Problem Set 1

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Overview

Problem set 1 for CS 4100 Summer II. Taught by assistant teaching professor, Rajagopal Venkat. [1]

1 Categorizing AI Environments

Q1 **Under what (implementation-related) assumptions is maze-solving a fully observable environment? What are the agent's percepts?**

We would assume that the agent is able to have all the information regarding the maze environment to be able to determine if the maze is fully observable. This would include the complete maze layout like the ways it can traverse, the walls, the starting position, and the exit position. Therefore, the agent's percept would be:

- The complete map of the maze.
- The agent's current location.
- The goal/exit location.
- The possible turns/moves it could make from its current position.

Q2 **Under what (implementation-related) assumptions is maze-solving a partially observable environment? What are the agent's percepts?**

A maze is partially observable if the agent has limited information. This would mean that the agent will not have all the information regarding the maze like all the walls and the ways it can traverse. Therefore, the agent's percept would be:

- An incomplete map of the maze (created when the agent moves around the maze).
- The agent's current location.
- The agent may only have limited information about the goal location like its general direction but not its exact coordinate.
- The agent can only sense/see the maze in its immediate surrounding.

Q3 **Is a known environment always fully observable? Explain with an example.**

No, consider the game of solitaire. The agent knows the rules of this environment where in this case would be the rules of the card game making it a known environment. However, it is only partially observable since the agent will not know which card will appear next because this information is hidden.

Q4 **Is a partially observable environment always an unknown environment? Explain with an example.**

No, consider an agent that delivers coffee within Google's headquarter. The agent has general information about the office building, like the number of floors, elevator locations etc. This environment is only partially observable because the agent has limited visibility due to its visual, and auditory sensors.

However, this is not an unknown environment since the agent has an understanding of the building layout and can navigate its way to a location following certain sets of rules such as not bumping into employees, moving on certain designated office highways, utilizing elevators to move up/down floors etc.

Q5 **What is the difference between an unknown environment and a non-deterministic environment? Explain with an example.**

An unknown environment is one where we do not know the rules. In a non-deterministic environment, there is the factor of randomness/uncertainty. A good example of this is to visualize a drone visiting an unexplored alien planet with the goal of finding life. This is an unknown environment since we do not have any prior maps or information about this planet.

It is also non-deterministic because the chances of discovering life is not certain. The creation of life is complicated, and although we do know what creates life, we cannot say for certainty that just because building blocks of life exists, that life will exist.

2 Search

In this problem set, we will be using Rook Jumping Mazes (RJMs) - a game based on Rook moves in Chess - to understand various search techniques. In a RJM, you are given a square $n \times n$ grid, with each cell containing a number between 1 to $n - 1$. From any cell, the player may move either horizontally or vertically but must take exactly the number of steps as shown on the player's current cell. Regardless of the length of the jump (i.e. number of cells passed), this action is considered a single jump (i.e., having a cost of 1) for purposes of our search. The objective is to get from an assigned start state to a pre-specified goal state. Consider this example:

	1	2	3	4	5
A	4	G	3	3	2
B	3	3	4	4	4
C	1	1	3	4	4
D	3	3	3	1	2
E	3	1	1	3	3

The cell D1 is the start state and the cell A2 is the goal state. The shortest solution to this RJM is: [D1, A1, A5, C5, C1, C2, D2, A2]. Such a game lends itself well to the various search algorithms we covered in class.

Q6 **Can both Breadth-First Search (BFS) and Depth-First Search (DFS) be used to solve RJMs? Will both yield the shortest path?**

Yes, BFS and DFS can be used to solve RJMs. This is because we can turn a RJM into an unweighted graph which BFS and DFS can be used for.

Breadth-First Search will find the shortest path because it searches all the surrounding nodes in the same level. Thus, BFS will be able to go down multiple branches and compare them allowing us to find the shortest path. However, DFS may not always find the shortest path depending on which branch DFS chooses.

Q7 **Assuming that each jump has a cost of 1, irrespective of the number of cells the rook passes while making that jump, we can use A* search to find the shortest path. Recall from class that A* search requires a consistent heuristic to guarantee an optimal solution. Show that for this problem, the Manhattan distance from any cell to the goal cell divided by (n - 1) is a consistent heuristic, where n is the number of cells in a row/column. In other words, show that**

$$H(node, goal, n) = \frac{|node_x - goal_x| + |node_y - goal_y|}{n-1}$$

is a consistent heuristic. Remember that a proof may not rely on a single example, but instead must hold true in general.

