T II (g)

$G: \; X_{min} < C$

$P(\vec{X}_n \in G \mid H_0) = \alpha$

$P(X_{min} < C \mid H_0) = \alpha$

$H_0: \; \xi \sim R(0,1) \quad \Rightarrow \quad F_0(x) = \int_0^x 1\,dx = X$

$\xi_{min} \sim 1 - (1 - F(x))^n$

$P(X_{min} < C) = 1 - (1 - \underset{\underset{C}{\parallel}}{F_0(C)})^n = \alpha. \quad \Rightarrow$

$1 - C = (1-\alpha)^{\frac{1}{n}}$

$C = 1 - (1-\alpha)^{\frac{1}{n}}$

$G: \; X_{min} \leq 1 - (1-\alpha)^{\frac{1}{n}}$

$\alpha_1 = \alpha$

$W = P(\vec{X}_n \in G \mid H_1) = P(X_{min} \leq C \mid H_1) =$

$\begin{cases} H_1: \; \xi \sim \frac{e}{e-1} e^{-x} \{(0,1)\} \\ F_1(x) = \int_0^x \frac{e}{e-1} e^{-t}\,dt = \frac{e}{e-1}\left(1 - e^{-x}\right) \end{cases}$

$= 1 - (1 - F_1(C))^n = 1 - \left(1 - \frac{e}{e-1}\left(1 - e^{-C}\right)\right)^n =$

$= 1 - \left(1 - \frac{e}{e-1}\left(1 - e^{(1-\alpha)^{\frac{1}{n}} - 1}\right)\right)^n$

$\alpha_2 = 1 - W$

Сосм.:

$W \to 1.$?

$$\cancel{e^{(1-\alpha)^{\frac{1}{n}}-1} = e^{e^{\ln((1-\alpha)^{\frac{1}{n}}-1)}} = e^{e^{\ln[-(1-(1-\alpha)^{\frac{1}{n}})]}} =}$$

$$\cancel{= e^{e^{\ln(1-(1-\alpha)^{\frac{1}{n}})+}}}$$

$$e^{-e^{\ln(1-(1-\alpha)^{\frac{1}{n}})}} = e^{-(1+\ln[1-(1-\alpha)^{\frac{1}{n}}+o(\frac{1}{n})])}$$

$$W = 1-\left[1-\frac{e}{e-1}\left(1-e^{-1}e^{-\ln[1-(1-\alpha)^{\frac{1}{n}}+o(\frac{1}{n})]}\right)\right]^n =$$

$$= 1-\left[1-\frac{e}{e-1}\left(1-e^{-1}\left(1-\ln[1-(1-\alpha)^{\frac{1}{n}}+o(\frac{1}{n})]\right)\right)\right]^n =$$

$$= 1-\left[1-\frac{e}{e-1}\left(\frac{e-1}{e}-\frac{\ln[\dots]}{e}\right)\right]^n = 1-\left[\frac{\ln[\dots]}{e-1}\right]^n =$$

$$= 1-\left(\frac{\ln[1-(1-\alpha)^{\frac{1}{n}}+o(\frac{1}{n})]}{e-1}\right)^n = 1-\left(\frac{-((1-\alpha)^{\frac{1}{n}}-o(\frac{1}{n}))+o(\frac{1}{n})}{e-1}\right)^n =$$

$$= 1-\left(\frac{-(1-\alpha)^{\frac{1}{n}}+o(\frac{1}{n})}{e-1}\right)^n \xrightarrow[n\to\infty]{} \cancel{e^{\frac{-(1-\alpha)^{\frac{1}{n}}}{e-1}}} \cancel{\neq 1.} \xrightarrow[n\to\infty]{} 1$$

Не сосм.

№ 12.

H_0: $\xi \sim p(x) = \frac{1}{4}\{1,2\} + \frac{1}{6}\{3\} + \frac{1}{3}\{4\}$

H_1: $\xi \sim p(x) = \frac{1}{4}\{1,2,3,4\}$

$\alpha = 0{,}2$, $n = 2$.

