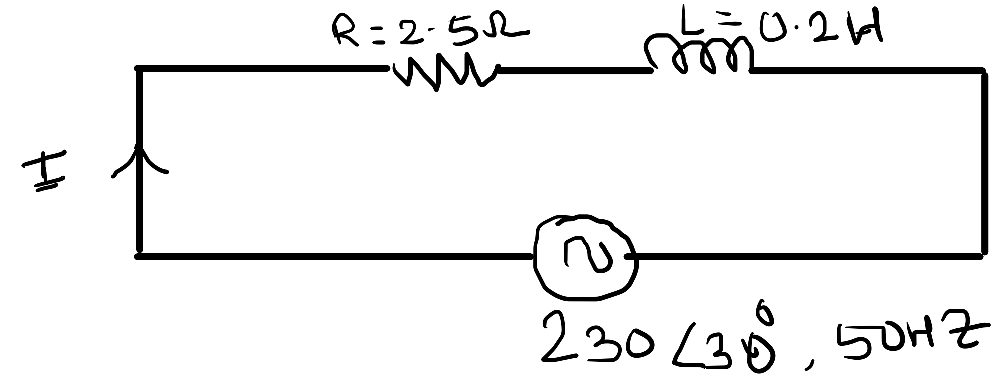
(v) Power factor $= \cos\phi = \cos 30^\circ = 0.866$ (lagging)

(iv) Active power $= VI\cos\phi = \frac{15}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \cos 30$

$$P = I\cancel{Z} \cdot I \frac{R}{\cancel{Z}} = I^2 R = 32.5\text{ W}$$

3). A R-L series circuit is connected across $230\angle 30^\circ$, 50Hz supply. The value of R is 2.5 ohms and inductor L=0.2 H. Find current flowing through the circuit and power factor of the circuit.

⇒



$$I = \frac{V}{Z} = \frac{230\angle 30^\circ}{Z}$$

$$Z = R + jX_L$$

Here $X_L = \omega L = 2\pi f L$

$$X_L = 2\times\pi\times 50\times 0.2$$

$$X_L = 62.8\,\Omega$$

$$Z = (2.5 + j62.8)$$

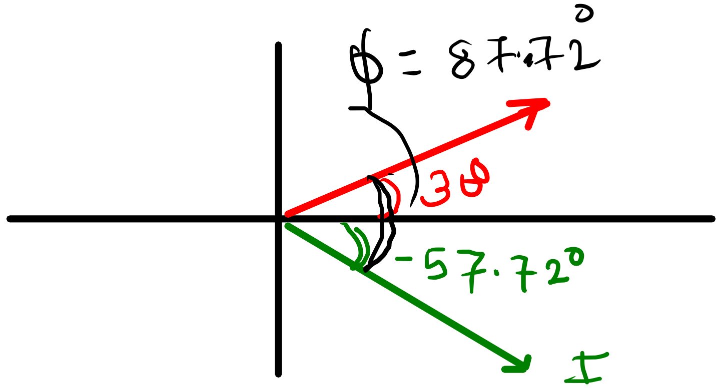
$$Z = 62.87\angle 87.72$$

$$\phi = 87.72$$

$$I = \frac{230\angle 30^\circ}{62.87\angle 87.72}$$

$$I = 3.65\angle -57.72^\circ$$

$$PF = \cos\phi = \cos 87.72 = 0.039 \text{ (lagging)}$$



4). Two elements series circuit is connected across ac source e=200$\sqrt{2}$ sin(341t + 20°) . The current flowing in the circuit is found to be 10$\sqrt{2}$ cos(314t-25°). Determine the paramerters of the circuit.

$\Rightarrow \quad E = \dfrac{200\sqrt{2}}{\sqrt{2}} \angle 20^{\circ}$

$i = 10\sqrt{2}\cos(314t - 25^{\circ}) = 10\sqrt{2}\sin(314t - 25 + 90^{\circ})$

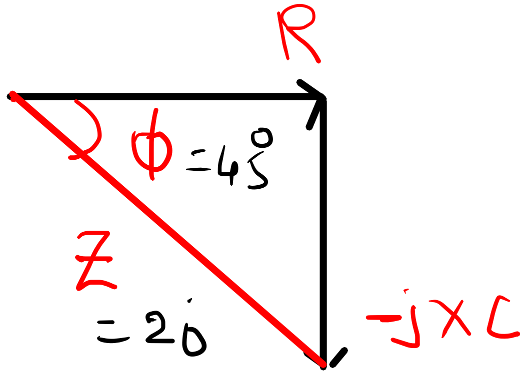
$= 10\sqrt{2}\sin(314t + 65^{\circ})$

$I = \dfrac{10\sqrt{2}}{\sqrt{2}} \angle 65^{\circ}$

$Z = \dfrac{E}{I} = \dfrac{200\angle 20^{\circ}}{10\angle 65^{\circ}}$

$Z = \underline{\underline{20}}\angle \underline{\underline{-45^{\circ}}} \qquad \phi = -45^{\circ}$

$PF = \cos\phi = \cos 45 =$



$\cos\phi = \dfrac{R}{Z}$

$\sin\phi = \dfrac{X_C}{Z}$

$\boxed{R = Z\cos\phi = 20\cos 45^{\circ} = 14.14\,\Omega}$

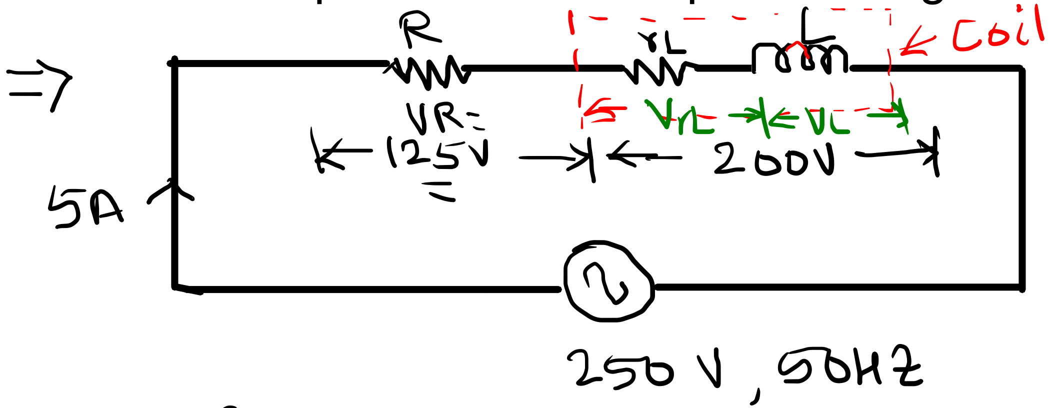
$X_C = Z\sin\phi = 20\sin 45^{\circ} = 14.14\,\Omega$

$X_C = \dfrac{1}{\omega C} \qquad C = \dfrac{1}{\omega \times X_C} = \dfrac{1}{314 \times 14.14}$

$\boxed{C = 225.4\,\mu F}$

5). A current of 5A flows through a non-inductive resistance in series with a choking coil when supplied at 250 V , 50Hz. If the voltage across the non-inductive resistance is 125 V and that of the coil 200 V. Calculate the impedance, reactance and resistance of the coil, power absorbed by the coil and total power. Draw the phasor diagram.

⇒



$r_L = ?$ $X_L = ?$, $Z_{coil} = ?$

$P_{coil} = ?$ $P_{total} = ?$

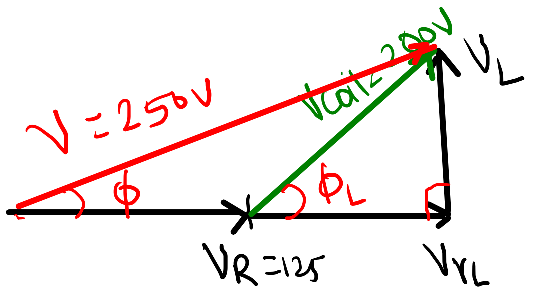
$250\, V_{rL} = (250)^2 - (200)^2 - (125)^2$

⇒ $Z_{coil} = \frac{200}{5} = 40\,\Omega$

$V_{rL} = 27.5V$ ∴ $r_L = \frac{V_{rL}}{5} = \frac{27.5}{5} = 5.5\,\Omega$

$\boxed{r_L = 5.5\,\Omega}$

⇒ Voltage triangle $\bar{V}_{coil} = \bar{V}_{rL} + \bar{V}_L$, $\bar{V} = \bar{V}_R + \bar{V}_{coil}$



$V_{rL}^2 + V_L^2 = (200)^2$ ---- (1)

$(125 + V_{rL})^2 + V_L^2 = (250)^2$ -- (1)

$125^2 + 2 \times 125 \times V_{rL} + V_{rL}^2 + V_L^2 = (250)^2$

$(125)^2 + 250 V_{rL} + (200)^2 = (250)^2$

$R = 125/5 = 25\,\Omega$

$(V_{coil})^2 = V_{rL}^2 + V_L^2$

$V_L^2 = (200)^2 - (27.5)^2$

$V_L = 198.1V$

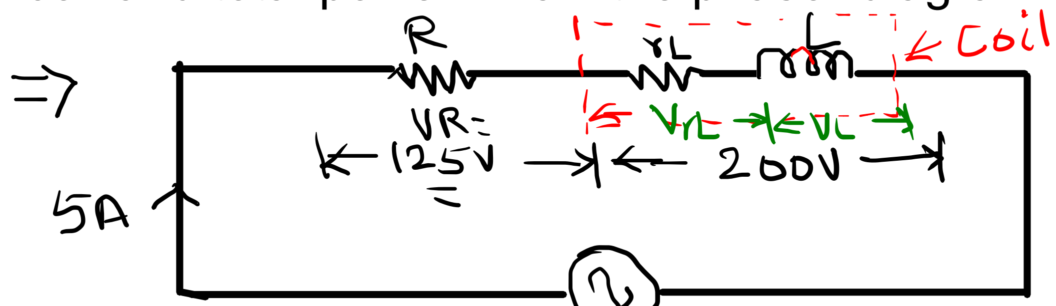
$X_L = \frac{V_L}{I} = \frac{198.1}{5} = 39.6\,\Omega$

$P_{coil} = V_{coil} \times I \cos\phi_L = I \times Z_{coil} \times I \frac{r_L}{Z_{coil}}$

$= I^2 r_L = 5^2 \times 5.5 = 137.5W$

$P_t = I^2 (R + r_L) = 25 \times (25 + 5.5) = 762.5W$

5). A current of 5A flows through a non-inductive resistance in series with a choking coil when supplied at 250 V , 50Hz. If the voltage across the non-inductive resistance is 125 V and that of the coil 200 V. Calculate the impedance, reactance and resistance of the coil, power absorbed by the coil and total power. Draw the phasor diagram.

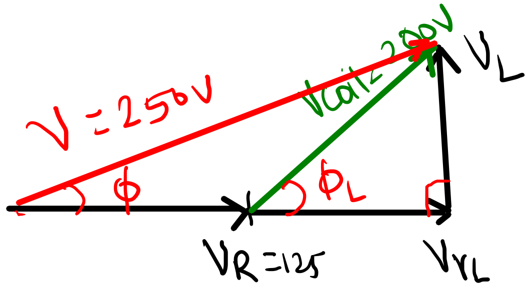


$r_L = ?$ $X_L = ?$, $Z_{coil} = ?$

$P_{coil} = ?$ $P_{total} = ?$

$\Rightarrow Z_{coil} = \frac{200}{5} = 40\,\Omega$

$\Rightarrow$ Voltage triangle $\bar{V}_{coil} = \bar{V}_{rL} + \bar{V}_L$, $\bar{V} = \bar{V}_R + \bar{V}_{coil}$



$V_{rL}^2 + V_L^2 = (200)^2$ ---- (1)

$(125 + V_{rL})^2 + V_L^2 = (250)^2$ -- (1)

$125^2 + 2 \times 125 \times V_{rL} + V_{rL}^2 + V_L^2 = (250)^2$

$(125)^2 + 250 V_{rL} + (200)^2 = (250)^2$

$R = 125/5 = 25\,\Omega$

$250 V_{rL} = (250)^2 - (200)^2 - (125)^2$

$V_{rL} = 27.5\,V$ $\therefore r_L = \frac{V_{rL}}{5} = \frac{27.5}{5} = 5.5\,\Omega$

$\boxed{r_L = 5.5\,\Omega}$

$(V_{coil})^2 = V_{rL}^2 + V_L^2$

$V_L^2 = (200)^2 - (27.5)^2$

$V_L = 198.1\,V$

$X_L = \frac{V_L}{I} = \frac{198.1}{5} = 39.6\,\Omega$

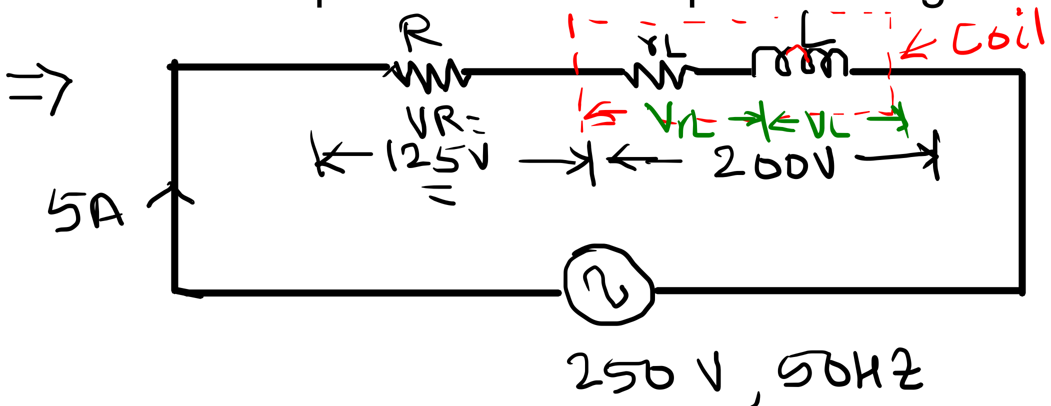
$P_{coil} = V_{coil} \times I \cos\phi_L = I \times Z_{coil} \times I \frac{r_L}{Z_{coil}}$

$= I^2 r_L = 5^2 \times 5.5 = 137.5\,W$

$P_t = I^2 (R + r_L) = 25 \times (25 + 5.5) = 762.5\,W$

5). A current of 5A flows through a non-inductive resistance in series with a choking coil when supplied at 250 V , 50Hz. If the voltage across the non-inductive resistance is 125 V and that of the coil 200 V. Calculate the impedance, reactance and resistance of the coil, power absorbed by the coil and total power. Draw the phasor diagram.

