Problem Set 4

OSM Lab-Math
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Problem 1 (6. 1)
The standard form is

$$\begin{aligned} \min \ & -e^{-w^T x} \\ \text{s.t.} \ & w^T A w - w^T A y - w^T x \leq -a \\ & y^T w - w^T x = b \end{aligned}$$

Problem 2 (6. 5)
Denote the quantity for knobs as x, the quantity for milk cartons as y.

$$\begin{aligned} \min \ & -0.05x - 0.07y \\ \text{s.t.} \ & 3x + 4y \leq 240,000 \\ & x + 2y \leq 100 \end{aligned}$$

Problem 3 (6. 6)
The Jaconbian matrix is

$$(6xy + 4y^2 + y, \ 3x^2 + 8xy + x)$$

Setting each entry to zero, we get the critical values are $(x, y) = (0, 0), (0, -\frac{1}{4}), (-\frac{1}{3}, 0), (-\frac{1}{9}, -\frac{1}{12})$.
The Hessian matrix is

$$\begin{pmatrix} 6y & 6x + 8y + 1 \\ 6x + 8y + 1 & 8x \end{pmatrix}$$

At $(0, 0)$, the determinant is zero, so the critical value is a saddle point.
At $(0, \frac{1}{4})$, $(-\frac{1}{3}, 0)$, and $(-\frac{1}{9}, -\frac{1}{12})$, the determinants are greater than zero, and traces are less than zero. So these three values are local maxima.

Problem 4 (6.11)
Suppose the first guess is x_0

$$\begin{aligned} f'(x_0) &= 2ax_0 + b \\ f''(x_0) &= 2a \end{aligned}$$

So the Newton Method gives us $-\frac{b}{2a}$. Plugging this value into the equation, we know that $-\frac{b}{2a}$ is a critical value. Moreover, since the second derivative is always greater than zero, we know the critical value is a minimizer.