Aalto University
School of Electrical Engineering

CS-C3240 – Machine Learning D

Feature Engineering

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Version 1.0, February 14, 2022

Variance and Covariance

For two features $\mathcal{X}_1, \mathcal{X}_2$, and the arithmetic mean $E[\cdot]$, consider sets of measurements with zero mean:

$$\overrightarrow{x_1} = \{x_1^{(1)}, \dots, x_1^{(n)}\}$$
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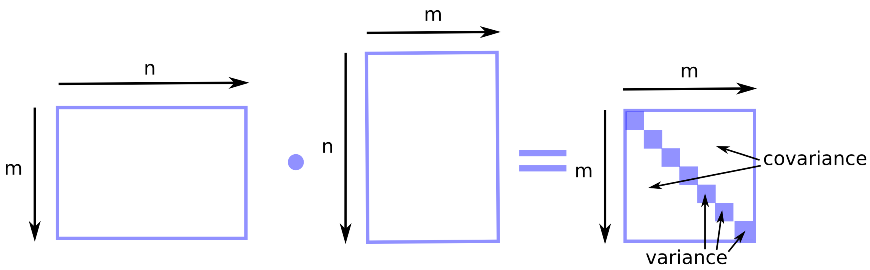
Covariance: $E[(\overrightarrow{x_i} - E[\overrightarrow{x_i}])(\overrightarrow{x_j} - E[\overrightarrow{x_j}])]$

Covariance matrix



Questions?

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Literature

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