Aalto University
School of Electrical Engineering

CS-C3240 – Machine Learning D

Round 3: From features to classification

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Version 2.3, January 23, 2022

Outline

Feature Engineering

Feature engineering

Example: Voiced vs. unvoiced audio

A way to detect voice in audio is to calculate the number of zero-crossing. A 100 Hz signal will cross zero 100 times per second; an unvoiced segments can have 3000 zero crossing per second.

Feature pre-processing

- → Domain knowledge available?
- → Normalisation
- → Overlapping windows
- → Detection of outliers
- → Are features independent?

Feature engineering

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Feature engineering

Simple normalization: Scaling

For each sample x_i from a set $\mathcal{X}$, compute the scaled value as

$$x_i' = \frac{x_i - \min(\mathcal{X})}{\max(\mathcal{X}) - \min(\mathcal{X})}$$

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after scaling, it is common to center the values around e.g. 0 or their arithmetic mean, median, centre of mass etc.

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Standardization to zero mean/unit variance

Given a set of values $x_i; i \in \{1..n\}$ from a set $\mathcal{X}$ with mean μ and standard deviation σ, we derive the standardized values x_i' as

$$x_i' = \frac{x_i - \mu}{\sigma}$$

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Using the variance σ^2 instead of σ is called variance scaling

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Important:

When normalizing on the training set input, this need to be applied identically ot the test set input. Do not normalize the test set input on the test set data.

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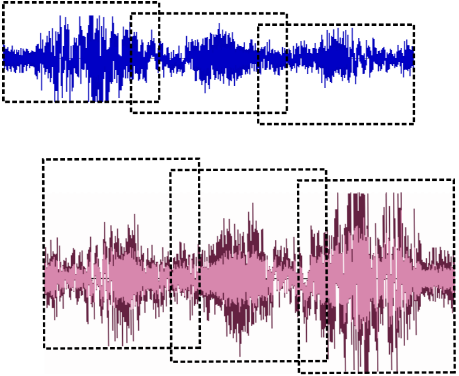
Feature engineering



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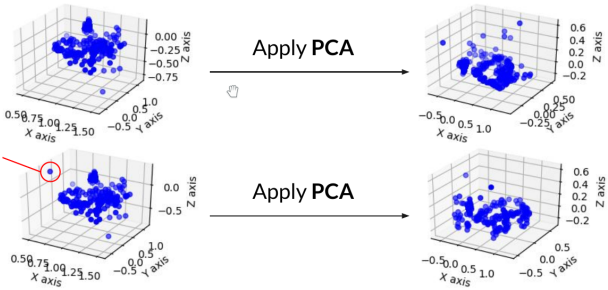

Feature pre-processing

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Feature engineering

Common pitfalls in outlier handling:

It is not unusual to find values that clearly depart from the rest.

Example: In insurance, most claims are small but a few are large. Removing the large claims will completely invalidate an insurance model.

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Feature engineering

Common pitfalls in outlier handling:

It is not unusual to find values that clearly depart from the rest.

Example: In insurance, most claims are small but a few are large. Removing the large claims will completely invalidate an insurance model.

Caution: Do not throw away outliers, unless you have evidence that they are errors

Darell Huff, How to lie with Statistics, 1954

Feature pre-processing

→ Domain knowledge available?
→ Normalisation
→ Overlapping windows
→ **Detection of outliers**
→ Are features independent?

Feature engineering

Common pitfalls in outlier handling:

It is not unusual to find values that clearly depart from the rest.

Approach: If outliers are present, use algorithms that are robust to outliers. For instance, covarianceor mean are sensitive to outliers. $\rightarrow$ replace mean with median.

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- $\rightarrow$ Domain knowledge available?
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- $\rightarrow$ Are features independent?

Feature engineering

Common pitfalls in outlier handling:

It is not unusual to find values that clearly depart from the rest.

$\rightarrow$ Outliers behave sometimes different than the rest $\rightarrow$ train separate model on outliers

Detection clustering, density estimation, one-class SVM

Feature pre-processing

$\rightarrow$ Domain knowledge available?

$\rightarrow$ Normalisation

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Feature engineering

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Examples for dependent features:

Feature pre-processing

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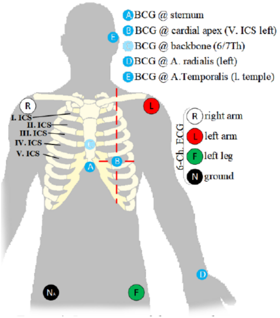
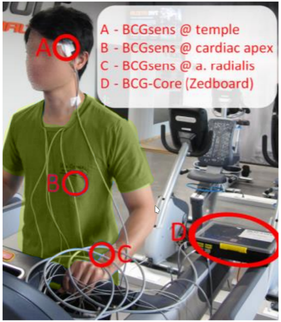
Feature engineering

Example: walking speed vs. heart rate



(a) Positioning of the sensors



(b) Subject performing the study

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Feature Selection

A large portion of the performance of Machine Learning algorithms is due to the right choice and processing of features.

