**COE-C2003 Basic course on fluid mechanics, S2021**

**Round 6: Pipe flows (return at the latest by Thu 28.10. at 13:00 o'clock)**

Each problem (1-4) will be assessed on a scale of 0-3. Remember to explain the different stages in the solution. More detailed information can be found from MyCourses.

1. Water flows from the container shown in the Fig. 1. Determine the loss coefficient needed in the valve if the water is to 'bubble up' 7.6 cm above the outlet pipe.

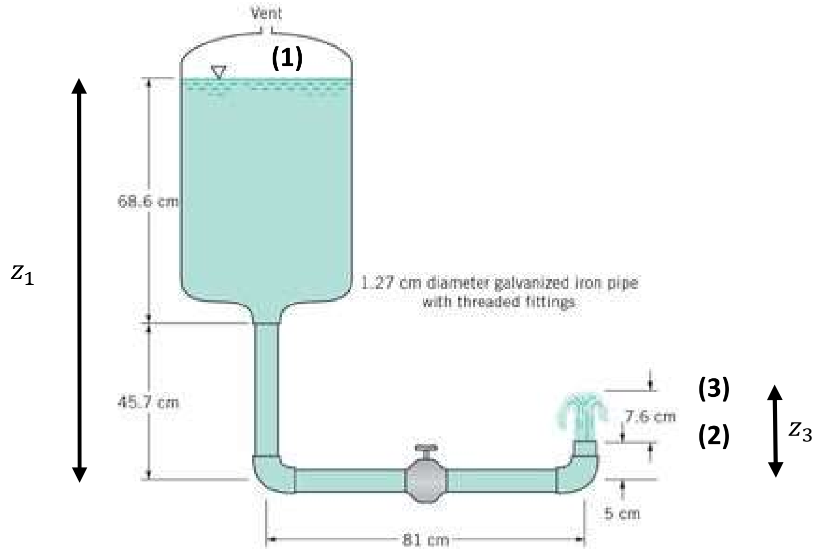



Figure 1. Problem 1

**Answer:**

First, $z_3 - z_2 = 7.6\ cm$, $p_1 = p_2 = p_3 = 0$, and $V_3 = 0$

Bernoulli between (2) and (3)

$$\frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 = \frac{p_3}{\rho g} + \frac{V_3^2}{2g} + z_3$$

Thus we get

$$V_2 = \sqrt{2g(z_3 - z_2)} = \sqrt{2 * 9{,}81 * 0.076} = 1.22\frac{m}{s}$$

Also we define $z_1 = 0.457\ m + 0.686\ m = 1.143\ m$, $z_3 = 0.05\ m + 0.076\ m = 0.126\ m$

$$\cancel{\frac{p_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + z_1 = \cancel{\frac{p_3}{\rho g}} + \cancel{\frac{V_3^2}{2g}} + z_3 + f\frac{l}{D}\frac{V_1^2}{2g} + \sum K_L \frac{V^2}{2g}$$

Thus we have

$z_1 = z_3 + f\frac{l}{D}\frac{V^2}{2g} + \sum K_L \frac{V^2}{2g}$ (1)

$$\sum K_L = K_{L\_Entrance} + 2 \times K_{L\_Elbow} + K_{L\_Valve} = 0.2 + 2(1.5) + K_{L\_Valve} = 3.2 + K_{L\_Valve}$$

From table B1, $\frac{\epsilon}{D} = \frac{0.00015\,m}{0.0127\,m} = 0.0118$

$$Re = \frac{1.22\frac{m}{s} * 0.0127m}{1 \times 10^{-6}\frac{m^2}{s}} = 15494$$

From Moody, we get $f = 0.043$

Hence from (1) we get

$$1.143\,m = 0.126\,m + \left(0.043\left(\frac{45.7 + 81 + 5}{1.27}\right) + 3.2 + K_{L\_Valve}\right)\frac{(1.22\,m/s)^2}{\left(2 * 9.81\frac{m}{s^2}\right)}$$

Or

$K_{L\_Valve} = 5.72$

2. The exhaust from a car's engine flows through a complex pipe system as shown in Fig. 2. Assume that the pressure drop through this system is $\Delta p_1$ when the engine is idling at 1000 rpm. Estimate the pressure drop (in terms of $\Delta p_1$) with the engine at 3000 rpm when driving on a highway. List all the assumptions that you made to arrive at your answer.

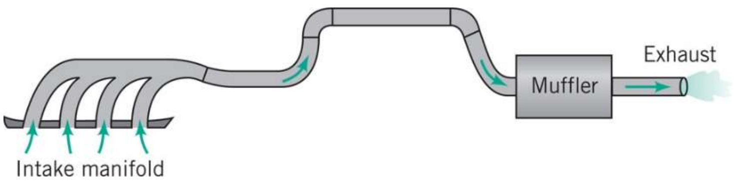



Figure 2. Problem 2

**Answer:**

For steady flow,

$$\frac{p_{out}}{\rho g}+\frac{V_{out}^2}{2g}+z_{out}=\frac{p_{in}}{\rho g}+\frac{V_{in}^2}{2g}+z_{in}-h_L \qquad (1)$$

We assume that $z_{out}=z_{in}$ and $V_{out}=V_{in}$, and $h_L=f\frac{l}{D}\frac{V^2}{2g}+\sum K_L\frac{V^2}{2g}$

We get from (1)

$$\Delta p_1=p_{in}-p_{out}=\rho g h_L=\rho g\left(f\frac{l}{D}\frac{V^2}{2g}+\sum K_L\frac{V^2}{2g}\right)=\frac{1}{2}\rho V^2\left(f\frac{l}{D}+K_L\right)$$

