INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

8th September 2021

Subject CS2A – Risk Modelling and Survival Analysis (Paper A)

Time allowed: 3 Hours 30 Minutes (9.30 - 13.00 Hours)

Total Marks: 100

Q. 1) Answer the following Questions

i) The Pareto Distribution is

a. Negatively skewed always
b. Sometimes positively skewed, sometimes negatively skewed. It depends on mean and median of the distribution.
c. Positively skewed always
d. None of the above

(2)

ii) Claims from a particular portfolio are normally distributed with mean 800 and standard deviation 100. An individual excess of loss arrangement with retention limit 800 is in place. Insurer's mean claim payment net of reinsurance is

a. 783.130
b. 547.278
c. 235.855
d. None of the above

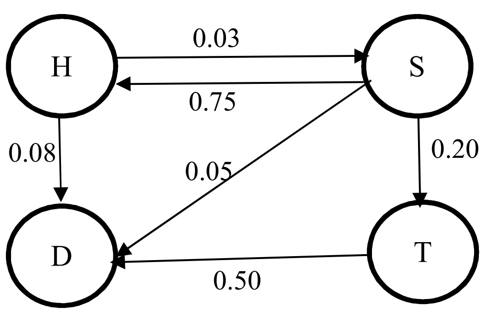
(3)

iii) Claims arising from a certain type of insurance policy are believed to follow an exponential distribution. The lower quartile claim is 250. The mean claim size is

a. 695.210
b. 869.015
c. 180.337
d. 509.520

(2)

iv) Consider the following Health (H), Sickness (S), Death (D) model with the addition of an extra 'Terminally ill' state (T). The rates associated with different transitions are as follows.



What is the expected future life time of a healthy life? (Choose the closest value)

a. 10.5 years
b. 11 years
c. 12 years
d. 12.5 years
e. 14 years

(4)

v) The number of cars that enter the parking area of a shopping complex are modelled as a Poisson process with a rate 40 per hour. What is the probability that 5 cars enter in first 15 minutes and then another 4 cars enter in next 15 minutes?

a. 0.072%
b. 3.783%
c. 1.892%
d. 0.291%
e. Less than 0.001%

(3)

vi) In a certain village, the electric fittings go out of order as soon as there is excessive usage of electricity. The probability of such event happening over a day is 20%. The village has to spend the night in darkness. To fix the electricity, an engineer from the Government agency is called the next day. There is only 75% probability that he is able to fix the electricity the same day. If he is unable to fix, he comes again the next day and tries to fix. The process is repeated till he is able to fix.

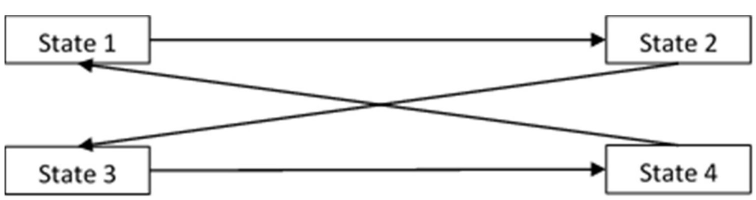
Calculate the long term probability that the village has to see dark nights. Select from the correct options below:

a. 23%
b. 77%
c. 50%
d. 21.05%
e. 78.95%

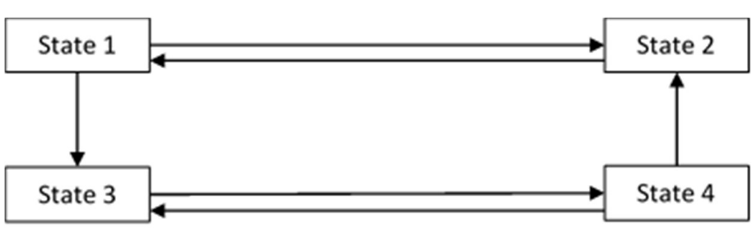
(4)

vii) The diagrams below show four Markov chains, where the arrows represent a non-zero transition probability

