40823117L 資工系 113 方國走.

4. Find ... for linear transformation $T: R^3 \to R^3$, $T(v_1, v_2, v_3) = (v_1+v_2, v_2+v_3, v_3)$

(a.) ~~$T(v_1+v_2, v_3)$~~ $T(v) = T(-2, 1, 2) = \overset{-2+1,\ 1+2,\ 2}{(-1, 3, 2)}$ #

(b.) $T(v_1, v_2, v_3) = (v_1+v_2, v_2+v_3, v_3) = (0, 1, 2)$

$v_1 + v_2 = 0$

$v_2 + v_3 = 1$ $\quad v_2 = -1,\ v_1 = 1$ $\quad (1, -1, 2)$ #

$v_3 = 2$

16. Let T be .... such that $T(1,-1) = (2,-3)$ and $T(0,2) = (0,8)$ Find $T(2,4)$

Because $(2,4) = 2(1,-1) + 3(0,2)$

$T(2,4) = 2T(1,-1) + 3T(0,2) = 2(2,-3) + 3(0,8)$

$= (4,-6) + (0,24) = (4,18)$ #

31. Define the linear transformation T by $T(v) = Av$. Find $\ker(T)$ ... rank(T)

(a.) $A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $\ker(T) = \{(0,0)\}$ #

(b.) $\dim(\ker(T)) = \text{nullity}(T) = 0.$ #

(c.) $A^T = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} \end{bmatrix}$ range(T) is span$\{(1,0,\frac{1}{2}), (0,1,-\frac{1}{2})\}$

d.) $\dim(\text{range}(T)) = \text{rank}(T) = 2$

36. Given $T: P_5 \to P_3$ and nullity$(T) = 4$, find Rank(T)

$\text{Rank}(T) = \dim P_5 - \text{nullity}(T) = 6 - 4 = 2$ #