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Pressure increase in a pump function: $\Delta p = f(D, \rho, \omega, Q)$, Q: flow rate

D: impeller diameter in the pump ρ: liquid density ω: rotational speed

A) List the phrases in definition of Π-terms

- Relevant variables $k = 5$

  $\Delta p = ML^{-1}T^{-2}$, $D = L$, $\rho = ML^{-3}$, $\omega = T^{-1}$, $Q = L^3T^{-1}$
- Basic dimensions $r = 3$: M, L, T
- Number of P_i-terms: $k - r = 5 - 3 = 2$
- Repeating variables: D, ρ, ω, since they cannot form dimensionless P_i-terms

B) Define a proper set of dimensionless variables for the problem

$r = 2 \Rightarrow \Pi_1 = \Delta p\, D^a \rho^b \omega^c = M^0L^0T^0 \Rightarrow ML^{-1}T^{-2}L^a M^b L^{-3b} T^{-c} = M^0L^0T^0$

$\Rightarrow a = -2, b = -1, c = -2$

$\Rightarrow \Pi_1 = \dfrac{\Delta p}{\rho D^2 \omega^2}$

- $\Pi_2 = Q\, D^a \rho^b \omega^c = M^0L^0T^0 \Rightarrow L^3T^{-1}L^aM^bL^{-3b}T^{-c}$

$\Rightarrow a = -3, b = 0, c = -1 \Rightarrow \Pi_2 = \dfrac{Q}{D^3\omega}$

$\Rightarrow \dfrac{\Delta p}{\rho D^2 \omega^2} = \phi\left(\dfrac{Q}{D^3\omega}\right)$