Quiz 1

Question 1: Formulation of linear optimisation problem

Maria wants to bake artisan sourdough bread for her sister's wedding. She estimates that she will need to bake 25 loaves of bread. Unfortunately, she has started too late with making the dough, so that she does not have enough time to follow her original recipe's rising time of 72 h. In order to shorten the rising time, she has two possibilities: She can add more sourdough starter to the dough, where for one loaf, each additional 50 g of starter will reduce the rising time by 3 h. Or she can speed up the bread rise by adding industrial yeast, but since yeast bread is less flavourful and dries out more quickly, she wants to use as little yeast as possible. For each 0.1 g of yeast she adds to the dough of one bread, the rising time will be reduced by 1 h. If she wants to finish the bread in time, the rising time can be at most 40 h. Also note that she only has 500 g of surplus starter in total for all 25 loaves.

How much yeast does she need to use for each loaf of bread in order to have the bread ready in time?

Formulate a corresponding LP and solve with Excel Solver or Julia. Which of the following is the optimal solution?

(a) 3.08 g of additional yeast.

(b) 0 g of additional yeast.

(c) 0.2 g of additional yeast.

(d) 30.8 g of additional yeast.

Solution

The correct solution is (a).

Let x_1 be the number of additional 50 g units of sourdough, and x_2 the additional units of 0.1 grams of yeast Maria uses for one loaf of bread.

The objective function is to minimize the use of yeast, hence the objective function is:

$$\min .\ x_2$$

For the constraints, Maria only has 40 h of rising time. Using no additional components, the rising time would be 72 h. That means, she must cause a difference of 32 h by using additional sourdough and yeast. For one loaf, we have

$$3x_1 + x_2 \geq 72 - 40$$

She also is limited by the amount of surplus sourdough starter she has:

$$50 \cdot 25 \cdot x_1 \leq 500$$

And obviously, we cannot have a negative amount of additional rising agents:

$$x_1, x_2 \geq 0$$

Entering this LP formulation into Julia gives the optimal solution: $x_1 = 0.4$ and $x_2 = 30.8$, which translates to $x_1 \cdot 50g = 20$ g of additional sourdough and $x_2 \cdot 0.1g = 3.08$ g of additional yeast per loaf.

Question 2: Solving LPs graphically

For each of the given directions, determine the z which is an objective of a linear program whose steepest ascent is in this direction.

a) Direction $[5, 3]^\top$

   (i) $z = 5x_1$

   (ii) $z = 3x_2$

   (iii) $z = 3x_1 + 5x_2$

   (iv) $z = 5x_1 + 3x_2$

b) Direction $[1, 6]^\top$

   (i) $z = 6x_1 + 1x_2$

   (ii) $z = -x_1 - 6x_2$

   (iii) $z = 1x_1 - 6x_2$

   (iv) $z = 2.5x_1 + 15x_2$

c) Direction $[-1, 1]^\top$

   (i) $z = x_1 - x_2$

   (ii) $z = x_1 + x_2$

   (iii) $z = -x_1 - x_2$

   (iv) $z = -x_1 + x_2$

Solution

The correct answers are a) (iv), b) (iv), c) (iv).

The gradient is $\nabla z = [\frac{\partial z}{\partial x_1}, \frac{\partial z}{\partial x_2}]^\top$, so one possible objective is

$$z = \frac{\partial z}{\partial x_1} \cdot x_1 + \frac{\partial z}{\partial x_2} \cdot x_2.$$

This, together with the fact that multiplying with positive numbers does not change the direction, gives us the correct solutions. Note that we can obtain infinitely many possible objectives for the same gradient, simply by multiplying z with a positive number.