A little book about matter

> "Science can amuse and fascinate us all, but it is engineering that changes the world."
>
> — Isaac Asimov

Lesson 6: Electrical circuits (part I)

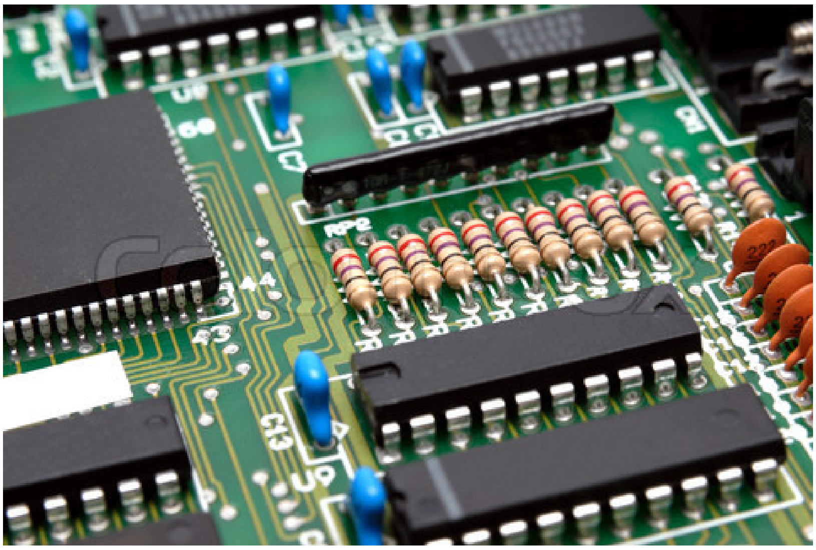


Figure 1: An electronic circuit is composed of individual electronic components, such as resistors, transistors, capacitors, inductors and diodes, connected by conductive wires or traces through which electric current can flow. Circuits can be constructed of discrete components connected by individual pieces of wire, but today it is much more common to create interconnections by photolithographic techniques on a laminated substrate (a printed circuit board or PCB as shown on this picture) and solder the components to these interconnections to create a finished circuit.

Our modern world is dominated by electrical technology and yet most people have no basic understanding of how it works: They don't know how electricity is generated and they don't know how basic circuits works. This probably has something to do with the difficulty of the subject[1]: It's *abstract* (you can't actually see electrons moving through a metal wire and the ugly circuitboards are often hidden from view), *technical* (the equipment is very advanced and the laws of electromagnetism use complicated mathematics) and *highly specialised* (the topic is divided up into a wide range of interrelated disciplines and it's difficult to be an expert on all of it).

[1] It also has something to do with the faults of our education system!

How we came this far is a fascinating story. The Institute of Electrical and Electronics Engineers (IEEE) is the world's largest technical professional organisation for the advancement of technology and they have produced a list of milestones that represent key historical achievements in electrical engineering. It would be impossible to go over all the important discoveries and inventions that are covered in this list, so you will have to go study electrical engineering if you want to know more than what we cover here. The rapid development of technology that we have witnessed over the past 200 years is far from over yet (just look at how quickly smartphones and wireless technologies are developing). By combining biology, chemistry, physics, and information technology, we are even approaching an understanding of how to build and program circuits and machines at the molecular level. This will ultimately lead to better knowledge about the basic building blocks of life and we will probably be able to create artificial intelligence - buckle up, the future has only just begun![2]

[2] Although the catastrophic effects of climate change (and, more importantly, our unsustainable modern way of life) will most likely prevent this continuous growth.

The following lessons are going to be an introduction to the three most basic concepts of electricity: **Current**, **voltage** and **resistance**. These concepts will most likely always be fundamental to understanding any type of electrical circuit, ranging from a doorbell to a smartphone to a human brain. These concepts emerged during the Enlightenment in Europe (roughly from 1750 to 1850) and many people contributed to this development. Here are a couple of important relevant milestones:

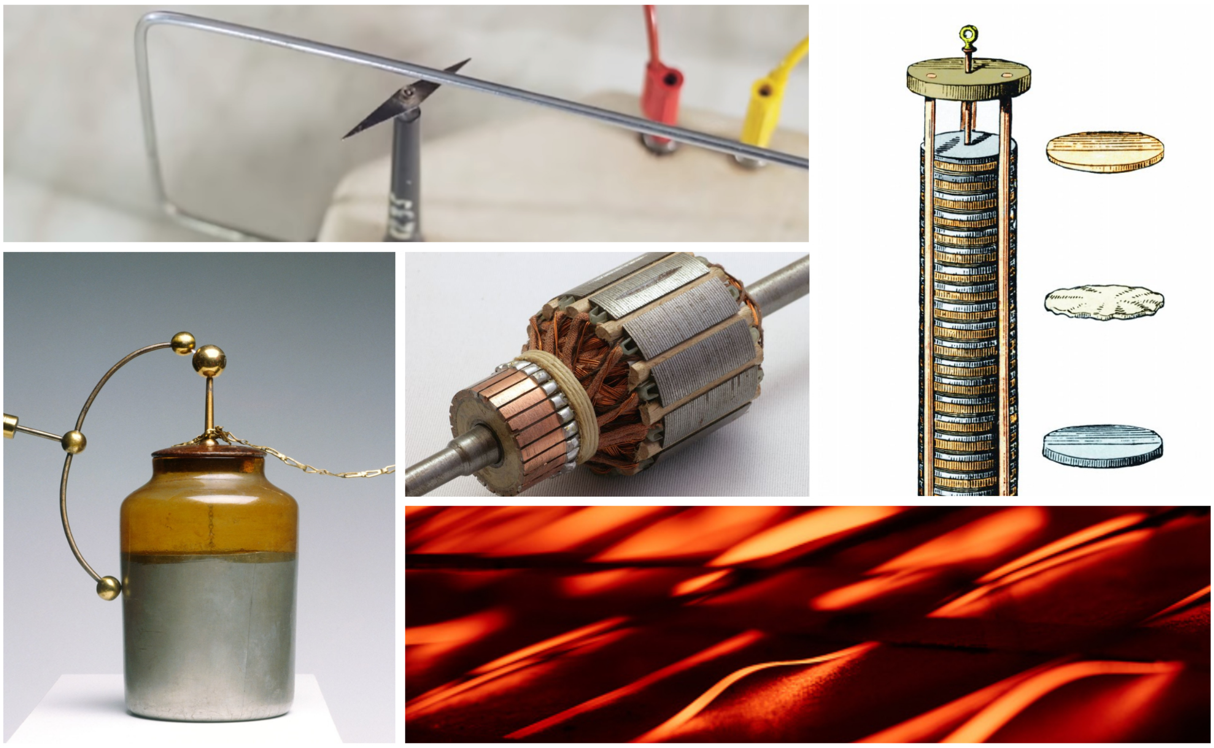


Figure 2: Lower left: The first capacitor: A Leyden jar (1745). Upper right: The first battery: A voltaic pile (1799) consisting of a series of **galvanic cells** made of copper and zinc (discussed in more detail later). Upper left: A compass needle deflects in the presence of an electric current, hence magnetism is related to electricity (1820). Center: A modern electric motor. Lower right: Metal filaments heating up due to an electric current passing through them (that's how your toaster works).

- The invention of the **Leyden jar**, the very first **capacitor** (a way of storing electric charge) in 1745.
- The invention of the **voltaic pile** (the first electrical battery that could continuously provide an electric current to a circuit) by Alessandro Volta in 1799.
- The discovery that **an electric current creates a magnetic field** (the first connection found between electricity and magnetism) by Hans Christian Ørsted in 1820.
- The invention of the first **electrical motor** by Michael Faraday in 1821 and his subsequent discovery of **electromagnetic induction** in 1831.

- The discovery of **resistive heating** (the process by which the passage of an electric current through a conductor produces heat) by James Prescott Joule in 1840.

Electric current

In a piece of metal, we can assume that free electrons are constantly moving around, bumping into other electrons (and the metal ions they came from) and thereby changing their direction of motion in a random way[3]. This motion is similar to that of particles in an ideal gas and in the absence of an external electric field, these electrons have no preferred direction of travel. This implies that there is *no net displacement of charge in any direction*, see figure 3.

[3] This free electron model is a quantum mechanical model developed in 1927 by Arnold Sommerfeld and despite its simplicity, it is surprisingly successful in explaining many experimental phenomena. You can think of the electrons as making up a 'gas' or 'sea' of electrons.

