Topology (Main Source: Munkres)

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Part I

Point-Set Topology

CHAPTER 1

TOPOLOGICAL SPACES AND CONTINUOUS FUNCTIONS

1 Topological Spaces

Definition 1.1. (Topology) A *topology* τ on a set X is a collection $\mathscr{B}$ of sets called *open sets* such that:

1. $\varnothing$ and X belong to $\mathscr{B}$
2. For any collection of sets $U_\alpha \in \mathscr{B}$, $\cup_\alpha U_\alpha$ is also in $\mathscr{B}$ (closed under arbitrary unions)
3. For any finite collection $\{U_1, U_2 \cdots U_k\}$ of sets of $\mathscr{B}$, $\cap_{i=1}^{k} U_i \in \mathscr{B}$ (closed under finite intersections)

Definition 1.2. (Discrete and Indiscrete topology)

1. The topology $(\varnothing, X)$ of a set X is called the *indiscrete topology.*
2. The topology in which every subset of X is an open set is called the *Discrete topology*

Definition 1.3. ((Finer and Coarser topologies)) Let τ and τ' be two topologies of space X. We say τ' is *finer* than τ if $\tau \subset \tau'$, or every open set in τ is one in τ'.

Remark 1.4. Of course, not all topologies are comparable, like the above definition suggests.