

$$a(\{x_i, y_i\})$$

$$b(\{x_i, y_i\})$$

$$\bar{Y} = \frac{1}{N} \sum_{i=1}^N y_i$$

$$\bar{X} = \frac{1}{N} \sum_{i=1}^N x_i$$

Chain Rule

$$\frac{\partial F}{\partial x}(g(x)) = F'(g(x)) g'(x)$$

$$g(x) = y - ax - bx$$

$$F(x) = x^2$$

$$\frac{\partial g}{\partial a} = -1$$

$$y_i = a + b x_i + \varepsilon_i$$

$$\text{Obj: min } \sum \varepsilon_i^2$$

$$\varepsilon_i = y_i - a - b x_i$$

$$\text{Obj fun: } \sum_{i=1}^N (y_i - a - b x_i)^2$$

$$\frac{\partial}{\partial a} = \sum_i -2(y_i - a - b x_i) = 0 \quad (1)$$

$$\frac{\partial}{\partial b} = \sum_i -2x_i(y_i - a - b x_i) = 0 \quad (2)$$

Step 1

Solve (1) for a

$$\sum_i (y_i - a - b x_i) = 0$$

$$\sum_i y_i - \sum_i a - \sum_i b x_i = 0$$

$$\sum_i y_i - N a - b \sum_i x_i = 0$$

$$\sum_i y_i - b \sum_i x_i = N a$$

$$\sum_i \frac{1}{N} \frac{1}{N} y_i - b \frac{1}{N} \sum_i x_i = a$$

$$\bar{Y} - b \bar{X} = a$$

Step 2  
Plug a into (2)  
Solve for b

$$\sum_i x_i (y_i - a - b x_i) = 0$$

$$\sum_i x_i \left[ y_i - \left( \frac{1}{N} \sum_i y_i - b \frac{1}{N} \sum_i x_i \right) - b x_i \right] = 0$$

$$\sum_i x_i y_i - \frac{1}{N} \sum_i x_i \sum_i y_i + x_i b \cdot \frac{1}{N} \sum_i x_i - b x_i x_i = 0$$

$$\sum_i x_i y_i - \frac{1}{N} \sum_i x_i \sum_i y_i + \frac{1}{N} \sum_i x_i b \cdot \frac{1}{N} \sum_i x_i - \frac{1}{N} b \sum_i x_i x_i = 0$$

$$\sum_i x_i y_i - \frac{1}{N} \sum_i x_i \sum_i y_i = b \sum_i x_i^2 - b \frac{1}{N} \sum_i x_i \sum_i y_i$$

$$N \sum_i x_i y_i - \sum_i x_i \sum_i y_i = b (N \sum_i x_i^2 - \sum_i x_i \sum_i y_i)$$

$$b^* = \frac{N \sum_i x_i y_i - \sum_i x_i \sum_i y_i}{N \sum_i x_i^2 - (\sum_i x_i)^2}$$

Step 3  
Plug  $b^*$  into  
step 1

$$a = \frac{1}{N} \sum_i y_i - b \frac{1}{N} \sum_i x_i$$

$$a^* = \frac{1}{N} \sum_i y_i - \left( \frac{N \sum_i x_i y_i - \sum_i x_i \sum_i y_i}{N \sum_i x_i^2 - (\sum_i x_i)^2} \right)$$