Matrix assignment

January 4, 2024

Questions

1. If $\mathbf{A}$ is a square matrix satisfying $A'A = I$, write the value of $\left|A\right|$.

2. If $y = x\left|x\right|$, find $\dfrac{dy}{dx}$ for $x < 0$.

3. $\dfrac{d^2y}{d^2x} + x\left(\dfrac{dy}{dx}\right)^2 = 2x^2\log\left(\dfrac{d^2y}{dx^2}\right)$.

4. Find the direction cosines of a line which makes equal angles with the coordinate axes.

5. A line passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction of the vector $\hat{i} + \hat{j} - 2\hat{k}$. Find the equation of the line in cartesian form.

6. Examine whether the operation defined on $\mathbf{R}$, the set of all real numbers, by $a * b = \sqrt{a^2 + b^2}$ binary operation or not, and if it is a binary operation, find whether it is associative or not.

7. $\mathbf{A} = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$, show that$(\mathbf{A} - 2\mathbf{I})(\mathbf{A} - 3\mathbf{I})$= 0

8. Find $\int \sqrt{3 - 2x - x^2}dx$

9. Find $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}dx$

10. Find $\int \frac{x-3}{(x-1)^3}e^x dx$

11. Find the differential equation of the family of curves $y = Ae^{2x} + Be^{-2x}$, where A and B are arbitrary constants.

12. If $|\overrightarrow{a}| = 2|\overrightarrow{b}| = 7$ and $\overrightarrow{a} \times \overrightarrow{b} = 3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$, find the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$

13. Find the volume of a cuboid whose edges are given by $-3\hat{\imath} + 7\hat{\jmath} + 5\hat{k}$,$-5\hat{\imath} + 7\hat{\jmath} - 3\hat{k}$ and $7\hat{\imath} - 5\hat{\jmath} - 3\hat{k}$

14. If $P(notA) = 0.7$, $P(B) = 0.7$ and $P(B \mid A) = 0.5$, then find $P(A \mid B)$.

15. A coin is tossed 5 times. What is the probability of getting

    (i) 3 heads

    (ii) at most 3 heads

16. Find the probability distribution of X, the number of heads in a simultaneous toss of two coins

17. Check whether the relation R defined on the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive.

18. Let $f : N \to Y$ be a function defined as $f(x) = 4x + 3$, where $Y = \{y \in N : y = 4x + 3, \text{ for some } x \in N\}$. Show that f is invertible. Find its inverse.

19. Find the value of $\sin\left(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right)$.

20. Using properties of determinants, show that $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$
$= 3(a+b+c)(ab+bc+ca)$

21. If $x\sqrt{1+y}+y\sqrt{1+x}=0$ and $x \neq y$, prove that $\dfrac{dy}{dx} = \frac{-1}{(x+1)^2}$.

22. If $(\cos x)^y = (\sin y)^x$, find $\dfrac{dy}{dx}$.

23. If $(x-a)^2+(y-b)^2=c^2$, for some $c>0$, prove that $\dfrac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}$ is a constant independent of a and b.

24. Find the equation of the normal to the curve $x^2=4y$ which passes through the point $\left(-1,4\right)$.

25. Find:

$$\int \frac{x^2+x+1}{(x+2)\left(x^2+1\right)}dx$$

26. prove that

$$\int_0^a f\left(x\right)dx=\int_0^a f\left(a-x\right)dx$$

and hence evaluate

$$\int_0^{\frac{\pi}{2}} \frac{x}{\sin x+\cos x}dx$$

27. Solve the differential equation:

$$x\frac{dy}{dx}=y-x\tan\left(\frac{y}{x}\right)$$

28. Solve the differential equation :

$$\frac{dy}{dx}=-\left[\frac{x+y\cos x}{1+\sin x}\right]$$

29. The scalar product of the vector $\overrightarrow{a}=\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of the vectors $\overrightarrow{b}=2\hat{i}+4\hat{j}-5\hat{k}$ and $\overrightarrow{c}=\hat{\lambda}+2\hat{j}+3\hat{k}$ and is equal to 1. Find the value of λ and hence find the unit vector along $\overrightarrow{b}+\overrightarrow{c}$.

30. If the lines $\frac{x-1}{-3}=\frac{y-2}{2\lambda}=\frac{z-3}{2}$ and $\frac{x-1}{3\lambda}=\frac{y-1}{2}=\frac{z-6}{-5}$ are perpendicular find the value of λ. Hence find weather the lines are intersecting or not.