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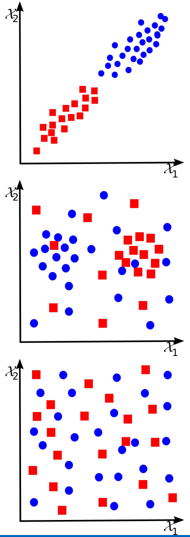
Avoid non-important features

- Noisy data
- Non-correlation between features and classes
- Correlated features
- Sometimes, less is better



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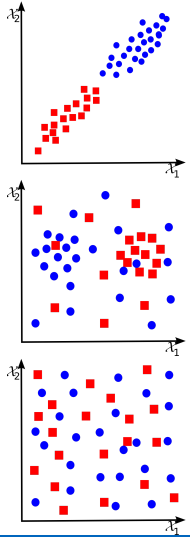
Choosing the most important features

- Reduces training and evaluation time
- Reduces complexity of a model (easier to interpret)
- Improves prediction/recall of a model
- Reduces overfitting



Feature selection algorithms

How to identify good/meaningful features?

Feature selection

For a set of features $\{\mathcal{X}\}$, how to find a good subset $\{\mathcal{X}\}_s \subseteq \mathcal{X}\}$ which is best suited to distinguish between the considered classes $\mathcal{C}_i \in \{\mathcal{C}\}$?

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Las Vegas Filter

Repeatedly generate random feature subsets $\{\mathcal{X}\}_s \subseteq \mathcal{X}\}$, train a classifier $\hat{h}_s(\overrightarrow{\hat{w}_s}, \cdot) = \min_{i \in \{\mathcal{X}_s\}} \mathcal{L}\left(h(\overrightarrow{w}, \overrightarrow{x}^{(i)}), y^{(i)}\right)$ and validate $\hat{h}_s(\overrightarrow{\hat{w}_s}, \cdot)$ for its classification performance

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Focus algorithm

1 Train and evaluate a classifier for singleton feature $\mathcal{X}_o$

2 Evaluate each set of two features $\mathcal{X}_o, \mathcal{X}_p$

⋮

Until consistent solution is found

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$\vdots$

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Complexity:

$$\binom{|\mathcal{X}|}{k} = \frac{|\mathcal{X}|!}{(|\mathcal{X}|-k)!(k!)} \rightarrow \mathcal{O}(2^{|\mathcal{X}|})$$

$$\binom{|\mathcal{X}|}{1} \cdot \binom{|\mathcal{X}|}{2} \cdots \binom{|\mathcal{X}|}{|\mathcal{X}|}$$

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Relief algorithm

Given a collection of values $x_i; i \in \{1..n\}$ of a feature $\mathcal{X}$, compute

$$\frac{\text{Closest distance to all other samples of the same class}}{\text{Closest distance to all samples not in that class}}$$

Rationale: Feature more relevant the more it separates a sample from samples in other classes and the less it separates from samples in same class

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Complexity:
$\mathcal{O}\left(|\mathcal{X}| \cdot n^2\right)$

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Pearson Correlation Coefficient

$$r(\mathcal{X}_1, \mathcal{X}_2) = \frac{\text{Cov}(\mathcal{X}_1, \mathcal{X}_2)}{\sqrt{\text{Var}(\mathcal{X}_1)\text{Var}(\mathcal{X}_2)}}$$

- Identifies linear relation between features $\mathcal{X}_i$

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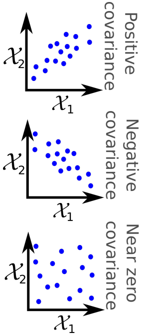

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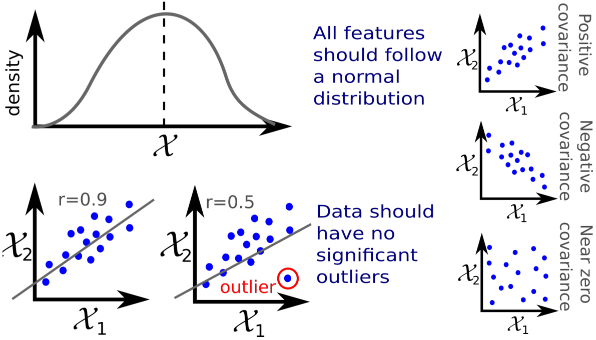

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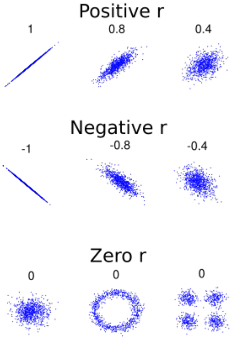

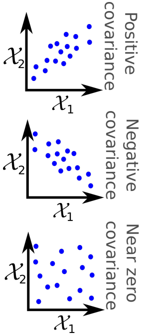

Variance and Covariance

For two features $\mathcal{X}_1, \mathcal{X}_2$, consider sets of measurements with zero mean:

$$\overrightarrow{x_1} = \{x_1^{(1)}, \dots, x_1^{(n)}\}$$
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$\Rightarrow \frac{1}{n}\overrightarrow{x_1}\,\overrightarrow{x_2}^T$

Covariance matrix

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Feature matrix:

$$X = \begin{bmatrix} \overrightarrow{x_1} \\ \vdots \\ \overrightarrow{x_m} \end{bmatrix}$$

Covariance matrix

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$$\Sigma = \frac{1}{n}XX^T$$

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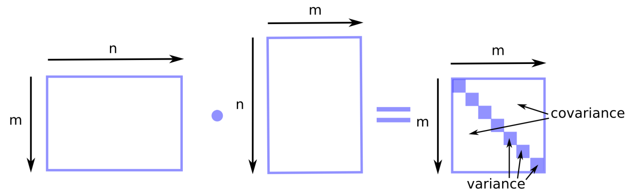

Questions?

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Literature

- C.M. Bishop: Pattern recognition and machine learning, Springer, 2007.
- R.O. Duda, P.E. Hart, D.G. Stork: Pattern Classification, Wiley, 2001.