$\Rightarrow$

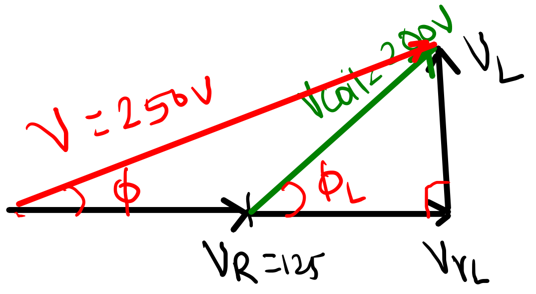


$r_L = ?$ $X_L = ?$, $Z_{coil} = ?$

$P_{coil} = ?$ $P_{total} = ?$

$\Rightarrow Z_{coil} = \frac{200}{5} = 40\,\Omega$

$\Rightarrow$ Voltage triangle $\bar{V}_{coil} = \bar{V}_{rL} + \bar{V}_L$, $\bar{V} = \bar{V}_R + \bar{V}_{coil}$



$V_{rL}^2 + V_L^2 = (200)^2$ ---- (1)

$(125 + V_{rL})^2 + V_L^2 = (250)^2$ -- (1)

$125^2 + 2 \times 125 \times V_{rL} + \underline{V_{rL}^2 + V_L^2} = (250)^2$

$(125^2) + 250 V_{rL} + (200)^2 = (250)^2$

$R = 125 / 5 = 25\,\Omega$

$250 V_{rL} = (250)^2 - (200)^2 - (125)^2$

$V_{rL} = 27.5V$ $\therefore r_L = \frac{V_{rL}}{5} = \frac{27.5}{5} = 5.5\,\Omega$

$\boxed{r_L = 5.5\,\Omega}$

$(V_{coil})^2 = V_{rL}^2 + V_L^2$

$V_L^2 = (200)^2 - (27.5)^2$

$V_L = 198.1V$

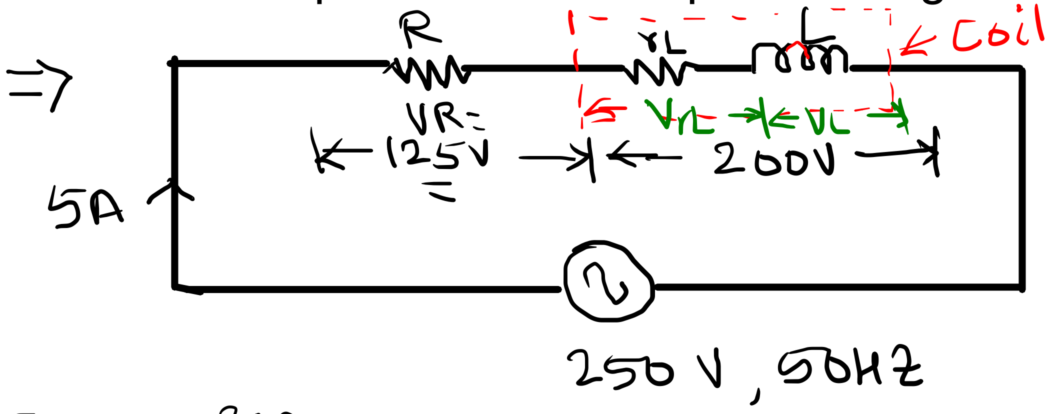
$X_L = \frac{V_L}{I} = \frac{198.1}{5} = 39.6\,\Omega$

$P_{coil} = V_{coil} \times I \cos\phi_L = I \times Z_{coil} \times I \frac{r_L}{Z_{coil}}$

$= I^2 r_L = 5^2 \times 5.5 = 137.5W$

$P_t = I^2 (R + r_L) = 25 \times (25 + 5.5) = 762.5W$

5). A current of 5A flows through a non-inductive resistance in series with a choking coil when supplied at 250 V , 50Hz. If the voltage across the non-inductive resistance is 125 V and that of the coil 200 V. Calculate the impedance, reactance and resistance of the coil, power absorbed by the coil and total power. Draw the phasor diagram.

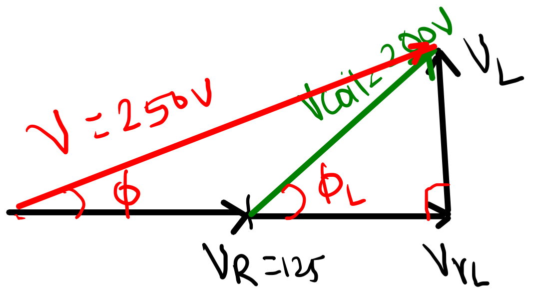


$r_L = ?$ $X_L = ?$, $Z_{coil} = ?$

$P_{coil} = ?$ $P_{total} = ?$

$\Rightarrow Z_{coil} = \frac{200}{5} = 40\,\Omega$ ✓

$\Rightarrow$ Voltage triangle $\bar{V}_{coil} = \bar{V}_{rL} + \bar{V}_L$, $\bar{V} = \bar{V}_R + \bar{V}_{coil}$



$V_{rL}^2 + V_L^2 = (200)^2$ ---- ①

$(125 + V_{rL})^2 + V_L^2 = (250)^2$ -- ①

$125^2 + 2 \times 125 \times V_{rL} + V_{rL}^2 + V_L^2 = (250)^2$

$(125)^2 + 250 V_{rL} + (200)^2 = (250)^2$

$R = 125/5 = 25\,\Omega$

$250\, V_{rL} = (250)^2 - (200)^2 - (125)^2$

$V_{rL} = 27.5\,V$ $\therefore r_L = \frac{V_{rL}}{5} = \frac{27.5}{5} = 5.5\,\Omega$

$\boxed{r_L = 5.5\,\Omega}$ ✓

$(V_{coil})^2 = V_{rL}^2 + V_L^2$

$V_L^2 = (200)^2 - (27.5)^2$

$V_L = 198.1\,V$

$X_L = \frac{V_L}{I} = \frac{198.1}{5} = 39.6\,\Omega$

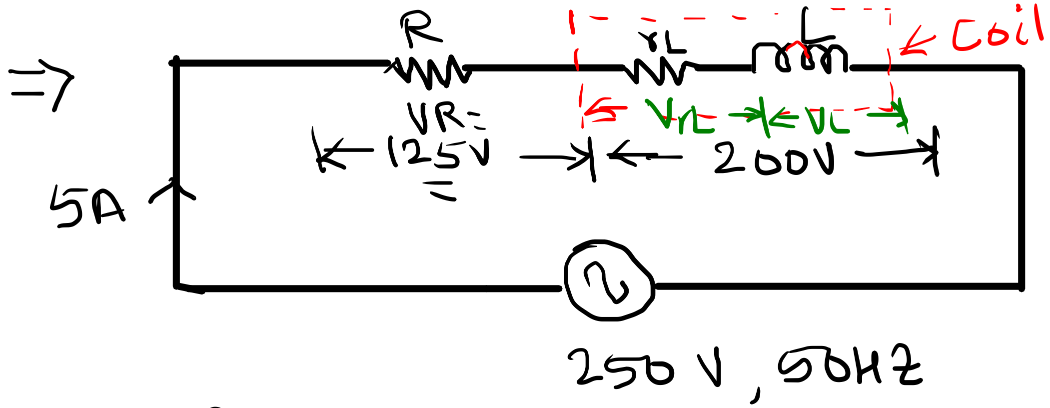
$P_{coil} = V_{coil} \times I \cos\phi_L = I \times Z_{coil} \times I \frac{r_L}{Z_{coil}}$

$= I^2 r_L = 5^2 \times 5.5 = 137.5\,W$

$P_t = I^2 (R + r_L) = 25 \times (25 + 5.5) = 762.5\,W$

5). A current of 5A flows through a non-inductive resistance in series with a choking coil when supplied at 250 V , 50Hz. If the voltage across the non-inductive resistance is 125 V and that of the coil 200 V. Calculate the impedance, reactance and resistance of the coil, power absorbed by the coil and total power. Draw the phasor diagram.

$\Rightarrow$

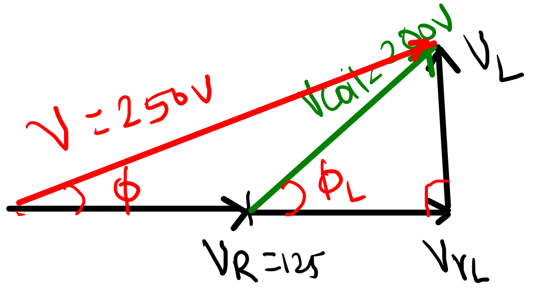


$r_L = ?$ $\quad X_L = ?$, $\quad Z_{coil} = ?$

$P_{coil} = ?$ $\quad P_{total} = ?$

$\Rightarrow Z_{coil} = \frac{200}{5} = 40\,\Omega$

$\Rightarrow$ Voltage triangle $\quad \bar{V}_{coil} = \bar{V}_{rL} + \bar{V}_L$, $\quad \bar{V} = \bar{V}_R + \bar{V}_{coil}$



$V_{rL}^2 + V_L^2 = (200)^2$ ---- ①

$(125 + V_{rL})^2 + V_L^2 = (250)^2$ -- ①

$125^2 + 2\times 125\times V_{rL} + V_{rL}^2 + V_L^2 = (250)^2$

$(125)^2 + 250V_{rL} + (200)^2 = (250)^2$

$R = 125/5 = 25\,\Omega$

$250\,V_{rL} = (250)^2 - (200)^2 - (125)^2$

$V_{rL} = 27.5V \quad \therefore r_L = \frac{V_{rL}}{5} = \frac{27.5}{5} = 5.5\,\Omega$

$\boxed{r_L = 5.5\,\Omega}$

$(V_{coil})^2 = V_{rL}^2 + V_L^2$

$V_L^2 = (200)^2 - (27.5)^2$

$V_L = 198.1V$

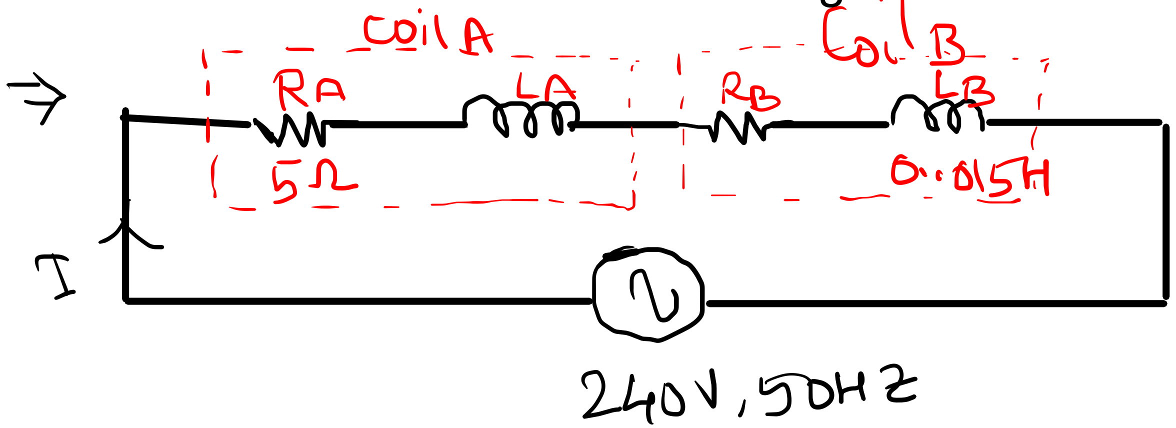
$X_L = \frac{V_L}{I} = \frac{198.1}{5} = 39.62\,\Omega$

$P_{coil} = V_{coil}\times I\cos\phi_L = I\times Z_{coil}\times I\,\frac{r_L}{Z_{coil}}$

$= I^2 r_L = 5^2\times 5.5 = 137.5W$

$P_t = I^2(R + r_L) = 25\times(25+5.5) = 762.5W$

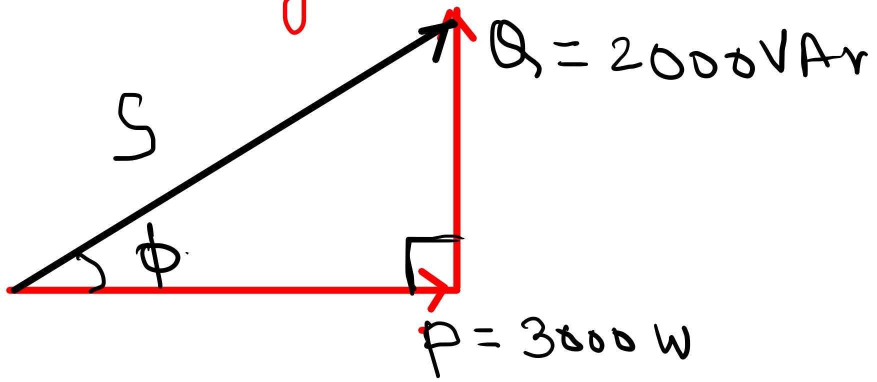
6. Two coils A and B are connected in series across a 240 V, 50 Hz supply. The resistance of A is 5Ω and inductance of B is 0.015 H. If the input from the supply is 3kWatts and 2kVAr. Find inductance of A and resistance of B. Calculate volatge across each coil.



