AMES Section Notes – Week 10, Setting Up population Growth

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Consider a population where the population level can be modeled:

$$N_t = (b - d)N_{t-1} \tag{1}$$

And survival rate is:

$$S_i j = 1 - d \tag{2}$$

Where S_{ij} is the survival rate of individuals i years old make it to j years old.

Everyone dies after 3 years.

We can model this as:

$$\overset{t+1}{N}_{0_{t+1}} = b_0 \overset{t}{N}_{0_t} + b_1 N_{1_t} + b_2 N_{3_t} + b_3 N_{3_t} \tag{3}$$
$$N_{1_{t+1}} = S_{0,1} N_{0_t} + 0 + 0 + 0 \tag{4}$$
$$N_{2_{t+1}} = 0 + S_{1,2} N_{1_t} + 0 + 0 \tag{5}$$
$$N_{3_{t+1}} = 0 + 0 + S_{2,3} N_{2_t} + 0 \tag{6}$$

Where we're looking at the number of individuals that are m years old in time period $t + 1$, $N_{m_{t+1}}$.

Can we write this more clearly with matrices?

$$\begin{bmatrix} N_{0_{t+1}} \\ N_{1_{t+1}} \\ N_{2_{t+1}} \\ N_{3_{t+1}} \end{bmatrix} = \begin{bmatrix} b_0 & b_1 & b_2 & b_3 \\ S_{0,1} & 0 & 0 & 0 \\ 0 & S_{1,2} & 0 & 0 \\ 0 & 0 & S_{2,3} & 0 \end{bmatrix} \begin{bmatrix} N_{0_t} \\ N_{1_t} \\ N_{2_t} \\ N_{3_t} \end{bmatrix} \tag{7}$$

We can then rewrite this in matrix notation by naming these matrices,

$$\underset{4\times 1}{N_{t+1}} = \underset{4\times 4}{A}\underset{4\times 1}{N_t} \tag{8}$$