COE-C2003 Basic course on fluid mechanics, S2021

Round 4: The energy equation and losses (return at the latest by Thu 14.10. at 13:00 o'clock)

Each problem (1-4) will be assessed on a scale of 0-3. Remember to explain the different stages in the solution. More detailed information can be found from MyCourses.

1. Water flows steadily from one location to another in the inclined pipe shown in Fig. 1. At one section, the static pressure is 55 kPa. At the other section, the static pressure is 34 kPa. Which way is the water flowing ? Explain.

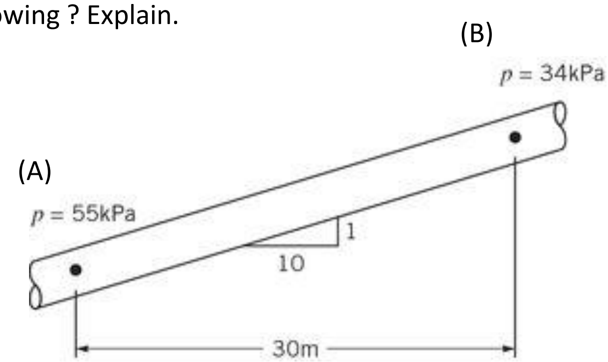


Figure 1: Problem 1

Answer:

To determine the direction of water flow we apply the energy equation (Eq. 5.56) for flow from sections (A) to (B) and flow from sections (B) to (A). The loss obtained with Eq. 5.56 is positive for the correct flow direction but negative for the incorrect flow direction.

For flow from sections (A) to (B) Eq. 5.56 leads to

$$loss = \frac{p_A - p_B}{\rho} + \overbrace{\frac{V_A^2 - V_B^2}{2}}^{=0} + g(z_A - z_B) + \overbrace{w_{shaft\_net\_in}}^{=0}$$

Nothing is said about the velocity, we assume it is the same between points (A) and (B).

Inserting values we get

$$loss = \frac{(55-34)kPa}{999\,\frac{kg}{m3}} + 9.81\,\frac{m}{s2} \times (-3m)\,\frac{N}{kg\,\frac{m}{s2}} = -8.41\,\frac{Nm}{kg}$$

For the flow between sections (B) to (A) 5.56 leads to

$$loss = \frac{p_B - p_A}{\rho} + g(z_B - z_A)$$

Or

$$loss = \frac{(34-55)kPa}{999\frac{kg}{m3}} + 9.81\frac{m}{s2} \times (3m)\frac{N}{kg\frac{m}{s2}} = +8.41\ \frac{Nm}{kg}$$

<u>The water flow is from section (B) to section (A)</u>

2 What is the maximum possible power output of the hydroelectric turbine shown in Fig. 2 ?

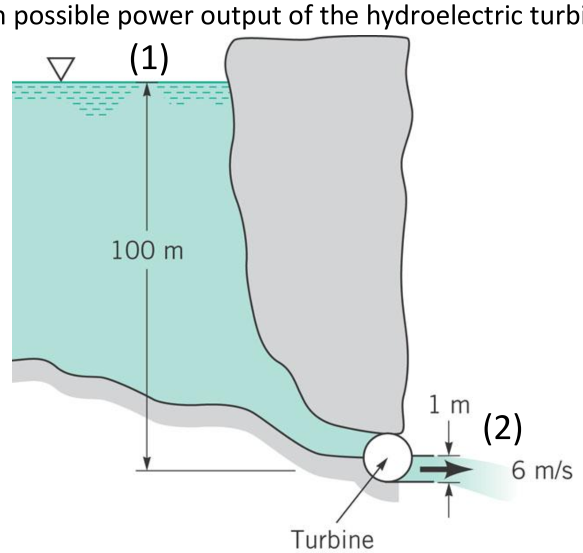


Figure 2. Problem 2.