P = active power = 3 kW

Q = Reactive power = 2 KVAR

⇒ power triangle



$S = 3605.5$ VA | $\cos\phi = \dfrac{3000}{3605.5} = 0.83$ (lag)

$\phi = 33.7^\circ$

$\cos\phi = \dfrac{P}{S}$

$S^2 = P^2 + Q^2 = (3000)^2 + (2000)^2$

$S = V \times I \qquad I = \dfrac{S}{V} = \dfrac{3605.5}{240}$

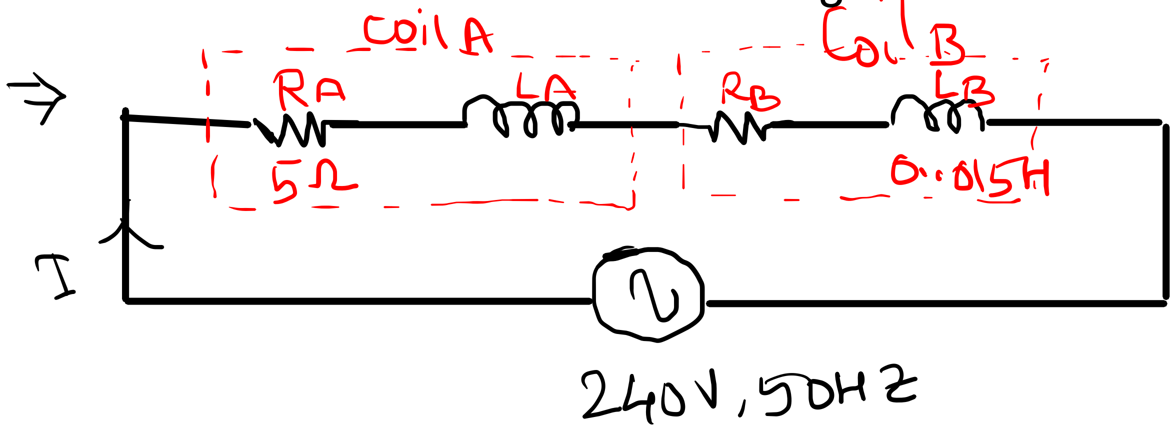
$I = 15.02\,A$

$|Z| = \dfrac{V}{I} = \dfrac{240}{15.02} = 15.98\,\Omega$

$Z = 15.98\angle 33.7^\circ$

$Z = Z_A + Z_B = R_A + jX_A + R_B + jX_B$

6. Two coils A and B are connected in series across a 240 V, 50 Hz supply. The resistance of A is 5Ω and inductance of B is 0.015 H. If the input from the supply is 3kWatts and 2kVAr. Find inductance of A and resistance of B. Calculate volatge across each coil.



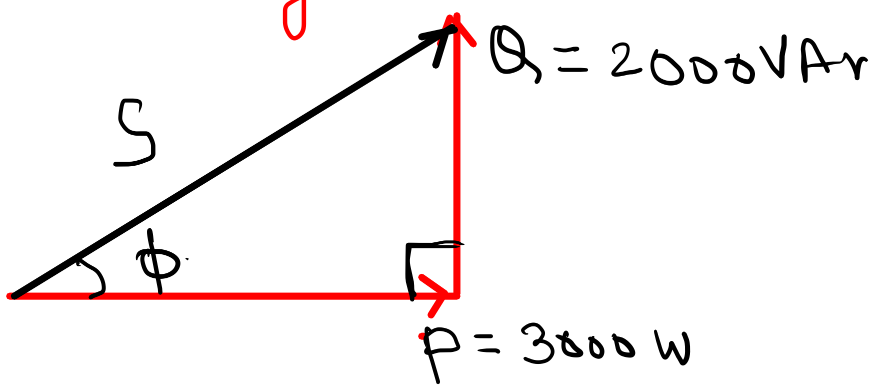
P = active power = 3 kW

Q = Reactive power = 2 KVAR

$S = 3605.5$ VA | $\cos\phi = \frac{3000}{3605.5} = 0.83$ (lag)

$\phi = 33.7^\circ$

⇒ power triangle



$S = V \times I$ $\quad I = \frac{S}{V} = \frac{3605.5}{240}$

$I = 15.02A$

$|Z| = \frac{V}{I} = \frac{240}{15.02} = 15.98\Omega$

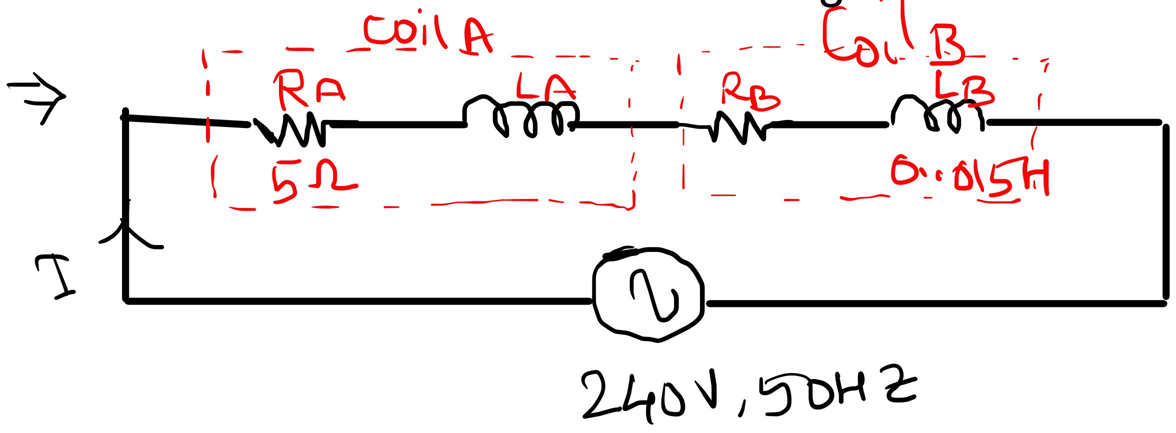
$Z = 15.98\angle 33.7^\circ$

$\cos\phi = \frac{P}{S}$

$S^2 = P^2 + Q^2 = (3000)^2 + (2000)^2$

$Z = Z_A + Z_B = R_A + jX_A + R_B + jX_B$

6. Two coils A and B are connected in series across a 240 V, 50 Hz supply. The resistance of A is 5 Ω and inductance of B is 0.015 H. If the input from the supply is 3kWatts and 2kVAr. Find inductance of A and resistance of B. Calculate volatge across each coil.



$$Z = R_A + jX_A + R_B + jX_B$$

$$Z = (R_A + R_B) + j(X_A + X_B)$$

$$Z = 15.98 \angle 33.7^\circ$$

$$P = 3000 \text{ Watts}$$

$$P = I^2 R = I^2 (R_A + R_B)$$

$$R_A + R_B = \frac{P}{I^2} = \frac{3000}{(15.02)^2}$$

$$R_A + R_B = 13.29\,\Omega$$

$$R_B = 13.29 - R_A = 13.29 - 5 = 8.3\,\Omega$$

$$X_B = 2\pi f L_B = 2\times\pi\times 50 \times 0.015 = 4.71\,\Omega$$

$$Z^2 = (R_A + R_B)^2 + (X_A + X_B)^2$$

$$(X_A + X_B) = Z \sin\phi$$

$$(15.98)^2 = (13.29)^2 + (X_A + X_B)^2$$

$$(X_A + X_B)^2 = (15.98)^2 - (13.29)^2 =$$

$$X_A + X_B = 8.86 \quad \therefore X_A = 4.16\,\Omega$$

$$X_A = 2\pi f L_A \quad \therefore L_A = \frac{X_A}{2\pi f} = \frac{4.16}{2\pi \times 50}$$

$$L_A = 0.013\,H$$

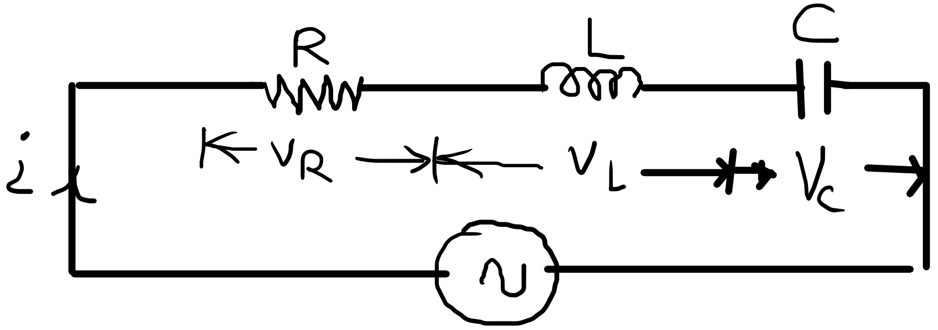
$$V_A = I \times Z_A = 15.02(R_A + jX_A) = 15.02(5 + j4.16)$$

$$V_A = (97.6 \angle 39.7^\circ) \leftarrow = 75.1 + j62.33)$$

$$V_B = I \times Z_B = 15.02(R_B + jX_B) = 15.02(8.3 + j4.71)$$

$$V_B = 143.17 \angle 29.45^\circ$$

R-L-C series Circuit

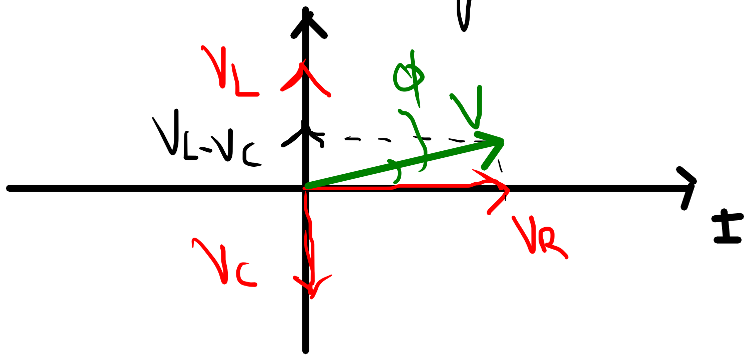


$v = V_m \sin \omega t$

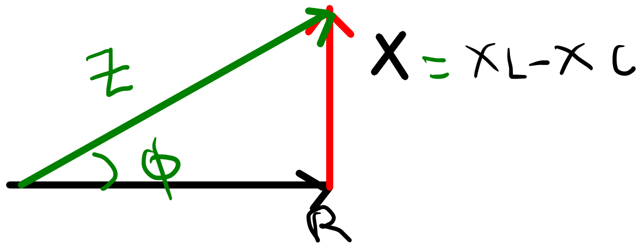
$\Rightarrow$ Case (I) $X_L > X_C$

$V_L > V_C$

$\rightarrow$ Phasor diagram



$\rightarrow$ Impedance triangle

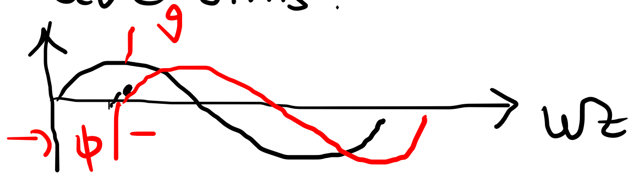


$\bar{V} = \bar{V_R} + \bar{V_L} + \bar{V_C}$

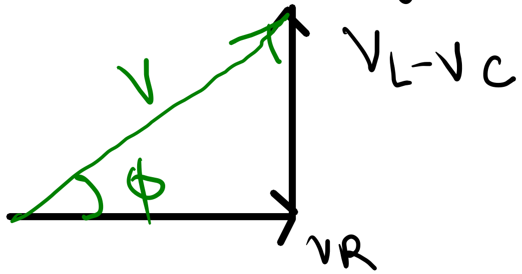
$V_R = IR$, $V_L = X_L \cdot I$, $V_C = X_C I$

$Z = R + j(X_L - X_C) = R \pm jX$ $\quad \boxed{X = X_L - X_C}$

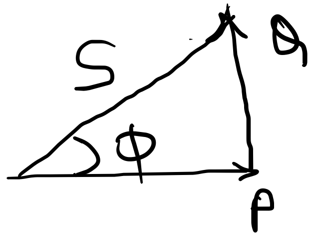
$\rightarrow$ Waveforms.



$\rightarrow$ Voltage triangle

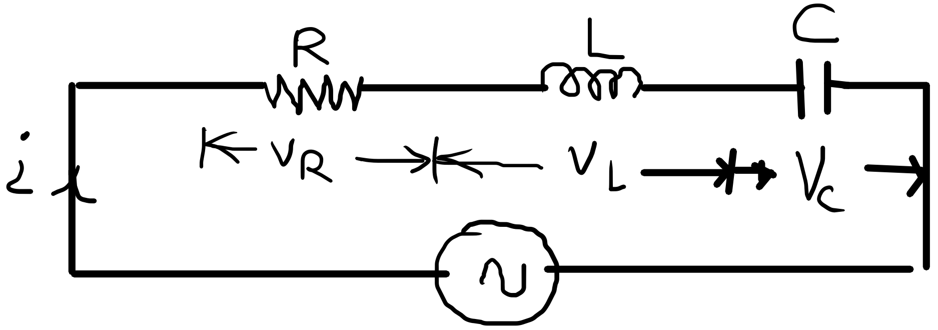


$\rightarrow$ power triangle



power factor $= \cos\phi = \frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$

R-L-C series Circuit

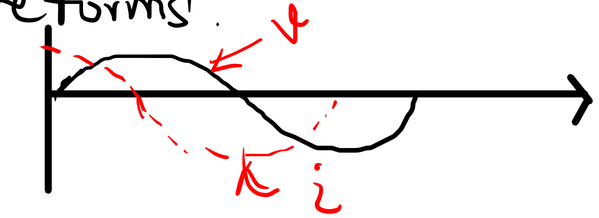


$v = V_m \sin \omega t$

$\bar{V} = \bar{V_R} + \bar{V_L} + \bar{V_C}$

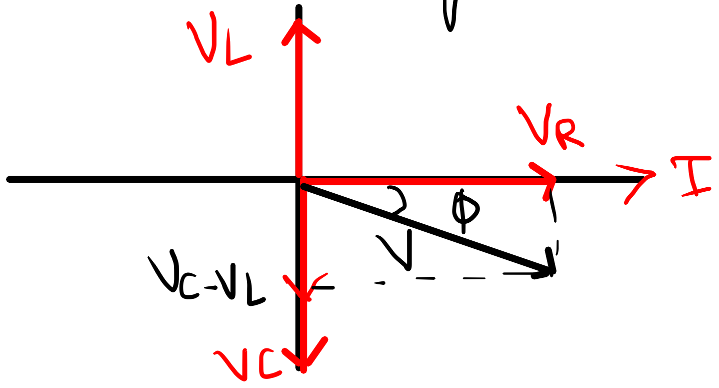
$V_R = IR$, $V_L = X_L \cdot I$, $V_C = X_C I$

→ Waveforms.

