

$$\begin{aligned}
& \sum_{x_a \in X} \omega(x_a) (\bar{\delta}_a(x_a) - \hat{\delta}_a(x_a))^2 \\
&= \frac{1}{2} \sum_{x_a \in X} 2\omega(x_a) \left[(\bar{\delta}_a(x_a))^2 + (\hat{\delta}_a(x_a))^2 - 2\hat{\delta}_a(x_a)\bar{\delta}_a(x_a) \right] \\
&= \frac{1}{2} \sum_{x_a \in X} 2\omega(x_a) (\bar{\delta}_a(x_a))^2 + 2\omega(x_a) (\hat{\delta}_a(x_a))^2 \\
&\quad - 2\bar{\delta}_a(x_a)\hat{\delta}_a(x_a)\omega(x_a) \\
&= \underbrace{\sum_{x_a \in X} \omega(x_a) (\bar{\delta}_a(x_a))^2}_{\text{constant}} + \frac{1}{2} \sum_{x_a \in X} 2\omega(x_a) (\hat{\delta}_a(x_a))^2 \\
&\quad - 2\bar{\delta}_a(x_a)\hat{\delta}_a(x_a)\omega(x_a)
\end{aligned}$$

The given problem now reduced to solving the non constant part.

$$\frac{1}{2} \sum_{x_a \in X} 2\omega(x_a) (\hat{\delta}_a(x_a))^2 - 2\bar{\delta}_a(x_a)\hat{\delta}_a(x_a)\omega(x_a)$$

Let say $X = \{x_{a_1}, x_{a_2}, x_{a_3}\}$.

The first half of the equation becomes

$$\begin{aligned}
& \frac{1}{2} (2\omega(x_{a_1})\hat{\delta}(x_{a_1})^2 + 2\omega(x_{a_2})\hat{\delta}(x_{a_2})^2 \\
& \quad + 2\omega(x_{a_3})\hat{\delta}(x_{a_3})^2)
\end{aligned}$$

Let $\hat{\sigma}(x_{a_1}) = x_1$, $\hat{\sigma}(x_{a_2}) = x_2$, $\hat{\sigma}(x_{a_3}) = x_3$

(Since $\hat{\sigma}(x_a)$ is the x we are trying to solve)

$$= \frac{1}{2} (2\omega(x_{a_1}) x_1^2 + 2\omega(x_{a_2}) x_2^2 + 2\omega(x_{a_3}) x_3^2)$$

$$= \frac{1}{2} [x_1^T (2\omega(x_{a_1})) x_1 + x_2^T (2\omega(x_{a_2})) x_2 + x_3^T (2\omega(x_{a_3})) x_3]$$

$$= \frac{1}{2} [x_1 \ x_2 \ x_3] \begin{bmatrix} 2\omega(x_{a_1}) & 0 & 0 \\ 0 & 2\omega(x_{a_2}) & 0 \\ 0 & 0 & 2\omega(x_{a_3}) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

This is of the form $\frac{1}{2} x^T H x$

$$\therefore H = \begin{bmatrix} 2\omega(x_{a_1}) & 0 & 0 \\ 0 & 2\omega(x_{a_2}) & 0 \\ 0 & 0 & 2\omega(x_{a_3}) \end{bmatrix}$$

The second half of the equation

$$\frac{1}{2} \sum_{x_a \in X} -2 \omega(x_a) \bar{\phi}(x_a) \hat{\phi}(x_a)$$

$$= - \sum_{x_a \in X} \omega(x_a) \bar{\phi}(x_a) \hat{\phi}(x_a)$$

Let $X = \{x_{a_1}, x_{a_2}, x_{a_3}\}$ and $\hat{\phi}(x_{a_1}) = x_1$
 $\hat{\phi}(x_{a_2}) = x_2, \hat{\phi}(x_{a_3}) = x_3$

$$= - \left[\omega(x_{a_1}) \bar{\phi}(x_{a_1}) x_1 + \omega(x_{a_2}) \bar{\phi}(x_{a_2}) x_2 \right. \\ \left. + \omega(x_{a_3}) \bar{\phi}(x_{a_3}) x_3 \right]$$

$$= \left[-\omega(x_{a_1}) \bar{\phi}(x_{a_1}) \quad -\omega(x_{a_2}) \bar{\phi}(x_{a_2}) \quad -\omega(x_{a_3}) \bar{\phi}(x_{a_3}) \right]$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

which is of the form $f^T x$

$$\therefore f^T = \left[-\omega(x_{a_1}) \bar{\phi}(x_{a_1}) \quad -\omega(x_{a_2}) \bar{\phi}(x_{a_2}) \quad -\omega(x_{a_3}) \bar{\phi}(x_{a_3}) \right]$$

constraints :

$$\hat{\delta}(x_a) \leq \hat{\delta}(y_a) \quad \forall (x_a, y_a) : x_a \leq y_a.$$

$$\text{let } x = \{x_{a_1}, x_{a_2}, x_{a_3}\}$$

$$\text{and } x_{a_1} \leq x_{a_2}$$

$$x_{a_2} \leq x_{a_3}$$

Now the constraints become

$$\hat{\delta}(x_{a_1}) \leq \hat{\delta}(x_{a_2}) \Rightarrow \hat{\delta}(x_{a_1}) - \hat{\delta}(x_{a_2}) \leq 0$$

$$\hat{\delta}(x_{a_2}) \leq \hat{\delta}(x_{a_3}) \Rightarrow \hat{\delta}(x_{a_2}) - \hat{\delta}(x_{a_3}) \leq 0$$

which is of the form

$$Ax \leq b$$

$$\begin{matrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} & \begin{matrix} (A) \end{matrix} & \begin{bmatrix} \hat{\delta}(x_{a_1}) \\ \hat{\delta}(x_{a_2}) \\ \hat{\delta}(x_{a_3}) \end{bmatrix} & \begin{matrix} (x) \end{matrix} & \leq & \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} & \begin{matrix} (b) \end{matrix} \end{matrix}$$