Answer:

For the flow from section (1) to section (2) Eq. 5.56 yields

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 + w_{shaft\_net\_in} - loss \qquad (1)$$

Since $p_1 = p_2 = p_{atm}$ and $w_{shaft\_net\_in} = -w_{shaft\_net\_out}$

Eq. 1 can be expressed as

$$w_{shaft\_net\_out} = g(z_1 - z_2) - \frac{V_2^2}{2} - loss$$

The maximum work or power output is achieved when loss = 0.
Thus

$$\dot{W}_{shaft\_net\_out\_maximum} = \dot{m}w_{shaft\_net\_out\_maximum} = \dot{m}[g(z_1 - z_2) - \frac{V_2^2}{2}]$$

Now

$$\dot{m} = \rho V_2 A_2 = \rho V_2 \frac{\pi D_2^2}{4} = 999\ \frac{kg}{m3}\left(6\frac{m}{s}\right)\frac{\pi(1m)^2}{4} = 4707.6\frac{kg}{s}$$

And

$$\dot{W}_{shaft\_net\_out\_maximum} = 4707.6\frac{kg}{s}\left[9.81\ \frac{m}{s2}100m - \frac{6\ m/s^2}{2}\right]$$

$$\dot{W}_{shaft\_net\_out\_maximum} = 4.53 \times 10^6\frac{Nm}{s} = 4.53 \times 10^6 W = 4.53\ MW$$

3 Gasoline (SG=0.68) flows through a pump at 0.12 m^3/s as indicated in Fig. 3. The loss between sections (1) and (2) is equal to $0.3V_1^2/2$. What will the difference in pressures between sections (1) and (2) be if 20 kW is delivered by the pump to the fluid ?

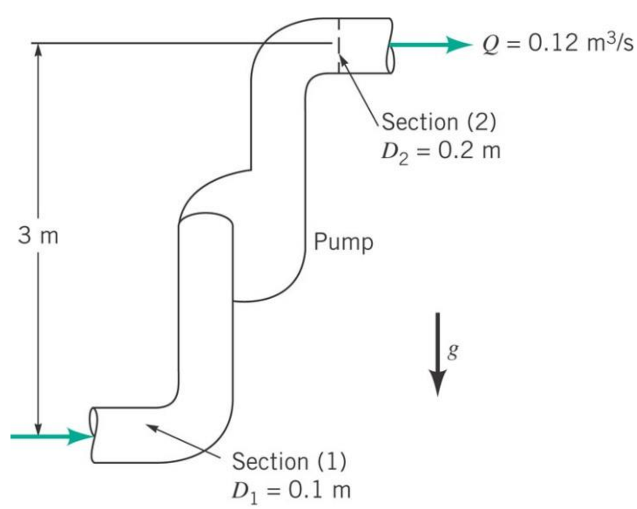


Figure 3. Problem 3

Answer:

From Eq. 5.56 we get for the flow from section (1) to section (2)

$$p_1 - p_2 = \rho\left[\frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) - w_{shaft\_net\_in} + loss\right] \qquad (1)$$

From the volume flowrate we obtain

$$V_2 = \frac{Q}{A_2} = \frac{Q}{\frac{\pi D_2^2}{4}} = \frac{0.12\,\frac{m^3}{s}}{\frac{\pi(0.2m)^2}{4}} = 3.819\ \frac{m}{s}$$

And from the conservation of mass it follows

$$V_1 = V_2\frac{A_2}{A_1} = V_2\frac{D_2^2}{D_1^2} = 3.819\ \frac{m}{s}\frac{(0.2m)^2}{(0.1m)^2} = 15.276\ \frac{m}{s}$$

Also

$$w_{shaft\_net\_in} = \frac{\dot{W}_{shaft\_net\_in}}{\rho Q} = \frac{20000\ \frac{Nm}{s}}{0.68 \times 999\ \frac{kg}{m3} \times 0.12\,\frac{m^3}{s}} = 245.3\ \frac{Nm}{kg}$$

And the loss is

$$loss = 0.3\frac{V_1^2}{2} = \frac{0.3 \times \left(15.276\,\frac{m}{s}\right)^2}{2} = 35.0\ \frac{Nm}{kg}$$

