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#61. Find the coordinate matrix of x in R^n relative to the basis B'

$B' = \{(1,2,3), (1,2,0), (0,-6,2)\}$, $x = (3,-3,0)$

To find $[x]_{B'} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$ solve equation

$c_1(1,2,3) + c_2(1,2,0) + c_3(0,-6,2) = (3,-3,0)$

$c_1 + c_2 = 3 \qquad 2c_1 + 2c_2 - 6c_3 = -3$

$3c_1 + 2c_3 = 0$

The solution of this system is $c_1 = -1$, $c_2 = 4$, $c_3 = \frac{3}{2}$

$[x]_{B'} = \begin{bmatrix} -1 \\ 4 \\ \frac{3}{2} \end{bmatrix}$ #

#66. Find the transition matrix from B to B'

$B = \{(1,-1), (3,1)\}$ $B' = \{(1,2), (-1,0)\}$

Begin by formatting $[B' \; B] = \begin{bmatrix} 1 & -1 & 1 & 3 \\ 2 & 0 & -1 & 1 \end{bmatrix}$

$[I_2 \; P^{-1}] = \left[\begin{array}{cc|cc} 1 & 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & -\frac{3}{2} & -\frac{5}{2} \end{array}\right]$

$P^{-1} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{3} \\ -\frac{3}{2} & \frac{5}{2} \end{bmatrix}$ #

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\# 22. Determine all vectors that are orthogonal to u.

$u = (1, -1, 2)$

$v = v_1 - v_2 + 2v_3 = 0$

$v = (s, t, \frac{1}{2}t - \frac{1}{2}s)$ ~~t, s~~ $t, s \in R$ #

\# 46. Let $f(x) = x+2$ and $g(x) = 15x-8$ be functions in the vector space.

$C[0,1]$ with inner product $\langle f, g \rangle = \int_0^1 f(x)g(x)\,dx$.

(a) Find $\langle f, g \rangle$.

$\langle f, g \rangle = \int_0^1 f(x)g(x)\,dx = \int_0^1 (x+2)(15x-8)\,dx = \int_0^1 (15x^2+22x-16)\,dx$

$= [5x^3 + 11x^2 - 16x]_0^1 = 0$ #

(b.) Find $\langle -4f, g \rangle$

$\langle -4f, g \rangle = -4\langle f, g \rangle = -4 \cdot 0 = 0.$

(c) Find $\|f\|$

$\|f\|^2 = \langle f, f \rangle = \int_0^1 (x+2)^2\,dx = \int_0^1 (x^2+4x+4)\,dx$

$= [\frac{x^3}{3} + 2x^2 + 4x]_0^1 = \frac{19}{3}$ $\quad \sqrt{\frac{19}{3}}$ #

(d.) Orthonormalize the set $B = \{f, g\}$.

$\|g\|^2 = \langle g, g \rangle = \int_0^1 (15x-8)^2\,dx = \int_0^1 (225x^2 - 240x + 64)\,dx = [75x^3 - 120x^2 + 64x]_0^1$

$= 19$ $\quad \|g\| = \sqrt{19}$ $\quad u_1 = \frac{1}{\|f\|} f = \sqrt{\frac{3}{19}}(x+2)$ $\quad u_2 = \frac{1}{\|g\|} g = \frac{1}{\sqrt{19}}(15x-8)$

$B' = \{(\sqrt{\frac{3}{19}}x + 2\sqrt{\frac{3}{19}}), (\frac{15}{\sqrt{19}}x - \frac{8}{\sqrt{19}})\}$ #