

Momentum equation:

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial z} \left[ (1 + \delta) \frac{Q^2}{S} \right] + \frac{S}{\rho} \frac{\partial p}{\partial z} = N \frac{Q}{S} + \nu \frac{\partial^2 Q}{\partial z^2}$$

Second term expanded:

$$(1 + \delta) \frac{\partial}{\partial z} \left[ \frac{Q^2}{S} \right] = (1 + \delta) \left( \frac{1}{S} 2Q \frac{\partial Q}{\partial z} + Q^2 \left( -\frac{1}{S^2} \right) \frac{\partial S}{\partial z} \right)$$

Equation rearranged; **blue terms** “neglected” & will be parametrized:

$$(1 + \delta) \frac{2Q}{S} \frac{\partial Q}{\partial z} = -\frac{\partial Q}{\partial t} + (1 + \delta) \frac{Q^2}{S^2} \frac{\partial S}{\partial z} + N \frac{Q}{S}$$

$$\frac{\partial Q}{\partial z} = -\frac{1}{2(1 + \delta)} \frac{S}{Q} \frac{\partial Q}{\partial t} + \frac{1}{2} \frac{Q}{S} \frac{\partial S}{\partial z} + \frac{N}{2(1 + \delta)}$$

