Institute and Faculty of Actuaries

EXAMINERS' REPORT

CS2 - Risk Modelling and Survival Analysis

Core Principles

Paper A

September 2023

Introduction

The Examiners' Report is written by the Chief Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

For some candidates, this may be their first attempt at answering an examination using open books and online. The Examiners expect all candidates to have a good level of knowledge and understanding of the topics and therefore candidates should not be overly dependent on open book materials. In our experience, candidates that spend too long researching answers in their materials will not be successful either because of time management issues or because they do not properly answer the questions.

Many candidates rely on past exam papers and examiner reports. Great caution must be exercised in doing so because each exam question is unique. As with all professional examinations, it is insufficient to repeat points of principle, formula or other text book works. The examinations are designed to test "higher order" thinking including candidates' ability to apply their knowledge to the facts presented in detail, synthesise and analyse their findings, and present conclusions or advice. Successful candidates concentrate on answering the questions asked rather than repeating their knowledge without application.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Sarah Hutchinson
November 2023

A. General comments on the *aims of this subject and how it is marked*

The aim of the Risk Modelling and Survival Analysis Core Principles subject is to provide a grounding in mathematical and statistical modelling techniques that are of particular relevance to actuarial work, including stochastic processes and survival models.

Some of the questions in this paper admit alternative solutions from those presented in this report, or different ways in which the provided answer can be determined. All mathematically correct and valid alternative solutions received credit as appropriate.

In cases where an error was carried forward to later parts of the answer, candidates were given full credit for those later parts.

In higher order skills questions, where comments were required, well-reasoned comments that differed from those provided in the solutions also received credit as appropriate.

Candidates are advised to take careful note of all instructions that are provided with the exam in order to maximise their performance in future CS2A examinations.

B. Comments on *candidate performance in this diet of the examination.*

CS2 has an extensive syllabus, and the examinations will cover many areas of that syllabus with problem questions that test understanding and the ability to apply statistical and modelling techniques in a range of scenarios. Therefore, exam preparation and in-assessment question answering strategies are both important components to success.

In preparation, candidates are reminded that covering the whole syllabus and practicing questions in all areas is important. Questions in Survival Analysis have consistently been better than those in other areas notably Stochastic Processes, Time Series and the later parts of the Core Reading on Loss Distributions. Candidates need to give themselves adequate time to prepare and revise this large syllabus.

Within the examination, careful reading of the questions followed by planning which leads to well-structured answers is important. Although the exam is open book, in some ways candidates might be advised to prepare as if it were closed book and there is more value in using examination time to plan and structure solutions to particular problem questions rather than research syllabus material in open books. The examiners hope that the solutions contained in this document might give examples of thorough and structured solutions to problem questions.

C. Pass Mark

The Pass Mark for this exam was 55.
993 presented themselves and 331 passed.

Solutions for Subject CS2A - September 2023

Q1

(i)

Gompertz Law is a relatively simple expression for the force of mortality [½]
It has been found to work well at older ages [½]
In 2020 this group were aged 64 to 72 [½]
A Makeham form with a non-age related element might be slightly better [½]
The formula contains and exponential element to mortality [½]
The value of *c* in this Gompertz expression is relatively small [½]
Hence mortality increases relatively slowly with age [½]

[Marks available 3½, maximum 2]

(ii)

assuming that the force of mortality is constant for each year of age [½]
we can find one year survival probabilities by exp(-mu_x) [½]
and four year survival probabilities by chaining together for 4 years [½]
then expected number at the next dinner
= 3 * 4_p_72 + 8 * 4_p_68 + 18 * 4_p_64 [1]

The mu_x and p_x calculations are:

age	mu_x	exp(-mu_x)
64	0.004617	0.995394
65	0.004619	0.9953921
66	0.00462	0.9953903
67	0.004622	0.9953885
68	0.004624	0.9953866
69	0.004626	0.9953848
70	0.004628	0.9953829
71	0.00463	0.9953811
72	0.004631	0.9953793
73	0.004633	0.9953774
74	0.004635	0.9953756
75	0.004637	0.9953737

[1½]

giving expected number
= 18* 0.981692 + 8 * 0.981663 + 3* 0.981634
= 28.47 people [1]

Alternative solution using the formula for t_p_x in the Gompertz found in the Core Reading (the formula can be stated and does not need to be derived)

Using the formula on page 32 of the Core Reading:
4p72 = g ^(c^72 (c^4-1))
where g = exp(-0.0045 / log(1.0004)) = 0.000012978
and c = 1.0004

