Firm Dynamics Exercises

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Due Wednesday, July 12, 8:00 a.m.

Exercise 1. *Solve for the firm's optimal decisions given stochastic z and quadratic adjustment costs. Productivity shocks will follow an AR(1) process:*

$$\ln(z_{t+1}) = \rho \ln(z_t) + (1-\rho)\mu + u_t, \tag{1}$$

where $u_t \sim N(0, \sigma_z)$ Further, assume that prices for output and new capital can both be normalized to one: $p = p^k = 1$. The parameterization you should use is summarized in the table below.

Table 1: Parameterization

Parameter	Description	Value
α_k	Capital's share of output	0.297
α_l	Labor's share of output	0.650
δ	Depreciation rate	0.154
ψ	Coefficient on quadratic adjustment costs	1.080
w	Wage rate	0.700
r	Interest rate	0.040
σ_z	Std. deviation of disturbances to z	0.213
μ	Mean of $ln(z)$ process	0.000
ρ	Persistence of z process	0.7605
`sizez`	Number of grid points in z space	9

Tips:

1. *Pay attention to grid sizes. What kind of range do want for the values of k? Of z?*
2. *You'll need to approximate the continuous AR(1) process with something discrete.*
3. *If you use `Numba` (and I recommend that you do), be careful how you use `Numpy` matrix operations in a loop - it can slow things down substantially.*

Exercise 2. *Consider and extension to the model in Exercise 1 where, instead of the quadratic costs of adjustment, the firm faces a fixed costs to adjusting their capita. In particular, assumed that for any non-zero investment amount, the firm must pay a fixed cost that is proportional to its capital stock:*

$$c(k', k) = \begin{cases} \psi_1 * k, & \textit{if } I \neq 0; \\ 0, & \textit{if } I = 0; \end{cases} \tag{2}$$

Note that in this case, there is no first order condition as the per period flow to the firms is discontinuous because of the fixed capital adjustment cost. In addition, we want to think about the value function slightly differently.

Use the same calibration as in Exercise 1, but let $\psi_1 = 0.03$.

$$V(z, k) = \max \left[\underbrace{\pi(z, k) - I - \psi_1 k + \beta E_{z'|z} V(z', k')}_{\textit{Value if make investment}}, \underbrace{\pi(z, k) + \beta E_{z'|z} V(z', (1-\delta)k)}_{\textit{Value if don't invest}} \right] \tag{3}$$

What do you think the policy function will look like? Plot the $\frac{I}{k}$ over k and again over z. Does this look like you thought it would?

Tips:

1. *Pay attention to the grid for the capital stocks (see the Jupyter Notebook illustrating VFI for a description of how we formed this) and how to determine where on the grid you go if no investment is made.*

2. *You'll need a couple max operators in the VFI loop. You had one in the continuous adjustment cost case, now you'll need another as you represent Equation (3).*

Exercise 3. *Use the model with stochastic profitability shocks and quadratic adjustment costs parameterized in Exercise 1. Solve this with the Coleman policy function iteration (PFI) method you learned from John Stachurski.*

- *Plot the value functions found from your VFI and PFI solutions together. How do they compare?*

- *Plot the policy functions from your VFI and PFI solutions together. How do they compare?*

- *How do these methods compare in terms of time to compute the model?*

Tips:

1. *I haven't done this before, but I *think* it can be done!*

BONUS:

Exercise 4. *Use the the model you've solved above (either the one with convex adjustment costs in Exercise 1 or with non-convex adjustment costs in Exercise 2) and solve for the steady state of a general equilibrium version of the model. Let the the household sector be described by a representative agent who solves:*

$$\max_{\{C_t,L_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \tag{4}$$

Assume the per period utility function is separable and given by:

$$u(C, L) = \ln(c) - \frac{hL^2}{2}, \tag{5}$$

where h is a scaling parameter on the disutility of labor. Assume that the household parameters are given by $\beta = 0.96$ and $h = 6.616$. The household's per-period budget constraint is thus:

$$C_t + B_{t+1} = (1 + r_t)B_t + w_t L_t, \tag{6}$$

where B_{t+1} are household claims on risk free bonds that earn interest r_{t+1}. These bonds are in zero net supply.

In general equilibrium, all markets will clear. That is $B_t = 0$ (since zero net supply of these bonds), labor demand from the firms equals labor supply from the representative household, and goods demand (for consumption by households and investment by firms) equals the supply of goods (i.e., total output produced). You can uses these market clearing conditions to help determine the factor prices in general equilibrium (in the steady-state, these will be $\bar{r}$ and $\bar{w}$). Remember that Walras' Law means that you only need to solve for market clearing in two of the three markets since if those clear, the third will as well.

What are these factor prices in equilibrium? What does the equilibrium stationary distribution look like?

Tips:

1. *You can solve for $\bar{r}$ quite easily from the household's necessary conditions.*
2. *Your general equilibrium will be found by some fixed point process. Be sure that your grid space for capital is not binding as you iterate over the factor price(s) to find a fixed point.*
3. *You'll want to modularize your code because functions will be called repeatedly through the solution algorithm.*
4. `Numba` *will be your friend here.*

5. *Recall the national accounting identify: $Y = C + I$ (where Y is output, C is consumption, and I invesmtent).*
6. *To find the zero for the market clearing condition(s) you'll want to use a root finder from `Scipy` or write your own algorithm.*