COE-C2003 Basic course on fluid mechanics, S2021

Round 5: Dimensional analysis (return at the latest by Thu 21.10. at 13:00 o'clock)

Each problem (1-4) will be assessed on a scale of 0-3. Remember to explain the different stages in the solution. More detailed information can be found from MyCourses.

1. The pressure increase Δp in a pump can be described with the connection Δp = f(D, ρ, w, Q), where *D* is the impeller diameter in the pump, ρ the liquid density, w the rotational speed, and *Q* is the flow rate. A) List the phases needed in the definition of the Π –terms, and B) Define a proper set of dimensionless variables for the problem.

Answer:

A) Using dimensional analysis and the method of the repeating variables, we obtain the parameters. The phases in determining the Pi-terms are:
   1) List the relevant variables
   2) Define the basic dimensions
   3) Define the number of Pi-terms
   4) Select the repeating variables
   5) Form the Pi-terms by selecting one non-repeating variable at a time. So each Pi-term is a product of one non-repeating variable and the repeating variables to proper powers.
   6) Check the result that it is non-dimensional
   7) Express the Pi-terms as a dimensionless connection

B)

1)

$\Delta p = f(D, \rho, w, Q)$ so that k=5.

2) Basic dimensions

   $\Delta p = ML^{-1}T^{-2}$
   $D = L$
   $\rho = ML^{-3}$
   $w = T^{-1}$
   $Q = L^3T^{-1}$

   So r=3

3) Number of the Pi-terms k – r = 5 – 3 = 2
4) Repeating variables, we choose $D, \rho, and\ w$

5)

$$\prod\nolimits_1 = \Delta p\, D^a \rho^b w^c = M^0 L^0 T^0$$
$$ML^{-1}T^{-2}L^1(ML^{-3})^b(T^{-1})^c = M^0 L^0 T^0$$

$$MM^b = M^0$$
$$L^{-1}L^a L^{-3b} = L^0$$
$$T^{-2}T^{-c} = T^0$$

$$1 + b = 0 \rightarrow b = -1$$
$$-1 + a - 3b = 0 \rightarrow a = -2$$
$$-2 - c = 0 \rightarrow c = -2$$

$$\prod\nolimits_1 = \Delta p\, D^{-2}\rho^{-1}w^{-2} = \frac{\Delta p}{D^2 \rho w^2}$$

6) Check the non-dimensionality

$$\frac{\frac{N}{m^2}}{m^2 \frac{kg}{m^3 \frac{1}{s^2}}} = \frac{\frac{kg\, m}{s^2}}{m^4 \frac{kg}{m^3} \frac{1}{s^2}} = 1$$

The other Pi-term

$$\prod\nolimits_2 = Q\, D^a \rho^b w^c = M^0 L^0 T^0$$

$$L^3 T^{-1} L^a (ML^{-3})^b (T^{-1})^c = M^0 L^0 T^0$$

$$L^3 L^a L^{-3b} = L^0$$
$$T^{-1}T^{-c} = T^0$$
$$M^b = M^0$$

$$3 + a - 3b = 0 \rightarrow a = -3$$
$$-1 - c = 0 \rightarrow c = -1$$
$$b = 0$$

$$\prod\nolimits_2 = Q\, D^{-3}\rho^0 w^{-1} = \frac{Q}{D^3 w}$$

Check the non-dimensionality

$$\frac{\frac{m^3}{s}}{\frac{m^3}{s}} = 1$$

7)

$$\prod_1 = \Phi\left(\prod_2\right)$$

$$\frac{\Delta p}{D^2 \rho w^2} = \Phi\left(\frac{Q}{D^3 w}\right)$$

2. To test the aerodynamics of a new prototype automobile, a scale model will be tested in a wind tunnel. For dynamic similarity, it will be required to match the Reynolds numbers between the model and the prototype. Assuming that you will be testing a $1/10$-scale model and both the model and the prototype will be exposed to standard air pressure, will it be better for the wind tunnel air to be colder or hotter than standard sea-level air temperature of $15°C$? Why ? Hint: you can use the book Table B4.

Answer:

For an external flow, we know that Re-number similarity is very important. Thus

$Re_m = Re_p$, where subscripts m and p indicate model and prototype, respectively.

We get $\left(\frac{Vl}{\nu}\right)_m = \left(\frac{Vl}{\nu}\right)_p$ or

$V_m = \frac{\nu_m}{\nu_p}\frac{l_p}{l_m}V_p = 10\frac{\nu_m}{\nu_p}V_p$ since $l_m = \frac{1}{10}l_p$

If the wind tunnel air is at standard sea-level conditions, then

$$\nu_m = \nu_p$$

and $V_m = 10V_p$.