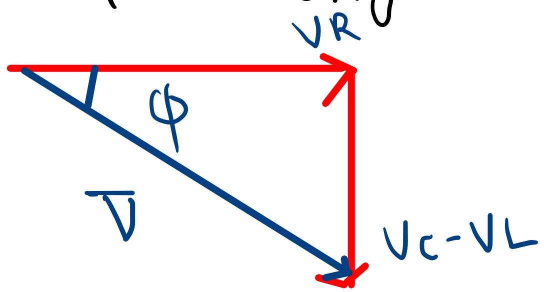


⇒ Case (I) $X_C > X_L$

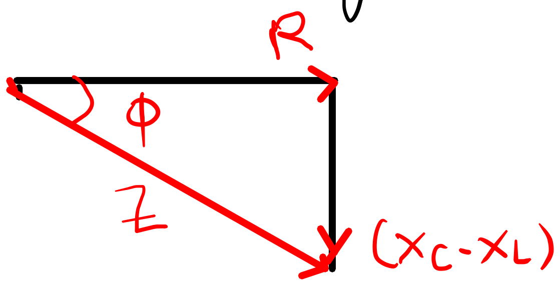
→ Phasor diagram



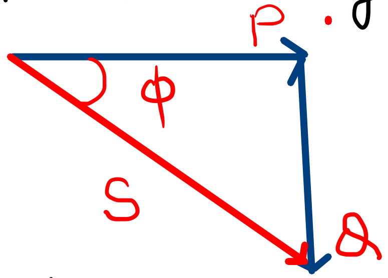
→ Voltage triangle



→ Impedance triangle

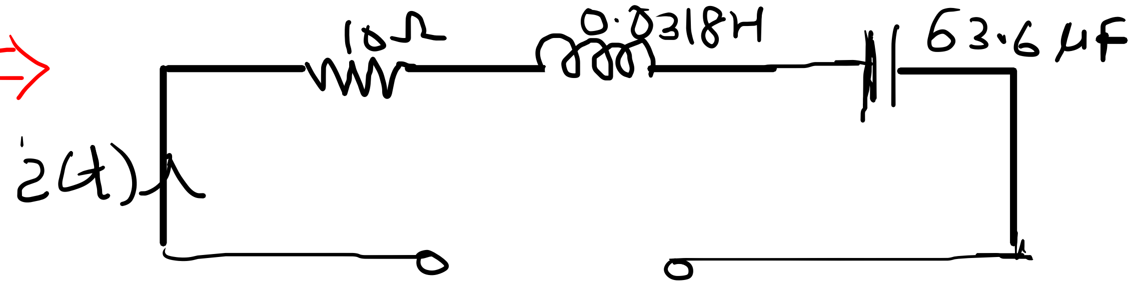


→ power triangle



power factor $= \cos\phi = \dfrac{V_R}{V} = \dfrac{R}{Z} = \dfrac{P}{S}$

7)A voltage e(t)=100sin(314t) is applied to a series circuit consisting of 10 ohm resistance, a 0.0318 H inductor and 63.6 uF capacitor. Calculate (i) expression for current (ii) Power factor (iii) Active power.



$$e(t) = 100 \sin 314t \qquad E = \frac{100}{\sqrt{2}} \angle 0^\circ$$

$$I = \frac{E}{Z}$$

$$Z = R + j(X_L - X_C)$$

$$X_L = \omega L = 314 \times 0.0318 = 9.98\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{314 \times 63.6 \times 10^{-6}} = 50.07\ \Omega$$

$$Z = 10 + j(9.98 - 50.07)$$

$$Z = 10 - j\,40.09$$

$$Z = 41.31 \angle -76^\circ$$

$$I = \frac{V}{Z} = \frac{(100/\sqrt{2}) \angle 0^\circ}{41.31 \angle -76^\circ}$$

$$\boxed{I = 1.71 \angle 76^\circ}$$

$$i(t) = I \times \sqrt{2} \sin(314t + 76^\circ)$$

$$= 1.71 \times \sqrt{2} \sin(314t + 76^\circ)$$

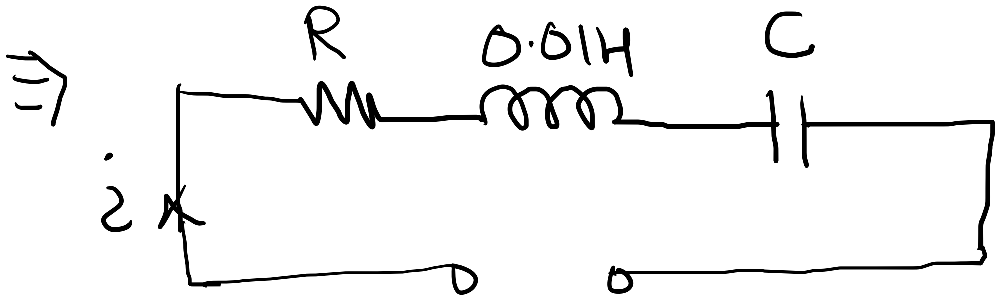
$$i(t) = 2.42 \sin(134t + 76^\circ)$$

$$PF = \cos\phi = \cos 76^\circ = 0.24 \text{ (leading)}$$

$$P = VI\cos\phi$$

$$= \frac{100}{\sqrt{2}} \times 1.71 \times 0.24 = 29.25 \text{ Watts}$$

8). A resistance R, an inductor L =0.01 H and a capacitance C are connected in series when voltage $v = 400\cos(3000t-10^\circ)$ is applid to a series combination. if the current is $i=10\sqrt{2}\cos(3000t-55^\circ)$. Find R and C.



$$v = 400\cos(3000t - 10^\circ)$$

$$i = 10\sqrt{2}\cos(3000t - 55^\circ)$$

$$v = 400\sin(3000t - 10^\circ + 90^\circ)$$

$$v = 400\sin(3000t + 80^\circ)$$

$$V = \frac{400}{\sqrt{2}}\angle 80^\circ$$

$$i = 10\sqrt{2}\sin(3000t - 55 + 90^\circ)$$

$$i = 10\sqrt{2}\sin(3000t + 35^\circ)$$

$$I = \frac{10\sqrt{2}}{\sqrt{2}}\angle 35^\circ$$

I lags V by $(\underline{80} - \underline{35}) = \underline{\underline{45^\circ}}$

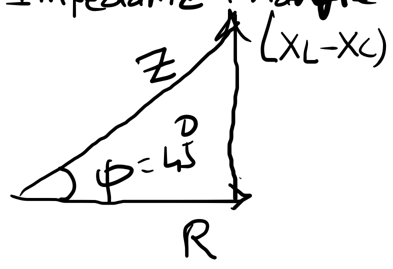
$$\phi = 45^\circ$$

$$PF = \cos\phi = \cos 45^\circ = 0.707 \text{ (lagging)}$$

$$Z = \frac{V}{I} = \frac{\frac{400}{\sqrt{2}}\angle 80^\circ}{10\angle 35^\circ}$$

$$\underline{\underline{Z = 28.28\angle 45^\circ}}$$

Impedance triangle



$$R = Z\cos\phi = 28.28\cos 45^\circ$$

$$\boxed{R = 19.99\,\Omega}$$

$$(X_L - X_C) = Z\sin\phi = 28.28\sin 45^\circ = 19.99\,\Omega$$

$$X_L = \omega L = 3000 \times 0.01 = 30\,\Omega$$

$$X_L - X_C = 19.99$$

$$-X_C = 19.99 - 30 = -10.01$$

$$X_C = \frac{1}{\omega C}$$

$$C = \frac{1}{\omega \times X_C}$$

$$C = \frac{1}{3000 \times 10.01}$$

$$C = 33.3\,\mu F$$

Concept Admittance (Y)

Admittance is Reciprocal of Impedance.

$$\bar{Y} = \frac{1}{Z} = \frac{1}{R \pm jX}$$

$$\bar{Y} = \frac{1}{R \pm jX} \times \frac{R \mp jX}{R \mp jX}$$

$$Y = \frac{R \mp jX}{R^2 + X^2}$$

$$Y = \frac{R}{R^2+X^2} \mp j\frac{X}{R^2+X^2}$$

$$Y = G \mp jB$$

Admittance = ($\mho$) (mho) Siemens

Conductance ($\mho$) (mho) siemens

Suceptance ($\mho$) (mho) simens

For R-L circuit

$$Z = R \pm jX_L \;\&\; Y = G - jB$$

for R-C circuit

$$Z = R - jX_C \;\&\; Y = G \pm jB$$

Concept Admittance (Y)

Admittance is Reciprocal of Impedance.

$$\bar{Y} = \frac{1}{Z} = \frac{1}{R \pm jX}$$

$$\bar{Y} = \frac{1}{R \pm jX} \times \frac{R \mp jX}{R \mp jX}$$

$$Y = \frac{R \mp jX}{R^2 + X^2}$$

$$Y = \frac{R}{R^2+X^2} \mp j\frac{X}{R^2+X^2}$$

$$Y = G \mp jB$$

Admittance = (℧)(mho) Siemens

Conductance (℧) (mho) Siemens

Suceptance (℧) (mho) Simens

For R-L circuit

$Z = R + jX_L$ & $Y = G - jB$

for R-C circuit

$Z = R - jX_C$ & $Y = G + jB$

Concept Admittance (Y)

Admittance is Reciprocal of Impedance.

$$\bar{Y} = \frac{1}{Z} = \frac{1}{R \pm jX}$$

$$\bar{Y} = \frac{1}{R \pm jX} \times \frac{R \mp jX}{R \mp jX}$$

$$Y = \frac{R \mp jX}{R^2 + X^2}$$

$$Y = \frac{R}{R^2+X^2} \mp j\frac{X}{R^2+X^2}$$

$$Y = G \mp jB$$

Admittance = (℧) (mho) Siemens

Conductance (℧) (mho) Siemens

Suceptance (℧) (mho) Simens

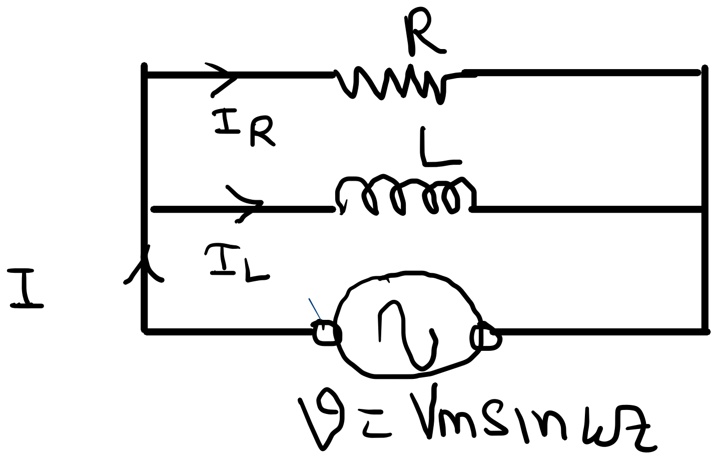
For R-L circuit

$Z = R \underline{+} jX_L$ & $Y = G - jB$

for R-C circuit

$Z = R - jX_C$ & $Y = G \underline{+} jB$

R-L Parallel Circuit



$\bar{I} = \bar{I}_R + \bar{I}_L$

$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{jX_L}$$

$Y_R = \frac{1}{R}$

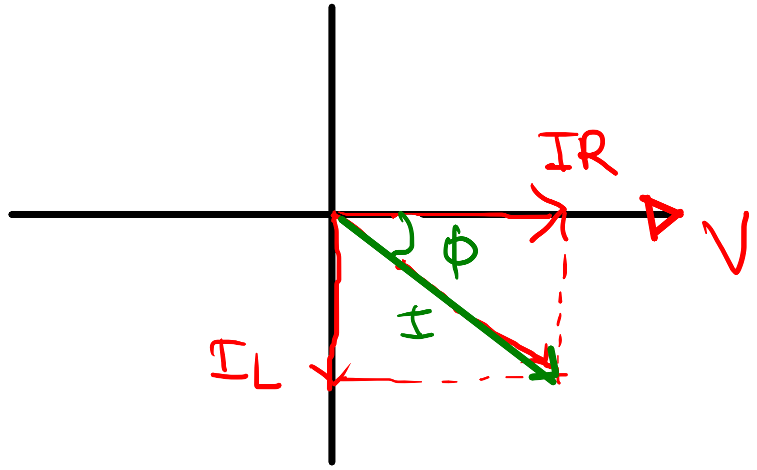
$Y_L = \frac{1}{jX_L}$

$Y = Y_R - j\,Y_L$

$$\boxed{Y = \frac{1}{R} - j\frac{1}{X_L}}$$

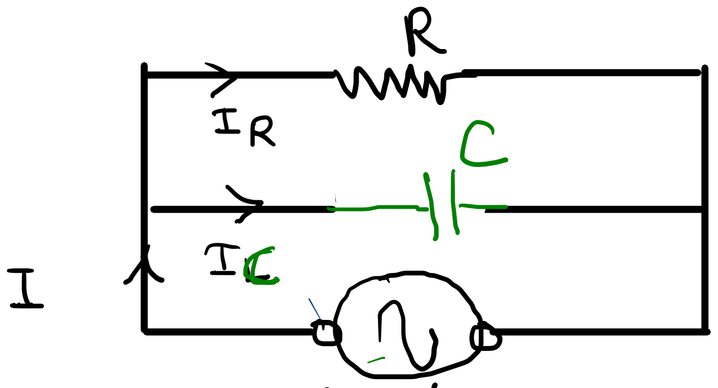
⇒ phasor diagram

since voltage across R & L is same so take V as reference.