So:
4p72 = 0.9816302

Similarly:
4p68 = g ^(c^72 (c^4-1)) = 0.9816593
4p64 = g ^(c^72 (c^4-1)) = 0.98168
So expected number is: 0.9816302 * 3 + 0.9816593 * 8 + 0.9816883 * 18
= 28.46855
(full marks are available for other alternative approaches correctly evaluated)

(iii)
the new member is age 60 in 2020
there are dinners in 2024 / 28 / 32 / 36 / 40 [1]
therefore expected cost
= 60 * (4_p_60 + 8_p_60 + 12_p_60 + 16_p_60 + 20_p_60) [1]
again assuming constant force of mortality during each year
and that the same Gompertz formula can be applied to the new entrant [1]

the mu_x and p_x are given by

age	mu_x	exp(-mu_x)
60	0.004609	0.9954013
61	0.004611	0.9953995
62	0.004613	0.9953977
63	0.004615	0.9953958
64	0.004617	0.995394
65	0.004619	0.9953921
66	0.00462	0.9953903
67	0.004622	0.9953885
68	0.004624	0.9953866
69	0.004626	0.9953848
70	0.004628	0.9953829
71	0.00463	0.9953811
72	0.004631	0.9953793
73	0.004633	0.9953774
74	0.004635	0.9953756
75	0.004637	0.9953737
76	0.004639	0.9953719
77	0.004641	0.99537
78	0.004643	0.9953682
79	0.004644	0.9953663

[1½]

then
4_p_60 = 0.981721
4_p_64 = 0.981692

4_p_68 = 0.981663
4_p_72 = 0.981634
4_p_76 = 0.981605

expected cost = 60 * (0.981721 + (0.981721)(0.981692) +
(0.981721)(0.981692)(0.981663) + (0.981721)(0.981692)(0.981663)(0.981634)
+ (0.981721)(0.981692)(0.981663)(0.981634)(0.981605) [1]
= 60 * 4.731859 = £283.91 [½]

[Total 13]

This question was generally well-answered.

There are a number of correct routes to solutions for parts (ii) and (iii) and all are able to receive full credit. The Gompertz calculations are all quite straightforward. Most candidates completed these in a spreadsheet and then pasted them into their answer document with the addition of appropriate explanations and assumptions.

Where candidates were not awarded many marks,, the most common reason was not reading the question carefully.

Q2

(i)(a)
(F(beta_n * x + alpha_n)) ^ n
= (beta_n * x + alpha_n) ^ n [1]
= (x / n + 1 – 1 / n) ^ n [½]
= (1 – 1 / n * (1 – x)) ^ n [½]
= (1 – 1 / n * (1 + C * (x – A) / B) ^ -(1 / C)) ^ n [½]

(i)(b)
(F(beta_n * x + alpha_n)) ^ n
= (1 – (lambda / (lambda + beta_n * x + alpha_n)) ^ delta) ^ n [1]
= (1 – ((lambda + beta_n * x + alpha_n) / lambda) ^ -delta) ^ n [½]
= (1 – ((lambda * n ^ (1 / delta) + n ^ (1 / delta) * x) / lambda) ^ -delta) ^ n [½]
= (1 – 1 / n * (1 + x / lambda) ^ -delta) ^ n [½]
= (1 – 1 / n * (1 + C * delta * x / lambda) ^ -(1 / C)) ^ n [½]
= (1 – 1 / n * (1 + C * (x – A) / B) ^ -(1 / C)) ^ n [½]

(For both (a) and (b) the candidate can obtain marks either by starting with LHS of the equations in the question and substituting in the CDF of the uniform / pareto to obtain the RHS or they can start with the RHS and apply the values of A,B,C to obtain the uniform / pareto CDFs. In either case at least two lines of working are required beyond the initial substitution to "demonstrate" as the question asks)