Hence if $V_p = 100\ km/h$, then $V_m = 1000\ km/h$. This is too large for any typical wind tunnel to handle. For one thing, at these velocities compressibility effects become important. At 100 km/h they are not.

Assume tests are conducted with $\frac{\nu_m}{\nu_p} < 1$ so that more realistic wind tunnel velocities are used. From the table B4 at T=15C, $\nu_p = 1.47 \times 10^{-5}\, {m^2}/{s}$. When T=100C, $\nu_m = 2.29 \times 10^{-5}\, {m^2}/{s}$. When T= -40C, $\nu_m = 1.04 \times 10^{-5}\, {m^2}/{s}$.

When looking at the ratio $\frac{\nu_m}{\nu_p}$, when it is < 1, then the model velocity would be lower. Hence it would be ***beneficial to have a cold wind tunnel.***

However, even at -40C, the ratio is only $\frac{\nu_m}{\nu_p} = 0.707$ indicating a (too high) velocity of

$V_m = 10\frac{\nu_m}{\nu_p}V_p = 10 \times 0.707 \times 100\frac{km}{h} = 707\ km/h.$

3. The drag on a sphere moving in a fluid is known to be a function of the sphere diameter, velocity, and the fluid viscosity and density. Laboratory tests on a 10-cm diameter sphere were performed in a water tunnel and some model data are plotted in Fig. 1. For these tests the viscosity of the water was $1 \times 10^{-3}\frac{kg}{ms}$ and the water density was $998\frac{kg}{m^3}$. Estimate the drag on an 2.0 m diameter balloon moving in air at a velocity of 1.5 m/s. Assume the air to have a viscosity of $1.8 \times 10^{-5}\frac{kg}{ms}$ and density of $1.2\frac{kg}{m^3}$.

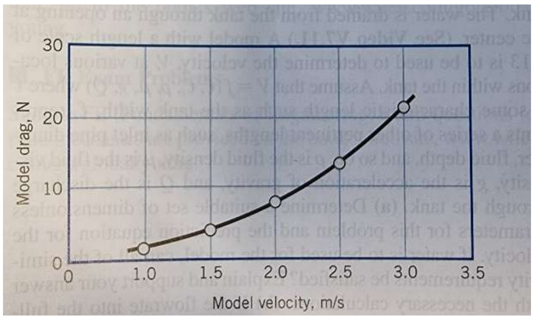


Figure 1.

<u>Answer:</u>

$$\mathcal{D} = f(d, V, \rho, \mu)$$

Where $\mathcal{D} = drag\ force$, d= sphere diameter, V=velocity, $\rho = density$, and $\mu = viscosity$. From the pi-theorem we get that 2 pi-terms are required which are (k=5, r=3)

$$\frac{\mathcal{D}}{\rho V^2 d^2} = \phi\left(\frac{\rho V d}{\mu}\right)$$

Or

$$V_m = \frac{\mu_m}{\mu}\frac{\rho}{\rho_m}\frac{d}{d_m}V$$

$$= \frac{1\times10^{-3}}{1.8\times10^{-5}}\frac{1.2\frac{kg}{m3}}{998\frac{kg}{m3}}\frac{2m}{0.1m}1.5\frac{m}{s} = 2.0\frac{m}{s}$$

From the Fig. 1, for $V_m = 2.0\ m/s$, $\mathcal{D}_m = 7.75\ N$.

Since

$$\frac{\mathcal{D}}{\rho V^2 d^2} = \frac{\mathcal{D}_m}{\rho_m V_m^2 d_m^2}$$

Or

$$\mathcal{D} = \frac{\rho}{\rho_m}\frac{V^2}{V_m^2}\frac{d^2}{d_m^2}\mathcal{D}_m$$

Thus

$$\mathcal{D} = \frac{1.2\ kg/m3}{998kg/m3}\frac{(1.5m/s)^2}{(2.0\ m/s)^2}\frac{(2m)^2}{(0.1m)^2}7.75\ N = 2.09\ N \sim 2.1\ N$$

4 In the nozzle of an internal combustion engine, the liquid pressure is transformed into liquid velocity. In the Figure 2, there is shown the sectional cut of a nozzle tip. By using the Bernoulli equation, calculate the liquid velocities right after the nozzle exit (Section (2), V_2). We can assume that the velocity before the nozzle is very low (Section (1), V_1,). By using Matlab, show the injection velocity by using injection pressures between 500-2000 bar with 100 bar steps. In addition, the backpressure (P_2, Section (2)) in the combustion chamber is either 10 bar, 100 bar, or 200 bar. **NOTE, here is a free jet that we typically assume to be at the atmospheric pressure of 1 bar. But now, the free jet enters into a combustion chamber of an engine with the pressure of either 10, 100, or 200 bars.** Using literature, find a realistic value for diesel fuel density at room temperature.

