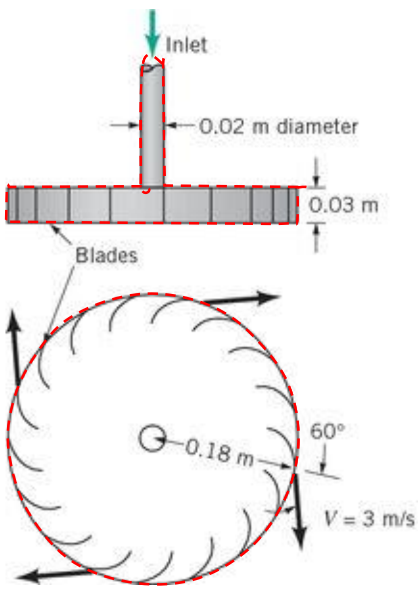
COE-C2003 Basic course on fluid mechanics, S2021

Round 3: The conservation principle (return at the latest by Thu 7.10. at 13:00 o'clock)

Each problem (1-4) will be assessed on a scale of 0-3. Remember to explain the different stages in the solution. More detailed information can be found from MyCourses.

1. Water flows out through a set of thin, closely spaced blades as shown in Fig. 1 with a speed of V= 3 m/s around the entire circumference of the outlet. Determine the mass flowrate through the inlet pipe.



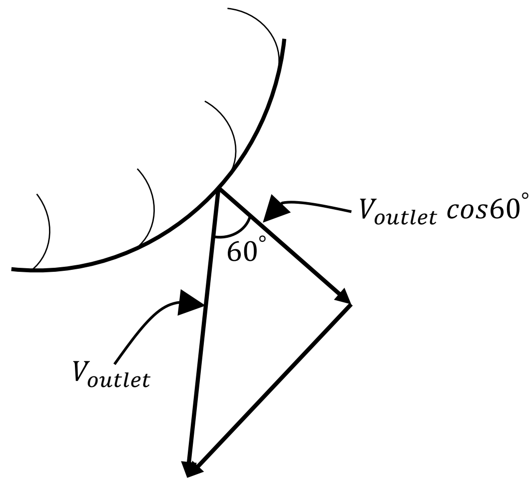


Figure 1. Problem 1

Answer:

Use the control volume contained within the broken lines shown in the sketch above. From the conservation of mass principle

$$\dot{m}_{inlet} = \dot{m}_{outlet}$$

Also

$$\dot{m}_{outlet} = \rho A_{outlet} V_{outlet} cos60^{\circ}$$

$$= \rho 2\pi r_{outlet} h\, V_{outlet} cos60^{\circ}$$

$$= 1000 \frac{kg}{m3} * 2 * \pi * 0.18m * 0.03m * 3 \frac{m}{s} * cos60$$

$$= 50.89\ kg/s$$

2 The four devices shown in Fig. 2 rest on frictionless wheels, are restricted to move in the x-direction only, and are initially held stationary. The pressure at the inlets and outlets of each is atmospheric, and the flow is incompressible. The contents of each device are not known. When released, which devices will move to the right and which to the left ? Explain.

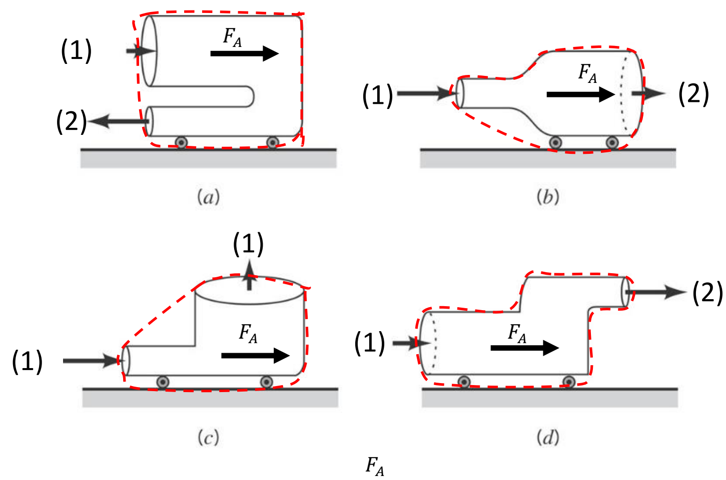


Figure 2. Problem 2.

