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The drag on a sphere moving in a fluid is a function of sphere diameter, velocity, fluid viscosity and density. Lab tests on 10-cm diameter sphere were performed in a water tunnel and model data are in Fig. 1. For these tests, $\mu_{water} = 1 \times 10^{-3}$ kg/m.s, $\rho_{water} = 999$ kg/m^3

Estimate the drag on 2m diameter balloon moving in air at velocity 1.5 m/s. Assume air viscosity is 1.8×10^{-5} kg/m.s and density of 1.2 kg/m^3

Drag force is a function: $\mathcal{D} = f(D, V, \mu, \rho)$

- Relevant variables: $k = 5$

  $\mathcal{D}: MLT^{-2}$, $D: L$, $V: LT^{-1}$, $\mu: ML^{-1}T^{-1}$, $\rho: ML^{-3}$
- Basic dimensions: $r = 3$: M, L, T
- Number of Pi-terms: $k - r = 5 - 3 = 2$
- Repeating variables: D, V, ρ
- Dimensionless variables:

$$\Pi_1 = \mathcal{D} D^a V^b \rho^c = MLT^{-2} L^a L^b T^{-b} M^c L^{-3c} = M^0 L^0 T^0$$

$$\Rightarrow a = -2, b = -2, c = -1 \Rightarrow \Pi_1 = \frac{\mathcal{D}}{D^2 V^2 \rho}$$

$$\Pi_2 = \mu D^a V^b \rho^c = ML^{-1}T^{-1} L^a L^b T^{-b} M^c L^{-3c} = M^0 L^0 T^0$$

$$\Rightarrow a = -1, b = -1, c = -1 \Rightarrow \Pi_2 = \frac{\mu}{DV\rho}$$ (Inverse of Reynold's number)

$$\Rightarrow \frac{\mathcal{D}}{D^2 V^2 \rho} = \tilde{\phi}\left(\frac{DV\rho}{\mu}\right)$$ $\Rightarrow$ We can find $\mathcal{D}$ if we know V by looking at the graph

Assume dimensionless parameters are similar between prototype and model sphere

$$\Rightarrow \frac{D_m V_m \rho_m}{\mu_m} = \frac{D V \rho}{\mu} \Rightarrow V_m = \frac{D}{D_m} \frac{\mu_m}{\mu} \frac{\rho}{\rho_m} V$$

$$\Rightarrow V_m = \frac{2m}{10 \times 10^{-2}\ m} \cdot \frac{1 \times 10^{-3}\ kg/ms}{1.8 \times 10^{-5}\ kg/ms} \cdot \frac{1.2\ kg/m^3}{998\ kg/m^3} \cdot 1{,}5\ m/s = \frac{1000}{499}\ m/s \approx 2m/s$$

According to the graph, at $V_m = 2m/s$, model drag $\mathcal{D}_m$ is approximately 8 N

Dimensionless parameters are similar $\Rightarrow \dfrac{\mathcal{D}_m}{D_m^2 V_m^2 \rho_m} = \dfrac{\mathcal{D}}{D^2 V^2 \rho}$

$$\Rightarrow \mathcal{D} = \frac{D^2}{D_m^2} \frac{V^2}{V_m^2} \frac{\rho}{\rho_m} \mathcal{D}_m = \frac{2^2\ m}{(10 \times 10^{-2})^2 m} \cdot \frac{(1.5)^2\ m/s}{(1000/499)^2\ m/s} \cdot \frac{1.2\ kg/m^3}{998\ kg/m^3} \cdot 8N$$

$\Rightarrow \mathcal{D} = 2.15568$ N (Drag force on prototype sphere - answer)