PF = $\cos\phi$ (lagging)

R-C Parallel Circuit



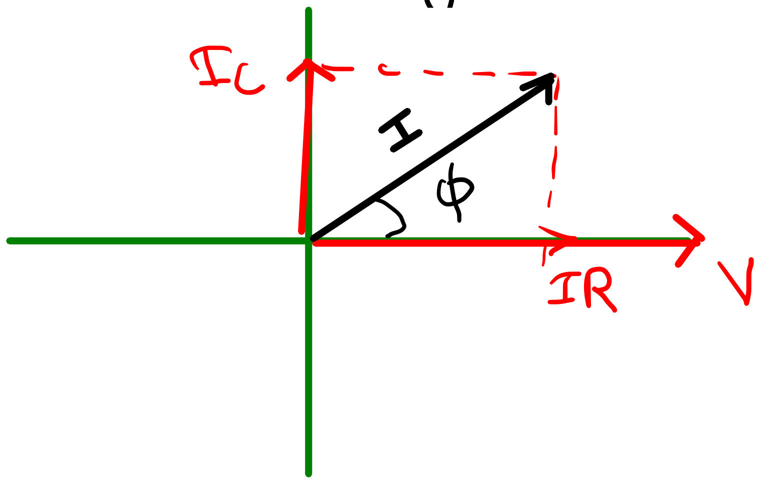
$\bar{I} = \bar{I}_R + \bar{I}_C$

$Y_R = \frac{1}{R}$

$Y_C = \frac{1}{-j/\omega C} = j\omega C$

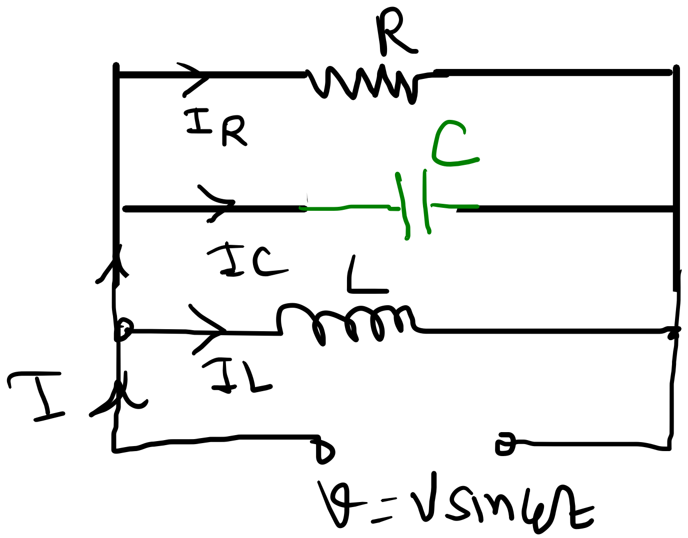
$Y = Y_R + Y_C = \frac{1}{R} + j\omega C$

$\Rightarrow$ phasor diagram



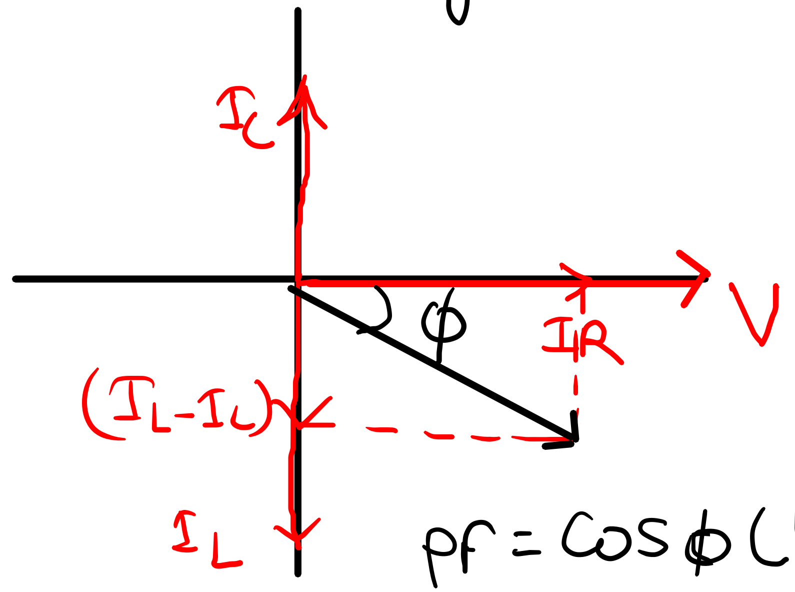
$PF = \cos\phi$ (leading)

R-L-C Parallel Circuit



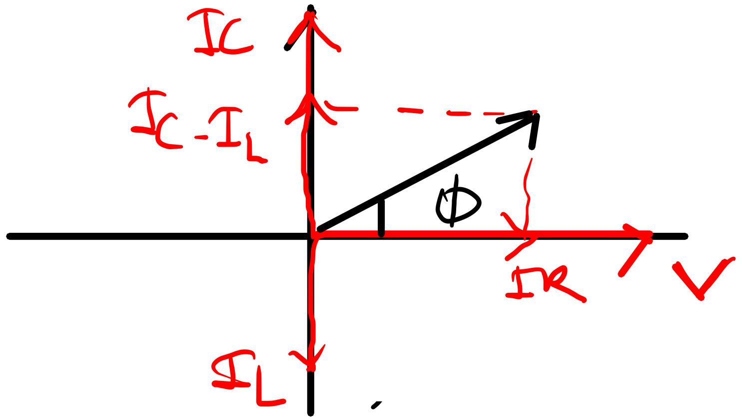
$$\bar{I} = \bar{I}_R + \bar{I}_L + \bar{I}_C$$

$\Rightarrow$ phasor diagram ($I_L > I_C$)



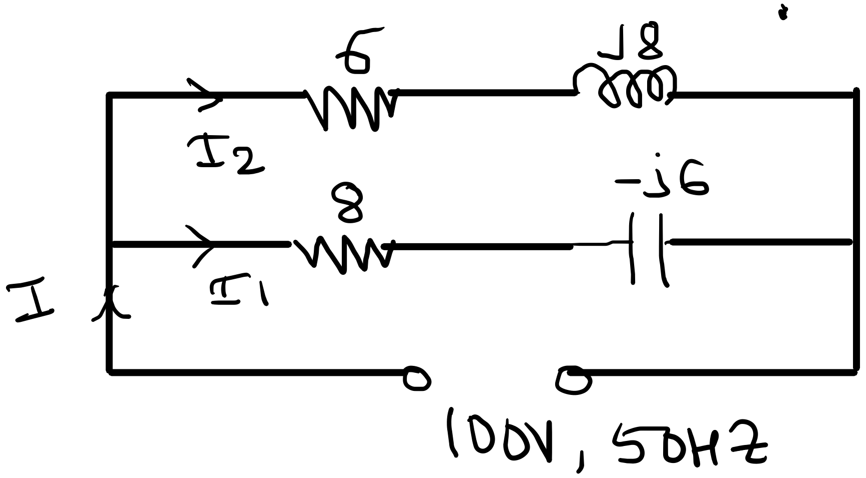
pf = $\cos\phi$ (lagging)

$\Rightarrow$ phasor diagram ($I_C > I_L$)



pf = $\cos\phi$ (leading)

1. Find current I1 and I2 in the following circuit. Find overall power factor of the circuit, Active power. Draw phasor diagram of the circuit.



$\Rightarrow V = 100\angle 0^\circ$

$Z_1 = 8 - j6 = 10\angle -36.86^\circ$

$Z_2 = 6 + j8 = 10\angle 53.13^\circ$

$I_1 = \frac{V}{Z_1} = \frac{100\angle 0^\circ}{10\angle -36.86^\circ} = 10\angle 36.86^\circ$

$= 8 + j5$

$I_2 = \frac{V}{Z_2} = \frac{100\angle 0^\circ}{10\angle 53.13} = 10\angle -53.13^\circ$

$= 6 - j8$

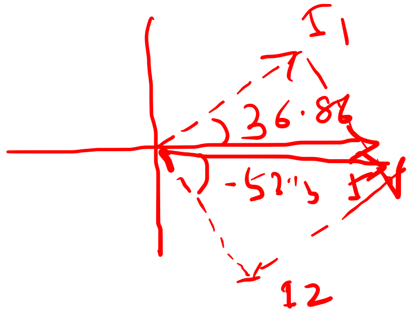
$\bar{I} = \bar{I}_1 + \bar{I}_2$

$I = 10\angle 36.86^\circ + 10\angle -53.13^\circ$

$I = 14.00 - j2 \simeq 14 - j2$

$\boxed{I = 14.14\angle -8.13^\circ}$

$V = 100\angle 0^\circ$

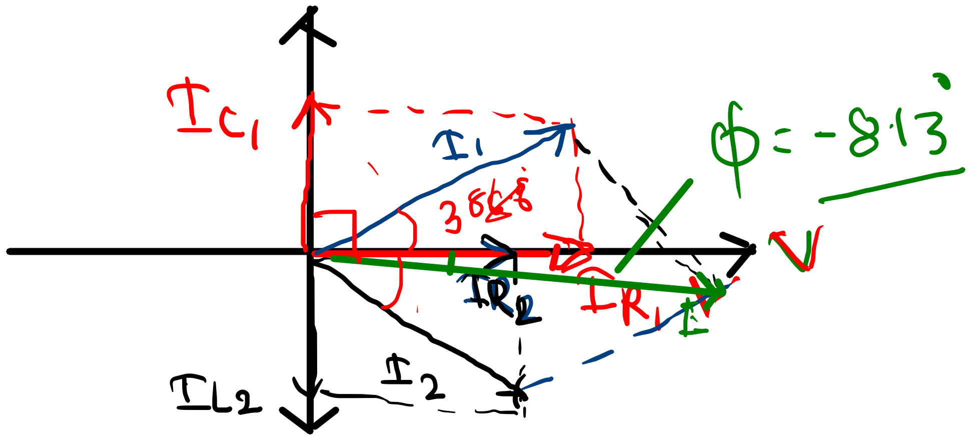


$PF = \cos\phi = \cos(8.13^\circ)$

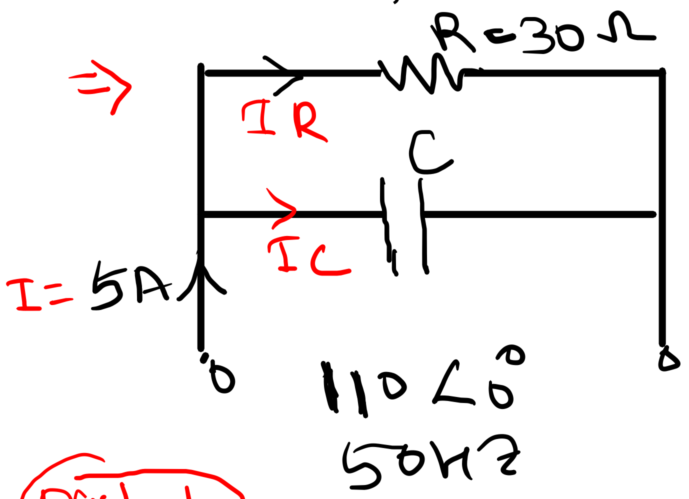
$\boxed{PF = 0.989\ (\text{lagging})}$

$P = VI\cos\phi = 100 \times 14.14 \times 0.989$

$P = 1398.44$ watts



2. A resistor of 30 ohm and a capacitor of unknown value are connected in parallel across a 110 V, 50Hz supply. The combination draws a current of 5A from the supply. Find the value of unknown capacitance. The combination is connected across a 110 V supply of unknown frequency, it is observed that total current drawn from the mains falls to 4 A , determine frequency of the supply.



Part-1

$\bar{I} = \bar{I}_R + \bar{I}_C$

$I^2 = I_R^2 + I_C^2$

$I = 5, \quad I_R = \frac{110}{30} = 3.67A$

$I_C^2 = I^2 - I_R^2$

$I_C^2 = (5)^2 - (3.67)^2$

$I_C = 3.39A$

$X_C = \frac{V}{I_C} = \frac{110}{3.39}$

$X_C = 32.45\,\Omega$

$X_C = \frac{1}{\omega C}$

$C = \frac{1}{\omega \times X_C}$

$C = \frac{1}{314 \times 32.45}$

$C = 98.15\,\mu F$

Part-2 $\quad I = 4A$

$I_C^2 = (4)^2 - (3.67)^2$

$I_C = 1.59A$

$X_C = \frac{110}{1.59} = 69.18\,\Omega$

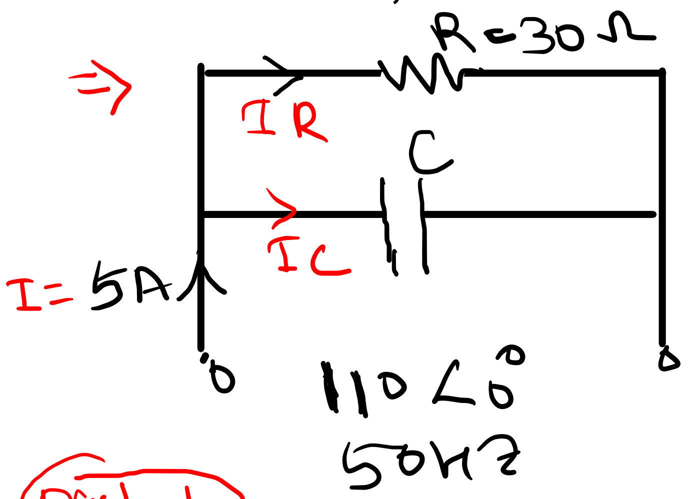
$X_C = \frac{1}{\omega C}$

$\omega = \frac{1}{C \times X_C} = \frac{1}{98.15 \times 10^{-6} \times 69.18} = 147.27\, rad$

$f = \frac{\omega}{2\pi} = \frac{147.27}{2\pi} = 23.47\, Hz$

2. A resistor of 30 ohm and a capacitor of unknown value are connected in parallel across a 110 V, 50Hz supply. The combination draws a current of 5A from the supply. Find the value of unknown capacitance. The combination is connected across a 110 V supply of unknown frequency, it is observed that total current drawn from the mains falls to 4 A , determine frequency of the supply.

⇒



Part-1

$$\bar{I} = \bar{I}_R + \bar{I}_C$$

$$I^2 = I_R^2 + I_C^2$$

$$I = 5, \quad I_R = \frac{110}{30} = 3.67A$$

$$I_C^2 = I^2 - I_R^2$$

$$I_C^2 = (5)^2 - (3.67)^2$$

$$I_C = 3.39A$$

$$X_C = \frac{V}{I_C} = \frac{110}{3.39}$$

$$X_C = 32.45\,\Omega$$

$$X_C = \frac{1}{\omega C}$$

$$C = \frac{1}{\omega \times X_C}$$

$$C = \frac{1}{314 \times 32.45}$$

$$C = 98.15\,\mu F$$

Part-2 $I = 4A$

$$I_C^2 = (4)^2 - (3.67)^2$$

$$I_C = 1.59A$$

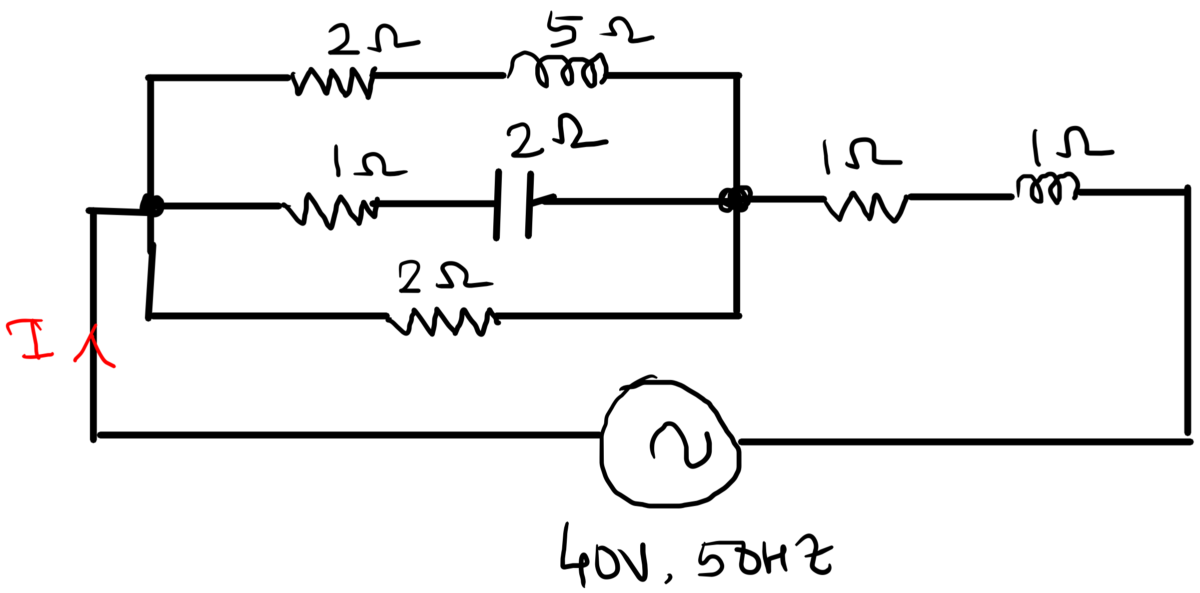
$$X_C = \frac{110}{1.59} = 69.18\,\Omega$$

$$X_C = \frac{1}{\omega C}$$

$$\omega = \frac{1}{C \times X_C} = \frac{1}{98.15\times10^{-6}\times 69.18} = 147.27\,rad$$

$$f = \frac{\omega}{2\pi} = \frac{147.27}{2\pi} = 23.47\,Hz$$

3. In the following circuit, calculate i) total impedance (ii) total current (iii) power factor (iv) Active and reactive power.