Here, we want to proof that using the Manhattan distance, this equation holds true for all cases:

$$h(n) \leq c(n \to n') + h(n')$$

Where $n = current\ node$ and $n' = neighbors\ of\ node\ n$. Starting at node n, we can move right, up, left or down:

1. $x = k, \quad y = 0$
2. $x = 0, \quad y = k$
3. $x = -k, \quad y = 0$
4. $x = 0, \quad y = -k$

Where $k \in \{1, \ldots, n-1\}$.

Case 1:

$$\frac{|node_x - goal_x| + |node_y - goal_y|}{n-1} \leq 1 + \frac{|(node_x + k) - goal_x| + |node_y - goal_y|}{n-1}$$

$$\frac{|node_x - goal_x| + |node_y - goal_y|}{n-1} - \frac{|(node_x + k) - goal_x| + |node_y - goal_y|}{n-1} \leq 1$$

$$\frac{|node_x - goal_x| - |(node_x + k) - goal_x|}{n-1} \leq 1$$

$$|node_x - goal_x| - |(node_x + k) - goal_x| \leq n - 1$$

Let $node_x - goal_x = f$

$$|f| - |f + k| \leq n - 1$$
$$-k \leq n - 1$$
$$k \geq -n + 1$$

Lower End of k = 1

$$1 \geq -n + 1$$
$$n \geq 0$$

Upper End of k = n - 1

$$n - 1 \geq -n + 1$$
$$2n \geq 2$$
$$n \geq 1$$

These inequalities will always hold true since n will always be at least 1.

Case 2:

$$\frac{|node_x - goal_x| + |node_y - goal_y|}{n-1} \leq 1 + \frac{|node_x - goal_x| + |(node_y + k) - goal_y|}{n-1}$$
$$\frac{|node_x - goal_x| + |node_y - goal_y|}{n-1} - \frac{|node_x - goal_x| + |(node_y + k) - goal_y|}{n-1} \leq 1$$
$$\frac{|node_y - goal_y| - |(node_y + k) - goal_y|}{n-1} \leq 1$$
$$|node_y - goal_y| - |(node_y + k) - goal_y| \leq n - 1$$

Let $node_y - goal_y = f$

$$|f| - |f + k| \leq n - 1$$
$$-k \leq n - 1$$
$$k \geq -n + 1$$

Lower End of k = 1
$$1 \geq -n + 1$$
$$n \geq 0$$

Upper End of k = n - 1
$$n - 1 \geq -n + 1$$
$$2n \geq 2$$
$$n \geq 1$$

These inequalities will always hold true since n will always be at least 1.

Case 3:

$$\frac{|node_x - goal_x| + |node_y - goal_y|}{n-1} \leq 1 + \frac{|(node_x - k) - goal_x| + |node_y - goal_y|}{n-1}$$
$$\frac{|node_x - goal_x| + |node_y - goal_y|}{n-1} - \frac{|(node_x - k) - goal_x| + |node_y - goal_y|}{n-1} \leq 1$$
$$\frac{|node_x - goal_x| - |(node_x - k) - goal_x|}{n-1} \leq 1$$
$$|node_x - goal_x| - |(node_x - k) - goal_x| \leq n - 1$$

Let $node_x - goal_x = f$

$$|f| - |f - k| \leq n - 1$$
$$k \leq n - 1$$

We know that k = [1, n - 1]
$$[1,\ n-1] \leq n - 1$$

Thus, we know that the statement above will always hold true because n will always be at least 1.

Case 4:

$$\frac{|node_x - goal_x| + |node_y - goal_y|}{n-1} \leq 1 + \frac{|node_x - goal_x| + |(node_y - k) - goal_y|}{n-1}$$
$$\frac{|node_x - goal_x| + |node_y - goal_y|}{n-1} - \frac{|node_x - goal_x| + |(node_y - k) - goal_y|}{n-1} \leq 1$$
$$\frac{|node_y - goal_y| - |(node_y - k) - goal_y|}{n-1} \leq 1$$
$$|node_y - goal_y| - |(node_y - k) - goal_y| \leq n-1$$

Let $node_y - goal_y = f$

$|f| - |f - k| \leq n - 1$
$k \leq n - 1$

We know that k = [1, n - 1]
$[1,\ n-1] \leq n-1$

Thus, we know that the statement above will always hold true because n will always be at least 1.

