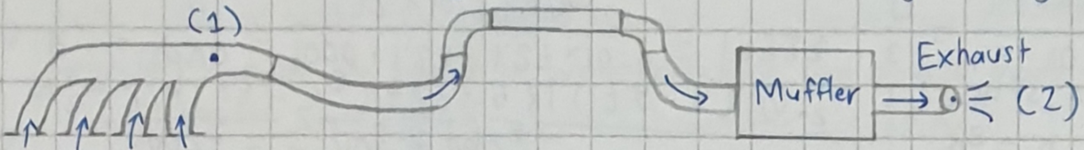
Nguyen Xuan Binh 887799 Round 6 Problem 2

The exhaust from a car's engine flows through a complex pipe system. Assume that pressure drop through this system is Δp_1 when the engine is idling at 1000 rpm. Estimate the pressure drop in terms of Δp_1 with the engine at 3000 rpm when driving on a highway. List all assumptions



Intake manifold

Bernoulli extended equation: $p_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2 + \frac{\rho V^2}{2}\left(f\frac{L}{D} - \Sigma K_L\right)$

Since (1) and (2) are near the intake and exhaust, they can approximate the external air flow $\Rightarrow V_1 \approx V_2 \approx V_{external}$ (since the car is at rest)

According to the figure, $z_1 \approx z_2$. The equation can be simplified as

$$p_1 = p_2 + \frac{\rho V^2}{2}\left(f\frac{L}{D} - \Sigma K_L\right) \Rightarrow p_1 - p_2 = \frac{\rho V_I^2}{2}\left(f_I\frac{L_I}{D_I} - \Sigma K_{L_I}\right) = \Delta p_1 \text{ at } 1000 \text{ rpm}$$

When the car runs at a steady speed on highway, its assumed that $V_1 \approx V_2 \approx V_{external}$

$$\Rightarrow \Delta p_2 = \frac{1}{2}\rho V_{II}^2\left(f_{II}\frac{L}{D} - \Sigma K_L\right) \Rightarrow \frac{\Delta p_1}{\Delta p_2} = \frac{\rho_I V_I^2}{\rho_{II} V_{II}^2}\left(f_I\frac{L_I}{D_I} - \Sigma K_{L_I}\right)\Big/\left(f_{II}\frac{L_{II}}{D_{II}} - \Sigma K_{L_{II}}\right)$$

Since at both cases, they go through the same pipe $\Rightarrow$ same bends and inlet $\Rightarrow \Sigma K_{L_I} = \Sigma K_{L_{II}}$ and $L_I = L_{II}$, $D_I = D_{II}$. Since it's the same air content going through the engine $\Rightarrow \rho_I = \rho_{II}$

$\Rightarrow \frac{\Delta p_1}{\Delta p_2} = \frac{V_I^2}{V_{II}^2}$

Also, the flow at inlet and exhaust is highly turbulent $\Rightarrow$ According to Moody diagram, f stays nearly constant $\Rightarrow f_I = f_{II}$

At 3000 rpm, the flow must be faster in the pipe compared to being idle at 1000 rpm

$\Rightarrow$ There's correlation between pipe flow velocity and revolutions per minute

- In the case of linear increase $\Rightarrow \frac{V_I}{V_{II}} = \frac{1000}{3000} = \frac{1}{3} \Rightarrow \frac{\Delta p_1}{\Delta p_2} = \frac{1^2}{3^2} = \frac{1}{9} \Rightarrow \Delta p_2 = 9\Delta p_1$ (answer)
- In the case of quadratic increase $\Rightarrow \frac{V_I}{V_{II}} = \frac{1000^2}{3000^2} = \frac{1}{9} \Rightarrow \frac{\Delta p_1}{\Delta p_2} = \frac{1^2}{9^2} = \frac{1}{81} \Rightarrow \Delta p_2 = 81\Delta p_1$ (answer)