Modeling Coupled 1D PDEs of Cardiovascular Flow with Spatial Neural ODEs

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Introduction & Motivation

- Uncertainties in boundary conditions (BCs) → need for rapid parametric blood flow modeling
- 3D model → cross-sectional averaging → 1D model → flow rate Q, area S, pressure p
- Coupled PDEs for 1D blood flow modeling in deformable wall vessels:

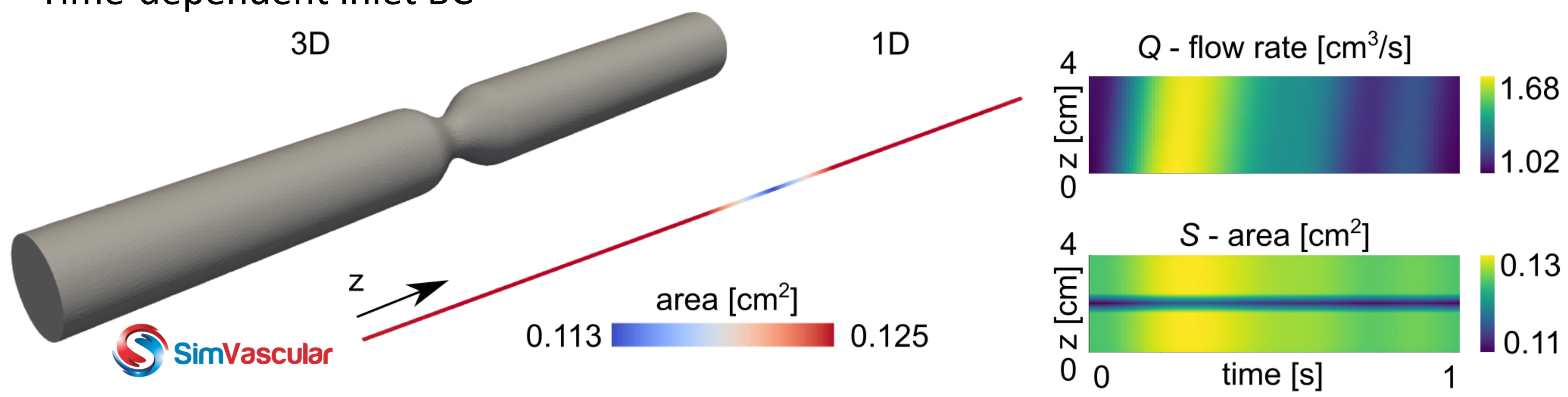
Continuity
$$\frac{\partial S}{\partial t} = -\frac{\partial Q}{\partial z}$$

Momentum
$$\frac{\partial Q}{\partial t} = -(1+\delta)\frac{\partial}{\partial z}\left(\frac{Q^2}{S}\right) - \frac{S}{\rho}\frac{\partial p}{\partial z} + N\frac{Q}{S} + \nu\frac{\partial^2 Q}{\partial z^2}$$

- Inlet BC: prescribed flow rate Q_{in}, outlet BC: resistance, initial condition: $Q(t=0)=0$
- Constitutive equation for pressure: $p = p(S, E, h)$
- **Objective:** Create a fast physics-based data-driven method for modeling coupled PDEs of blood flow in idealized stenosed arteries with deformable walls and realistic inlet flow rate waveforms