$\therefore$ By proving all 4 cases, we can say that $H(node, goal, n) = \frac{|node_x - goal_x| + |node_y - goal_y|}{n-1}$ is a consistent heuristic. ■

Q8 **Consider the following RJM:**

	1	2	3	4
A	2	3	2	3
B	1	2	1	1
C	3	2	2	3
D	1	1	3	G

On this maze, using the heuristic from Q7, complete the A* search process shown below, starting with Step 2. Terminate your search when the Goal node (cell D4) is popped from the priority queue. Double check your calculations!

Initialization:

→ Starting at node B2

$B2$ Priority = cost + $H(B2) = 0 + \frac{4}{3} = \frac{4}{3}$

B2 Cost = 0
→ Push B2 to PQ

PQ = { $(v = B2, priority = \frac{4}{3}, cost = 0, path = [])$ }

Step 1:

→ Pop B2
Neighbor(s) of B2 = D2, B4

$D2$ Priority = cost + wt(B2, D2) + $H(D2) = 0 + 1 + \frac{2}{3} = \frac{5}{3}$
$B4$ Priority = cost + wt(B2, C4) + $H(B4) = 0 + 1 + \frac{2}{3} = \frac{5}{3}$

D2 Cost = 1, B4 cost = 1
→ Push D2, B4 to PQ

$$\text{PQ} = \left\{ \begin{array}{l} (v = D2, priority = \frac{5}{3}, cost = 1, path = [B2]), \\ (v = B4, priority = \frac{5}{3}, cost = 1, path = [B2]) \end{array} \right\}$$

Step 2:

→ Pop D2
Neighbor(s) of D2 = D1, C2, D3

$D1$ Priority = cost + wt(D2, D1) + $H(D1) = 1 + 1 + 1 = 3$
$C2$ Priority = cost + wt(D2, C2) + $H(C2) = 1 + 1 + 1 = 3$
$D3$ Priority = cost + wt(D2, D3) + $H(D3) = 1 + 1 + \frac{1}{3} = \frac{7}{3}$

D1 Cost = 2, C2 cost = 2, D3 cost = 2
→ Push D1, C2, D3 to PQ

$$\text{PQ} = \left\{ \begin{array}{l} (v = B4, priority = \frac{5}{3}, cost = 1, path = [B2]), \\ (v = D3, priority = \frac{7}{3}, cost = 2, path = [B2, D2]), \\ (v = D1, priority = 3, cost = 2, path = [B2, D2]), \\ (v = C2, priority = 3, cost = 2, path = [B2, D2]) \end{array} \right\}$$

Step 3:

→ Pop B4
Neighbor(s) of B4 = A4, B3, C4

$A4$ Priority = cost + wt(B4, A4) + $H(A4) = 1 + 1 + 1 = 3$
$B3$ Priority = cost + wt(B4, B3) + $H(B3) = 1 + 1 + 1 = 3$
$C4$ Priority = cost + wt(B4, C4) + $H(C4) = 1 + 1 + \frac{1}{3} = \frac{7}{3}$

A4 Cost = 2, B3 cost = 2, C4 cost = 2
→ Push A4, B3, C4 to PQ

$$\text{PQ} = \left\{ \begin{array}{l} (v = D3, priority = \frac{7}{3}, cost = 2, path = [B2, D2]), \\ (v = C4, priority = \frac{7}{3}, cost = 2, path = [B2, B4]), \\ (v = A4, priority = 3, cost = 2, path = [B2, B4]), \\ (v = C2, priority = 3, cost = 2, path = [B2, D2]), \\ (v = D1, priority = 3, cost = 2, path = [B2, D2]), \\ (v = B3, priority = 3, cost = 2, path = [B2, B4]) \end{array} \right\}$$

Step 4:

