补充题: **场算符的Heisenberg方程**

实标量场理论的拉氏密度为

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2.$$

正则量子化的等时对易关系为:

$$[\phi(\vec{x},t),\pi(\vec{y},t)] = i\delta^{(3)}(\vec{x}-\vec{y}),$$
$$[\phi(\vec{x},t),\phi(\vec{y},t)] = [\pi(\vec{x},t),\pi(\vec{y},t)] = 0.$$

写出哈密顿量H的表达式. 然后利用Heisenberg方程 $i\dot{\phi}(x) = [\phi(x),H]$ 验证场算符满足

$$(\partial_\mu\partial^\mu + m^2)\phi(x) = 0,$$

这和经典的Klein-Gordon方程形式一致.