1 \ 2	1	2	3	4
1	$\left(\frac{1/4}{1/4}\right)^2 = 1$	$\left(\frac{1/4}{1/4}\right)^2 = 1$	$\frac{1/4}{1/4}\,\frac{1/4}{1/6} = \frac{3}{2}$	$\frac{1/4}{1/4}\cdot\frac{1/4}{1/3} = \frac{3}{4}$
2	$\frac{1/4}{1/4}\cdot\frac{1/4}{1/4} = 1$	$\frac{1/4}{1/4}\,\frac{1/4}{1/4} = 1$	$\frac{1/4}{1/4}\,\frac{1/4}{1/6} = \frac{3}{2}$	$\frac{1/4}{1/4}\cdot\frac{1/4}{1/3} = \frac{3}{4}$
3	$\frac{1/4}{1/6}\,\frac{1/4}{1/4} = \frac{3}{2}$	$\frac{3}{2}$	$\frac{9}{4}$	$\frac{3}{2}\cdot\frac{3}{4} = \frac{9}{8}$
4	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{9}{8}$	$\frac{9}{16}$

H_0:

	1	2	3	4
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{24}$	$\frac{1}{12}$
2	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{24}$	$\frac{1}{12}$
3	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{36}$	$\frac{1}{18}$
4	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{9}$

H_1:

	1	2	3	4
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
2	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
3	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
4	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$

$G: \ell \geq C$

$P(\ell \geq C \mid H_0) \leq 0{,}2$

Наиб. $\ell = 9/4 \rightarrow p = \frac{1}{36} < 0{,}2.$

$\ell = \frac{3}{2} \rightarrow p = \frac{1}{36} + \frac{4}{24} = \frac{7}{36} \approx 0{,}194 < 0{,}2.$

$\ell \geq \frac{3}{2}, \qquad \alpha_1 = \frac{7}{36}.$

$\alpha_2 = P(\ell < 3/2 \mid H_1) = \frac{5}{16}.$

$W = 1 - \alpha_2 = \frac{11}{16}.$

№13.

a, φ^2 b, ψ^2

$\xi \sim N(\cancel{\varphi_1, \sigma_x^2})$ $\eta \sim N(\cancel{\psi_1, \sigma_2^2})$

$n = 139, \varphi_x = 5{,}722$ $m = 1000, \psi_2 = 6{,}161$

$H_0: \cancel{\sigma_1^2 = \sigma_2^2}\ \varphi^2 = \psi^2$

$H_1: \varphi^2 \neq \psi^2$

$S^2 = D^2 \Rightarrow$

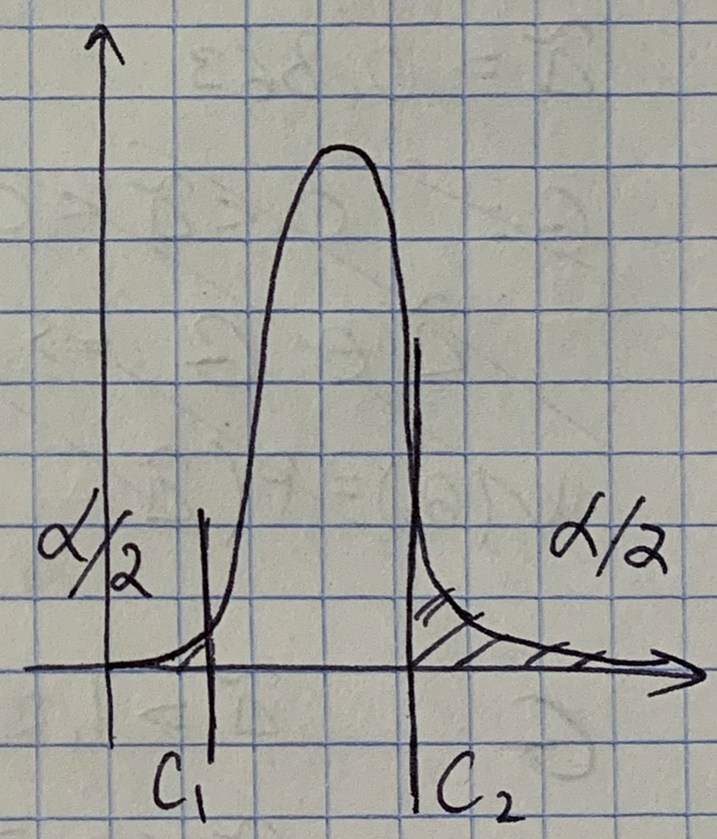
$S_\xi^2 = \varphi^2 = 32{,}741$

$S_\eta^2 = \psi^2 = 37{,}957$

$\tilde{\Delta} = \frac{S_\xi^2}{S_\eta^2} = 9863$

по Теор.