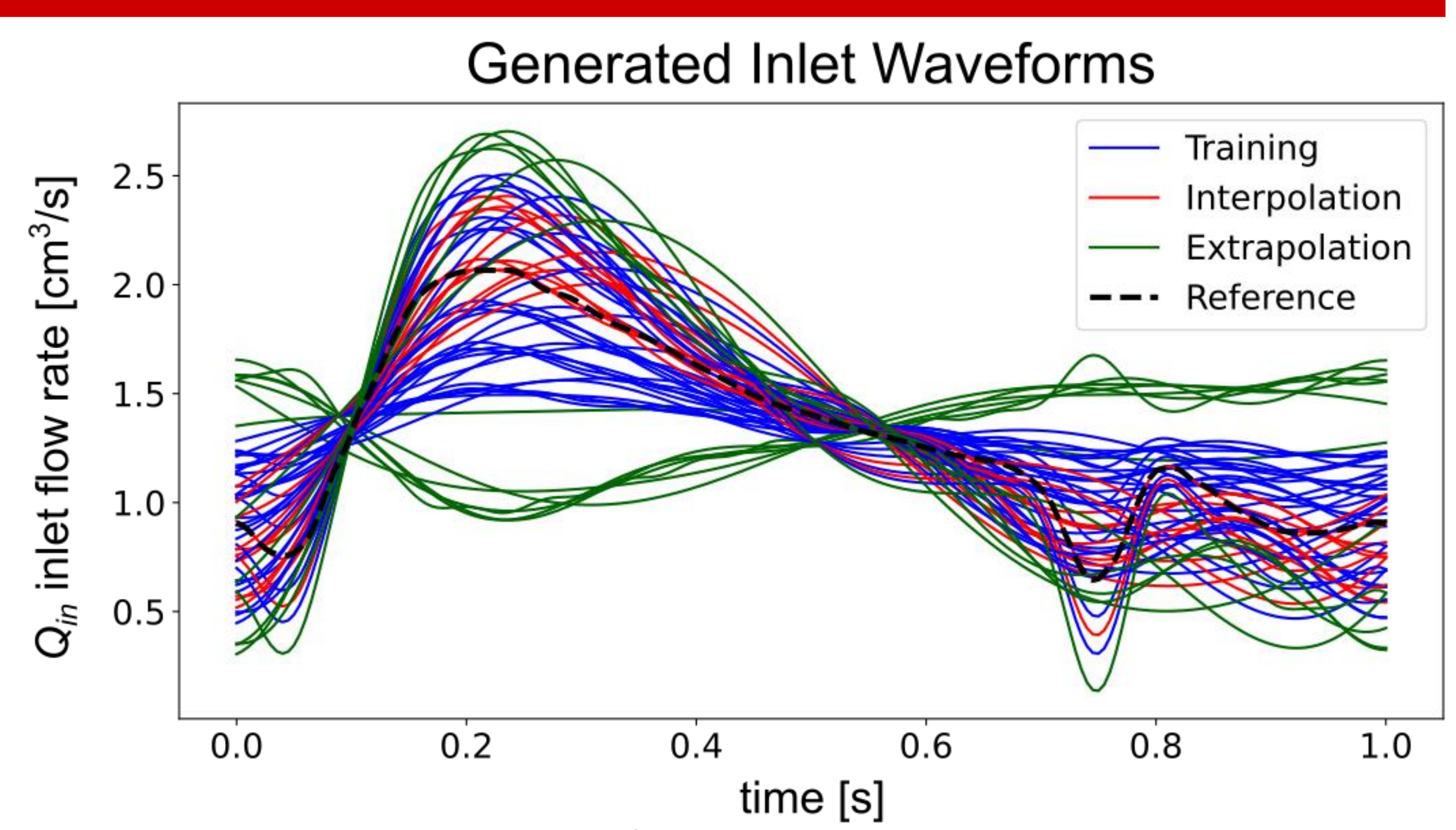
Challenges:

- PDE coupling → stability & smoothness
- Time-dependent inlet BC



Methods & Data

- Ground-truth Q_{GT} is 1D FEM simulations from *SimVascular*
- 54 simulations with different realistic inlet flow rate waveforms generated from Fourier series fit
- Neural ODEs → parametrize part of the 1D momentum PDE with a NN
- Use Neural ODE for the momentum equation and finite-differences for the continuity equation
- NN → 5 layers, **101**-10-10-10-**101**
- Training, interpolation, extrapolation sets.

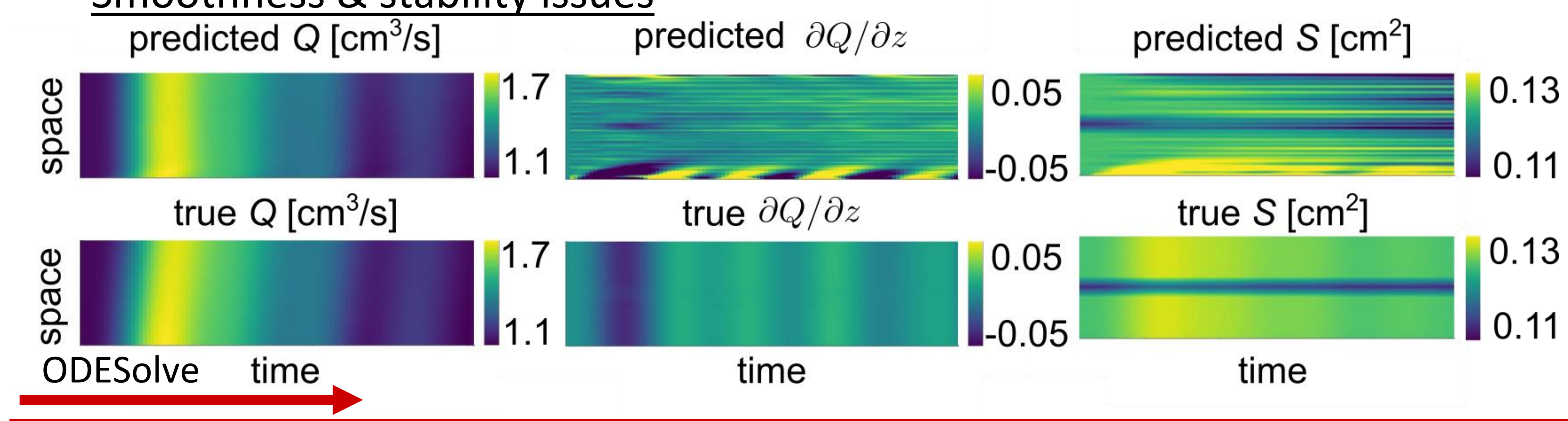


$$Q_{in}(t) = A_0 + \sum_1^k \left(aA_k \cos\left(\frac{2\pi kt}{T}\right) + B_k \sin\left(\frac{2\pi kt}{T}\right)\right)$$