$\Rightarrow \quad Z_1 = 2+j5 \qquad Z_4 = 1+j1$

$Z_2 = 1-j2$

$Z_3 = 2\Omega$

$Y_1 = \frac{1}{Z_1} = \frac{1}{(2+j5)} = 0.068 - j0.172$

$Y_2 = \frac{1}{Z_2} = \frac{1}{1-j2} = 0.2+j0.4$

$Y_3 = \frac{1}{Z_3} = \frac{1}{2} = 0.5$

$Y_p = Y_1 + Y_2 + Y_3 = 0.768 + j0.228$

$Z_p = \frac{1}{Y_p} = \frac{1}{0.768+j0.228}$

$Z_p = 1.196 - j0.355$

$Z_T = Z_p + Z_4 = 2.196 + j0.645$

$\boxed{Z_T = 2.28\angle 16.36^\circ}$

$I = \frac{V}{Z_T} = \frac{40\angle 0}{2.28\angle 16.36} = 17.54\angle -16.36^\circ$

$PF = \cos\phi = \cos(16.36) = 0.95$ (lagging)

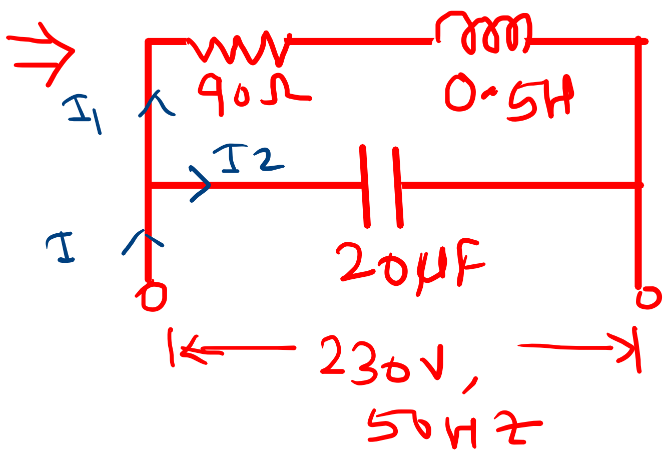
$P = VI\cos\phi \qquad Q = VI\sin\phi$

$P = 40 \times 17.54 \times 0.95 \qquad Q = 40 \times 17.54 \sin(16.36^\circ)$

$P = 666.62$ Watts $\qquad Q = 197.62$ VAR

4) A series combination of 0.5 H inductor and 90 ohm resistor are connected in parallel across 20 uF. Find (i) the total current (ii) power factor of the circuit (iii) total power taken from the source. Draw phasor diagram. A voltage of 230 V , 50 Hz is maintained across the circuit.

⇒



$Z_1 = 90 + j\, 314 \times 0.5$

$Z_1 = 90 + j157 = 180.9\angle 60.17^\circ$

$Z_2 = -j \dfrac{1}{314 \times 20 \times 10^{-6}}$

$Z_2 = -j159.15 = 159.15\angle -90^\circ$

$I_1 = \dfrac{V}{Z_1} = \dfrac{230\angle 0^\circ}{180.9\angle 60.17} = 1.27\angle -60.17^\circ$

$I_2 = \dfrac{V}{Z_2} = \dfrac{230\angle 0^\circ}{159.15\angle -90^\circ} = 1.44\angle 90^\circ$

$\bar{I} = \bar{I}_1 + \bar{I}_2 = 1.27\angle -60.17 + 1.44\angle 90^\circ$

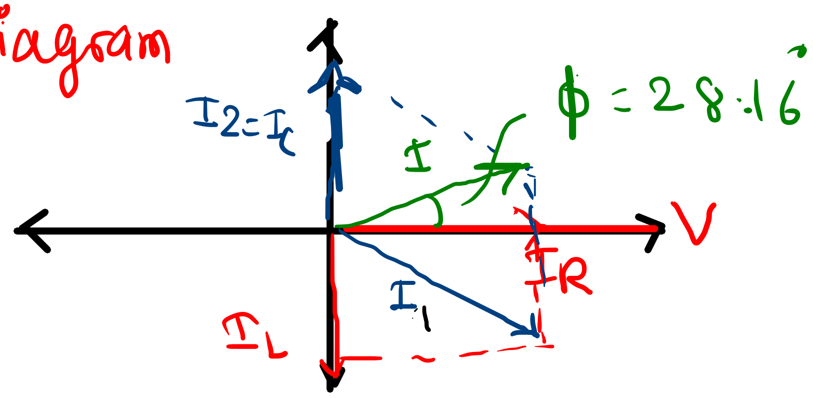
$\boxed{I = 0.716\angle 28.16^\circ}$

$pf = \cos\phi = \cos(28.16^\circ) = 0.88$ (leading)

Power $P = VI\cos\phi = 230 \times 0.716 \times 0.88$

$P = 144.91$ watts.

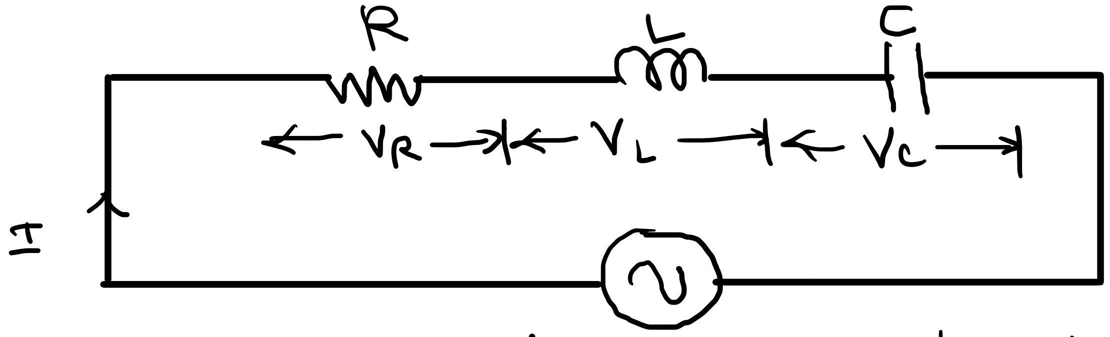
⇒ Phasor diagram



Resonance

Resonance is said to occur when the current is either maximum or minimum for a particular frquecncy of the AC input. At resonance circuit impedance becomes minimum or maximum. The power factor of the circuit beomes unity.

Series Resonance



$V = V_m \sin \omega t \mid \omega \updownarrow$

$Z = R + j(X_L - X_C)$

$X_L = \omega L = 2\pi f L \quad f\uparrow \; X_L\uparrow$

at $f=0$, $X_L=0$

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \quad f\uparrow \; X_C\downarrow$

at $f=0$, $X_C=\infty$

At resonance ($pf=1$)

$\rightarrow X_L = X_C$

$Z = R + j(X_L - X_C)$

$Z_r = R + j0 = R$ (minimum)

$I_r = \frac{V}{Z} = \frac{V}{R}$ (maximum)

[Acceptor circuit]

$\rightarrow X_L = X_C$ at $\omega = \omega_r$

$\omega_r L = \frac{1}{\omega_r C}$

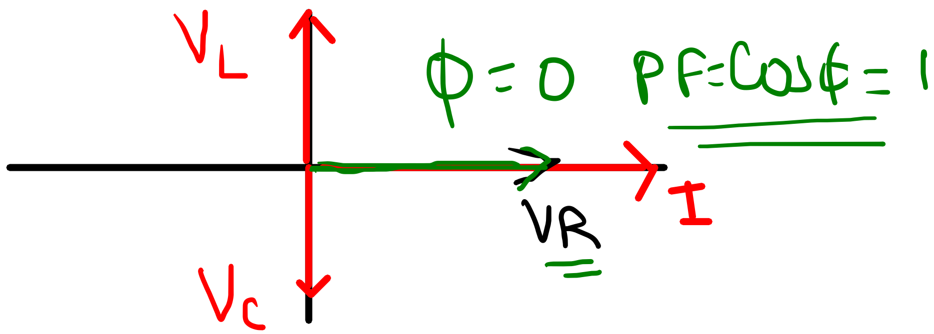
$\omega_r^2 = \frac{1}{LC}$ So $\boxed{\omega_r = \frac{1}{\sqrt{LC}} \text{ rad/sec}}$

$2\pi f_r = \frac{1}{\sqrt{LC}}$ So $f_r = \frac{1}{2\pi\sqrt{LC}}$ Hz

$f_r \rightarrow$ Resonance frequency.

⇒ Series Resonance

→ phasor diagram.



$X_L = X_C$

$I_r \cdot X_L = I_r \cdot X_c$

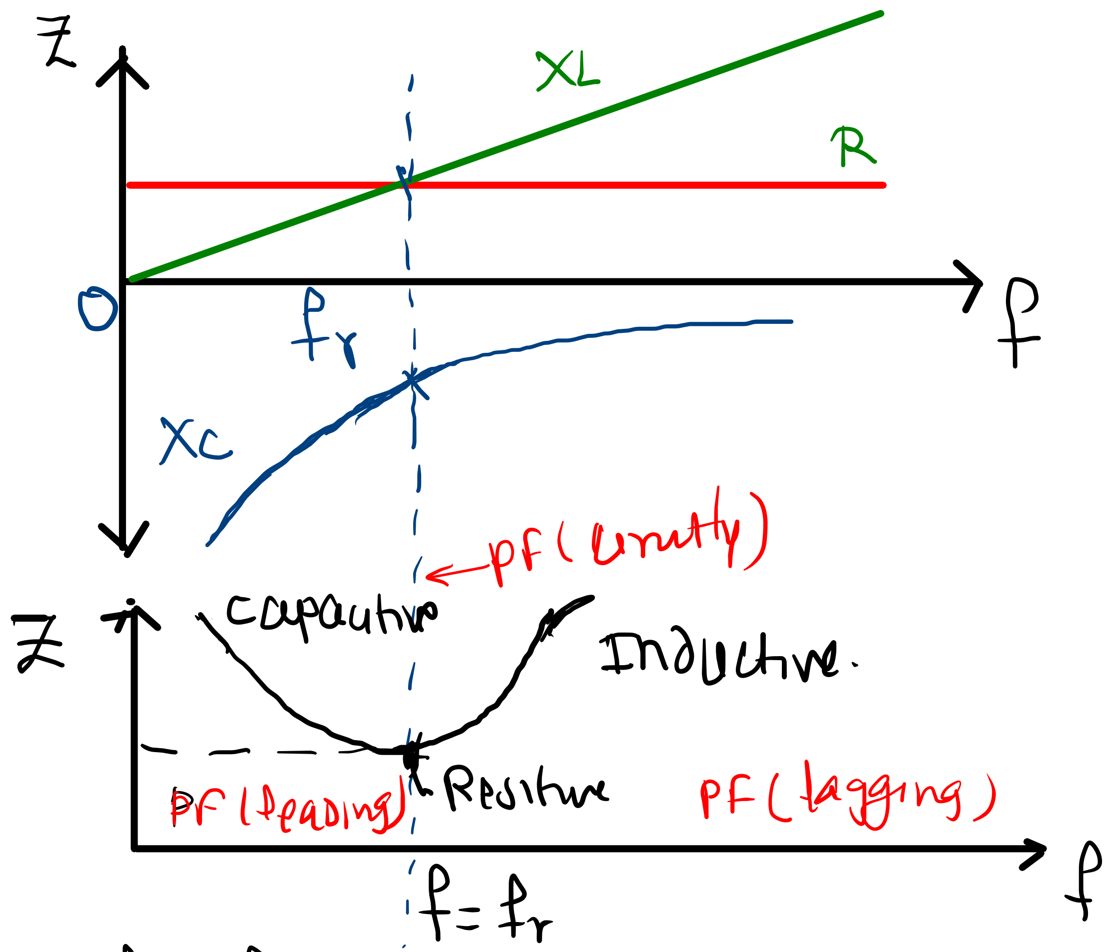
$(V_L = V_C) > V$

$V_R = I_r \cdot R$

$(V_L = V_C) > V$ so

→ Voltage magnification circuit

⇒ Variation of R, X_L & X_c and Z



→ $f < f_r$ → impedance capacitive, pf leading

→ $f = f_r$ → impedance Resistive, pf = unity

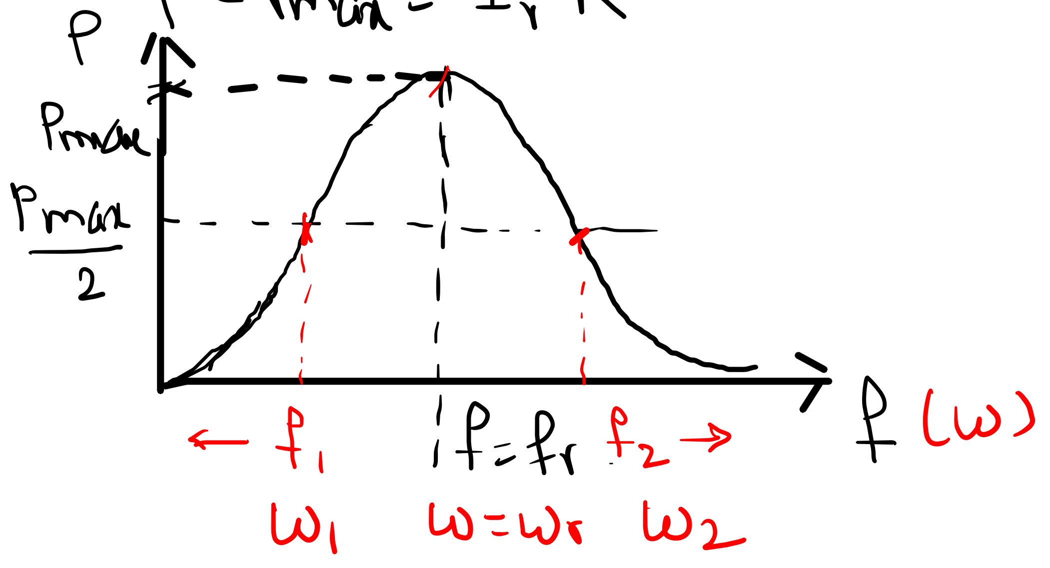
→ $f > f_r$ → impedance inductive, pf lagging

