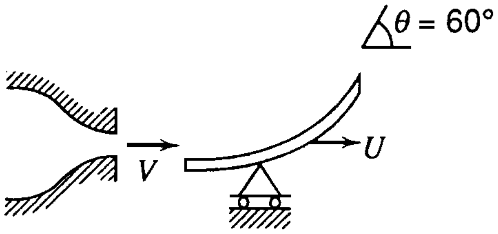
P1. The figure below shows a vane with a turning angle of 60°, mounted on a movable carriage. The vane deflects a horizontal water jet that comes from a stationary nozzle. The nozzle has an exit area of 30 cm^2, the exit velocity being 30 m/s. The water temperature is 40°C.



Determine the horizontal braking force needed to maintain a constant carriage velocity of $U = 10$ m/s. Effects of gravity and fluid friction can be neglected. (10p)

Given: water, 40°C; $\theta = 60°$; $A = 30$ cm^2; $V = 30$ m/s (jet); $U = 10$ m/s (vane).

Sought: horizontal braking force, R_x

Consider a control volume (CV) that is fixed to the moving vane, which means that the flow through CV can be considered to be stationary; liquid flow means incompressible flow. Let coordinate x be in the direction of movement. The lower surface of CV is between the wheels and the horizontal ground, surface (1) at the inlet, section (2) at the outlet. Neglect any velocity variations across (1) and (2). Linear x-momentum equation, one inlet, one outlet:

$$\dot{m}(V_{2,x} - V_{1,x}) = \Sigma F_x$$

Fluid friction can be neglected, i.e., no fluid drag and same ambient pressure all around the control surfaces. The only resulting force on the control volume then is the reaction (braking) force from the ground, $\sum F_x = R_x$.

$V_{1,x} = V_1 = V - U$; $V_{2,x} = V_2 \cos\theta$. $V_2 =?$

Since effects of gravity and fluid friction can be neglected, and the pressures at inlet and outlet are equal, it follows from the Bernoulli equation along a streamline that the (absolute) fluid velocities at section 1 and 2 are equal as well, $V_2 = V_1 = V - U$.

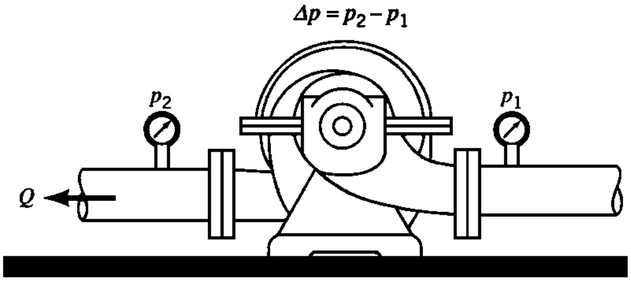
Mass flow rate, $\dot{m} = \rho A(V - U)$, i.e.,

$$R_x = -\rho A(V - U)^2(1 - \cos\theta)$$

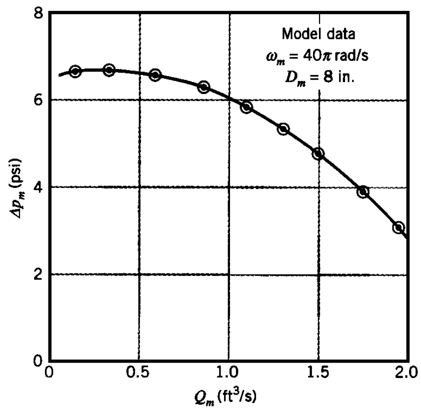
Table A.1: $\rho = 992$ kg/m^3 $\Rightarrow$ $R_x = -595$ N.

Answer: 0.60 kN (to the left)

P2. The pressure rise, Δp, across a certain type of a centrifugal pump (see figure) can, for non-cavitating conditions, be expressed as $\Delta p = p_2 - p_1 = f(\rho, D, \omega, Q)$, where ρ is the fluid density, D the impeller diameter, ω the angular velocity of the impeller, and Q the volume rate of flow through the pump.



Centrifugal pump



1 psi = 6895 Pa, 1 ft^3/s = 0.02832 m^3/s

A model pump having an impeller diameter of 8.0 in (1 in = 25.4 mm) is tested in the laboratory using water. When operated at an angular velocity of $40\,\pi$ rad/s, the model pressure rise as a function of the volume flow rate is shown on the graph to the right. Use this graph to predict the pressure rise across a geometrically similar pump (prototype) for a water flowrate of 6.0 ft^3/s. The prototype has an impeller diameter of 12 in and operates at an angular velocity of $60\,\pi$ rad/s. The water densities can be set equal. (10p)

<u>Given:</u> Dimensional relation, $\Delta p = f(\rho, D, \omega, Q)$; model: $D_m = 8$ in, $\omega_m = 40\pi$ rad/s, Δp_m *vs.* Q_m (figure); prototype: $D_p = 12$ in, $\omega_p = 60\pi$ rad/s, $\rho_p = \rho_m$.

