Quiz 3

Question 1: Formulation of a dual problem

$$\begin{aligned}
\text{max.} \quad & z_P = x_1 + 4x_2 \\
\text{s.t.} \quad & x_1 + x_2 \leq 4 \\
& 2x_1 - x_2 \leq -2 \\
& x_1, x_2 \geq 0
\end{aligned}$$

Transform the above primal optimization problem to its dual problem.

a

Which of the following is the correct dual objective function

(a) min. $z_D = -x_1 - 4x_2$

(b) max. $z_D = 4y_1 + 2y_2$

(c) min. $z_D = 4y_1 - 2y_2$

(d) min. $z_D = 4x_1 - 2x_2$

b

Which of the following is a constraint of the dual problem?

(a) $y_1 - y_2 \leq 4$

(b) $y_1 + 2y_2 \geq 1$

(c) $y_1 - y_2 \geq -2$

(d) $y_1 + y_2 \leq 4$

Solution

The correct solution to the first question is c and the correct solution to the second question is b.

The dual problem is as follows:

$$\begin{aligned}
\text{min.} \quad & z_D = 4y_1 - 2y_2 \\
\text{s.t.} \quad & y_1 + 2y_2 \geq 1 \\
& y_1 - y_2 \geq 4 \\
& y_1, y_2 \geq 0
\end{aligned}$$

Question 2: An integer optimization problem

You're an executive at a local beverage store and you've noticed that sale of the items increases sharply at the beginning of May. The three most affected beverages are water, homemade essential juice and gold flaked mineral soda.

A week before May you're certain that there is demand for precisely 30 bottles of water and 30 bottles of soda with endless demand for juice. You can buy water, juice and soda at 1€/btl, 5€/btl and 15€/btl respectively. You can sell said beverages at 3€/btl, 10.39€/btl and 25€/btl respective to the previous list.

Your supplier has difficulties due to excessive consumption and you can buy only up to 50 bottles of juice and 20 bottles of soda, but there is plenty of water. You have a single delivery person at your disposal, who can retrieve 50 litres of beverages at a time, but they demand a wage of 100€ per trip. The bottle sizes for water, juice and soda are 1l, 0.75l and 0.75l respectively. Your objective is to maximize profits.

Formulate the problem in Julia and answer the following questions.

a

How many trips should the delivery person make?

  (a) 0

  (b) 1

  (c) 2

  (d) 3

b

How much water should you order?

  (a) 0 bottles

  (b) 3 bottles

  (c) 7 bottles

  (d) 10 bottles

c

How much juice should you order

  (a) 0 bottles

  (b) 20 bottles

  (c) 40 bottles

  (d) 50 bottles

d

How much soda should you order?

(a) 0 bottles

(b) 7 bottles

(c) 20 bottles

(d) 30 bottles

d

How much profit do you expect?

(a) 0€

(b) 256€

(c) 383.5€

(d) 429.39€

Solution

The problem was solved with Julia and the solution is as follows:

Trips: 1

Water: 7 bottles

Juice: 50 bottles

Soda: 20 bottles

Profit: 383.5€