AMES Class Notes

Andie Creel

7th November, 2024

1 What is the goal of this course?

This course aims to give you the tools to formalize problems and "speak math", so that you can understand and engage more fully with interdisciplinary problems elsewhere.

This class will help you choose which "math" approach should be applied to a given problem. We don't require you to memorize equations for regurgitation. Instead, you are familiarized with solution strategies and trained in how to apply them to environmental problems.

2 Introduction: Science vs Math

Science is evidence (data) base. Math is proof-based. In science, you observe things many times and can reject the hypothesis based on that evidence. However, in science, one cannot prove/conclude anything. In math, you do not need much evidence. Instead, you can do a proof and conclude something.

Math builds conclusions off of a few simple assumptions. Science rejects alternative explanations based on many observations.

3 Sets

Sets are collections of elements.
The **element** x is in the **set** X:

$$x \in X$$

You may indicate different elements as $x_1, x_2, x_3...$ and all are in the set X:

$$x_1, x_2, x_3... \in X$$

Sets can be any collections of things

$$dogs \in Pets.$$

You may hear the term **class** which is a set of sets. For instances, the "set" of pets is in the "class" of animals.

Let's say you have set X and set Y. If every element of X is contained in Y then we X a **subset** of Y,

$$X \subset Y.$$

Your set X may equal set Y in certain cases, in which you can indicate a subset as

$$X \subseteq Y. \tag{1}$$

You can have two different sets, X and Y, and can talk about the **intersection** of the two sets which would be all the elements are in X AND in Y

$$X \cap Y$$

You could also talk about the **union** of two sets, which would be all elements in X OR in Y

$$X \cup Y$$

there is also the **complement** to set, which are all the elements in the *space* you are working with but NOT in your set.

$$X^{\complement}$$

4 Spaces of Numbers

Note: The symbol $\simeq$ means that two sets are equal or the same.

Real numbers: $\mathbb{R} \simeq [-\infty, \infty]$
Positive real numbers: $\mathbb{R}^{+} \simeq [0, \infty]$
Positive real numbers in (x, y) space: $\mathbb{R}^{++} \simeq x \in [0, \infty]$ and $y \in [0, \infty]$
Counting Numbers/Integers: $[1, 2, 3, 4, ...) \in \mathbb{N}$, $\mathbb{N} \subset \mathbb{R}$
Rational numbers: can be written as a fraction.
Irrational numbers: cannot be written as a fraction. Examples are π, e.

5 Properties

Communicative:

$$a + b = b + a$$

Associative:

$$a + (b + c) = (a + b) + c$$

Distributive:

$$a(b + c) = ac + bc$$

An application of the distributive property is

$$(a+b)(c+d) = ac + ad + bc + bd.$$

When we learned this, we all shrugged and said "sure". But if we return to how we were originally taught multiplication,

$$\begin{array}{r} 15 \\ \underline{\times 12} \\ 10 \\ 20 \\ 50 \\ \underline{+100} \\ 180 \end{array}$$

this is actually an application of the distributive property,

$$(2+10)(5+10) = 10 + 20 + 50 + 100 = 180. \tag{2}$$

Rules with zero:

$$a + 0 = a$$
$$0a = 0$$

Rules with one:

$$1 * a = a$$

Inverse property:

$$a * \frac{1}{a} = 1$$

A note on equal signs: The equal sign means that the two things on either side of the equal sign will evaluate to the same thing. It is not like a function where you put elements in and get something out. It is an identity, the two items on each side of the "=" literally are the same thing.

6 Convex set

A set is **convex** if you are able to draw a line between *any two* elements in the set and all elements intersected by the line are elements in the set. A circle is a convex set. A cresent is not a convex set.

Convex sets are important if you're operating with a **non-convex** set, you can fall into tipping points where you arrive at states of the world that are completely unfamiliar. The new state of the world was not in your original set of possible states of the world. In climate change, we don't want to go into unknown states of the world. We'd like to stay in our current states of the world and move smoothly from state to state.

However, there are some issues where you may want to cross a tipping point. For instance, when addressing environmental justice or systemic discrimination issues, you may want to tip into a state of the world (where there are no inequities) and have it be difficult to return to your original state of the world.

7 Functions

7.1 Functions map to ONE output

A function is something that returns a single element from it's inputs. For example

$$f(x) = x + 2$$
$$x \in \mathbb{R}^1 \text{ (domain)}$$

is a function because we give it one element x and it will return a single element.

Domain is what x is permissible to be. The **co-domain** is anything that $f(x)$ may be equivalent to. The **range** is what $f(x)$ can take the value of given the domain. Therefore, the range is a subset of the co-domain (and may be equal to the co-domain):

$$\text{range} \subseteq \text{co-domain}$$

The domain of $f(x) = x + 2$ was given as $x \in \mathbb{R}^1$. The co-domain will be $\mathbb{R}^1$ because $f(x)$ could be real number in 1-dimensional space. In this case, the range of $f(x)$ will also be the co-domain.

Something that is is NOT a function would be some bizarre thing where if you defined the function g as

$$g(x) = x + 2$$

but $g(2) = 4 \;\;\&\;\; 5$. This operator g is saying 2+2 = 4 and 2+2 = 5. This makes no sense, it's not a functions.

7.2 Functions can map many inputs to one output

We've established that functions output ONE element. However, they can have *many* inputs,

$$f(x,y) = x + y$$
$$\mathbb{R}^2 \to \mathbb{R}^1$$

the function above takes two inputs $(x,y) \in \mathbb{R}^2$ and maps those inputs into a single element (one dimensional real number, $\mathbb{R}^1$)

8 More vocab

8.1 Explicit functions

$$y = a + bx$$
$$x = c + dy$$

8.2 Implicit function

$$G(x,y;a,b,c,d) = g$$

where G is a function that has **endogenous** variables x and y and **exogenous** variables a,b,c,d.

Endogenous variables are determined by the system. **Exogenous** variables are **parameters** that we take as "given" in the system.