AMES Class Notes

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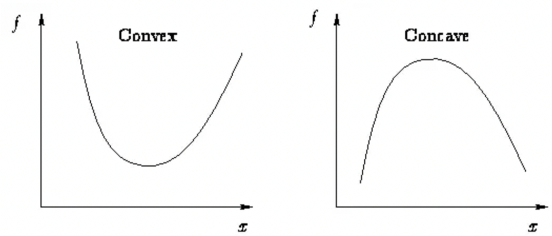
1 Concavity/Convexity



$$Y = f(x)$$

Note: $F(x)$ is R^1 (*i.e.,* 1-dimensional) because there is only one input. But we're drawing it in R^2 (*i.e.,* 2-dimensional) because we're using an x-y graph. To visualize a n dimensional function you need a $n+1$ dimensions because you need an additional dimension for the output.

2 Quasi-Concavity/Convexity

If a function is in R^2 (*i.e.* it takes two inputs), then you must evaluate whether the function is convex or concave in R^3. That will determine if the function is convex or concave. But then if you graph the function in the R^2 space (meaning you are not using an additional dimension for the output), the function may look convex or concave in the lower dimension. Despite how it looks in the lower dimension, you must determine if it's quasi-convex or quasi-concave from the higher dimension.

The lower-dimension level sets of a quasi-concave or quasi-convex function will be convex. You determine if the 3-D function is quasi-concave or quasi-convex but determining if the 3-D function looks like a dome when graphed (quasi-concave) or a bowl (quasi-convex). In other words, as the level sets move away from the origin, if the value of the level sets increases, then the function is quasi-concave. If the value of the level sets decreases, the function is quasi-convex.

3 Sequences and Series

Sequence: The result of a function where the only inputs are integers. Any real number can be represented as a sequence. The output of a sequence is usually a set (or list) of numbers.

Series: A summation of a sequence. The output of a series is typically a single number.

Infinity: Arbitrarily long unit. Infinity is not a real number, it's a concept.

Example one: Consider the following sequence,

$$-1, -1/2, -1/3, -1/4, -1/5, ...$$

The limit of the sequence goes to zero

$$lim \to 0$$

Zero: Zero is also a unique number because (unless you're being so careful) it can lose it's unit. If you say Eli has 6 apples, then you take 6 apples from him, you may not say Eli has zero *apples*. You might instead say he has *nothing* (which has no unit).

Monotone: The function is *only* increasing or *only* decreasing. The function is always going in the same direction of increasing or decreasing.

Bound: If the output of a function never increases above a certain number, then it is bounded from above. If it's output never decreases below a certain number, it's bounded from below.

Example two: Consider the following function

$$f(x) = -1^x$$

output: -1, 1, -1, 1, -1, 1, ...

This is an example of a sequence that is *bounded* by -1 and 1. However, it has *no limit* and is *not monotonic.*

It's important to understand bounds, limits and monotonicity so that when you collect data you can understand the general ways two things are related to one another.

Method of exhaustion: Find examples of methods of exhaustion videos and post by October 2nd.

Some infinities are bigger than others: Some infinities are *countable* meaning you can write down an expression that can denote any number in your sequence. However, some infinities are not countable.

Examples: Let a be a real number, $0 < a < \infty$,

$$\lim_{N\to\infty} \frac{a}{N} = \infty$$
$$\lim_{N\to\infty} \frac{N}{N+a} = 1$$

Building intuition on a complicated limit:

$$\lim_{N\to\infty} \left(1 + \frac{1}{N}\right)^N = e$$

$$\begin{aligned} N = 1 &\implies 2 \\ N = 2 &\implies (1+1/2)(1+1/2) = 2.25 \\ N = 3 &\implies (1+1/3)(1+1/3)(1+1/3) = 2\frac{4}{9} \\ N \to \infty &\implies e \approx 2.7182... \end{aligned}$$

Here we built intuition here by looking at the difference in output between N=1 and N=2, then N=2 and N=3. The difference between outputs became smaller as we went forward, so we intuited that this limit would be bounded above by *something* (*i.e.,* the limit).

The number e is *irrational* and *real.*

3.1 Why do we care? Future generations

If we think about sustainable development (non-declining welfare for future generations), then how many future generations do we actually care about?? *All* of the future generations? Realistically, that is probably not true. You would probably prefer to keep your own children healthy more than to care about your great-great-great-grandchildren.

If you do not prefer the immediate generation at least a little more than the future, you would let everyone starve today in order to save all resources for future generations. That doesn't seem right either. This leads us to the reason why we use discount rates, which we will discuss more later in class.

For example, consider a fish population that we are trying to conserve forever. If the value of that fish population is *infinite*, then we would give up everything and anything else just to save the fish. That is insane.

4 Science

Consider two positive real value numbers,

$$s \in S \in \mathbf{R}^{++}$$
$$w \in W \in \mathbf{R}^{++}.$$

If S, W are two variables that you are collecting data on, then you may be measuring them with error. Say, W is water, and S is the survival rate of a species.

What we're going to learn in this class is that, often times, the "best" way to approximate the relationship between two variables is a line.

4.1 Intermediate Value Theorem

The more data you have, the better approximation you have of real relationships.

Intermediate value theorem tells us that we should want more data. Assuming x is continuous, we can always find another x that will help us approximate the relationship between x and y more accurately.

Let's consider the slope. Consider the function:

$$S = F(w) \tag{1}$$

The slope is

$$\frac{F(w_2) - F(w_1)}{w_2 - w_1} = \frac{\Delta S}{\Delta w}$$

Now consider if w_2 is arbitrarily smaller than w_1. $w_2 = w_1 + \epsilon$. We can write our slope equation as

$$\frac{F(w_1 + \epsilon) - F(w_1)}{\epsilon}$$

This is the definition of a derivative,

$$\frac{ds}{dw} \equiv \lim_{\epsilon \to 0} \frac{F(w_1 + \epsilon) - F(w_1)}{\epsilon}$$