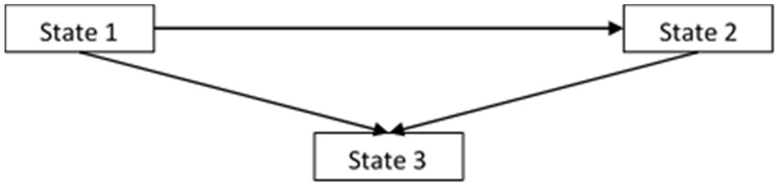
1) Markov Chain 1



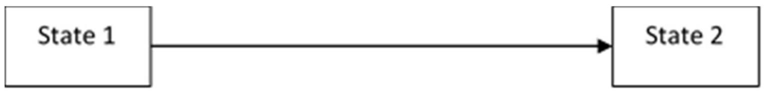
2) Markov Chain 2



3) Markov Chain 3



4) Markov Chain 4



Based on the above, select the correct option

a. None of the four chains are Irreducible
b. Only Markov chains 1,2 and 3 are Irreducible
c. Only Markov chains 1 and 2 are Irreducible
d. Only Markov chains 2 and 3 are Irreducible
e. All four Markov chains are Irreducible

(2)
[20]

Q. 2) **i)** State why it is important to divide data into homogeneous classes when undertaking mortality investigations. (2)

ii) List down certain reason explaining the importance of graduating the crude mortality rates before using them in financial projections. (2)

iii) What do you understand by central exposed to risk? (2)

iv) Specify the data needed for the exact calculation of a central exposed to risk (waiting time) depending on age and sex. (2)
[8]

Q. 3) Individual claim amounts on a particular insurance policy can take the values 150, 200 or 250. There is at most one claim in a year. Annual premiums are 60. The insurer must choose between three reinsurance arrangements:

1. No Reinsurance
2. individual excess of loss with retention 150 for a premium of 10
3. proportional reinsurance of 25% for a premium of 20

i) Complete following table of loss

Loss Table	Reinsurer's loss			Insurer's Loss		
Claims	1	2	3	1	2	3
0						
150						
200						
250						

(4)

ii) Determine whether any of the reinsurance arrangements is dominated from the viewpoint of the insurer. (2)

iii) Determine the minimax solution for the insurer. (1)

[7]

Q. 4) An insurance portfolio contains policies for three categories of policyholder: A, B and C. The number of claims in a year, N, on an individual policy follows a Poisson distribution with mean λ. Individual claim sizes are assumed to be exponentially distributed with mean 4 and are independent from claim to claim. The distribution of λ, depending on the category of the policyholder, is

Category	Value of λ	Proportion of policyholders
A	2	0.2
B	3	0.6
C	4	0.2

If the total claim amount denoted by a policyholder in one year is S

i) Prove that $E(S)=E[E(S|\lambda)]$ (2)

ii) Show that $E(S|\lambda)=4\lambda$ and $Var(S|\lambda)=32\lambda$ (2)

iii) Calculate E(S) (2)

iv) Calculate Var(S) (2)

[8]

Q. 5) **i)** Define an ARCH model and explain what particular properties of the model would make it appropriate for modelling a time series asset.

(4)

Consider the time series

$Y_t=0.1+0.4Y_{t-1}+0.9e_{t-1}+e_t$

Where e_t is white noise process with variance σ^2

ii) Identify the model as an ARIMA(p,d,q) Process (1)

iii) Determine whether Y_t is

a) A stationary process (1)

b) An invertible process (1)

iv) Calculate E(Y_t) and find auto co-variance function for Y_t (6)

v) Determine that MA(∞) representation for Y_t (4)

[17]

