Lab 0: Gaussian Elimination Optimization

In the Lab 0 assignment, we were tasked with optimizing the Gaussian elimination algorithm down to versions with two loops and one loop, starting from a version with three loops.

ForwardThreeLoops

The 'ForwardThreeLoops' version of the algorithm is a straightforward implementation of Gaussian elimination.

It uses three nested loops to perform row operations and transform the augmented matrix into upper triangular form. The loops iterate over the columns (for pivoting), the rows (for row replacements), and the elements of each row (for row operations).

ForwardTwoLoops

To reduce the algorithm to two loops, we precompute the elements of the pivot row divided by the pivot element and store them in a vector. Then, we use a single loop to iterate over all rows below the pivot row. For each of these rows, we update its elements using the precomputed vector and the corresponding

multiplier.

ForwardOneLoop

The 'ForwardOneLoop' version takes this a step further by precomputing all necessary row operations and
storing them in a list. Each entry in the list is a tuple containing the pivot row index, the target row index, and the factor by which the pivot row needs to be multiplied before subtracting from the target row.
We then use a single loop to iterate through this list, applying the row operations directly.

By precomputing the necessary row operations and their corresponding multipliers, we were able to reduce
the number of loops in the algorithm while still accurately performing Gaussian elimination.

Matrix Derivation for ForwardOneLoop

The matrices used in the 'ForwardOneLoop' function are derived by precomputing the necessary row operations.
For each pivot row, we calculate the factor by which it needs to be multiplied before subtracting from each
target row. These factors are calculated as (Element in target row's pivot column) / (Pivot element).

The precomputed row operations are then stored as tuples in a list. The tuple contains:
- Pivot row index
- Target row index
- Factor

During the single loop, we directly use these precomputed tuples to perform the row operations. This is equivalent to transforming the augmented matrix into an upper triangular form, enabling us to solve the system of equations using back-substitution.