DUALITY

Duality is the property of two mathematical structures being equivalent in the sense that truth in one automatically implies another truth in the other one.

GENERALISED SIMPLEX TABLEAU IN MATRIX FORM

Consider : Maximise $z = c^T x$ subject to $Ax = b$,
$x \geq 0$

Matrix form:
$$\begin{pmatrix} 1 & -c^T \\ 0 & A \end{pmatrix} \begin{pmatrix} z \\ x \end{pmatrix} = \begin{pmatrix} 0 \\ b \end{pmatrix}$$

Let c_B and x_B be associated with the basic variables.

The solution is then (with all $x_N = 0$)

$$\begin{pmatrix} z \\ x_B \end{pmatrix} = \begin{pmatrix} 1 & -c_B^T \\ 0 & B \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ b \end{pmatrix} = \begin{pmatrix} 1 & c_B^T B^{-1} \\ 0 & B^{-1} \end{pmatrix} \begin{pmatrix} 0 \\ b \end{pmatrix}$$

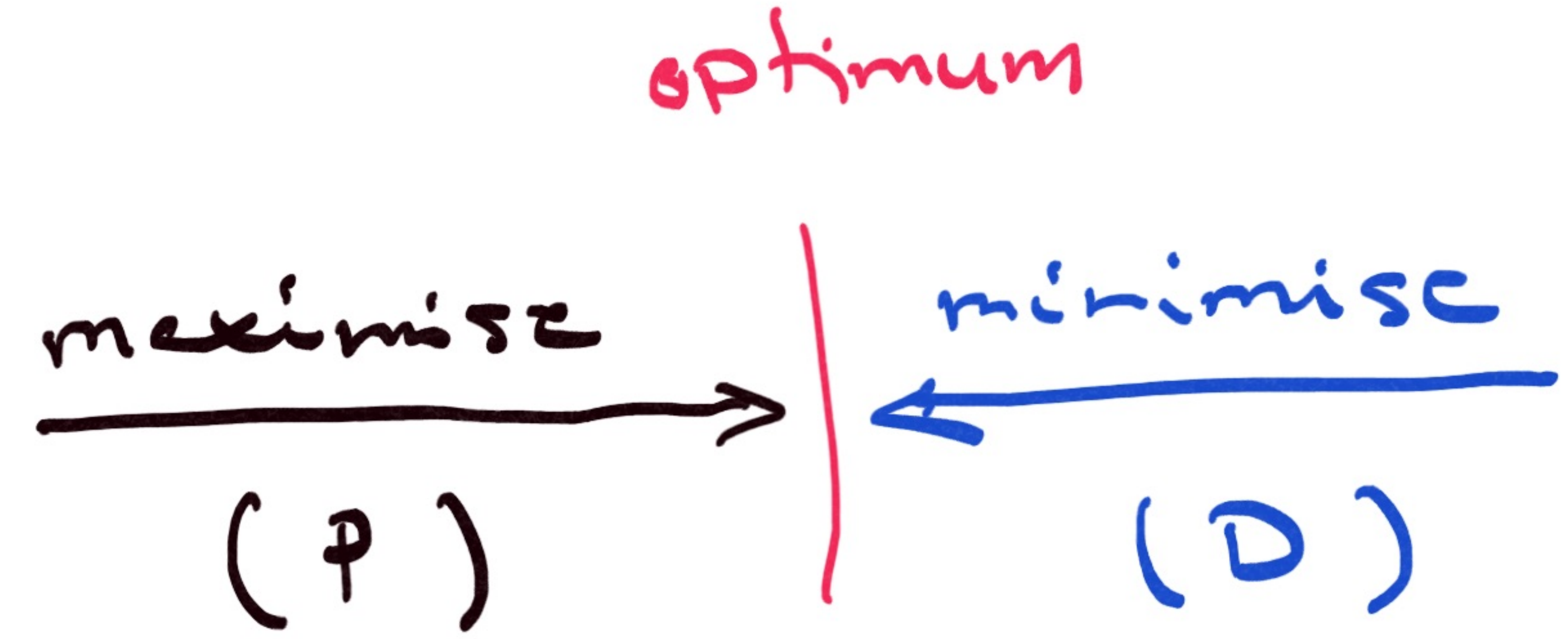
$$= \begin{pmatrix} c_B^T B^{-1} b \\ B^{-1} b \end{pmatrix}$$

The general version is thus:

$$\begin{pmatrix} 1 & c_B^T B^{-1} \\ 0 & B^{-1} \end{pmatrix} \begin{pmatrix} 1 & -c^T \\ 0 & A \end{pmatrix} = \begin{pmatrix} 1 & c_B^T B^{-1} A - c^T \\ 0 & B^{-1} A \end{pmatrix} = \begin{pmatrix} c_B^T B^{-1} b \\ B^{-1} b \end{pmatrix}$$

DUALITY IN LINEAR PROGRAMMING (LECTURE 5 NOTES)

$$(P)\quad \max\ z_p = 4x_1 + 3x_2$$
$$\text{s.t.}\quad 2x_1 + x_2 \le 4$$
$$x_1 + 2x_2 \le 4$$
$$x_1, x_2 \ge 0$$



$$(2x_1 + x_2)y_1 + (x_1 + 2x_2)y_2 \le 4y_1 + 4y_2$$

$$\Longleftrightarrow \underbrace{(2y_1 + y_2)}_{\ge 4}x_1 + \underbrace{(y_1 + 2y_2)}_{\ge 3}x_2 \le \underbrace{4y_1 + 4y_2}_{\text{minimise}}$$

(Invisible transpose)

THE DUAL PROBLEM (D)

$$\min \quad z_D = 4y_1 + 4y_2$$

$$\text{s.t.} \quad 2y_1 + y_2 \geq 4$$

$$y_1 + 2y_2 \geq 3$$

$$y_1, y_2 \geq 0$$

$$A^T y \geq c$$

Same optimum, different coordinates!

THE TWO TABLEAUX

(P)	x_1	x_2	x_3	x_4	sol
z	0	0	$5/3$	$2/3$	$28/3$
x_1	1	0	$2/3$	$-1/3$	$4/3$
x_2	0	1	$-1/3$	$2/3$	$4/3$

(D)	y_1	y_2	y_3	y_4	sol
z	0	0	$4/3$	$4/3$	$28/3$
y_1	1	0	$-2/3$	$1/3$	$5/3$
y_2	0	1	$1/3$	$-2/3$	$2/3$

$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ from constraints ; Here $B = A$:

$B^{-1} = \frac{1}{3}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$; $B^{-1}\begin{pmatrix} 4 \\ 4 \end{pmatrix} = \begin{pmatrix} 4/3 \\ 4/3 \end{pmatrix}$

$B^{-1}\begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 5/3 \\ 2/3 \end{pmatrix}$

$-B^{-1}$ for dual
due to negative slack
variables.