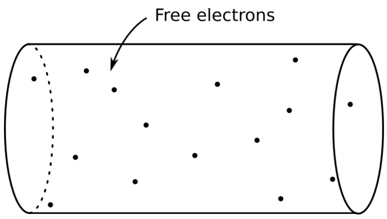
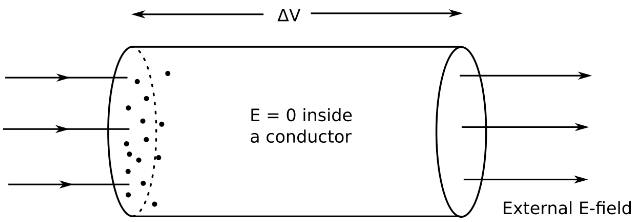


Figure 3: Think of the free electrons in this image as moving around in random directions, but with no overall displacement in any one particular direction.

If an external electric field (or, in other words, a potential difference) is applied across an isolated piece of metal, the electrons distribute themselves under the influence of this field such that they cancel out the field's effect in the metal's interior and come to rest (this was the idea behind the Faraday Cage mentioned in lesson 5, recall this nice animation.



However, if the metal is part of a closed, conducting circuit containing an electric **cell** (= the technical word for what you call a battery), then electrons can continue to flow to the **positive terminal** of the cell where they get absorbed in chemical reactions. At the same time, new electrons are released into the circuit due to chemical reactions at the **negative terminal** of the cell. *This allows an electric field to remain in the conductor and cause a continuous flow of electrons in one preferred direction*, see figure 4 (as long as the chemical reactions continue). The electrons are still moving around erratically but they now have a net displacement which defines what is called the **drift velocity** (or speed) of the electrons. The drift velocity in normal electrical wiring is surprisingly low, of the order $10^{-4}\,\mathrm{m/s} = 0.1\,\mathrm{mm/s}$.

The cell is responsible for creating an electric field by maintaining a constant potential difference (a voltage) across its terminals,

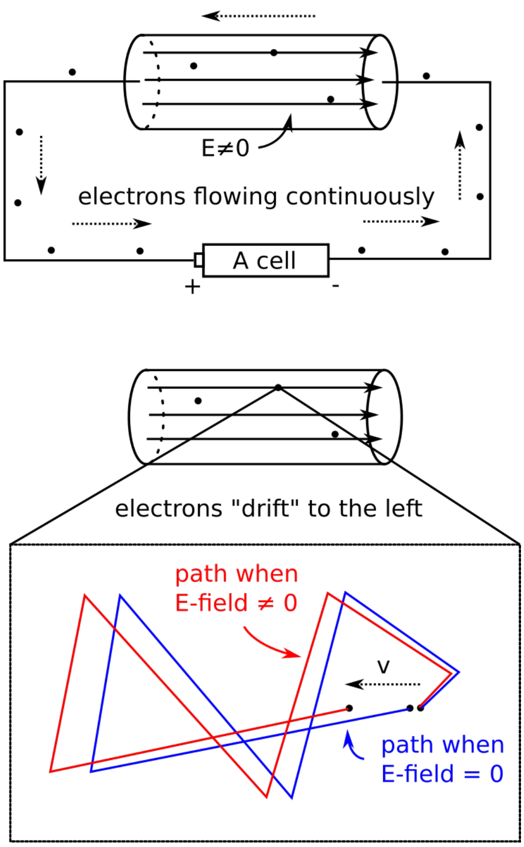


Figure 4: A closer look at how electrons drift towards the positive terminal.

e.g. a 9 V cell maintains a $9\,\mathrm{V} = 9\,\mathrm{J/C}$ potential difference across its terminals. We'll briefly discuss how a cell actually works in a minute. We think of the potential difference as 'forcing' the electrons round the circuit and giving them energy to spend, which leads to the following terminology: The **electromotive**[4] **force** or **emf**, $\mathcal{E}$, of a cell is *is the energy per unit charge delivered to the circuit when a given charge travels around the whole circuit*. The emf is of course just a voltage, so it is typically expressed in volts (V).

[4] A word combining 'electron' and 'motion'.