→ Pop D3
Neighbor(s) of D3 = A3

$A3$ Priority = cost + wt(D3, A3) + $H(A3) = 2 + 1 + \frac{4}{3} = \frac{13}{3}$

A3 Cost = 3
→ Push A3 to PQ

$$\text{PQ} = \left\{ \begin{array}{l} (v = C4, priority = \frac{7}{3}, cost = 2, path = [B2, B4]), \\ (v = A4, priority = 3, cost = 2, path = [B2, B4]), \\ (v = C2, priority = 3, cost = 2, path = [B2, D2]), \\ (v = D1, priority = 3, cost = 2, path = [B2, D2]), \\ (v = B3, priority = 3, cost = 2, path = [B2, B4]), \\ (v = A3, priority = \frac{13}{3}, cost = 3, path = [B2, D2, D3]) \end{array} \right\}$$

Step 5:

→ Pop C4
Neighbor(s) of C4 = C1

$C1$ Priority = cost + wt(C4, C1) + $H(C1) = 2 + 1 + \frac{4}{3} = \frac{13}{3}$

C1 Cost = 3
→ Push C1 to PQ

$$\text{PQ} = \left\{ \begin{array}{l} (v = A4, priority = 3, cost = 2, path = [B2, B4]), \\ (v = C2, priority = 3, cost = 2, path = [B2, D2]), \\ (v = D1, priority = 3, cost = 2, path = [B2, D2]), \\ (v = B3, priority = 3, cost = 2, path = [B2, B4]), \\ (v = A3, priority = \frac{13}{3}, cost = 3, path = [B2, D2, D3]), \\ (v = C1, priority = \frac{13}{3}, cost = 3, path = [B2, B4, C4]) \end{array} \right\}$$

Step 6:

→ Pop A4
Neighbor(s) of A4 = A1, D4

$A1$ Priority = cost + wt(A4, A1) + $H(A1) = 2 + 1 + 2 = 5$
$D4$ Priority = cost + wt(A4, D4) + $H(D4) = 2 + 1 + 0 = 3$

A1 Cost = 3, D4 Cost = 3
→ Push A1, D4 to PQ

$$\text{PQ} = \left\{ \begin{array}{l} (v = D4, priority = 3, cost = 3, path = [B2, B4, A4]) \\ (v = C2, priority = 3, cost = 2, path = [B2, D2]), \\ (v = D1, priority = 3, cost = 2, path = [B2, D2]), \\ (v = B3, priority = 3, cost = 2, path = [B2, B4]), \\ (v = A3, priority = \frac{13}{3}, cost = 3, path = [B2, D2, D3]), \\ (v = C1, priority = \frac{13}{3}, cost = 3, path = [B2, B4, C4]), \\ (v = A1, priority = 5, cost = 3, path = [B2, B4, A4]) \end{array} \right\}$$

Step 7:

→ Pop D4 $(v = D4, priority = 3, cost = 3, path = [B2, B4, A4])$
→ Goal is found!
→ Append v = D4 to path
→ return path = [B2, B4, A4, D4]
Terminate Program

3 Local Search

Your professor is a coffee snob, and since he knows a thing or two about AI, he tries to build an espresso machine that will learn a user's preferences over time to pull the perfect shot. He lets the machine control the following values:

- Weight of coffee beans (15-21 grams)
- Grind size (250 - 400 microns)
- Water Pressure (7 - 11 bars)
- Brew time (27 - 33 seconds)

Q9 **Based on our discussions in class, is this problem well-suited to local search algorithms? Explain your reasoning.**

This problem aims to learn the user's preferences to create the most optimal coffee mix tailored to the user. In other words, the focus is on the final solution rather than the specific optimizations or paths taken to achieve the perfect mix. Conveniently, local search algorithms are designed for this type of problem, making them well-suited for finding the ideal solution.

Q10 **How you would implement such a program? What would be a reasonable objective function? (This question is somewhat open-ended, so don't worry too much about the 'right' answer. I am really looking for whether your thought process reflects a solid understanding of how local search works.)**

I would use a hill-climbing algorithm. I would define the objective function to be based on user feedback. Lets say for each cup of coffee this machine brews, the user rates the coffee from 1 to 10 where 10 would mean perfect. The objection function $f(x)$ would try to maximize the user rating. However, if we want to use gradient descent, we can manipulate the user rating by doing

$$f(x) = 10 - UserRating(x)$$

This would mean that a user rating of 10 becomes:

$$f(x) = 10 - 10 = 0 \quad \xleftarrow{\text{best value}}$$

and a user rating of 1 becomes:

$$f(x) = 10 - 1 = 9 \quad \xleftarrow{\text{worst value}}$$

If we do this, we can frame the objective function from maximizing to minimizing $f(x)$ and thus perform gradient descent to find the optimal coffee blend.