From Eq. (1) then

$$p_1 - p_2 = 0.68 \times 999\frac{kg}{m3}\left[\frac{\left(3.819\,\frac{m}{s}\right)^2 - \left(15.276\,\frac{m}{s}\right)^2}{2} + \left(9.81\,\frac{m}{s^2}\right)3m - 245.3\frac{Nm}{kg} + 35.0\frac{Nm}{kg}\right]$$

Or

$$\underline{p_1 - p_2 = -197176.5\ \frac{N}{m^2} = -197\ kPa}$$

4. You will obtain numerical data from the course web pages (MyCourses: Exercises, Round 4) In the text file, there are 2 columns and 201 rows of data. Read this file into Matlab. Multiply the 2^{nd} column data with your student number's last digit (if it is zero, then multiply with 10).

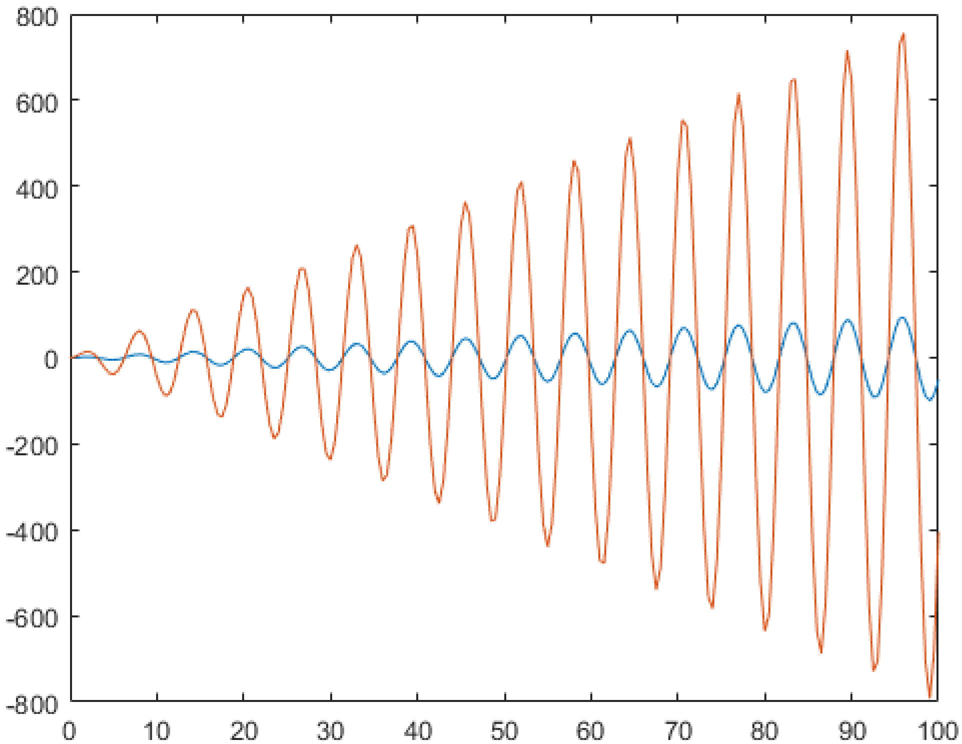
Plot this new data to the same figure with the original data (plot, hold on). Save the new data to a new text file (e.g. dlmwrite('newdata.txt',M,'delimiter','\t'). If needed, google 'matlab dlmwrite'. In the answer, show the coding, the figure, and list the new data (201 lines will take about 3 pages depending on the font size). **NOTE: again, use the New script when you make the code. Do not do the coding in the command window (a typical mistake made by many).** This will also help the assistant to run the code if necessary.

Answer:

```
clear all
% Load data
aa=load('data.txt');

plot(aa(:,1),aa(:,2))
hold on
% Multiply 2. column with the last digit in the student number (8)
d=aa(:,2).*8;
plot(aa(:,1),d);
M(:,1)=aa(:,1);
M(:,2)=d;

dlmwrite('data_ossi.txt',M,'delimiter','\t');
```



Original and the new data plotted.