$\tilde{\Delta} = \frac{S_\xi^2}{S_\eta^2} \sim F(n-1, m-1)$



$\tilde{\Delta} > C_2$

$\tilde{\Delta} < C_1$

G: $\cancel{|\tilde{\Delta}| \geq c}$ $\cancel{C_1 < \tilde{\Delta} < C_2}$

p-value $= P(|\Delta| \geq |\tilde{\Delta}| \mid H_0) = 2P(\Delta \geq 0{,}863 \mid H_0) =$

$=$

4

$\frac{1}{16}$

$\frac{1}{16}$

$\frac{1}{16}$

$\frac{1}{16}$

$\cancel{P(|\Delta| \geq c \mid H_0) = \alpha}$

$\cancel{2P(\Delta \geq c \mid H_0) = \alpha}$

$\cancel{c = u_{1-\frac{\alpha}{2}} = 1{,}27.}$

$\cancel{G: |\Delta| \geq u_{1-\frac{\alpha}{2}} = 1{,}27}$

$P(\Delta \leq C_1 \mid H_0) + P(\Delta \geq C_2 \mid H_0) = \alpha.$

$C_1 = F_{\alpha/2}(n-1, m-1)$ $\qquad$ $C_2 = F_{1-\alpha/2}(n-1, m-1)$

$C_1 = F_{0{,}025}(138, 999) = 0{,}767$

$C_2 = F_{0{,}975}(138, 999) = 1{,}272$ $\Rightarrow$

$\tilde{\Delta} = 0{,}863$

$\cancel{G: C_1 < \tilde{\Delta} < C_2}$

$\cancel{\tilde{\Delta} \in G \Rightarrow \text{отвергаем } H_0}$

$\cancel{W(\theta_1) = P(\tilde{\Delta}}$

$G: \quad \tilde{\Delta} > 1{,}272$

$\tilde{\Delta} < 0{,}767.$

$\tilde{\Delta} = 0{,}863 \notin G \Rightarrow$

Нет оснований отвергнуть H_0

$W(H_1) = P(\tilde{\Delta} > 1{,}272 \mid H_1) + P(\tilde{\Delta} < 0{,}767 \mid H_1) =$

$= \int_{1{,}272}^{\infty} q(t)\,dt + \int_{-\infty}^{0{,}767} q(t)\,dt = 0{,}05$

Ширина черепа

$\xi \sim N(a, \varphi^2)$ $\qquad$ $\eta \sim N(b, \psi^2)$

$n = 139, \ \varphi = 4{,}612$ $\qquad$ $m = 1000, \ \psi = 5{,}055$

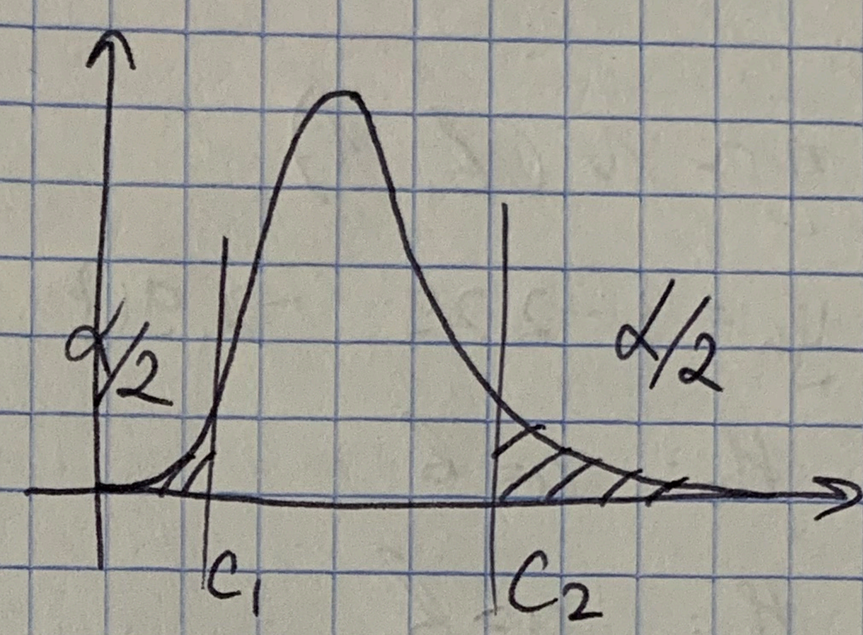
$H_0: \ \varphi^2 = \psi^2$

$H_1: \ \varphi^2 \neq \psi^2$

$S_\xi^2 = 21{,}271$

$S_\eta^2 = 25{,}553$

$\Delta = \frac{S_\xi^2}{S_\eta^2} = 0{,}832$



$\Delta \sim F(n-1, m-1)$

$G: \quad \Delta < C_1$

$\quad\quad \Delta > C_2$

$P(\Delta \leq C_1 \mid H_0) + P(\Delta \geq C_2 \mid H_0) = \alpha.$

$C_1 = F_{\frac{\alpha}{2}}(n-1, m-1)$ $\qquad$ $C_2 = F_{1-\frac{\alpha}{2}}(n-1, m-1)$