(ii)
The LHS is the probability that the block maximum of *n* observations is less than or equal to *beta_n * x + alpha_n* [1]

since the block maximum is less than or equal to *beta_n* * *x* + *alpha_n* if and only if all the observations are less than or equal to *beta_n* * *x* + *alpha_n* [1]
As *n* tends to infinity, the RHS tends to exp(-1 / *n* * (1 + *C* * (*x* – *A*) / *B*) ^ -(1 / *C*)) [1]
which is the distribution function of the generalised extreme value (GEV) distribution [1]
We have therefore shown that the distribution of a linear function of the block maximum approaches a GEV distribution as *n* tends to infinity [1]
we can use (1+x/n)^n tends to exp(x) as n tends to infinity [1]
In part (i)(a), the value of *C* is negative [½]
which indicates that the GEV distribution is of Weibull type [½]
This is as expected since the uniform distribution has a finite upper limit [½]
In part (i)(b), the value of *C* is positive [½]
which indicates that the GEV distribution is of Fréchet type [½]
This is as expected since the Pareto distribution has a heavy tail [½]
[Marks available 9, maximum 8]
(½ mark available for other generalised sensible comments, and 1 mark available for other GEV related comments)

[Total 14]

This question was poorly answered and in terms of average marks was the question that attracted lowest marks as a percentage of those available.

In part (i) candidates who scored well set out a clear structure that aligned with the question's request to "demonstrate" the equality given.

For both parts (i) (a) and (b) the marks could be obtained either by starting with LHS of the equations in the question and substituting in the CDF of the uniform / pareto to obtain the RHS or by starting with the RHS and applying the values of A,B,C to obtain the uniform / pareto CDFs. In either case, clarity of structure to the answer is key to demonstrating understanding of the loss distributions in question.

Part (ii) was particularly poorly answered and candidates are reminded of the importance of revising the whole subject syllabus including Extreme Value.

Q3
(i)
Let X_1, X_2 …X_5 be the choice of meal in the days 1-Monday, 2-Tuesday, through to … 5-Friday.
P(X_1=Pizza, X_3=Salad)= P(X_3=Salad|X_1=Pizza)* P(X_1=Pizza) [1]
=P^2(2,1) * 1/3 [1]
=0.24*1/3 [½]
=0.08 [½]

Where P^2= $\begin{pmatrix} 0.3 & 0.2 & 0.5 \\ 0.1 & 0.3 & 0.6 \\ 0.3 & 0.2 & 0.5 \end{pmatrix} * \begin{pmatrix} 0.3 & 0.2 & 0.5 \\ 0.1 & 0.3 & 0.6 \\ 0.3 & 0.2 & 0.5 \end{pmatrix} = \begin{pmatrix} 0.26 & 0.22 & 0.52 \\ 0.24 & 0.23 & 0.53 \\ 0.26 & 0.22 & 0.52 \end{pmatrix}$ [1]

(Full marks also available where the candidate uses the relevant direct probability calculations rather than using the P^2 matrix)

(ii)
P(X_3=Sushi, X_5=Sushi)= P(X_1=Pizza , X_3=Sushi,X_5=Sushi)+
P(X_1=Salad, X_3=Sushi,X_5=Sushi) + P(X_1=X_3 = X_5 =Sushi) [1]
For the first term:
P(X_1=Pizza , X_3=Sushi,X_5=Sushi)= [½]
P(X_1=Pizza) P(X_3=Sushi,X_5=Sushi| X_1=Pizza) = [½]
P(X_1=Pizza) P(X_3=Sushi | X_1=Pizza)* P(X_5=Sushi | X_3=Sushi)= [½]
1/3 * P^2(2,3) * P^2(3,3) = 1/3*0.53*0.52 [½]
=0.09187 [½]

Similarly the second term is
P(X_1=Salad , X_3=Sushi,X_5=Sushi)= [½]
1/3* P^2(1,3)* P^2(3,3)=1/3*0.52*0.52 [½]
=0.09013 [½]

and the final term is
P(X_1 = X_3 = X5 = Sushi) = 1/3*052*0.52 = 0.09013 [½]
And so the final figure is 0.09187+0.09013+0.09013 = 0.272 [½]

[Total 10]

This question was generally well answered and is a relatively straightforward application of stochastic processes. The matrix approach, correctly applied, provides the most straightforward route to answering the whole question whereas the combination of individual direct-route probability calculations was more prone to error.

