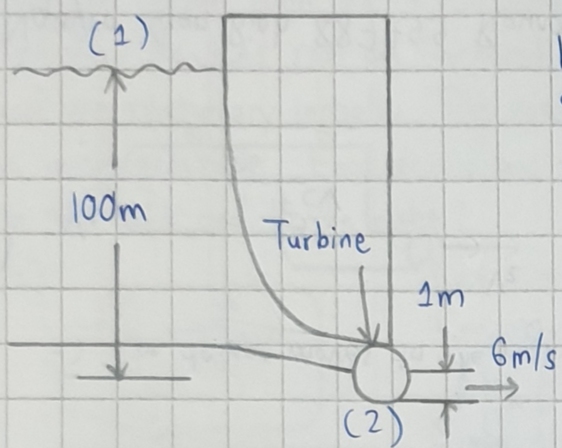
Nguyen Xuan Binh 887799 Round 4 Problem 2



What is the maximum possible power output of the hydroelectric turbine ?

Since both head pump and head loss are created by the turbine, the extended Bernoulli equation is

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_p + h_\ell$$

At (1), water is static $\Rightarrow v_1 = 0$

Both (1) and (2) are under same atmospheric pressure

$\Rightarrow p_1 = p_2$

Let $z_1 = 0 \Rightarrow z_2 = -100m$. The extended Bernoulli equation is simplified as :

$$0 = \frac{v_2^2}{2g} + z_2 + h_p + h_\ell \Rightarrow h_p = -\frac{v_2^2}{2g} - z_2 - h_\ell \Rightarrow h_p = -\frac{6^2}{2 \times 9.81} + 100 - h_\ell$$

$\Rightarrow h_p = 98.165 - h_\ell \Rightarrow$ Power of the turbine : $\dot{W} = \dot{m} g h_p = \dot{m} g (98.165 - h_\ell)$

$\Rightarrow \dot{W}_{max}$ when $h_\ell = 0$, although in reality zero friction rarely occurs

$$\dot{m} = \rho A v_2 = \rho \frac{1}{4} \pi \, m^2 \cdot v_2 = 1000 \, kg/m^3 \times 1/4 \pi \, m^2 \cdot 6 m/s = 1500\pi \; (kg/s)$$

$$\Rightarrow \dot{W}_{max} = 98.125 \, \dot{m} g = 98.125 \times 1500\pi \times 9.81 \, m/s^2 = 4536175.08 \, W \text{ (answer)}$$