⇒ Bandwidth

$P = I^2 R$

at resonance $I = I_r$ (max)

$P = P_{max} = I_r^2 R$



→ Half power frequencies $(\omega_1 \& \omega_2)$ or $(f_1 \& f_2)$

Bandwidth $= (f_2 - f_1)$ or $(\omega_2 - \omega_1)$

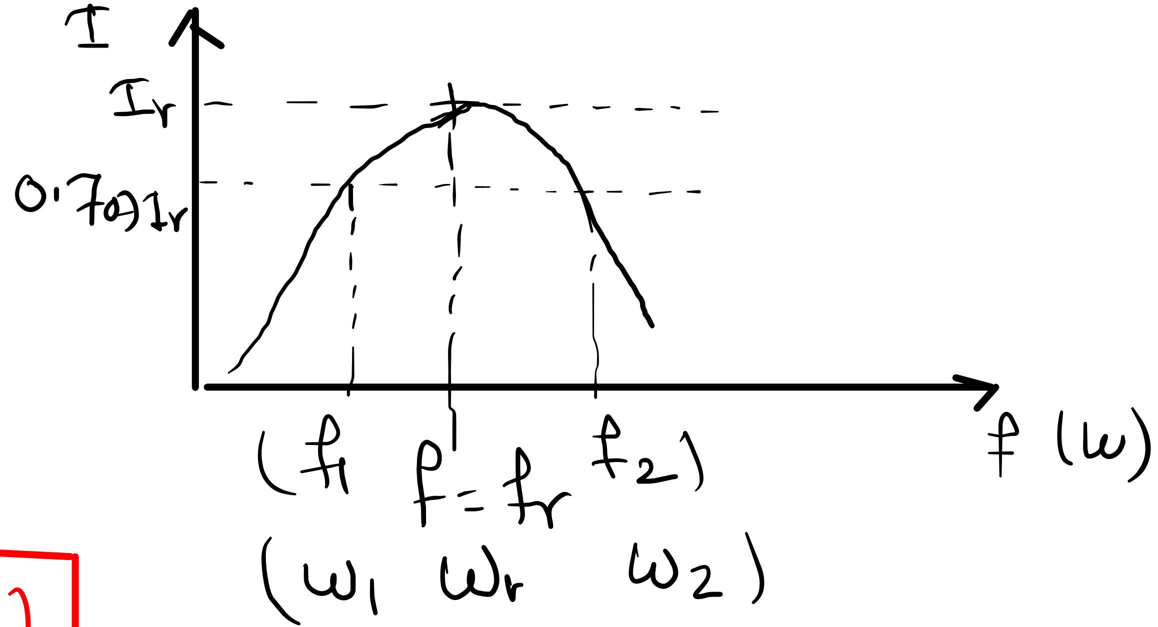
$P_{max} = I_r^2 \cdot R$

$P_{Half} = \frac{P_{max}}{2} = \frac{I_r^2 R}{2}$

Let at $P = P_{half}$, $I = I_H$

$I_H^2 \cdot \cancel{R} = \frac{I_r^2 \cdot \cancel{R}}{2}$

$I_H = \frac{I_r}{\sqrt{2}} = 0.707 I_r$



⇒ Bandwidth

At any freqn $I = \frac{V}{Z} = \frac{V}{\sqrt{R^2+(X_L-X_C)^2}}$ — (1)

$$= \frac{V}{\sqrt{R^2+(\omega L - \frac{1}{\omega C})^2}}$$

At half power $I = \frac{I_r}{\sqrt{2}} = \frac{V}{R\sqrt{2}} = \frac{V}{\sqrt{2}\cdot R}$ -- (2)

put (2) in eqn (1)

$$\frac{\cancel{V}}{\sqrt{2}R} = \frac{\cancel{V}}{\sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}}$$

$$\sqrt{2}R = \sqrt{R^2+(\omega L - \frac{1}{\omega C})^2}$$

sequaring both sides

$$2R^2 = R^2 + (\omega L - \frac{1}{\omega C})^2$$

$$(\omega L - \frac{1}{\omega C})^2 = R^2$$

$$(\omega L - \frac{1}{\omega C}) = \pm R$$

$$\frac{(\omega^2 LC - 1)}{\omega C} = R$$

$$\omega^2 LC - 1 = \pm \omega CR$$

$$\omega^2 LC - 1 \pm \omega CR = 0$$

$$LC[\omega^2 \pm \frac{R}{L}\omega - \frac{1}{LC}] = 0$$

$$\omega^2 \pm \frac{R}{L}\omega - \frac{1}{LC} = 0$$

Solving

ax^2+bx+c roots $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

finding roots.

$$\omega = \frac{\mp\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2}+\frac{4}{LC}}}{2} = \mp\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2}+\frac{1}{LC}}$$

→ Bandwidth

$$\omega = \pm \frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

for Low values of R

$$\frac{R^2}{4L^2} << \frac{1}{LC}$$

$$\omega = \pm \frac{R}{2L} \pm \sqrt{\frac{1}{LC}}$$

$$\omega = \pm \frac{R}{2L} \pm \frac{1}{\sqrt{LC}}$$

$$\omega = \pm \frac{R}{2L} \pm \omega_r$$

Considering ω_r as positive

$$\boxed{\omega_1 = \omega_r - \frac{R}{2L}} \text{ or } f_1 = f_r - \frac{R}{4\pi L}$$

$$\boxed{\omega_2 = \omega_r + \frac{R}{2L}} \qquad f_2 = f_r + \left(\frac{R}{4\pi L}\right)$$

$$BW = \omega_2 - \omega_1$$

$$= \cancel{\omega_r} + \frac{R}{2L} - \cancel{\omega_r} + \frac{R}{2L}$$

$$BW = \frac{2R}{2L} = \frac{R}{L} \text{ rad/sec}$$

Bandwidth in HZ

$$BW = f_2 - f_1 = \frac{\omega_2}{2\pi} - \frac{\omega_1}{2\pi}$$

$$= \frac{1}{2\pi}(\omega_2 - \omega_1)$$

$$\boxed{BW = \frac{R}{2\pi L} \text{ HZ}}$$

⇒ Quality Factor (Q)

$$Q = \frac{\text{potential drop across } L \text{ at resonance}}{\text{potential drop across } R \text{ at resonance}}$$

$$Q = \frac{X_L \times \cancel{I_r}}{R \times \cancel{I_r}} = \frac{\omega_r L}{R} = \frac{1}{\sqrt{LC}} \times \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

OR

$$Q = \frac{\text{potential drop across } C \text{ at resonance}}{\text{potential drop across } R \text{ at resonance}}$$

$$Q = \frac{X_C \cdot \cancel{I_r}}{R \cdot \cancel{I_r}} = \frac{1}{\omega_r C \cdot R} = \frac{\sqrt{LC}}{RC} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$Q = \frac{\omega_r \cdot L}{R} \quad \text{—— (i)}$$

$$BW = \frac{R}{L} \quad \text{—— (ii)}$$

$$BW = \frac{R}{L} \text{ rad/sec}$$

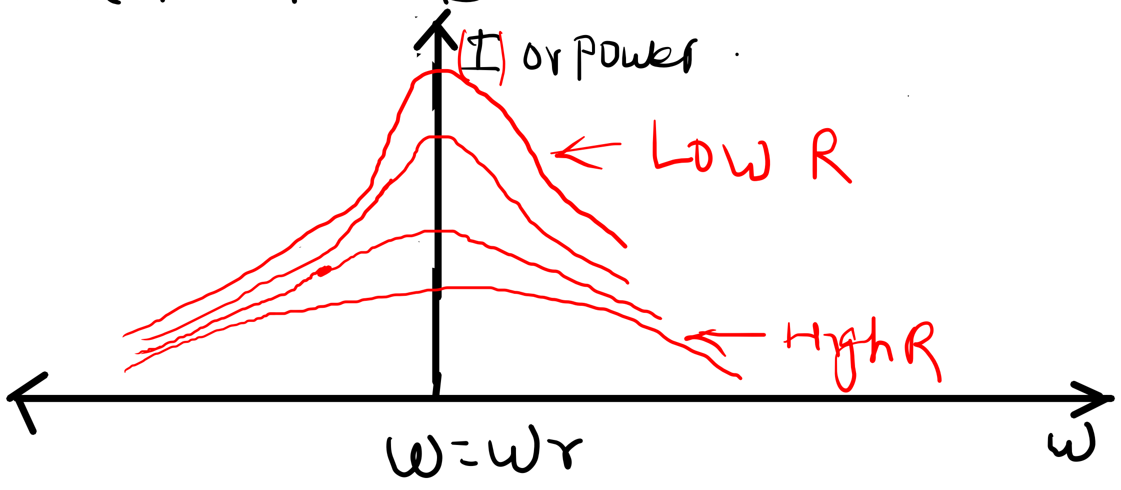
Comparing (i) & (ii)

$$Q = \frac{\omega_r}{BW(\text{rad/sec})} = \frac{\cancel{2\pi} f_r / \cancel{2\pi} \text{ Hz}}{\cancel{2\pi}\, BW_{Hz} / \cancel{2\pi}}$$

$$Q = \frac{f_r}{BW(Hz)}$$

$$\boxed{BW = \frac{f_r}{Q}} \quad \text{—— (3)}$$

R ↑ Q ↓ BW ↑



$\Rightarrow$ Frequency at which V_L & V_C are maximum :-

$V_C = I \cdot X_C$ or $\boxed{V_L = I \times X_L}$ $\quad Z = R + j(X_L - X_C)$

$$V_C = I X_C = \frac{V}{Z} X_C = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} X_C = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \times \frac{1}{\omega C}$$

sequcring both sides

$$V_C^2 = \frac{V^2}{R^2 + (\omega L - \frac{1}{\omega C})^2 (\omega C)^2} = \frac{V^2}{\omega^2 C^2 R^2 + \left[\frac{(\omega^2 LC - 1)}{\omega C}\right]^2 \cdot \frac{1}{\omega^2 C^2}}$$

$$V_C^2 = \frac{V^2}{\omega^2 C^2 R^2 + (\omega^2 LC - 1)^2}$$

$$\frac{dV_C^2}{d\omega} = 0$$

$$0 = \frac{(\omega^2 C^2 R^2 + (\omega^2 LC - 1)) \times 0 - V^2 (2 R^2 C^2 \omega + 2(\omega^2 LC - 1) 2\omega}{[(\omega^2 C^2 R^2 + (\omega^2 LC - 1)]^4}$$

$$2\omega R^2 C^2 + 2(\omega^2 LC - 1) 2\omega LC = 0$$

$$C \neq 0, \ \omega \neq 0$$

$$2\omega C\left(CR^2 + 2\omega^2 L^2 C - 2L\right) = 0$$

$$2L^2 C\omega^2 = 2L - CR^2$$

$$\omega^2 = \frac{2L}{2L^2C} - \frac{CR^2}{2L^2C}$$

$$\omega^2 = \frac{1}{LC} - \frac{R^2}{2L^2}$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

$$\frac{1}{LC} >> \frac{R^2}{2L^2}$$

$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$

$$2\omega C\left(CR^2 + 2\omega^2 L^2 C - 2L\right) = 0$$

$$2L^2 C\omega^2 = 2L - CR^2$$

$$\omega^2 = \frac{2L}{2L^2 C} - \frac{CR^2}{2L^2 C}$$

$$\omega^2 = \frac{1}{LC} - \frac{R^2}{2L^2}$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

$$\frac{1}{LC} \gg \frac{R^2}{2L^2}$$

$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$

$$2\omega C\left(CR^2 + 2\omega^2 L^2 C - 2L\right) = 0$$

$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$

$$2L^2 C\omega^2 = 2L - CR^2$$

$$\omega^2 = \frac{2L}{2L^2 C} - \frac{CR^2}{2L^2 C}$$

$$\omega^2 = \frac{1}{LC} - \frac{R^2}{2L^2}$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

$$\frac{1}{LC} >> \frac{R^2}{2L^2}$$

$$2\omega C\left(CR^2 + 2\omega^2L^2C - 2L\right) = 0$$

$$2L^2C\omega^2 = 2L - CR^2$$

$$\omega^2 = \frac{2L}{2L^2C} - \frac{CR^2}{2L^2C}$$

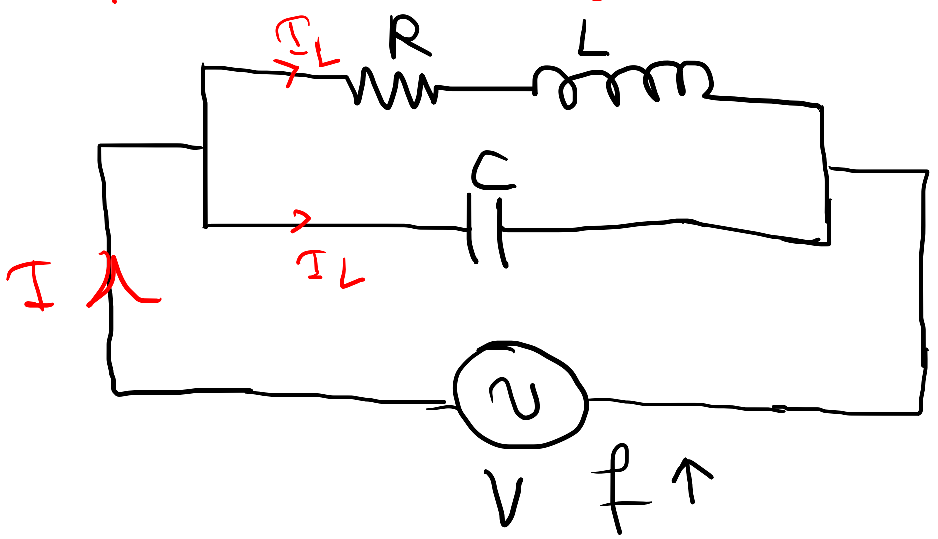
$$\omega^2 = \frac{1}{LC} - \frac{R^2}{2L^2}$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

$$\frac{1}{LC} >> \frac{R^2}{2L^2}$$

$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$

⇒ Parallel Resonance



$Z_1 = R + jX_L$

$Z_2 = -jX_C$

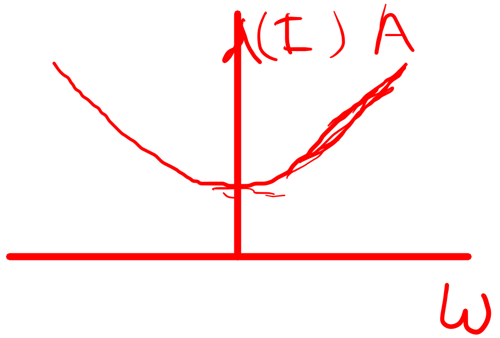
$$Y_1 = \frac{1}{Z_1} = \frac{1}{R + jX_C} \times \frac{R - jX_L}{R - jX_L}$$

$$Y_1 = \frac{R - jX_L}{R^2 + X_L^2}$$

$$Y_2 = Z_2 = \frac{1}{-jX_L} = j\frac{1}{X_L}$$

$$Y = Y_1 + Y_2 = \frac{R - jX_L}{R^2 + X_L^2} + \frac{j}{X_C}$$

$$Y = \frac{R}{R^2 + X_L^2} - j\left[\frac{X_L}{R^2 + X_L^2} - \frac{1}{X_L}\right] \text{ - - - - } (A)$$