Q4
(i)
A_t = sum(i=1 to t) Z_i [½]
E[A_t] = sum(i=1 to n) E[Z_i] = 0 [½]
Var(A_t) = sum(i=1 to n) Var(Z_i)
= sum(i=1 to t) { 2^2 x 0.5 + 2^2 x 0.5) = 4t [1]

(ii)
There are 2^n paths the score may take and each one has equal probability [1]
In order for A_n = k there must be non-negative integers p and m
where p is the number of '+2' and m the number of '-2' such that
p + m = n, and

2(p – m) = k [1]
that is
p = (2n + k)/4
m = (2n – k)/4 [½]

The number of ways to arrange these p and m results is
$\binom{p+m}{p} = \binom{n}{(2n+k)/4}$ [1]
Hence
Pr(A_n = k) = $\binom{n}{(2n+k)/4}/2^n$ [½]

(iii)
Pr(A_1 = 2 | A_n = 0) = $\frac{\Pr(A_n=0\ |A_1=2)\ x\ \Pr\ (A_1=2)}{\Pr(A_n=0)}$ [½]
now Pr(A_1 = 2) = ½ and [½]
Pr(A_n = 0) = $\binom{n}{\frac{n}{2}}/2^n$ [½]

for Pr(A_n = 0 | A_1 = 2) there are 2^(n-1) paths the score may take after A_1 = 2
we need non-negative p' and m' where p' is the number of +2 and m' the number of -2
then
p' + m' = n -1
m' – p' = 1
so
m' = n/2 and p' = n/2 – 1 [1]

The number of ways to arrange these p' and m' results is
$\binom{p\prime+m\prime}{p\prime} = \binom{n-1}{\frac{n}{2}-1}$ [1]

so
Pr(A_n = 0 | A_1 = 2) = $\frac{\frac{1}{2}x\binom{n-1}{\frac{n}{2}-1}/2^{n-1}}{\binom{n}{\frac{n}{2}}/2^n} = \frac{\binom{n-1}{\frac{n}{2}-1}}{\binom{n}{\frac{n}{2}}}$ [1]

= ½ n /n = 0.5 [½]

[Total 11]

Alternatively full marks are available for a full reasoning out of 0.5 in words: This reasoning would include the following or similar,

- *the first outcome is either a pass or a fail.*
- *the number of routes to a final score of 0 at time n if the first outcome is pass,*
- *is the same as the number of routes to a final score of 0 at time n if the first is a fail.*
- *due to symmetry of the success / fail pathways.*
- *the success / failure probability for the first outcome is ½.*
- *therefore the required probability must be 0.5.*

Overall, this question was very poorly answered, particularly parts (ii) and (iii).

Part (i) was well answered.

In part (ii) candidates that recognised the Binomial nature of the problem were rewarded with more marks available the more reasoning was given. This question is an excellent example of the application of statistical techniques covered in the Core Reading to a novel problem. These questions do require candidates spend some time thinking through the problem and structuring an answer and the examination papers.

Q5

(i)

$p = 0$, $d = 1$, $q = 1$ [1]

(ii)

var(Y_t) = var($e_t + b * e_{t-1}$)
= var(e_t) + b ^ 2 * var(e_{t-1}) + 2 * b * cov(e_t, e_{t-1}) [1]
= *sigma* ^ 2 + b ^ 2 * *sigma* ^ 2 + 2 * b * 0 [½]
= (1 + b ^ 2) * *sigma* ^ 2 [½]

cov(Y_t, Y_{t-1}) = cov($e_t + b * e_{t-1}$, $e_{t-1} + b * e_{t-2}$)
= cov(e_t, e_{t-1}) + b * cov(e_t, e_{t-2}) + b * cov(e_{t-1}, e_{t-1}) + b ^ 2 * cov(e_{t-1}, e_{t-2}) [1]
= 0 + b * 0 + b * *sigma* ^ 2 + b ^ 2 * 0 [½]
= b * *sigma* ^ 2 [½]

(iii)

Note that var(X_t) is the same as cov(X_t, X_{t-k}) for $k = 0$ [½]

cov(X_t, X_{t-k}) = cov(sum(1, t): Y_i, sum(1, $t - k$): Y_j)
= sum(1, t): sum(1, $t - k$): cov(Y_i, Y_j) [1]

The double sum contains $t - k$ terms with $j = i$ [½]
$t - k - 1$ terms with $j = i + 1$ [½]
and $t - k$ terms with $j = i - 1$ [½]
unless $k = 0$, in which case there are $t - k - 1$ terms with $j = i - 1$ [½]

Hence if $k > 0$, then cov(X_t, X_{t-k}) = ($t - k$) * (1 + b ^ 2) * *sigma* ^ 2
+ ($t - k - 1$) * b * *sigma* ^ 2 + ($t - k$) * b * *sigma* ^ 2 [1]
= ($t - k$) * (1 + b ^ 2) * *sigma* ^ 2 + (2 * ($t - k$) - 1) * b * *sigma* ^ 2 [½]

