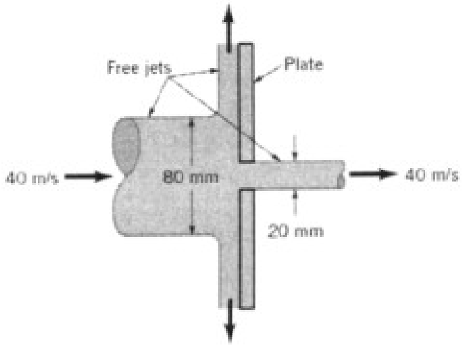
1. (Problem 5.38 in the Book) A circular plate having a diameter of 300 mm is held perpendicular to an axisymmetric horizontal jet of air having a velocity of 40 m/s and a diameter of 80 mm as shown below. A hole at the center of the plate results in a discharge of jet of air having a velocity of 40 m/s and a diameter of 20 mm. Determine the horizontal component of force requires to hold the plate stationary.



The control volume contains the plate and flowing air as indicated in the sketch above. Application of the horizontal or x direction component of the linear momentum equation yields

$$-u_1 \rho u_1 A_1 + u_2 \rho u_2 A_2 = -F_{A,x}$$

or

$$F_{A,x} = u_1^2 \rho \frac{\pi D_1^2}{4} - u_2^2 \rho \frac{\pi D_2^2}{4} = u_1^2 \rho \frac{\pi}{4}\left(D_1^2 - D_2^2\right)$$

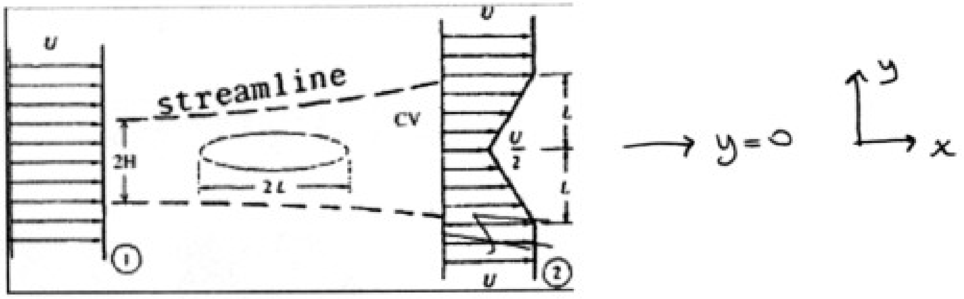
Thus

$$F_{A,x} = \left(40 \ \frac{m}{s}\right)^2 \left(1.23 \ \frac{kg}{m^3}\right) \frac{\pi}{4} \left[\frac{(80 \ mm)^2 - (20 \ mm)^2}{\left(1000 \ \frac{mm}{m}\right)^2}\right] \left(1 \ \frac{N}{kg \cdot \frac{m}{s^2}}\right)$$

and

$$F_{A,x} = \underline{\underline{9.27 \ N}}$$

2. (W-P3.44) When a uniform stream flows past an immersed thick cylinder, a broad low-velocity wake is created downstream, idealized as a V shape in the figure below. Pressures p_1 and p_2 are approximately equal. If the flow is two-dimensional and incompressible, with width b into the paper, derive a formula for the drag force F on the cylinder. Rewrite your result in the form of a dimensionless *drag coefficient* based on body length $C_D = F/(\rho U^2 bL)$.



Velocity profile at section ② is :

$$u(y) = \frac{U}{2}\left(1+\frac{|y|}{L}\right) \quad \text{where } y \text{ is between } (-L, L)$$

From mass conservation :

$$\int_{②} \rho u \, dA - \int_{①} \rho u \, dA = 0 \Rightarrow \int_{-L}^{L} \rho \frac{U}{2}\left(1+\frac{|y|}{L}\right) dA = (2H)\rho b U$$

Due to symmetry ; $\quad 2\int_0^L \rho \frac{U}{2}\left(1+\frac{y}{L}\right) dA = 2H\rho b U$

$$\Rightarrow 2\rho U \int_0^L \left(1+\frac{y}{L}\right) b \, dy = 2H\rho b U \Rightarrow \int_0^L \left(1+\frac{y}{L}\right) dy = H$$

$$y\Big|_0^L + \frac{1}{2L} y^2\Big|_0^L = H \Rightarrow \quad H = 3L/4$$

Linear momentum relation: (in x-direction)

$$\Sigma F_x = \int_{②} u \rho u \, dA - \int_{①} u \rho u \, dA = F_{drag}$$

$$\rho \, 2\int_0^L \left(1+\frac{y}{L}\right)\left(1+\frac{y}{L}\right) b dy - \rho \int_{-H}^{H} U.U \; b \, dy = F_{drag}$$

2. continued:

$$2\rho b \int_0^L \left(1+\frac{y^2}{L^2}+\frac{2y}{L}\right) dy - 2H\rho U^2 b = F_{drag}$$

$$2\rho b \left\{ y\Big|_0^L + \frac{1}{3L^2} y^3\Big|_0^L + \frac{1}{L} y^2\Big|_0^L \right\} - 2H\rho U^2 b = F_{drag}$$