31. If $\mathbf{A} = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 1 & 1 \\ 5 & 1 & 1 \end{pmatrix}$, Find A^{-1}. Hence solve the system of equations

$$\begin{aligned} x + 3y + 4z &= 8 \\ 2x + y + 2z &= 5 \\ 5x + y + z &= 7 \end{aligned}$$

32. Find the inverse of the following matrix, using elementary transformations :

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & 1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$

33. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

34. Using method of integration, find the area of the triangle whose vertices are $\big(1,0\big),\big(2,2\big),\big(3,1\big)$.

35. Using method of integration, find the area of the region enclosed between two circles $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 4$.

36. Find the vector and cartesian equations of the plane passing through the points having position vectors $\hat{\imath} + \hat{\jmath} - 2\hat{k}$, $2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\hat{\imath} + 2\hat{\jmath} + \hat{k}$. Write the equation of a plane passing through a point $\big(2,3,7\big)$ and parallel to the plane obtained above. Hence, find the distance between the two parallel planes.

37. Find the equation of the line passing through $(2,-1,2)$ and $(5,3,4)$ and of the plane passing through $(2,0,3)$, $(1,1,5)$ and $(3,2,4)$. Also, find their point of intersection.

38. There are three coins. One is a two-headed coin, another is a biased coin that comes up heads 75% of the time and the third is an unbiased coin. One of the three coins is chosen at random and tossed. If it shows heads, what is the probability that it is the two-headed coin ?

39. A company produces two types of goods, A and B, that require gold and silver. Each unit of type A requires 3 g of silver and 1 g of gold while that of type B requires 1 g of silver and 2 g of gold. The company can use at the most 9 g of silver and 8 g of gold. If each unit of type A brings a profit of ₹40 and that of type B ₹50,find the number of units of each type that the company should produce to maximize profit. Formulate the above LPP and solve it graphically and also find the maximum profit.

40. Find $\left|AB\right|$, if $\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 0 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 & 5 \\ 0 & 0 \end{pmatrix}$.

41. Differentiate $e^{\sqrt{3x}}$, with respect to x.

42. If $\mathbf{A} = \begin{pmatrix} p & 2 \\ 2 & p \end{pmatrix}$ and $\left|A^3\right| = 125$, then find the values of p

43. Find the general solution of the differential equation $\dfrac{dy}{dx} = e^{x+y}$.

44. If $(a + bx)\, e^{\frac{y}{x}} = x$,then prove that $x^3 \dfrac{d^2y}{dx^2} = \left(x\dfrac{dy}{dx} - y\right)^2$.

45. The volume of a cube is increasing at the rate of $8cm^3/s$. How fast is the surface area increasing when the length of its edge is $12cm$?

46. Find the cartesian and vector equations of the plane passing through the points $A(2, 5, -3)$, $B(-2, -3, 5)$, $C(5, 3, -3)$.

47. Find $\int_1^3 (x^2 + 2 + e^{2x})dx$ as the limit of sums.

48. Using integration, find the area of the triangular region whose sides have the equations $y = 2x + 1$, $y = 3x + 1$, and $x = 4$.

49. Find the differential equation representing the family of curves $y = ae^{2x} + 5$, where a is an arbitrary constant.

50. If $y = \cos\left(\sqrt{3x}\right)$, then find $\dfrac{dy}{dx}$.

51. Show that the points $A\left(-2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}\right)$, $B\left(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}\right)$ and $C\left(7\hat{\imath} - \hat{k}\right)$ are collinear.

52. Find $\left|\overrightarrow{a} \times \overrightarrow{b}\right|$, if $\overrightarrow{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\overrightarrow{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$.

53. Find:

$$\int \frac{x-5}{(x-3)^3} e^x dx$$

54. Solve for x:

$$\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\left(\frac{8}{31}\right)$$

55. If $x = ae^t(\sin t + \cos t)$ and $y = ae^t(\sin t - \cos t)$,and prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

56. Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ with respect to x.

57. Find:

$$\int \frac{2\cos x}{(1-\sin x)(2-cos^2 x)} dx$$

58. Show that for the matrix $\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix}$, $A^3 - 6A^2 + 5A + 11I = 0$

Hence,find $\mathbf{A^{-1}}$.

59. Using matrix method, solve the following system of equations :

$$\begin{aligned} 3x - 2y + 3z &= 8 \\ 2x + y - z &= 1 \\ 4x - 3y + 2z &= 4 \end{aligned}$$

60. A bag contains 5 red and 4 black balls, a second bag contains 3 red and 6 black balls. One of the two bags is selected at random and two balls are drawn at random (without replacement) both of which are found to be red. Find the probability that the balls are drawn from the second bag.