Plot the injection velocity as a function of injection pressure with Matlab into a single figure. Make three curves into the plot, one for each backpressure (combustion chamber pressure) of 10, 100, or 200 bar. Add also axis labels (xlabel, ylabel) and legend explaining the different backpressures.

Attach the code you used to the answer. NOTE, again use the Editor window (m-file), not the Command window. This will allow the assistant to run the code. You may also use the greek style in the legend: `legend('\rho_{2}=10 bar',...).`

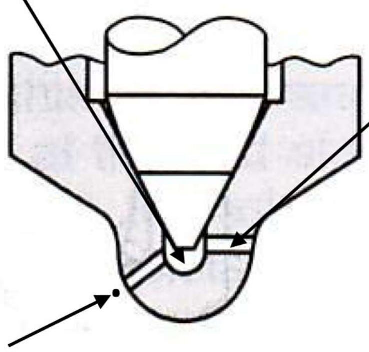
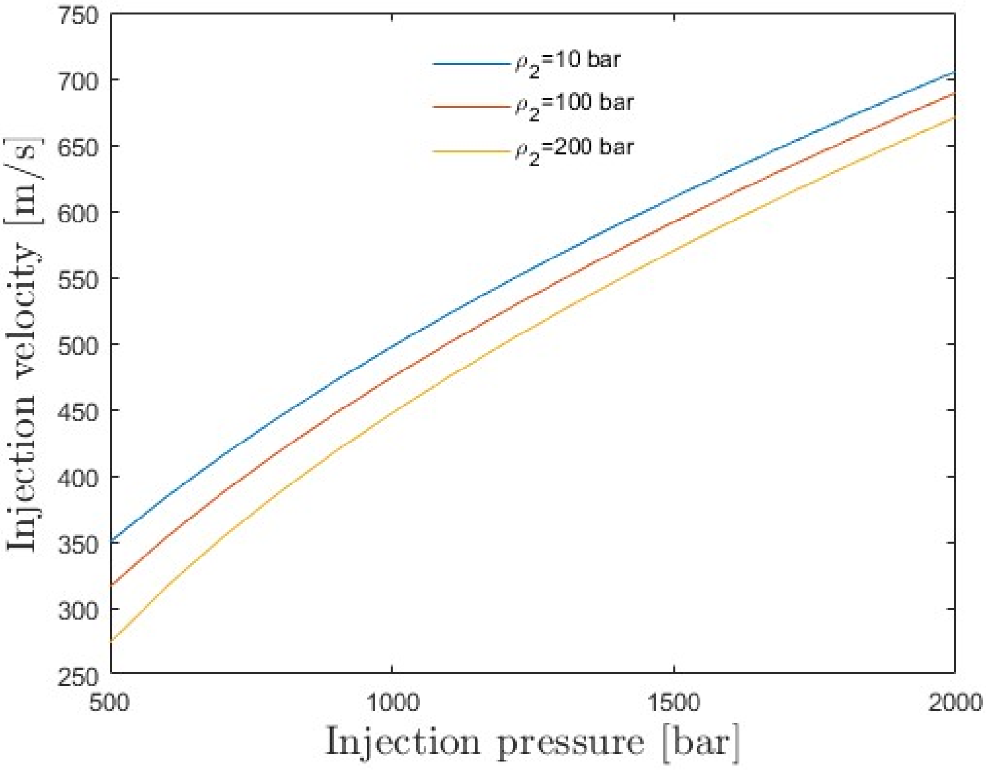


Figure 2. A section cut from an engine nozzle tip.

Answer:

```
clear all;


x=[500:100:2000];

v1=((2*(x-10)*100000)/800).^0.5;
v2=((2*(x-100)*100000)/800).^0.5;
v3=((2*(x-200)*100000)/800).^0.5;

plot(x,v1)
hold on
plot(x,v2)
plot(x,v3)

xlabel('Injection pressure [bar]','Fontsize',16, 'Interpreter','latex');
ylabel('Injection velocity [m/s]','Fontsize',16, 'Interpreter','latex');

legend('\rho_{2}=10 bar','\rho_{2}=100 bar','\rho_{2}=200
bar','Location',...
'North');
legend('boxoff')
```



750
700
650
600
550
500
450
400
350
300
250
500
1000
1500
2000
Injection velocity [m/s]
Injection pressure [bar]
ρ_2=10 bar
ρ_2=100 bar
ρ_2=200 bar