Q11 **Which of the optimizations and local search variants discussed in class (random restarts, varying step size, simulated annealing and genetic algorithms) are applicable to this problem, and why?**

All 4 techniques are applicable to this solution. For random restarts, lets assume we are trying to minimize the objective function, the optimization algorithm we are using may get stuck inside a valley which may not be the global minimum. By using random restarts, we will be able to move to a different spot and hopefully find a better minimum.

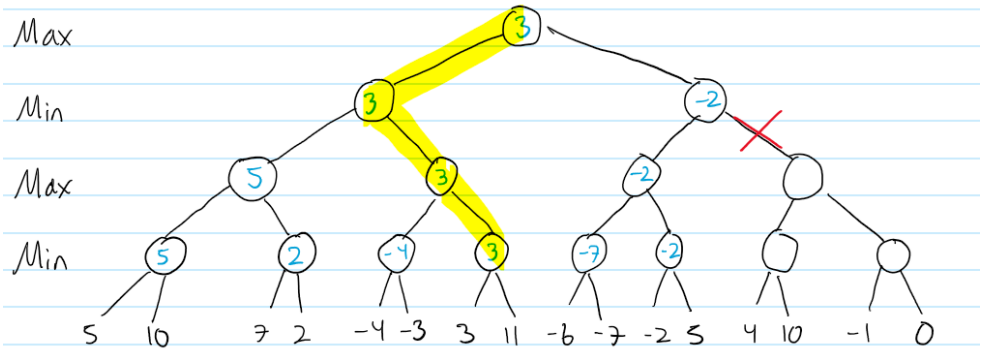
Varying step size can also be useful in finding the optimal solution. We can start with a larger step size to allow the algorithm to traverse a greater area of the search space and gradually reduce the step size as we hone in on a solution.

For simulated annealing, this technique accepts a worse solution with a probability that decreases over time. This encourages the algorithm to explore a larger area that surrounds the initial starting position in the hopes of finding a better local minimum and avoid being trapped inside a valley.

Lastly, genetic algorithms will allow the algorithm to explore multiple solutions that evolve/change over time. For example, it is common for a person's taste to change over time, and since genetic algorithms thrive based on the concept of change, it is particularly suited to this problem as it can adapt the espresso's machine mixture according to the user's changing tastes.

4 Adversarial Search

Q12 **Construct a game tree example with a depth of 4 and a branching factor of 2, that illustrates the best-case for alpha-beta pruning, except for the trivial solution. Hand drawn figures accepted for this question, but please ensure clarity.**



Q13 **Why is it desirable to combine iterative deepening with alpha-beta pruning from a practical standpoint?**

Iterative deepening allows us to use DFS's low memory requirement combined with BFS's ability to explore multiple solutions. Alpha-beta pruning, on the other hand, reduces the number of nodes/branches needed to be evaluated and explored speeding up the search by eliminating branches that cannot influence the final decision.

By combining these two together, the search can hone-in on more promising paths due to iterative deepening and skip exploring paths that are redundant. This will allow us to perform the search on a much bigger tree due to the low memory usage but at the same time, find a relatively good solution in a timely manner.

5 Academic Integrity

Q14 **Review, and copy/paste the academic integrity acknowledgement in your final submission as the answer to Q14.**

I have read and understood the academic integrity policy as outlined in the course syllabus for CS4100. By pasting this acknowledgement in my submission, I declare that all work presented here is my own, and any conceptual discussions I may have had with classmates have been fully disclosed. I declare that generative AI was not used to answer any questions in this assignment. Any use of generative AI to improve writing clarity alone is accompanied by an appendix with my original, unedited answers.

References

[1] R. Venkatesaramani, "Personal website." `https://rajagopalvenkat.com/`. Accessed: 2024-07-10.