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40923117L 資工系 113級 方圓王.

47. Find the inverse of the linear transformation

$T: R^2 \to R^2,\ T(x,y) = (x,-y)$

standard matrix: $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (invertible)

Inverse $A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ #

53. $T: R^2 \to R^3,\ T(x,y) = (-x, y, x+y),\ v = (0,1)$

$B = \{(1,1), (1,-1)\},\ B' = \{(0,1,0), (0,0,1), (1,0,0)\}$

Find $T(v)$

(a) standard: $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$

$T(v) = A(v) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = (0,1,1)$ #

(b) The image of each vector in B is as follows

$T(1,1) = (-1,1,2) = (0,1,0) + 2(0,0,1) - (1,0,0)$

$T(1,-1) = (-1,-1,0) = -(0,1,0) + 0(0,0,1) - (1,0,0)$

$A' = \begin{bmatrix} 1 & -1 \\ 2 & 0 \\ -1 & -1 \end{bmatrix}$ $T(v) = \begin{bmatrix} 1 & -1 \\ 2 & 0 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$T(v) = (0,1,0) + (0,0,1) + 0(1,0,0) = (0,1,1)$ #

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55. Find the matrix A' for T relative to the basis B'

standard : $A = \begin{bmatrix} 1 & -3 \\ -1 & 1 \end{bmatrix}$

$B = \{(1,0), (0,1)\}$ $P = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

$$A' = P^{-1}AP = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & -1 \end{bmatrix}$$

#

57. Use the matrix P to show that the matrices A and A' are similar

$P = \begin{bmatrix} 3 & -5 \\ 1 & -4 \end{bmatrix}$, $A = \begin{bmatrix} 18 & -19 \\ 11 & -12 \end{bmatrix}$ $A' = \begin{bmatrix} 5 & -3 \\ -4 & 1 \end{bmatrix}$.

$$A' = P^{-1}AP = \begin{bmatrix} \frac{4}{7} & -\frac{5}{7} \\ \frac{1}{7} & -\frac{3}{7} \end{bmatrix} \begin{bmatrix} 18 & -19 \\ 11 & -12 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ 1 & -4 \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ -4 & 1 \end{bmatrix}$$