Hence

$$\frac{\Delta p_{3000}}{\Delta p_{1000}}=\frac{\frac{1}{2}\rho_{3000}V_{3000}^2\left(f_{3000}\frac{l}{D}+K_L\right)}{\frac{1}{2}\rho_{1000}V_{1000}^2\left(f_{1000}\frac{l}{D}+K_L\right)}$$

Assume that $\rho_{3000}=\rho_{1000}$ and $f_{3000}=f_{1000}$, i.e. this means that we assume high Re-number (independent of surface roughness)

$$\frac{\Delta p_{3000}}{\Delta p_{1000}}=\left(\frac{V_{3000}}{V_{1000}}\right)^2$$

But $V=\frac{Q}{A}$ where $Q$ is assumed to be proportional to engine rpm. That is $V_{3000}=3V_{1000}$ so that

$$\frac{\Delta p_{3000}}{\Delta p_{1000}}=3^2=9$$

So the pressure drop is 9-times higher when driving at 3000 rpm compared to idling at 1000 rpm.

## Flow around immersed bodies

3. A golf ball has dimples on its surface, Fig. 3. Explain why they are used, what is their effect, and what is the physics behind this.

Figure 3. The surface of a golf ball.

**Answer:**

The answer needs to include topics such as:

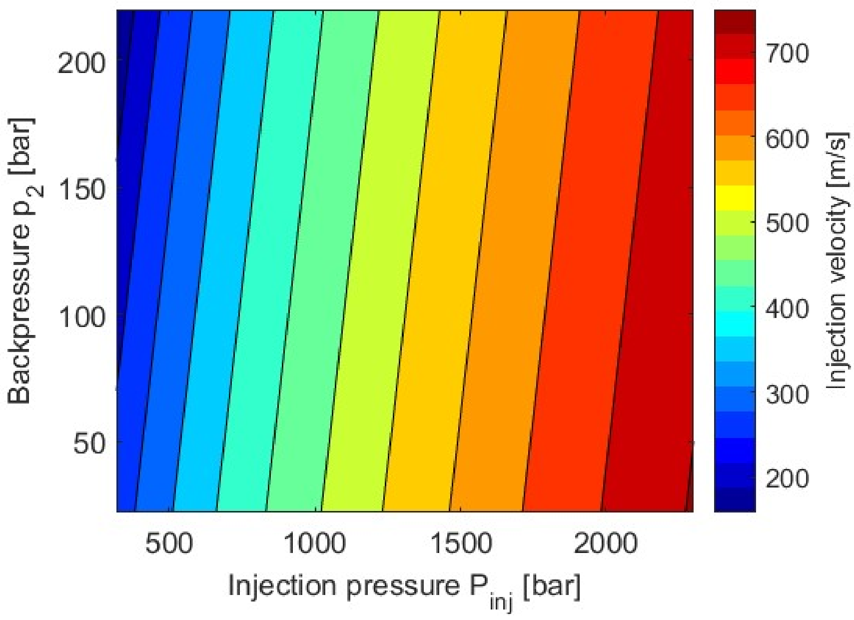
- (friction and pressure drag)
- (-flow separation increases pressure drag)
- higher turbulence leads to thinner wake region behind the golf ball compared to laminar case
- the dimples reduce the drag coefficient of the golf ball because they cause earlier transition into turbulence. Smooth surface would need higher velocity of the ball to reach turbulent boundary layer.
- the result is a considerable drop in pressure drag with a slight increase in friction drag.

4. We continue from Round 4 Matlab exercise where we calculated the injection velocity with the Bernoulli equation. Now, make a 2D contour plot of the injection velocity as a function of injection pressure and backpressure. Use the contour –command (google 'Matlab contourf'). Injection pressure is between 300 – 2300 bar and the backpressure is between 22 – 220 bar. The fuel density is 800 kg/m$^3$.
The basic idea is now that **two for *-loops are made in Matlab.*** One for the injection pressure and one for the backpressure (google 'Matlab for loop'). From this we get the injection velocity as a function of the injection pressure and backpressure ($V(j,i)$). Plot the figure so that the injection pressure is in the x-axis and the backpressure is in the y-axis (vertical axis). Plot also the axis texts (xlabel, ylabel). In the answer, show the coding and the figure (in a single pdf file).

**Answer:**

Code:

```
clear all;
set(0,'DefaultAxesFontWeight','default')
set(0,'DefaultAxesFontSize',[14])
set(0,'DefaultTextFontSize',[14])

rhoo_l=800; % liquid density

for i=1:100 % Loop over the injection pressure
    for j=1:100 % Loop over the backpressure

 Pinj=300+(i*20); % Injection pressure (320-2300bar in 100 bar steps)
 p_2=20+(j*2); % Backpressure (22-220bar in 2 bar steps)

 v(j,i)=((2*(Pinj-p_2)*100000)/rhoo_l).^0.5; % Injection velocity

    end
end

[x,y]=meshgrid(320:20:2300,22:2:220) % This was given in the exercise

contourf(x,y,v)
colormap jet(20)


xlabel('Injection pressure P_{inj} [bar]','Fontsize',14);
ylabel('Backpressure p_{2} [bar]','Fontsize',14);

% The color bar scale and axis label
c = colorbar;
c.Label.String = 'Injection pressure [m/s]';
c.Label.FontSize = 14;
```



Backpressure p2 [bar]
200
150
100
50
500
1000
1500
2000
Injection pressure Pinj [bar]
700
600
500
400
300
200
Injection velocity [m/s]