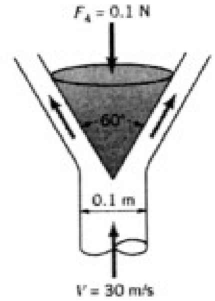
$$2\rho b \left\{ L + \frac{L}{3} + L \right\} - 2\left(\frac{3L}{4}\right) \rho U^2 b = F_{drag}$$

$$F_{drag} = -\frac{1}{3}\rho U^2 L b$$

(−) indicates drag force is in $-\hat{i}$ direction.

and $$C_D = \frac{F}{\rho U^2 L b} = \frac{1/3\, \rho U^2 L b}{\rho U^2 L b} = \frac{1}{3}$$

3. (Problem 5.50 in the Book) A vertical, circular cross-sectional jet of air strikes a conical deflector as indicated in the figure. A vertical anchoring force of 0.1 N is required to hold the deflector in place. Determine the mass in kg of the deflector. The magnitude of velocity of the air remains constant after deflection.



To determine the mass of the conical deflector we use the stationary, non-deforming control volume shown in the sketch above. Application of the vertical direction component of the linear momentum equation (Eq. 5.22) to the contents of this control volume yields

$$\dot{m}(-V_1 + V_2 \cos 30°) = -F_A - W_{cone}$$

or

$$W_{cone} = m_{cone}\, g = \dot{m}(V_1 - V_2 \cos 30°) - F_A = \rho A_1 V_1 (V_1 - V_2 \cos 30°) - F_A \quad (1)$$

However

$$V_1 = V_2$$

and

$$A_1 = \frac{\pi D_1^2}{4}$$

Thus Eq. 1 can be expressed as

$$m_{cone} = \rho \frac{\pi D_1^2}{4g} V_1 (V_1 - V_1 \cos 30°) - \frac{F_A}{g}$$

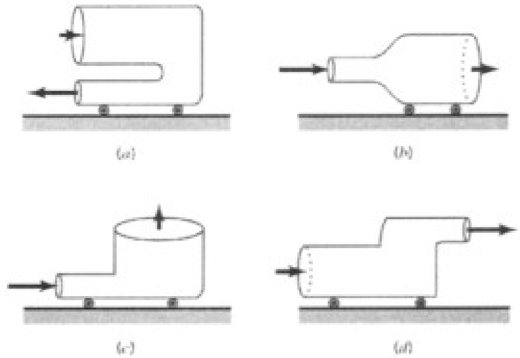
or

$$m_{cone} = \left(1.23 \frac{kg}{m^3}\right) \frac{\pi (0.1\,m)^2 (30 \frac{m}{s}) \left[30 \frac{m}{s} - (30 \frac{m}{s}) \cos 30°\right]}{(4)(9.81 \frac{m}{s^2})} - \frac{0.1\,N}{(9.81 \frac{m}{s^2})\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)}$$

and

$$m_{cone} = \underline{\underline{0.108\ kg}}$$

4. (Problem 5.58 in the Book) The four devices shown below rests on frictionless wheels, are restricted to move in the x direction only and are initially held stationary. The pressure at the inlets and outlets of each is atmospheric, and the flow is incompressible. The content of each device is not known. When realized, which devices will move to the right and which to the left? Explain.



we apply the horizontal component of the linear momentum equation to the contents of the control volume (broken lines) and determine the sense of the anchoring force F_A. If F_A is in the direction shown in the sketches, motion will be to the left. If F_A is in a direction opposite to that shown, the motion is to the right. If $F_A = 0$, there is no horizontal motion.

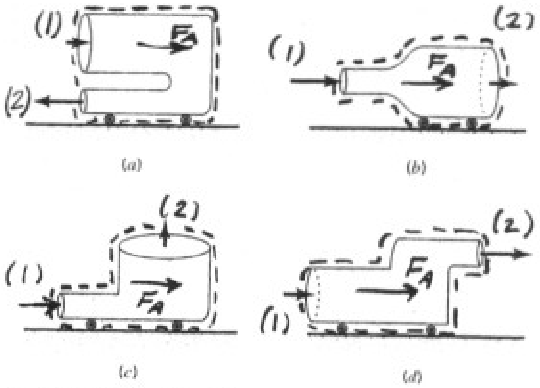


FIGURE P5.58

For sketch (a)

$$-V_1 \rho V_1 A_1 - V_2 \rho V_2 A_2 = F_A$$

Since F_A is to the left, motion is to the right.

For sketch (b)

$$-V_1 \rho V_1 A_1 + V_2 \rho V_2 A_2 = F$$

and from conservation of mass

$$\rho V_1 A_1 = \rho V_2 A_2$$

and since $V_1 > V_2$, then F_A is to the left and motion is to the right.

For sketch (c) (note: flow is into cv at (1))

$$-V_1 \rho V_1 A_1 = F_A$$

and F_A is to the left so motion is to the right.

For sketch (d)

$$-V_1 \rho V_1 A + V_2 \rho V_2 A_2 = F_A$$

and from conservation of mass

$$\rho V_1 A_1 = \rho V_2 A_2$$

and $V_1 < V_2$

so F_A is to the right and motion is to the left.