Q. 6) Answer the following questions

i) Following will be true if the characteristic equation of the process

$X_n = (11/6)X_{n-1} - X_{n-2} + (1/6)X_{n-3} + e_n$ has one root as $\lambda=2$

a. The process is not stationary and other roots are 3,4
b. The process is stationary and other roots are 3,4
c. The process is not stationary and other roots are 3,1
d. The process is stationary and other roots are 3,1

(4)

ii) When joint distributions $X_{t_1}, X_{t_2}, \dots, X_{t_n}$ *and* $X_{k+t_1}, X_{k+t_2}, \dots, X_{k+t_n}$ are identical for all $t_1, t_2, \dots, t_n$ *and* $k+t_1, k+t_2, \dots, k+t_n$ in J and all integer n

a. X is weakly stationary and statistical properties remain unchanged as time elapses
b. X is weakly stationary and statistical properties change as time elapses
c. X is strictly stationary and statistical properties remain unchanged as time elapses
d. X is strictly stationary and statistical properties change as time elapses

(2)

iii) When X and Y denote two random variables , equation Cov(X+Y,W)=Cov(X+W)+ Cov(Y+W)

a. Will be true only if W is strictly stationary
b. Will be true always
c. Will not be true
d. None of the above

(2)

[8]

Q. 7) As per English Life Tables No. 17 (ELT17), number of lives L(x) at ages 30 and 35 is given below:

Age (x)	L(x)
30	98600.4
35	98193.0

Calculate the number of L(32) assuming:

i) Constant force of mortality between exact ages 30 and 35 years (4)

ii) Uniform distribution of deaths between exact ages 30 and 35 years (2)

iii) It is known that the force of mortality from age 35 to 40 years is double of what was observed during age 30 to 35 years. Using the result from (i); Calculate the probability that a life alive at the age of 35, is still alive at age 38 (2)

[8]

Q. 8) A sports training team is set up of 50 members. From the training, the candidates either get injured or are moved out of the training (injured), or they moved to some other training unit (relocated).

These candidates are observed for a period of 24 months, from the date on which these 50 were admitted into the training team. The table below shows the number of injuries and relocations at different months.

Month	Number of injuries	Number of relocations
2	0	2
4	0	1
7	3	0
8	0	2
10	3	1
11	0	2
14	2	0
15	0	3
17	0	1
18	4	0
20	5	1
24	3	0

The coach of the team wants to estimate the survival function. Calculate the survival function at different time intervals using Kaplan-Meier estimate. **[8]**

Q. 9) A new insurance company launched certain insurance products in year 2015. As this was a new line of business for it then, it has been using the standard mortality table rates to price its product and continues to do so since then. Over the years, it has gathered mortality claims data on its business, which can be used to set its future mortality assumptions. However, the products team is insisting on keeping the mortality assumptions unchanged (same as used since 2015) to price its new products. The Chief Actuary has asked to perform an analysis and validate the suitability of these standard rates.

The table below summarises the experience analysis numbers from the policies sold by the insurance company and the standard mortality rates it has been using so far

Age group	Central exposed to risk	Number of Deaths in the sample	Standard mortality rate used
20-24	56655	80	0.000937
25-29	61220	78	0.000934
30-34	64908	80	0.001042
35-39	62052	85	0.001358
40-44	58751	120	0.001969
45-49	54900	150	0.003168
50-54	48679	295	0.005550
55-59	41699	366	0.008925

i) Carry out Chi square test at 5% confidence interval to check whether the underlying mortality for the insurer is same as the standard mortality rates (8)

ii) Based on (i) above, state your recommendation. (2)
[10]

Q. 10) An insurance company has a portfolio of two-year policies. Aggregate annual claims from the portfolio follow an exponential distribution with mean 25. Claims are independent of each other. Annual premium of 30 is received under the policies. Premiums under the policy are received at the start of the year and all claims are paid at the end of the year. Calculate the probability that the insurer is not ruined by the end of the second year, assuming the insurer started with zero capital at the start of year 1. **[6]**

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