Answer:

We apply the horizontal component of the linear momentum equation to the contents of the control volume (broken lines) and determine the direction of the anchoring force F_A.

If F_A is in the direction shown in the sketches, motion will be to the left. If F_A is in a direction opposite to that shown, the motion is to the right. If $F_A = 0$ there is no horizontal motion. We can start from the Eq. 5.17 for linear momentum

$$\sum F_{CV} = \sum V_{out}\rho_{out}A_{out}V_{out} - \sum V_{in}\rho_{in}A_{in}V_{in}$$

For sketch (a)

$$-V_1\rho V_1 A_1 - V_2\rho V_2 A_2 = F_A$$

Since F_A is to the left, motion is to the right.

For sketch (b)

$$-V_1\rho V_1 A_1 + V_2\rho V_2 A_2 = F_A$$

and from conservation of mass

$$\rho V_1 A_1 = \rho V_2 A_2$$

and since $V_1 > V_2$ then F_A is to the left, motion is to the right.

For sketch (c)

$$-V_1\rho V_1 A_1 = F_A$$

And F_A is to the left so motion is to the right.

For sketch (d)

$$-V_1\rho V_1 A_1 + V_2\rho V_2 A_2 = F_A$$

And from conservation of mass

$$\rho V_1 A_1 = \rho V_2 A_2$$

And $V_1 < V_2$

So F_A is to the right and motion is to the left.

3 Five liters per second of water enters the rotor shown in the Fig. 3 along the axis of rotation. The cross-sectional area of each of the three nozzle exits normal to the relative velocity is 18 mm^2. How fast will the rotor spin steadily if the resisting torque is reduced to zero and (a) $\theta = \theta^{\circ}$, (b) $\theta = 30^{\circ}$, or (c) $\theta = 60^{\circ}$?

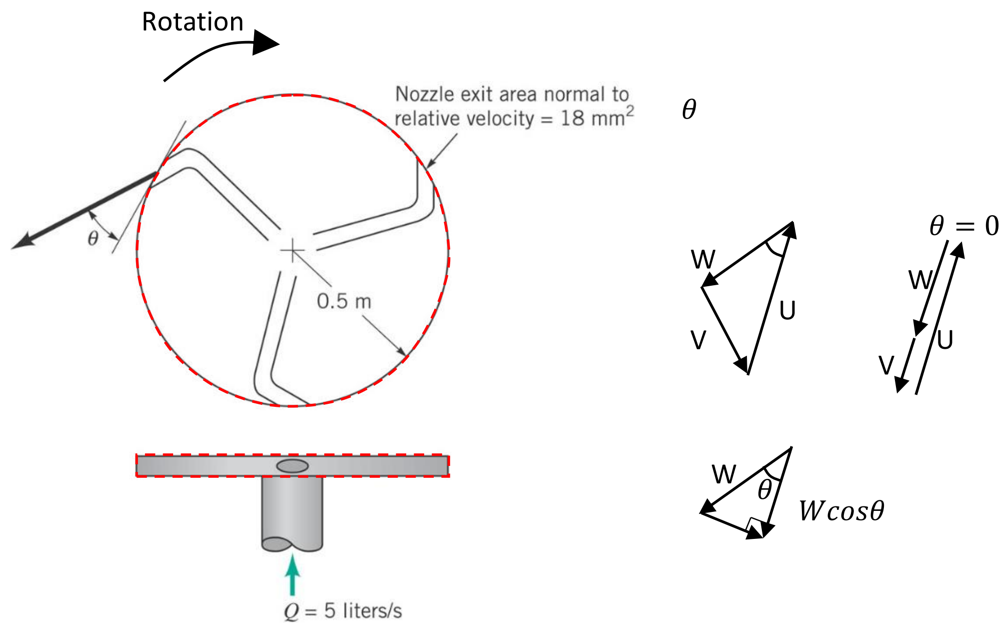


Figure 3. Problem 3.

Answer:

To determine the rotor angular velocity associated with zero shaft torque we can use the moment-of –momentum equation (Eg. 5.29). Because water is entering axially through the hollow stem of the rotor, there is no axial moment of momentum flow at the inlet.

$$T_{shaft} = \dot{m} r_{out} V_{out} \cos\theta$$

We know that $V = W + U$, where U is the velocity of the moving nozzle, W is the relative velocity, and V is the absolute velocity.