If $k = 0$, then cov(X_t, X_{t-k}) = var(X_t) = ($t - k$) * (1 + b ^ 2) * *sigma* ^ 2
+ ($t - k - 1$) * b * *sigma* ^ 2 + ($t - k - 1$) * b * *sigma* ^ 2 [½]
= t * (1 + b ^ 2) * *sigma* ^ 2 + 2 * ($t - 1$) * b * *sigma* ^ 2 [½]

(iv)
The autocorrelation at lag k is $cov(X_t, X_{t-k}) / var(X_t)$ [1]
Hence the results show that the autocorrelation decreases [1]

[Total 13]

This question was reasonably well answered. Following recent examination sessions where Time Series questions in paper A have been very poorly answered, the examiners were pleased to see improvement in this area.

Parts (i) and (ii) were well answered and are straightforward applications of the Core Reading on ARIMA() models.

Part (iii) was less well answered and as with comments on earlier questions, a major differentiator between stronger and weaker answers was the clarity of structure in the solution set out.

Q6

(i)
X1 and X2 be the random variables representing cash withdrawal per customer from the two ATM vestibules respectively [½]
'X' be the random variable representing combined cash withdrawal per customer [½]
Let lambda1 and lambda2 be the parameters of the Compound Poisson distributions [½]
As per page 13 of Unit 2 of the core reading, sum of 2 compound Poisson distributions follows a Compound Poisson distribution with
Parameter lambda_c= lambda1 + lambda2 [1]
And the cdf of the combined withdrawn amount per customer is;
P (X < 1400) = 1/ lambda_c *{lambda * P (X1 < 1400) + lambda2 * P (X2 < 1400) }
[1½]
= 1/70{20* phi((1400-1500)/300) +50* phi((1400-1000)/200) } [1½]
=0.80359 [½]

(ii)
Let S1 and S2 be the random variables representing the 2 compound distributions from the two ATM vestibules respectively [½]
Let A= S1+ S2 [½]

Exp(A) = lambda_c * EXP(X) [½]
Variance(A) = lambda_c * (Var(X) + (EXP(X))^2) [½]

MGF (X) = EXP(e^tx) = 1/ lambda_c *{lambda1 * MGF (X1) + lambda2 * MGF (X2) } [1]

Deriving once and taking t= 0 gives EXP(X) = 1/ lambda_c *{lambda1 * EXP (X1) + lambda2 * EXP (X2) } [1½]

Deriving TWICE and taking t= 0 = EXP(X^2) = 1/ lambda_c *{lambda1 * EXP (X1^2) + lambda2 * EXP (X2^2) } [1½]

Therefore;
Exp(A) = lambda_c * EXP(X) = 20* 1500+50*1000=$80000 [½]
Variance(A) = lambda_c * EXP(X^2)
= 20* (1500^2+ 300^2) +50*(1000^2+ 200^2)
= $ 98,800,000
[1]
S.D (A) = $ 9939.82 [½]

[Total 14]

Alternative solution
using the formula on p.16 of the Core Reading
E[A] = E[S1 + S2]
= E[S1] + E[S2]
= E[X] * E[N] + E[Y] * E[M]
= 1500*20 + 1000*50
= 80,000
Var[A] = Var[S1+S2] = Var[S1] + Var[S2]
= 20* E[X^2] + 50* E[Y^2]
= 20 * (1500^2 + 300^2) + 50 * (1000^2 + 200^2)
= 98,800,000
so standard deviation = sqrt(98800000) = 9939.82

Part (i) was poorly answered whereas part (ii) was reasonably well answered, meaning that the shape of marks given to many candidates was quite unusual for this question.

The main issue with part (i) was the tendency of many candidates to use a 'sum of normal distributions' rather than a compound distribution approach. Where this was done answers were either incorrect or unnecessarily approximate. The necessary equations for compound Poisson distributions are all found in the Core Reading and the examiners would expect candidates to be familiar with these.

