BRANCH & BOUND

Yet another divide & conquer.

Standard form : minimisation

↳ max can be handled with a change of sign

BOUNDS : GOOD OR BAD

→ If no bounds are available, the algorithm degenerates to an exhaustive search

EXAMPLE (SEE MATHEMATICA)

$$\min \quad z = x_1 - 2x_2$$

subject to

$$2x_1 + x_2 \leq 5$$

$$-4x_1 + 4x_2 \leq 5$$

$$x_1, x_2 \geq 0 \quad \& \quad x_1, x_2 \in \mathbb{Z}$$

Solve as a standard LP:

$$z_{min} = -\frac{15}{4} \quad \Longrightarrow \quad \geq -3 \quad (x_1, x_2 \in \mathbb{Z})$$

$$\left.\begin{aligned} x_1 &= \frac{5}{4} \quad \notin \mathbb{Z} \\ x_2 &= \frac{5}{2} \quad \notin \mathbb{Z} \end{aligned}\right\} \text{need to be determined}$$

TIME TO BRANCH

Let us select x_1 and start building our tree: (LEVEL 2)

(2A) : $x_1 \leq 1$

(2B) : $x_1 \geq 2$

What to do? Add constraints & solve again!

(Yes, this in not cheap!)

(2B) : $z_{min} = 0$

$x_1 = 2$

$x_2 = 1$

THIS IS A GOOD CANDIDATE!

$z_{min}^{opt} \in [-3, 0]$

AND BRANCH ...

(2A): $z_{min} = -\frac{7}{2}$ $\quad (\leq -3)$

$x_1 = 1$

$x_2 = \frac{9}{4}$

(3A): $x_2 \leq 2$

$z_{min} = -\frac{13}{4}$

$x_1 = \frac{3}{4}$

$x_2 = 2$

(3B): $x_2 \geq 3$

$\Longrightarrow$ not feasible

AND ONE MORE!

(4A): $x_1 = 0$

$z_{min} = -\frac{5}{2}$

$x_1 = 0$

$x_2 = \frac{5}{4}$

(4B): $x_1 = 1$

$z_{min} = -3$ (OPTIMAL!)

$x_1 = 1$

$x_2 = 2$