The new data listed below:

0	0
0.5	1.9177
1	6.7318
1.5	11.97
2	14.549
2.5	11.97
3	3.3869
3.5	-9.8216
4	-24.218
4.5	-35.191
5	-38.357
5.5	-31.044
6	-13.412
6.5	11.186
7	36.791
7.5	56.28
8	63.319
8.5	54.297
9	29.673
9.5	-5.7115
10	-43.522
10.5	-73.894
11	-88
11.5	-80.544
12	-51.511
12.5	-6.6322
13	43.698
13.5	86.808
14	110.95
14.5	108.45
15	78.034
15.5	25.602
16	-36.852
16.5	-93.952
17	-130.75
17.5	-136.58
18	-108.14
18.5	-50.687

19	22.782
19.5	94.464
20	146.07
20.5	163.48
21	140.56
21.5	81.12
22	-1.5578
22.5	-87.688
23	-155.7
23.5	-187.64
24	-173.87
24.5	-115.9
25	-26.47
25.5	73.248
26	158.62
26.5	207.62
27	206.58
27.5	153.83
28	60.683
28.5	-51.016
29	-153.96
29.5	-222.08
30	-237.13
30.5	-193.52
31	-100.2
31.5	21.162
32	141.17
32.5	229.81
33	263.98
33.5	233.46
34	143.91
34.5	15.866
35	-119.89
35.5	-229.76
36	-285.63
36.5	-272.06
37	-190.49
37.5	-59.34
38	90.096
38.5	221.14
39	300.7
39.5	307.67
40	238.44
40.5	108.26
41	-52.028
41.5	-203.37
42	-307.95
42.5	-338.67
43	-286.13
43.5	-161.41
44	6.231
44.5	176.18
45	306.33
45.5	363.49
46	331.86
46.5	217.32
47	46.463
47.5	-139.58
48	-295.01
48.5	-380.67
49	-373.87
49.5	-274.38
50	-104.95
50.5	93.88
51	273.46
51.5	388.92
52	410.44
52.5	330.84
53	167.87
53.5	-39.714
54	-241.4
54.5	-387.16
55	-439.9
55.5	-384.84
56	-233.66
56.5	-21.989
57	198.89
57.5	374.53
58	460.7
58.5	434.52
59	300.54
59.5	90.016
60	-146.31
60.5	-350.47
61	-471.46
61.5	-478.02
62	-366.63
62.5	-162.9

63	84.344
63.5	314.72
64	471.06
64.5	513.56
65	429.95
65.5	238.92
66	-14.019
66.5	-267.36
67	-458.56
67.5	-539.47
68	-488.47
68.5	-316.19
69	-63.362
69.5	208.79
70	433.38
70.5	554.29
71	540.2
71.5	392.66
72	146.2
72.5	-139.77
73	-395.23
73.5	-556.76
74	-583.21
74.5	-466.21
75	-232.67
75.5	61.367
76	344.19
76.5	545.91
77	615.7
77.5	534.63
78	320.72
78.5	24.998
79	-280.68
79.5	-521.07
80	-636.09
80.5	-595.79
81	-408.17
81.5	-117.63
82	205.48
82.5	481.92
83	642.99
83.5	647.6
84	492.7
84.5	214.58
85	-119.73
85.5	-428.5
86	-635.34
86.5	-688.1
87	-571.98
87.5	-313.64
88	24.921
88.5	361.22
89	612.37
89.5	715.55
90	643.68
90.5	412.49
91	77.159
91.5	-280.88
92	-573.69
92.5	-728.44
93	-705.52
93.5	-508.65
94	-184.43
94.5	188.66
95	519.28
95.5	725.56
96	755.39
96.5	599.59
97	294.58
97.5	-86.112
98	-449.53
98.5	-706.03
99	-791.37
99.5	-682.81
100	-405.1