We obtain

$$T_{shaft} = \dot{m} r_{out} (W_{out} \cos\theta - U) \quad (1)$$

We also note that

$$U_{out} = r_{out} w \quad (2)$$

and

$$W_{out} = \frac{Q}{3A_{nozzle\ exit}} \tag{3}$$

Combining Eqs. (1), (2), (3) we obtain

$$T_{shaft} = \rho Q r_{out}\left(\frac{Q\ cos\theta}{3A_{nozzle\ exit}} - r_{out}w\right) \tag{4}$$

(a) For $\theta = \theta^{\circ}$

From the Eq. (4) we obtain for T_{shaft}=0

$$w = \left(\frac{Q\ cos\theta}{3A_{nozzle\ exit}\ r_{out}}\right) = \frac{0.005\ \frac{m^3}{s}}{3 \times 0.000018\ m^2\ \times 0.5\ m} = 185\ \frac{rad}{s}$$

(b) For $\theta = 30^{\circ}$

From the Eq. (4) we obtain for T_{shaft}=0

$$w = \left(\frac{Q\ cos\theta}{3A_{nozzle\ exit}\ r_{out}}\right) = \frac{0.005\ \frac{m^3}{s}\ cos30}{3 \times 0.000018\ m^2\ \times 0.5\ m} = 160\ \frac{rad}{s}$$

(c) For $\theta = 60^{\circ}$

From the Eq. (4) we obtain for T_{shaft}=0

$$w = \frac{0.005\ \frac{m^3}{s}\ cos60}{3 \times 0.000018\ m^2\ \times 0.5\ m} = 92.5\ \frac{rad}{s}$$

4 Using Matlab, plot the following functions to the same figure : $\sin x$, $\cos x$ ja $log(x)$. Define x between -10 and 10, with 0.5 steps.

- Make all the curves with different colors. Use different markers for all the curves (e.g. square, triangle…). Use also different line types (solid line, dashed line, dotted line) and line thicknesses. Note: if needed google 'matlab plot'.
- Put also the axis labels (xlabel, ylabel) and data explanations (legend). Define the x- and y-axis scales between -10 < x < 10 and -2 < y < 3 (axis-command).

Below is an example from the expected outcome (Figure 4). In the answer, show the figure and also the coding.

Answer:

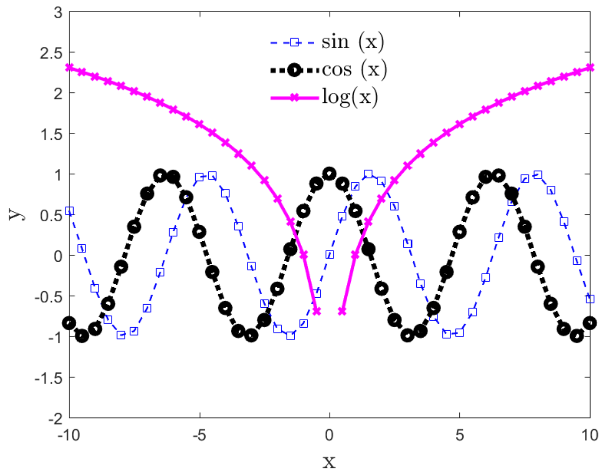
```
% Define x between an interval
x=[-10:0.5:10];
%Functions to be plotted
b=sin(x);
c=cos(x);
d=log(x);
%Plot the curves. Colors, markers, and line thicknesses are voluntary
%(as long as different options are used)
plot(x,b,'bs','LineStyle','--','Linewidth',1)
hold on;
plot(x,c,'ko','LineStyle',':','Linewidth',3)
plot(x,d,'mx','LineStyle','-','Linewidth',2)

%Scale the axis
axis([-10 10 -2 3]);

%Put names to the x- and y-axis
xlabel('x', 'Fontsize',16, 'Interpreter','latex');
ylabel('y','Fontsize',16, 'Interpreter','latex');

%Define legend
leg=legend('sin (x)','cos (x)','log(x)');
    set(leg,'FontSize',14,'Interpreter','latex','Location','North');
    legend('boxoff');

%Save fig
fig1=figure(1);
saveas(fig1,'Round3','png');
```


sin (x)
cos (x)
log(x)
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
y
-10
-5
0
5
10
x