<u>Sought:</u> Δp_p

The dimensional relation relates $n = 5$ quantities, and by inspection of units it can be seen that three primary dimensions are involved, M (mass), L (length), and T (time).

Table 5.1: $\{\Delta p\} = \mathrm{ML^{-1}T^{-1}}$, $\{\rho\} = \mathrm{ML^{-3}}$, $\{D\} = \mathrm{L}$, $\{\omega\} = \mathrm{T^{-1}}$, $\{Q\} = \mathrm{L^3T^{-1}}$.

Since, for instance, (ρ, D, ω) cannot be combined to a dimensionless group, only ρ contains M and only ω contains T, the reduction is $r = 3$; $n - r = 2$ implies $\Pi_1 = g(\Pi_2)$.

$\Pi_1 = \Delta p\, \rho^a D^b \omega^c \Rightarrow a = -1,\ b = c = -2$; $\Pi_1 = \Delta p/(\rho\omega^2 D^2)$.

$\Pi_2 = Q\rho^a D^b \omega^c \Rightarrow a = 0,\ b = -3,\ c = -1$; $\Pi_2 = Q/(\omega D^3)$.

$(\Pi_2)_m = (\Pi_2)_p \Rightarrow (\Pi_1)_p = (\Pi_1)_m$.

Insertion of data gives $(\Pi_2)_p = (10\pi)^{-1}$ $(= 0.03181)$;
$D_m = (8/12)$ ft $\Rightarrow Q_m = (10\pi)^{-1}\omega_m D_m^3 = 1.185$ ft^3/s.

A figure reading gives $\Delta p_m = 5.6$ psi, which means that

$\Delta p_p = (\rho_p/\rho_m)(\omega_p/\omega_m)^2(D_p/D_m)^2\Delta p_m = 28.4$ psi $= 196$ kPa.

<u>Answer:</u> 0.20 MPa

P3. Consider a Laval nozzle (a converging-diverging nozzle) with throat area 1.00 cm^2 and exit area 2.70 cm^2. The nozzle is connected to a large pressurized tank of dry air ($k = 1.40$) at constant temperature 500 K. The mass flow rate is 0.0517 kg/s. A Pitot-static probe placed in the exit plane reads $p_0 = 251$ kPa and $p = 240$ kPa. The nozzle flow can be considered as adiabatic and one-dimensional. Wall friction can be neglected. Determine the exit velocity. (10p)

Hint: The flow is choked.

Given: dry air, $k = 1.40$; $A_t = 1.00\ \text{cm}^2$; $A_e = 2.70\ \text{cm}^2$; $T_0 = 500$ K; $\dot{m} = 0.0510$ kg/s; Pitot-static at exit plane: $p_0 = 251$ kPa, $p = 240$ kPa.

Sought: V_e

$V_e = \text{Ma}_e\sqrt{kRT_e}$. Adiabatic flow means that $T_{0e} = T_0 = const.$; eq. (9.34) $\Rightarrow T_e = T_0/(1 + 0.2\text{Ma}_e^2)$. Find Ma_e.

Choked flow means that $\dot{m} = \dot{m}_{\max}$; eq. (9.46): $\dot{m}_{\max} = 0.6847 p_0 A^*/\sqrt{RT_0}$. Choked flow also means that the throat is sonic, $A^* = A_t$, and that p_0 in eq. (9.46) is the stagnation pressure in the tank. Using $R = 287\ \text{J kg}^{-1}\,\text{K}^{-1}$ this stagnation pressure is $p_0 = 282$ kPa. Since this pressure is higher than the stagnation pressure reading at the exit plane there must be a shock wave upstream of the Pitot-static tube. There are two possibilities, either there is (i) a normal shock wave standing in the diverging section, or there is (ii) a curved shock wave just upstream of the Pitot-static tube (supersonic outlet). Assume that it is alternative (i). In this case the flow downstream of the shock wave is isentropic,

$p_{0e}/p_e = 251/240 = 1.046 = (1 + 0.20\text{Ma}_e^2)^{3.5} \Rightarrow \text{Ma}_e = 0.254$, $T_e = 493.6$ K, $V_e = 113$ m/s. Is this really the correct velocity?

Check alternative (ii). In this case and along the stagnation line upstream the ratio of stagnation pressures must be $p_{02}/p_{01} = 251/282 = 0.89$, which implies that $\text{Ma}_1 = \text{Ma}_e \approx 1.6$ (Table B.2). However, a supersonic outlet with $A_e/A^* = A_e/A_t = 2.7$ from Table B.1 must have $\text{Ma}_e \approx 2.5$. Thus, this alternative can be ruled out.

Answer: $V_e = 113$ m/s

C. Norberg