$C_1 = F_{0{,}025}(138, 999) = 0{,}767$

$C_2 = F_{0{,}975}(138, 999) = 1{,}272.$

$G: \quad \Delta > 1{,}272$

$\quad\quad \Delta < 0{,}767.$

$\Delta \notin G \Rightarrow$ нет оснований отвергнуть H_0

$W(H_1) = 0{,}05$

№14

$\xi \sim N(a, 2)$

$x_n = \{-1{,}11;\ -6{,}10,\ 2{,}42\}$ $\quad \tilde{S}_x^2 = 12{,}217 \quad \overline{x} = -1{,}597$

$\eta \sim N(b, 1)$

$y_n = \{-2{,}29,\ -2{,}91\}$ $\quad \tilde{S}_y^2 = 0{,}096 \quad \overline{y} = -2{,}6$

$H_0: \ a = b$

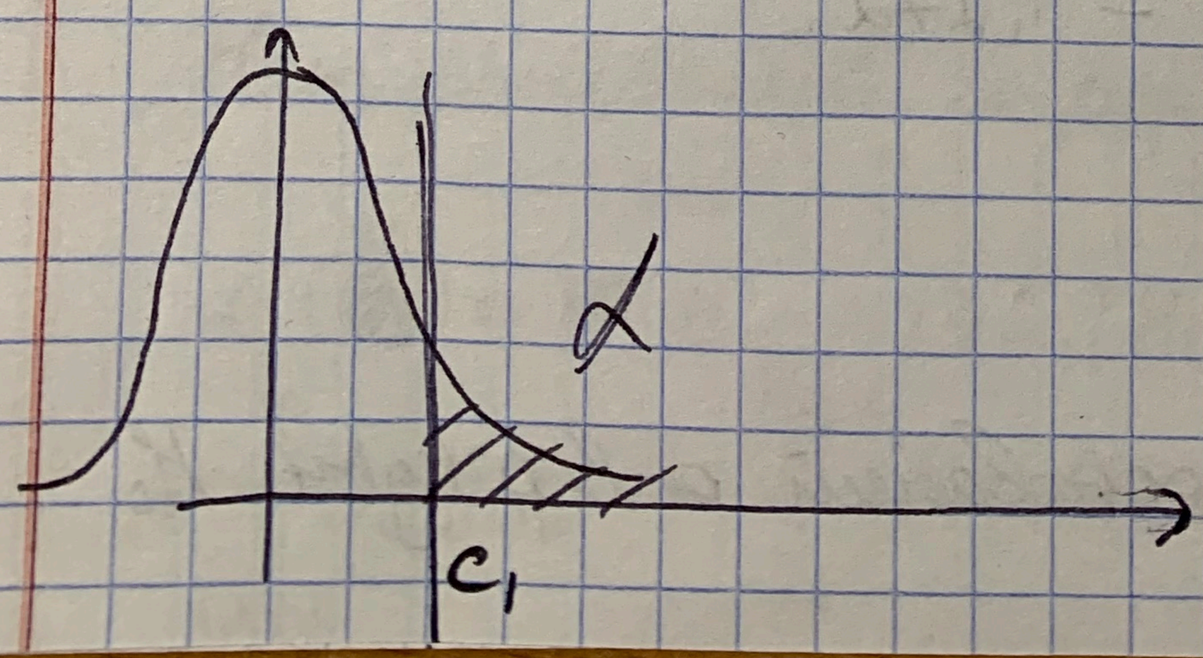
$H_1: \ a > b$

$$\Delta = \frac{\overline{x} - \overline{y}}{\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}} \rightsquigarrow N(0,1)$$

т.к. n и $m \to \infty$ $\quad S_x^2 = 2, \quad S_y^2 = 1$

$$\Delta = \frac{-1{,}597 + 2{,}6}{\sqrt{\frac{2}{3} + \frac{1}{2}}} \rightsquigarrow N(0,1)$$

$\Delta = 0{,}929$



$G: \Delta > C_1$

$P(\Delta > C_1 \mid H_0) = \alpha.$

$C_1 = u_{1-\alpha} = N_{0,95}(0,1) = \cancel{0,17}\ 1,64$

$G: \Delta > 1,64$

Δ & G => нет оснований отвергнуть H_0.

$W = P(\Delta > C_1 \mid H_1) = \int_{1,64}^{\infty} q(t)\,dt = 0,05$