At resonance the circuit is purely resistive so the reactive term must be zero.

$$\frac{X_L}{R^2 + X_L^2} - \frac{1}{X_C} = 0$$

$$\frac{X_L}{R^2 + X_L^2} = \frac{1}{X_L}$$

$$\boxed{X_L X_C = R^2 + X_L^2 \text{ .. } (1)}$$

$$\cancel{\omega_r} L \cdot \frac{1}{\cancel{\omega_r} C} = R^2 + \omega_r^2 L^2$$

$$\frac{L}{C} = R^2 + \omega_r^2 L^2$$

$$\omega_r^2 L^2 = \frac{L}{C} - R^2$$

$$\omega_r^2 = \frac{\cancel{L}}{C \cdot L^{\cancel{2}}} - \frac{R^2}{L^2}$$

$$\omega_r^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$\frac{1}{LC} >> \frac{R^2}{L^2}$ so $\omega_r = \frac{1}{\sqrt{LC}}$

Y is minimum so $Z = \frac{1}{Y}$ will be maximum

if Z is maximum so current I will be minimum.

Impedance at Resonance for parallel Resonance circuit

⇒ At Resonance the real part of admittance (from eqn A)

$$\frac{R}{R^2+X_L^2}$$

↑ from eqn 1

So impedance at resonance $Z_r = \frac{R^2+X_L^2}{R} = \frac{1}{Y_{real}}$

$$Z_r = \frac{X_L X_C}{R}$$

$$Z_r = \frac{\cancel{\omega_r} L \times \frac{1}{\cancel{\omega_r} C}}{R}$$

$$\boxed{Z_D = \frac{L}{CR} \quad \text{dynamic impedance}}$$

⇒ Comparision of series and parallel R-L-C Resonant circuits

Parameter	Series RLC circuit	Parallel RLC Circuit
Current	$I = \frac{V}{R}$ (maximum)	$I = \frac{V}{Z_D}$ (minimum)
Impedance	$Z = R$	$Z = L/CR$
Power factor	Unity (1)	Unity (1)
Resonant frequency	$f_r = \frac{1}{2\pi\sqrt{LC}}$	$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
Quality factor	$Q = \frac{\omega_r L}{R} = \frac{2\pi f_r L}{R}$	$Q = \frac{2\pi f_r L}{R}$
Magnification	Voltage (V_L, V_C)	Current (I_L, I_C)
	Acceptor circuit	Rejector circuit

Example:1 : A 5 uF capacitor is connected in series with the coil having an inductance of 50 mH. Calculate the frequency of resonance and resistance of the coil if a 60V source operating at resonance causes current of 1.5 A . What is power factor of the coil.

Resonant Frequency

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$f_r = \frac{1}{2\pi\sqrt{50\times10^{-3}\times5\times10^{-6}}}$$

$$f_r = 318.3\ Hz$$

$$f_r \simeq 318\ Hz$$

Impedance at resonance

$$Z = \frac{60}{1.5} = 40\,\Omega$$

$$Z^2 = R^2 + (X_L - X_C)^2$$

But at resonance

$$Z = R = 40\,\Omega$$

So $R = 40\,\Omega$

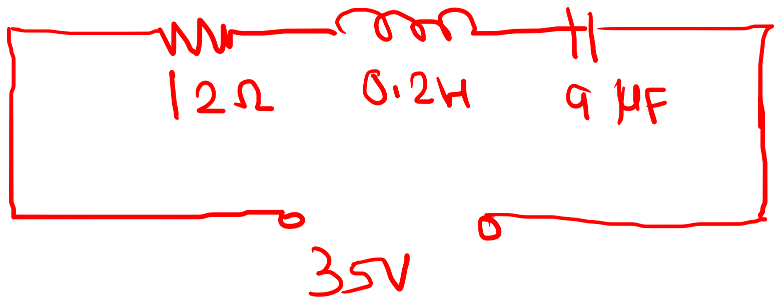
Quality factor (Q) $= \frac{X_L}{R} = \frac{2\pi f_r L}{R}$

$$Q = \frac{2\pi\times318\times50\times10^{-3}}{40}$$

$$\boxed{Q \simeq 2.5}$$

Ex:2 : An non-inductive resistance of 12 ohms, an inductance of 0.2 H and capacitor of 9 uF are connected in series calculate (i) resonance frequency (ii) current at resonance (iii) voltage across each component , when a voltage of 35 V at resonance frequency is applied to the whole circuit.

$\Rightarrow$



i) $$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{0.2 \times 9 \times 10^{-6}}}$$

$$f_r = 118.6 \simeq 119\,Hz$$

ii) Current at resonance (I_r)

Z = R at resonance

$Z = R = 12 \quad \therefore I_r = \frac{35}{12} = 2.9A$

iii) Voltage across each component

$$V_R = I_r \times R = 2.9 \times 12 = 34.8V$$

$$V_L = I_r \times X_L = 2.9 \times (2\pi \times 0.2 \times 119)$$

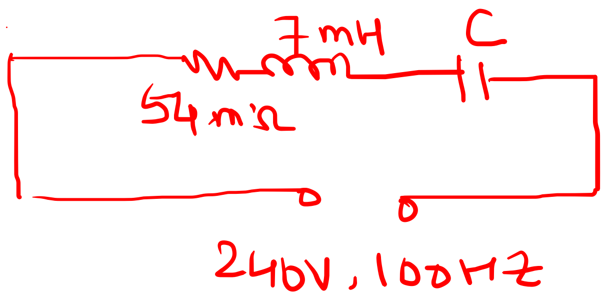
$$\boxed{V_L = 433.65V}$$

$$V_C = I_r \cdot X_C = 2.9 \times \frac{1}{2\pi \times 119 \times 9 \times 10^{-6}}$$

$$\boxed{V_C = 432.35V}$$

Ex-3: A series R-L-C circuit consisting of capacitor and a coil connected across 240 V, 100 Hz AC supply. If the coil has 54 milli-ohm resistance and 7 mH inductance. Calculate (i) value of the capacitor at 100 Hz resonance frequency (ii) quality factor of the coil (III) Half power frequencies

$\Rightarrow$



i) $\rightarrow$ $f_r = 100\,Hz$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$f_r^2 = \frac{1}{4\pi^2 LC}$$

$$C = \frac{1}{4\pi^2 f_r^2 L}$$

$$C = \frac{1}{4\pi^2 \times (100)^2 \times (7\times10^{-3})}$$

$$C = 3.62\times10^{-4} \simeq 362\,\mu F$$

ii) $$Q = \frac{\omega_r L}{R} = \frac{2\pi\times100\times7\times10^{-3}}{54\times10^{-3}}$$

$$Q = 81.45$$

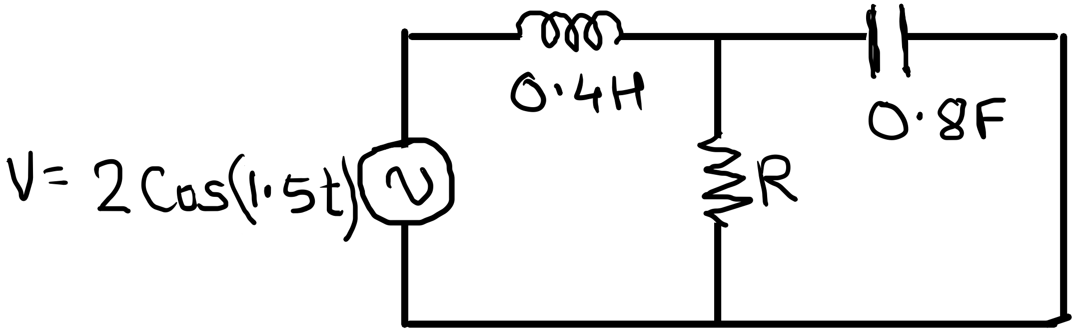
iii) Half power Frequencies

$$f_1 = f_r - \frac{R}{4\pi L} = 100 - \frac{54\times10^{-3}}{4\pi\times7\times10^{-3}} = 99.38\,Hz$$

$$f_2 = f_r + \frac{R}{4\pi L} = 100 + \frac{54\times10^{-3}}{4\pi\times7\times10^{-3}} = 100.61\,Hz$$

$$BW = f_2 - f_1 = 100.61 - 99.38 = 1.23\,Hz$$

Ex-4: Find value of R such that circuit below is at resonance.



Impedance of the circuit

⇒

$$Z = X_L + \frac{R X_C}{R + X_C}$$

$$Z = j\omega(0.4) + \frac{R \times \frac{1}{j\omega(0.8)}}{R + \frac{1}{j\omega(0.8)}}$$

given $\omega = 1.5$

$$Z = j(1.5 \times 0.4) + \frac{R \times \frac{1}{j1.5 \times 0.8}}{R + \frac{1}{j1.5 \times 0.8}}$$

$$Z = j0.6 + \frac{R/\cancel{j1.5 \times 0.8}}{\frac{jR\,1.5 \times 0.8 + 1}{\cancel{j1.5 \times 0.8}}}$$

$$Z = j0.6 + \frac{R}{1 + j1.2R}$$

$$Z = j0.6 + \frac{R(1 - j1.2R)}{1 + (1.2R)^2}$$

$$Z = \frac{R}{1 + (1.2R)^2} + j\left[0.6 - \frac{1.2R^2}{1 + 1.44R^2}\right]$$

At resonance Reactive terms is zero.

At resonance

$$0.6 - \frac{1.2R^2}{1+1.44R^2} = 0$$

$$\frac{0.6 + 0.86R^2 - 1.2R^2}{1+1.44R^2} = 0$$

$$0.6 - 0.34R^2 = 0$$

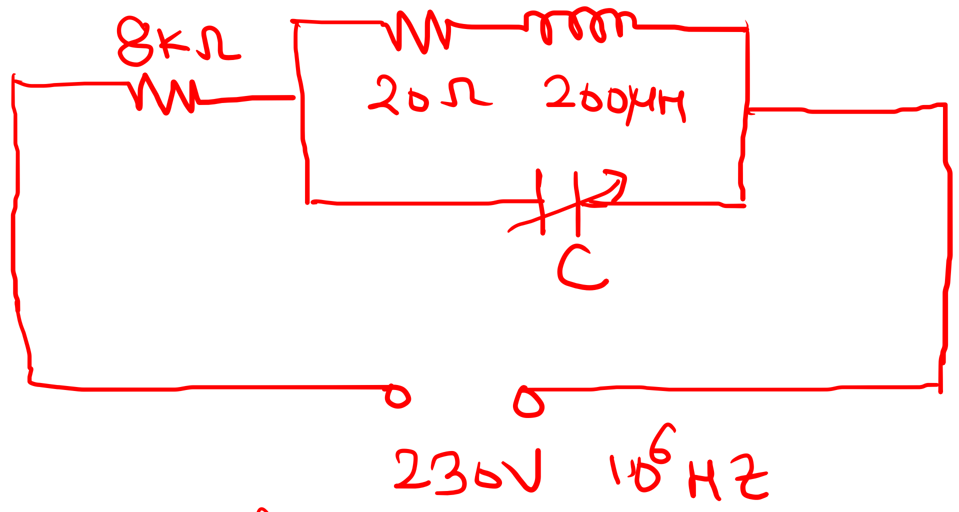
$$0.34R^2 = 0.6$$

$$R^2 = \frac{0.6}{0.34} =$$

$$\boxed{R = 1.33\,\Omega}$$

Q5. Find i) Value of capacitor (ii) quality factor (iii) Impedance at resonance (iv) Total current , in the network below is at resonance.

$\Rightarrow$



i) $\Rightarrow f_r = 10^6$ HZ

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

$$10^6 = \frac{1}{2\pi}\sqrt{\frac{1}{200\times10^{-6}\,C} - \frac{(20)^2}{2(200\times10^{-6})^2}}$$

$$(10^6)^2 = \frac{1}{4\pi^2}\left[\frac{1}{200\times10^{-6}\times C} - \frac{20^2}{2(200\times10^{-6})^2}\right]$$

$C \simeq 126.5$ PF

ii) $Q = \dfrac{2\pi f L}{R} = \dfrac{2\pi\times10^6\times200\times10^{-6}}{20}$

iii) Impedance at resonance

dynamic impedance $= Z_D = \dfrac{L}{CR}$

$$Z_D = \frac{200\times10^{-6}}{127\times10^{-12}\times20}$$

$Z_D = 78740\,\Omega$

$Z_T = 8000 + 78740 = 86740\,\Omega$

iv) Total Current

$$I_t = \frac{230}{86740} = 2.65\text{ mA}$$