$k \in \{6, 9, 12, 15, 18, 59\}$ $a \in \{-2, 0.5, 0, 0.5, 1, 1.5, 1.75, 2, 2.5\}$

Issues with Temporal Neural ODE

- Developed differentiable RK4 solver with inlet BC enforced
- Good prediction for Q, but $\partial Q/\partial z$ is incorrect → area S wrong too!
- Smoothness & stability issues



$$\frac{\partial S}{\partial t} = -\frac{\partial Q}{\partial z}$$

$$\frac{\partial Q}{\partial t} = NN_\theta(Q)$$

$$Loss = MSE(Q, Q_{GT})$$

Inlet BC: $Q(z=0) = Q_{in}$

IC: $Q(t=0) = Q_0$

Spatial Neural ODE

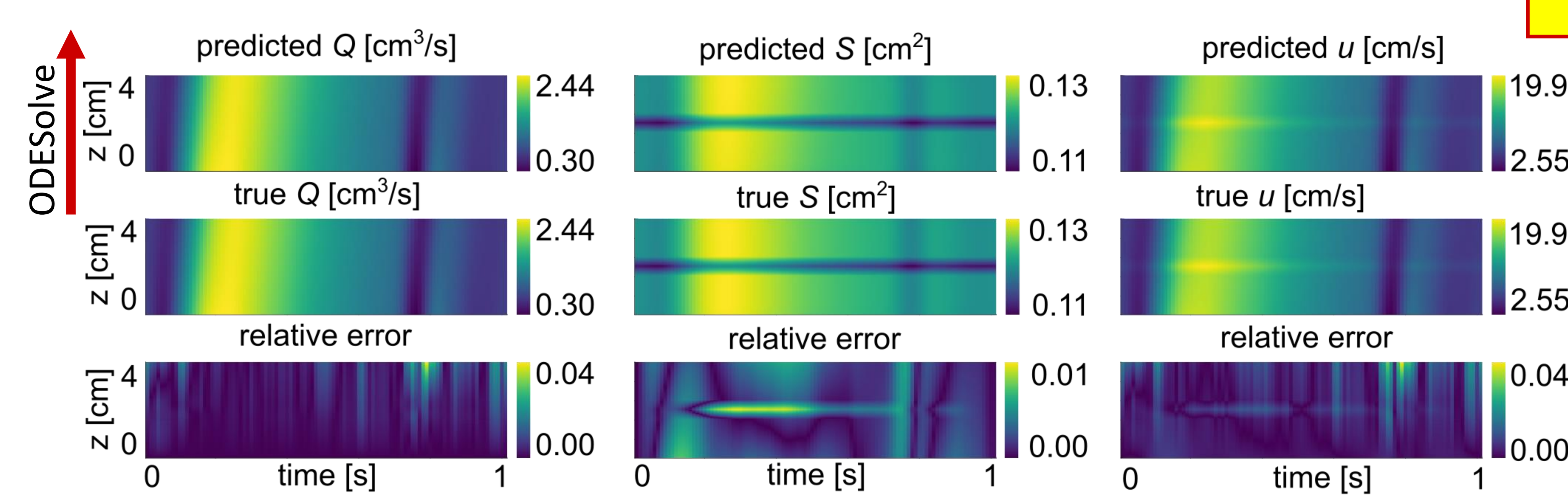
- Historically neural ODEs used for space periodic PDEs (Burgers, Kuramoto-S., isotropic turbulence)
- Blood flow problems are periodic in time!
- **Flip space & time** → Solve neural ODE in space!
- Inlet BC becomes initial condition
- Solution is **implicitly smooth** in spatial z direction

IC: $Q(z=0) = Q_{in}$

- Accurate predictions for Q and S **for unseen waveforms**, both interpolation and extrapolation
- Train with ground-truth Q only, ground-truth S values are not used

$$\frac{\partial Q}{\partial z} = NN_\theta(Q)$$

$$\frac{\partial S}{\partial t} = -\frac{\partial Q}{\partial z} \Rightarrow S$$



$\frac{\partial Q}{\partial z} = NN_\theta(Q)$ $\frac{\partial S}{\partial t} = -\frac{\partial Q}{\partial z}$ $u = \frac{Q}{S}$

Relative error	Q	S
Training	0.44%	0.19%
Interpolation	0.48%	0.22%
Extrapolation	0.66%	0.34%

Conclusion

- Casting neural ODE in space helps with inlet BC and with PDE coupling stability and solution smoothness.
- Next goal: Train with 3D models to develop low-dimensional surrogates
- Generic approach, applicable to a larger class of unsteady transport problems beyond cardiovascular flow.

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