If the electrons were traveling through empty space, all this electric potential energy would be converted into kinetic energy, however since they are constantly bumping into things along their way, the energy is instead mainly converted into heat, light, and sound (all forms of energy that have to do with motion at the atomic level). We are now ready to define what we exactly mean by **electric current**: *Electric current, I, is defined as the amount of charge flowing through a cross-sectional area per unit time*:

$$I \equiv \frac{\Delta Q}{\Delta t}$$

When charge is measured in coulombs (C) and time in seconds (s), then current will be expressed in C/s which we call one **ampere**[5] (A)

$$\mathrm{A} = \frac{\mathrm{C}}{\mathrm{s}} = \mathrm{C} \cdot \mathrm{s}^{-1}$$

[5] Strictly speaking, the ampere is a base SI unit with a specific definition, so in fact the coulomb is a derived unit given by $\mathrm{C} \equiv \mathrm{As}$.

Before scientists had discovered electrons, they defined the **conventional current direction** to flow from the positive terminal to the negative terminal, and this is still the preferred convention when discussing electric circuits in general. Remember that this is opposite to the actual flow of electrons!

Drift speed formula

We will now derive a simple formula that relates the current in a wire to the drift speed of the charge carriers. We will forget about electrons for a moment and assume the charges are all positive charges with a charge $+q$. First, consider a piece of wire with a constant circular cross-sectional area A. The number of charges per unit volume in a given piece of metal, the *charge density*, is sometimes denoted n (unit = m^{-3}). Assume the drift speed is v.

1. During a small time interval Δt, how far does a charge move? Express this in terms of the drift speed and Δt.
2. In that time, a certain number of charges flow across area A, filling up a new cylindrical volume. What is the length of this cylinder? What is its volume?
3. All the charges that crossed the area A now reside in this new cylindrical volume. How many charges are in this volume? (Hint: n)
4. What is the total amount of charge in this volume?
5. Now insert the total amount of charge that crossed the area, ΔQ, into the definition of current and you should arrive at the formula $I = qnAv$.
6. If the charge carriers in the current are known to be electrons (which they typically are), how would the formula change?

When current flows in a single, closed circuit, the current everywhere is the same. If this was not true, we would witness charge accumulate at certain points in the circuit, which would then result in the creation of new electric fields that would work against such buildups. The formula above thus implies that if you connect different conductors with different cross-sectional areas together in a single, closed circuit that is carrying the same current, then the widest areas will have the lowest drift speeds and the thinnest areas have the highest drift speeds. Situations like this lead us to think of electric current as a kind of 'fluid' flowing through the wires, but one should be careful with such analogies since they eventually break down.

The galvanic (or voltaic) cell

We mentioned earlier that a cell undergoes chemical reactions in order to absorb and release electrons into a circuit. Let's go into a bit of detail with that here. In 1780, Luigi Galvani discovered that when two different metals (e.g., copper and zinc) are in contact and then both are touched at the same time to two different parts of a muscle of a frog leg (to make a closed circuit), the frog's leg contracts. He called this "animal electricity".

A year after Galvani published his work, Alessandro Volta showed that the frog was not necessary. He invented what is now called the **galvanic (or voltaic) cell**, which derives electrical energy from spontaneous chemical reactions taking place between the metals. It generally consists of two different metals connected by a salt bridge (or separated by a porous membrane, as shown in figure 6). The function of such a cell is an interesting dive into the world of **electrochemistry** – a field that bridges physics and chemistry. Here's a good YouTube video that covers the electrochemistry of a galvanic cell in detail.

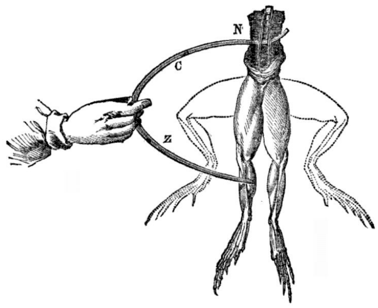


Figure 5: Luigi Galvani discovered that frog legs twitch when electricity is passed through the muscles, a phenomenon called galvanism that lead to the subject of electrophysiology and treatment by electrotherapy.

