Aalto University
School of Electrical Engineering

CS-C3240 – Machine Learning D

Round 3: From features to classification

Stephan Sigg

Department of Communications and Networking
Aalto University, School of Electrical Engineering
stephan.sigg@aalto.fi

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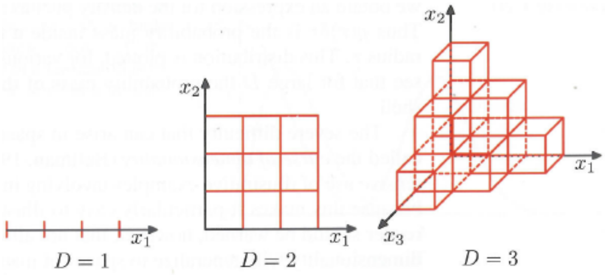
Issues related to high dimensional input data

Exponential growth Volume of the space grows exponentially with dimension



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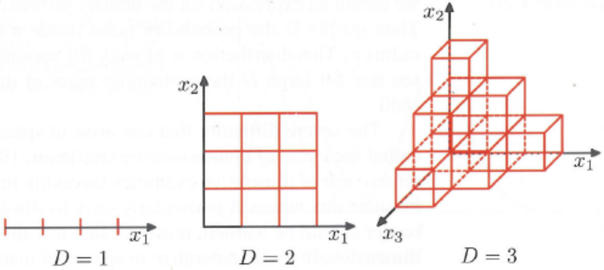
Curse of dimensionality

Too sparse samples across regions to estimate a distribution in that space
(Problematic for methods that require statistical significance)



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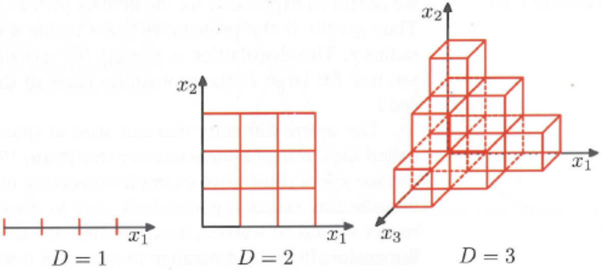
Hughes (peaking) phenomenon

Predictive power of classifier first increases with dimension, then decreases



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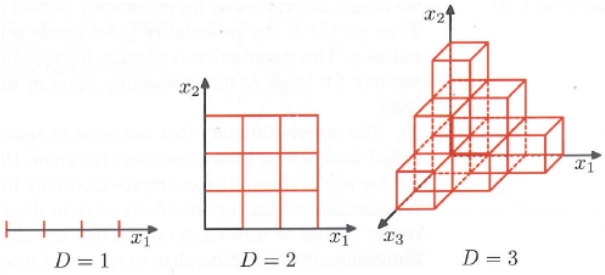
Counter-intuitive properties in higher dimensional spaces



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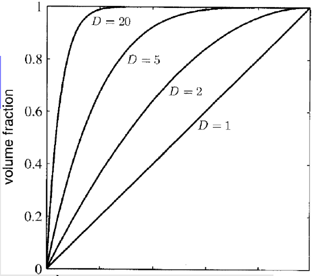

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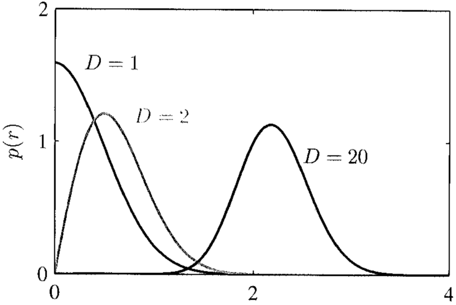
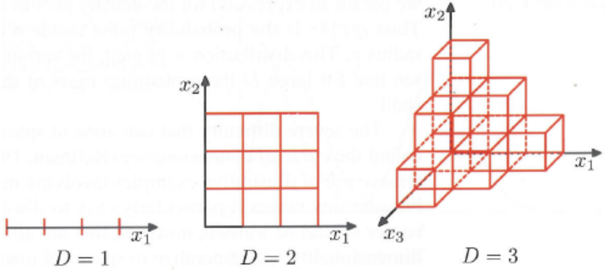
Probability mass concentrated in a thin shell
(here plotted as distance from the origin in a polar coordinate system)





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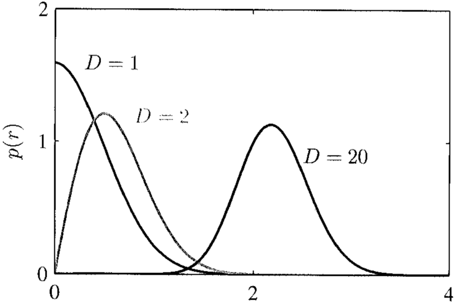

Curse of Dimensionality

Mechanisms to efficiently reduce dimensions or classifiers that respect properties of high-dimensional spaces required.

Questions?

Stephan Sigg
stephan.sigg@aalto.fi

Si Zuo
si.zuo@aalto.fi

Literature

- C.M. Bishop: Pattern recognition and machine learning, Springer, 2007.
- R.O. Duda, P.E. Hart, D.G. Stork: Pattern Classification, Wiley, 2001.