Spacetime Distance

For x(t)= $\sqrt{b^2+(ct)^2}$,if particle gets at x=2b, what is the spacetime distance traversed by it

Solving for t when euclidean distance = 2b, to get bounds of the integral,

In[∘]:= $\text{Solve}\left[2\,b-\sqrt{b^2+(c*t)^2}==0,\ t\right]$

Solve: There may be values of the parameters for which some or all solutions are not valid.

Out[∘]= $\left\{\left\{t\to-\frac{\sqrt{3}\,b}{c}\right\},\left\{t\to\frac{\sqrt{3}\,b}{c}\right\}\right\}$

Taking positive time, t= $\frac{\sqrt{3}}{c}\,b$,

$$\frac{ds^2}{dt^2}=-c^2+\left(\frac{dx}{dt}\right)^2$$

In[∘]:= $x' = D\left[\sqrt{b^2+(c\,t)^2},\ t\right]$

Out[∘]= $\frac{c^2\,t}{\sqrt{b^2+c^2\,t^2}}$

In[∘]:= $ds2dt2 = -c^2+(x')^2$

Out[∘]= $-c^2+\frac{c^4\,t^2}{b^2+c^2\,t^2}$

Spacetime length= $\int\frac{ds}{dt}dt$ from t=0 to $t=\frac{\sqrt{3}}{c}\,b$

In[∘]:= `dsdt = Sqrt[ds2dt2]`

Out[∘]= $\sqrt{-c^2+\frac{c^4\,t^2}{b^2+c^2\,t^2}}$

In[∘]:= `Simplify[dsdt]`

Out[∘]= $\sqrt{-\frac{b^2\,c^2}{b^2+c^2\,t^2}}$

In[•]:= `Integrate[dsdt, {t, 0, Sqrt[3]/c b}]`

Out[•]= $\frac{b\sqrt{-c}\,\mathrm{Log}\left[2+\sqrt{3}\right]}{\sqrt{c}}$

`FullSimplify[%, c > 0]`

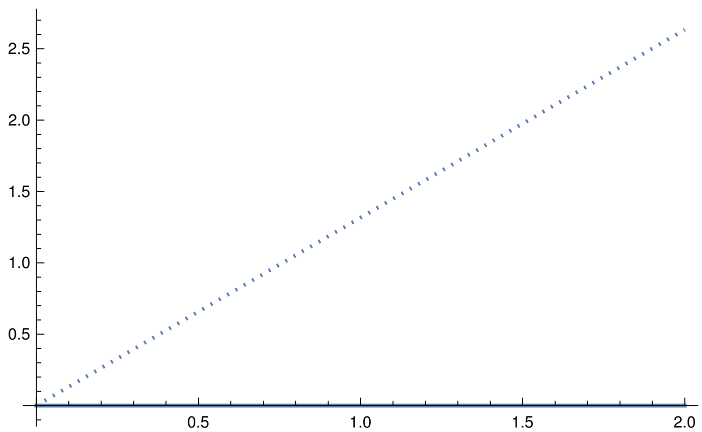
Out[•]= $i\, b\, \mathrm{Log}\left[2+\sqrt{3}\right]$

In[•]:= `L = i b Log[2 + Sqrt[3]]`

Out[•]= $i\, b\, \mathrm{Log}\left[2+\sqrt{3}\right]$

In[•]:= `ReImPlot[L, {b, 0, 2}]`

Out[•]=



Hence, spacetime length between the events for euclidean lengths b and 2b increases linearly as b increases