

40823119L 梁工系 方國平

11. Find the determinant of the matrix

$$A = \begin{bmatrix} 2 & 0 & -1 & 4 \\ -1 & 2 & 0 & 3 \\ 3 & 0 & 1 & 2 \\ -2 & 0 & 3 & 1 \end{bmatrix}$$

$$\det(A) =$$

$$2 \begin{vmatrix} -1 & 2 & 0 & 3 \\ 3 & 0 & 1 & 2 \\ -2 & 0 & 3 & 1 \end{vmatrix} - 1 \begin{vmatrix} 2 & 0 & 4 \\ -1 & 2 & 3 \\ 3 & 0 & 1 \end{vmatrix} + 4 \begin{vmatrix} -1 & 2 & 0 \\ 3 & 0 & 1 \\ -2 & 0 & 3 \end{vmatrix}$$

$$= 2 \cdot (39 + 10 + (-8)) - 1 \cdot (-2 + 12 + 12) + 4 \cdot (-6 - 8)$$

$$= 2 \cdot (39 + 10 + (-8)) = 82$$

18. Find the determinant of the matrix

$$A = \begin{bmatrix} 0 & 0 & 0 & 6 & 2 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\det(A) =$$

$$2 \cdot (-1)^{1+5} \cdot 2 \cdot (-1)^{1+4} \cdot 2 \cdot (-1)^{1+3} \cdot \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix}$$

$$= -8 \cdot (-4)$$

$$= 32$$

26. Find $|A^T|$, $|A^3|$, $|A^T A|$ and $|5A|$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 2 & -1 & 2 \end{bmatrix}$$

$$|A| = (-1)^{1+2} \cdot 1 \cdot (-1) = -1$$

$$a) |A^T| = |A| = -1$$

$$b) |A^3| = |A|^3 = -1$$

$$c) |A^T A| = |A^T| |A| = 1$$

$$d) |5A| = 5^3 \cdot (-1) = -125$$

34. Solve the system ... by each of the following methods

(a)
$$\begin{bmatrix} 2 & 1 & 2 & 6 \\ -1 & 2 & -3 & 0 \\ 3 & 2 & -1 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 3 & 0 \\ 2 & 1 & 2 & 6 \\ 3 & 2 & -1 & 6 \end{bmatrix} \xrightarrow{-2} \begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & 5 & -4 & 6 \\ 0 & 8 & -10 & 6 \end{bmatrix} \xrightarrow{-\frac{8}{5}}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & 1 & -\frac{4}{5} & \frac{6}{5} \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\begin{cases} x_1 = 1 \\ x_2 = 2 \\ x_3 = 1 \end{cases}$$

H

(b)
$$\begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & 1 & -\frac{4}{5} & \frac{6}{5} \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{2} \begin{bmatrix} 1 & 0 & \frac{7}{5} & \frac{12}{5} \\ 0 & 1 & -\frac{4}{5} & \frac{6}{5} \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{-\frac{7}{5}} \begin{bmatrix} 1 & 0 & 0 & \frac{12}{5} - \frac{7}{5} \\ 0 & 1 & -\frac{4}{5} & \frac{6}{5} \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{-\frac{4}{5}}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \begin{cases} x_3 = 1, x_2 = 2, x_1 = 1 \end{cases}$$

(c.) $A = \begin{bmatrix} 2 & 1 & 2 \\ -1 & 2 & -3 \\ 3 & 2 & -1 \end{bmatrix} \quad A_1 = \begin{bmatrix} 6 & 1 & 2 \\ 0 & 2 & -3 \\ 6 & 2 & -1 \end{bmatrix}$

$$A_2 = \begin{bmatrix} 2 & 6 & 2 \\ -1 & 0 & -3 \\ 3 & 6 & -1 \end{bmatrix} \quad A_3 = \begin{bmatrix} 2 & 1 & 6 \\ -1 & 2 & 0 \\ 3 & 2 & 6 \end{bmatrix}$$

$$\det(A) = -18$$

$$\det(A_1) = -18$$

$$\det(A_2) = -36$$

$$\det(A_3) = -18$$

$$\begin{cases} x_1 = 1 \\ x_2 = 2 \\ x_3 = 1 \end{cases}$$