Q7
(i)
First calculate 1- λ at each event
1-λ =0.9 (trivial)
1-λ =0.7/0.9 = 0.7778
1-λ =0.56/0.7 = 0.8
1-λ =0.373/0.56 = 0.66607 *but accept rounding to 2/3 hereafter*
[1½]

Thus lambda:

Time since operation (days)	S(t)	1-lambda	lambda
0≤t<2	1		
2≤t<4	0.900	0.900	0.100
4≤t<5	0.700	0.778	0.222
5≤t<7	0.56	0.800	0.200
7≤t<10	0.373	0.667	0.333

[2]

$\lambda = 0.1$
The only combination is d=1, n=10 [1]
$\lambda = 0.222$
The only combination is d=2, n=9 [½]
$\lambda = 0.2000$
The only combination is d=1, n=5 [½]

$\lambda = 0.33333$
The only combination, given there must be less than 4 lives at this point is d=1, n=3
[½]

Need to account censored events:
Must be 2 censoring events before time = 2 as n=10 at that point [½]
Must be 2 censoring events at time = 4 to fall from n=9 to n=5 [½]
Must be 1 censoring event *at time = 5 or 6* to fall from n=5 to n=3 [1]

So in summary

Time (days)	deaths	Censoring events
<2	0	2
2	1	0
4	2	2
5	1	1 or
6	0	1
7	1	0

(ii)
Unlikely for there to have been random censoring [½]
Very sick patient in bed all the time; it's difficult to see how censoring could have occurred, but data indicates this [½]

(iii)
Collect more data [½]
Have more frequent observations eg hourly [½]
Extend the period of observation [½]
Patients dying in last 2 intervals implies significant lost important data [½]
Reduce heterogeneity and obtain several different S(t) estimates [½]
Separate for male/female, existing health conditions, other sensible classes [½]

[Marks available 3, maximum 2]
[Total 11]

This question was very well answered. In terms of the average mark as a percentage of the available marks, this question was the best answered on the paper. Throughout this session in both A and B papers, candidates have often done best in survival models questions.

In part (i) again there is a new presentation of a familiar topic (Kaplan Meier) and once again the best answers showed a clear structure: starting with the 1-lambda terms implied by the survival function, moving to derive lambda and finally considering censoring.

In part (ii) a large range of comments about censoring were given credit.

Q8
(i)
mean(y) = exp[mu+0.5*Sigma^2] - - (Eqn 1) [½]
Var(y) = exp[2*mu+ Sigma^2] [exp(Sigma^2)-1] - - (Eqn 2) [1]
squaring Eqn 1
mean(y)^2 = exp[2*mu+Sigma^2] - - (Eqn 3) [½]
dividing Eqn 2 by Eqn 3
var(y)/ mean(y)^2 = [exp(Sigma^2)-1] [½]

Hence:
Sigma = (log((sd(y)/mean(y))^2+1))^0.5 = 0.3852 [½]
Mu = log(mean(y))-0.5*sigma^2 = 7.5267 [½]
P(X>x) = 1-psi(ln(x)-mu/sigma)=0.01 gives x = 4550.2 [1]
Using Invpsi(0.99) = 2.33 [½]

(ii)
P(X<1000) = psi(ln(1000)-mu/sigma)= 5.41%
So 541 claims [1]

(iii)
The probability of very large claims may be significantly underestimated [½]
leading to potential solvency issues [½]
This is particularly the case for long, fat-talied distributions (leptokurtic) [1]
The distribution gives the theoretical possibility of negative claims [½]
It may be suitable under some conditions [½]
where the claims distribution is not skewed [½]
and has thin-tails [½]
It should be left to the insurer's judgement. [1]
[Marks available 5, maximum 3]

(iv)

Weibull is potential candidate distribution	[1]
and Gamma is potential candidate distribution	[1]
both can model skewed observation data,	[½]
and are non-negative	[½]
Exponential will not be suitable,	[½]
as it is a decreasing function of x.	[½]
	[Marks available 4, maximum 3]

Decision criteria:

Use AIC/BIC scores	[½]
or calculate the (log) Likelihood	[½]
or carry out a Chi squared test	[½]
or use QQ plots	[½]
May apply Extreme Value Thery to test the tails	[½]
may depend on the model used in previous years	[½]
or on what is typically used by insurers	[½]
	[Marks available 3½, maximum 2]
	[Total 14]

This question was reasonably well answered.

The calculations in parts (i) and (ii) are relatively straightforward applications of loss distributions.

In part (iii) candidates successfully used a number of different approaches to gain marks. Some applied the results from (i) to calculate a probability of large claim and then commented on that probability whilst others used the portfolio of policies given in the question and centred their comments on that.

In part (iv) many candidates gave full answers on the candidate distributions but wrote too little on potential decision criteria.

[Paper Total 100]

END OF EXAMINERS' REPORT

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