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59. Find the orthogonal complement S^\perp of the subspace of \mathbb{R}^3 spanned by the two column vectors of the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 0 & -1 \end{bmatrix}$$

$S^\perp = N(A^T)$, the orthogonal complement of S is the nullspace of A^T .

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & -3 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -\frac{2}{3} \\ 0 & 1 & \frac{1}{3} \end{bmatrix}$$

the S^\perp is spanned by $\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

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60. Find the projection of the vector $v = [1 \ 0 \ -2]^T$ onto the subspace.

$$S = \text{span} \left\{ \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

$$\text{orthonormal basis is } \left\{ \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \right\}$$

$$\text{proj}_S v = (v \cdot u_1) u_1 + (v \cdot u_2) u_2 = \left(-\frac{2}{\sqrt{2}} \right) \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} + \left(-\frac{2}{\sqrt{2}} \right) \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$$

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