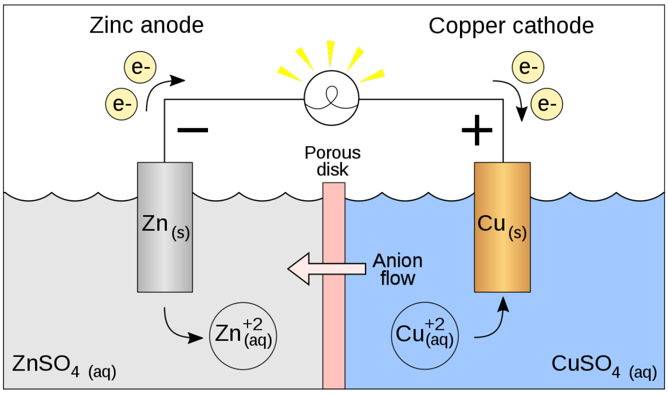
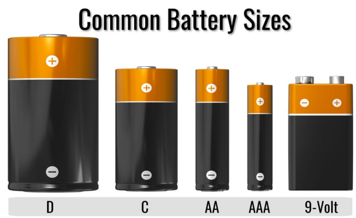


Figure 6: A galvanic cell or voltaic cell, named after the scientists Luigi Galvani and Alessandro Volta, respectively, is an electrochemical cell in which an electric current is generated from spontaneous reactions. A common apparatus generally consists of two different metals, each immersed in separate beakers containing their respective metal ions in solution that are connected by a salt bridge (or separated by a porous membrane). (Wikipedia)

The galvanic cell is an example of a **wet cell** where the electrolyte (the stuff containing ions) is liquid. A **dry cell** uses a paste electrolyte, with only enough moisture to allow current to flow. Unlike a wet cell, a dry cell can operate in any orientation without spilling. Most of the batteries from your everyday life are dry cells. In common usage, the word "battery" has come to mean a single galvanic cell like the one shown in figure 6, but the word was originally used to refer to multiple cells put together (as in the voltaic pile). A **primary cell** is a galvanic cell that is designed to be used once and discarded, since the chemical reactions occurring in the cell are not reversible. In a **secondary cell** (= a rechargeable battery) the reaction can be reversed by running a current into the cell to recharge it, regenerating the chemical reactants.



Resistance

Given a conductor, *the amount of current, I, flowing through it depends on the potential difference, V, applied across it*. This is very important to understand, so go back and read the last sentence one more time! We quantify this observation by defining the **resistance** of a conductor as *the ratio of the applied voltage to the current flowing* *through it*:

$$R \equiv \frac{V}{I} \tag{1}$$

When voltage is measured in volts (V) and current in ampere (A), then resistance is expressed in V/A which we call one **ohm** (Ω)

$$\Omega = \frac{\mathrm{V}}{\mathrm{A}} = \mathrm{V} \cdot \mathrm{A}^{-1}$$

It's absolutely crucial to understand that equation (1) is a definition and it always applies to any given situation where there is an applied voltage and a resulting current. Students very often get this important detail wrong and it prevents them from properly understanding circuits.

In an attempt to make circuits as clear as possible to you, I've put together this series of small experiments for you to perform in class. It's important to do them in the given order because in that way you will build up a proper understanding of what is going on. *Don't try to skip the steps and try to forget everything you have previously learnt about circuits*. Students often have a lot of misconceptions related to circuits and it's best to try and relearn everything from scratch.

Lesson 6: Exercises

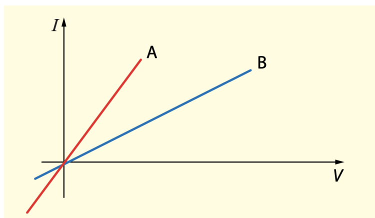
Make sure you have completed this series of small experiments before you attempt the exercises below.

1. What is the typical drift speed in a normal copper wire ($r = 1\,\mathrm{mm}$) carrying a current of 1 A? (Hint: You will need to find the charge density n by using the density and molar mass of copper.)

2. An ohmic resistor has a resistance of $12\,\Omega$ when a current of 3.0 A flows through it. Find the resistance when the current is 4.0 A.

3. The heating element of an electric kettle has a current of 15 A when connected to a potential difference of 220 V. Calculate the resistance of the heating element.

4. The graphs on the right show the IV characteristics of the same piece of metal wire kept at two different temperatures.



   (a) Discuss whether the wire obeys Ohm's 1st law.

   (b) Suggest which of the two lines on the graph corresponds to the higher temperature.

Answers to all the exercises.

Lesson 6 Quiz

Check your understanding of this lesson: Here is a quiz.