Computer Science Foundations
Puzzle-Solving Workshop and Seminar

Episode 5—October 28
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Welcome to the Puzzle-Solving Workshop and Seminar for Computer Science Foundations. You will notice there are two parts to this thread: *workshop* and *seminar*. Workshop is meant to strengthen your problem-solving skills, to help you in the Discrete Math thread as well as future math and computer science courses. Seminar is meant to work on your discussion, writing, and creativity skills. We will alternate between the two different modes.

Today we will be in workshop mode only.

1 Quantified Love

"Let us talk about love," Ada says suddenly as she and Carl are sitting on the grass, enjoying one of the few sunny afternoons left to them that autumn.

"Love?" asks Carl, "or looooooove?" he adds with a cheesy smile. As usual, he thought this was supposed to be a date.

"Consider," Ada says, raising her finger as she does when she's about to pose a logical problem. "Let's use the logical symbol $P(x, y)$ to mean that x is y's soulmate."

"Hmmm, do you believe that people have soulmates?" Carl tries to steer the conversation away from discrete math, which is how all their conversations end. Perhaps some philosophy would be romantic... "Or that people even have souls?"

"This is just logic. I'll let other people worry about belief and souls." Ada opens her notebook and begins to write. "We will consider the statement $P(x, y)$ over the domain of all people in the world."

Carl stews quietly in his disappointment.

Regardless, satisfy Ada's curiosity by translating the following quantified propositional logic statements into English and decide whether they are true, based on your opinion. Justify it, using normal, non-logical arguments.

(a) $\forall x \exists y P(x, y)$

(b) $\exists x \forall y P(x, y)$

(c) $P(x, y) \leftrightarrow P(y, x)$

2 A Puzzle About Fidelity

Carl wants to show Ada that it is ridiculous to reduce human relationships to logical statements. To do so, he makes up a fable that is so absurd, Ada will have to give up in frustration and see his point.[1]

This is what he says:

> A village consists of 100 married couples of a specific form. Each couple consists of a Jealous Partner and a Sketchy Partner. Every Jealous Partner in the village knows if a Sketchy Partner other than his or her partner has cheated, but does *not* know if her or his own partner has cheated. The village has a law that does not allow for adultery. Any Jealous Partner who can prove that her or his partner is unfaithful must chase her or him out of the village that very day. The people of the village would never disobey this law. One day, the fabulous drag queen of the village visits and announces that exactly one Sketchy Partner has been unfaithful (this counts as evidence, or legal proof, in the village.) At the end of that day, the law must be enforced.

"So, what happens?" Carl asks, half-smiling.

Ada taps her finger to her lips in curiosity, "Hmm, that's very interesting." Instead of thinking the fable is absurd, she immediatedly beings scribbling in her notebook and launches into it. "Here are some things we should consider."

(a) Make up a propositional logic symbol with two variables for the sentence "Sketchy Partner x and Jealous Partner y are married."

(b) Translate the sentence "Every Sketchy Partner x in the village is married to a Jealous Partner y and not to any other Jealous Partner in the village." into quantified propositional logic using your previous symbol. (We will consider the domain of the sentence over the 100 couples in the village, in this and every other problem below, but you don't need to write anything special for this. Just keep it in mind).

(c) Translate the sentence "Every Jealous Partner x in the village is married to a Sketchy Partner y and not to any other Sketchy Partner in the village." into quantified propositional logic, again using your symbol in the first part.

(d) Are the previous two logical statements equivalent?

[1] Note that Google and Microsoft have asked a hetero-normative version of this puzzle in past job interviews.

(e) Make up a propositional logic symbol with one variable for the sentence "Sketchy Partner x has cheated on his or her Jealous Partner."

(f) Write a quantified propositional logic statement using the above problems for the sentence "Jealous Partner y has evidence that her or his Sketchy Partner has cheated."

(g) Write a quantified propositional logic statement, using the above problems, for the sentence "Jealous Partner y chases her or his Sketchy Partner out of the village."

(h) Express the rule of the village using the previous two problems.

(i) Make up a quantified propositional logic symbol for the drag queen's incriminating statement: "Exactly one Sketchy Partner in the village has cheated on her or his Jealous Partner."

(j) Using all the previous problems, write a quantified propositional logic statement that describes what happens at the end of the day, in answer to Carl's question.

3 Sets

"But now," Ada continues, despite Carl's exasperation, "let's assume that the female impersonator—"

"The what?" Carl interrupts, confused about what is even going on anymore.

"The drag queen. Drags queens are also called female impersonators. Some of them can be very convincing."

"You don't mean that..." Carl trails off in disbelief, his eyes widening.

"I don't know *what* you're talking about." Ada scoffs. "Let's assume that the drag queen says that *more than one* Sketchy Partner has cheated."

(a) Make up a symbol to represent the set of all cheating Sketchy Partners, and make up another symbol for its size (the number of cheating partners).

"We can also consider the set of sets of the Cheaters that a Jealous Partner knows about. I wonder what we can describe using set notation." Ada stares into the sky.

Carl decides he must put his brain to use in order to impress Ada. After all, he added up all the integers from 1 to 100, surely he could figure something out in this case. Help Carl with the following problems.

(b) Make up a different symbol with a subscript x to represent the set of Cheating Partners that a Jealous Partner x knows about.

(b) Express the set in part (a) using the sets in part (b), using the symbols you defined and the symbols for set operations (difference, union, or complement)

"I wonder," says Carl, "if we can represent the *non*-cheating Sketchy Partners using my notation."

"That's a pretty good question, Carl," Ada says, looking at Carl as if noticing him for the first time. Carl opens his mouth to say something charming, just as Ada's eye is caught by Elise De Morgan waving from across the campus.

"Hey Ada," Elise says, as she walks up. "Hey Carl. What are you guys working on?"

"Nothing you'd care about, De Morgan, really, see you later," Carl tries to get rid of the newcomer.

"Haha, always joking, Carl. But De Morgan is my dad's name. Just call me Elise."

"Hey, Elise. Maybe you can help us," Ada says. "Since your dad is so good at rules of inference and Boolean logic. Did he teach you anything about it?"

"I might remember a thing or two," Elisa says, "let's take a look."

(d) Express the set of all *non*-Cheating Partners using your set notation from all previous problems.

Will Carl or Elise win Ada's affection using logic and your help? Will Ada ever stop thinking about logic? Unlock the next episode to find out! To be continued...