AMES Week 12 Section - pset 5

Andie Creel

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1 Q1 on PSet 5

In fisheries management it is common to assume that fish mortality is the sum of two constant instantaneous per capita mortality rates; "natural" mortality, m, and fishing mortality, f, (natural mortality is everything that is not fishing mortality).

1.1 a

Consider a shellfish population, $N(t)$. Assume a constant level of reproduction per unit time, A, independent of the current population (assume that larva drift in from a larger external population due to currents), write an ordinary differential equation showing the change in the fish stock over time.

What is an ODE? It's the derivative of some stock of interest with respect to time,

$$\begin{aligned}\frac{\partial N(t)}{\partial t} &= \text{births minus deaths}\\ &= A - (m+f)N\end{aligned}$$

Note that A has a unit of constant reproduction per unit of *time*. This is different than a lot of *birth rates* we've seen in class.

1.2 b

Solve the ordinary differential equation to get $N(T)$, where T is an arbitrary future time.

Solution intuition: We need to get the population level in time period T, which is $N(T)$. To get this, we could integrate the *changes* in population through time to get $N(T)$.

Problem set up:

$$\frac{\partial N(t)}{\partial t} = A - (m+f)N$$

Separate the variables

$$\frac{1}{A-(m+f)N}dN = 1dt$$

Integrate both sides from $N(0)$ to $N(T)$

$$\int_{N(0)}^{N(T)} \frac{1}{A-(m+f)N}dN = \int_{N(0)}^{N(T)} 1dt$$

You will need to do integration by substitution here. Then evaluate the integral from $N(0)$ to $N(T)$ and solve for $N(T)$. Assume $N(0)$ is known.

1.3 c

Find the equilibrium solution for N.

Solution intuition: What level of $N(t)$ leads to the population to in equilibrium *i.e.* not changing? When $\frac{dN(t)}{dt} = 0$

$$\frac{dN(t)}{dt} = A - (m+f)N = 0$$

Solve for N^*.

1.4 d

What is the sensitivity of the equilibrium to fishing mortality?

Solution intuition: What is the equilibrium? N^* from part c. What sensitivity mean in math? How N^* changes with respect to fishing mortality, f. Therefore, what you want to look at is $\frac{dN^*}{df}$

2 Q2 on PSet 5

Let $y(t)$ be the reserves of oil left in an oil well at time t. Suppose extraction takes place at a *constant continuous rate per unit of time* of α.

2.1 a

Write a differential equation for this problem.

What's an ODE? Derivative of stock wrt to time.

$$\frac{dy(t)}{dt} = -\alpha$$

2.2 b

If there were initially 500 million barrels of oil, then solve for the amount of oil at arbitrary time T.

Intuition: This is the same as problem 1b. We need to get the stock level in time period T , which is $y(T)$. To get this, we could integrate the changes in the stock through time.

2.3 c

If $\alpha = 2.5$, then when will 75% of the oil in the well be used up? Note from AC: continue to assume $y(0) = 500$.

Solution intuition: We want to know when 500 minus the changes in reserve is 25% of the original amount. Take the ODE and do separation of variables. Which can work with whichever variable we want (*i.e.* dy or dt). Because we're interested in the time period that 75%, I'm going to use dt.

$$\frac{dy(t)}{dt} = -\alpha \tag{1}$$
$$dy(t) = -\alpha dt \tag{2}$$
$$\int_0^T dy(t) = \int_0^T -\alpha dt \tag{3}$$

At this stage, we can see that the change in y from 0 to T is equal to $\int_0^T -\alpha dt$

$$\Delta y = \int_0^T -\alpha dt \tag{4}$$
$$= -\alpha T \tag{5}$$

So now we can set up our intuitive problem where we have 25% is left and we see how many time steps it takes to get there.

$$0.25y(0) = y(0) + \Delta y$$
$$0.25 * 500 = 500 - \alpha T$$

Solve for T.