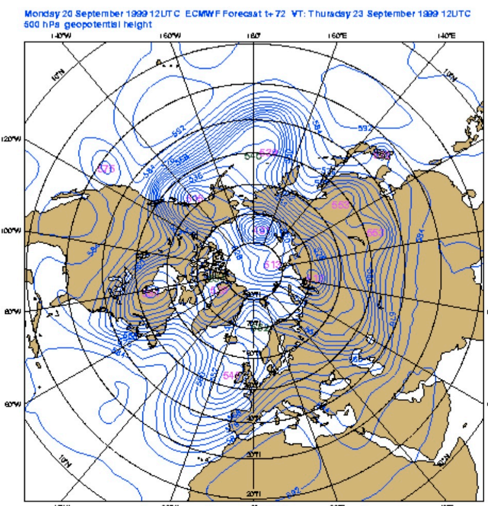
# Project 4 - Rossby waves

## I. BACKGROUND

Rossby waves are important, characteristic features of flows in the ocean and atmosphere.



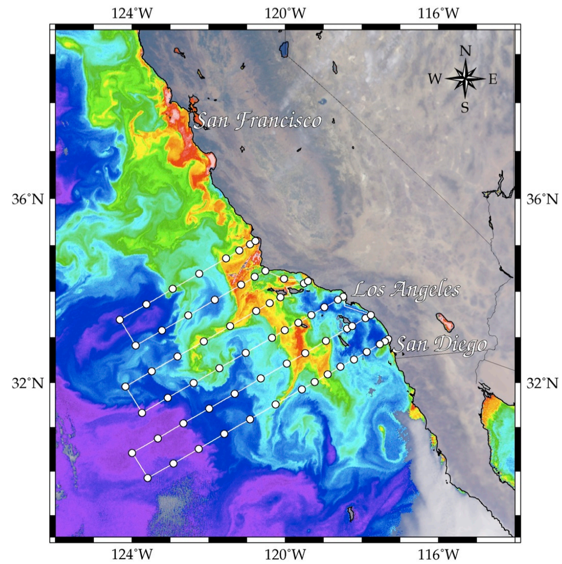




FIG. 1: ECMWF geopotential height map, showing atmospheric Rossby waves (left) and ocean eddies visible in the phytoplankton concentration off the California coast (right).

## II. INTRODUCTION

Here, we will examine some features of Rossby waves by running two-dimensional simulations in which we vary the rotation rate.

Let's consider flow in a 2D plane, normal to the axis of rotation. Consider small perturbations to a constant flow in the $x$-direction, $U$. Linearizing about the constant background flow, we have the following equations:

$$u'_t + U u'_x - f(y) v' = -\frac{1}{\rho_0} p_x, \tag{1}$$

$$v'_t + U v'_x + f(y) u' = -\frac{1}{\rho_0} p_y, \tag{2}$$

$$u'_x + v'_y = 0. \tag{3}$$

Here, we will consider flow on the so-called '$\beta$-plane', where the Coriolis parameter is approximated as $f \simeq f_0 + \beta y$. Since the flow is two-dimensional, it can be described by a streamfunction, $\psi'$, where $u' = -\psi'_y$, $v' = \psi'_x$. Taking the curl of the momentum equations to eliminate pressure gives

$$\nabla^2\psi'_t + U\nabla^2\psi'_x + \beta\psi'_x = 0. \tag{4}$$

By looking for plane-wave solutions of the form:

$$\psi' = \psi_0 e^{i(kx+ly-\omega t)}, \tag{5}$$

we can derive the Rossby wave dispersion relation:

$$\omega - kU = -\frac{k\beta}{k^2+l^2}. \tag{6}$$

## III. SIMULATIONS OF LINEAR ROSSBY WAVES

To simulate Rossby waves, we will use what is known as a $\beta$-plane channel geometry. This consists of free-slip walls at the edges of the $y$-domain, periodic boundary conditions in $x$, and a rotation vector coming out of the 2D plane in the $z$-direction. For this problem, it is easier to work in non-dimensional parameters. In the Diablo set-up scripts, set the domain size ($LX = 1$, $LY = 1$). For this problem, we don't need to solve for U3, or any scalars (we will add this later). The Rossby number controls the relative size of the nonlinear and Coriolis terms in the momentum equations. Since we want the nonlinear terms to be small, pick a small Rossby number (say 1/Ro=I_RO=100). Set up the boundary conditions with free-slip walls in $y$, and force the pressure gradient to vanish at both walls ($\partial p/\partial y = 0$). Then, modify the definition of $CORI\_Z$ so that its average is 1, but it varies linearly in $y$ to represent the beta-effect. Since the $y$-coordinate is not defined in *set_params.m* where $CORI\_Z$ is first defined, add the beta term in another script like *create_flow.m*.

To initialize the flow, create a streamfunction to mimic a plane-wave solution. To minimize edge effects, we can use a Gaussian envelope:

$$\psi' = \frac{1}{k^2+l^2} sin(kx) sin(ly) exp\left(\frac{-(y-LY/2)^2}{(LY/4)^2}\right). \tag{7}$$

In *create_flow.m*, initialize the velocity field based on the definition of $\psi'$: $u' = -\psi'_y$, $v' = \psi'_x$. Chose the wavenumbers $k$, and $l$ to be large enough so that we can fit several wavelengths into the envelope. $k = 4\pi/LX$, $l = 4\pi/LY$, $Re = 5000$, and $\beta = 0.5$ seems to work reasonably well as a starting point (although you will see strong dispersion of the waves).

When you run the simulation, you should see the wave crests slowly propagate. Which direction do they move? After the simulation has progressed so the waves have moved by about one wavelength, stop the simulation. Estimate the phase speed of the waves by creating a Hovmöller diagram by plotting U1 at $y = LY/2$ as a function of $x$ and $t$ (you will need U1_save for this). How does the phase speed compare to what you estimate from the dispersion relation, given above? Try varying $\beta$ and/or $k$ and $l$ and repeat the simulation and analysis.

## IV. SUGGESTED FURTHER INVESTIGATIONS

### A. Transport by linear and nonlinear Rossby waves

An important difference between linear Rossby waves and nonlinear eddies in the ocean and atmosphere is their ability to transport fluid properties. First, examine transport by linear Rossby waves by introducing a passive scalar in the calculations above. Try initializing the scalar with the same structure as the streamfunction, and then visualize both the scalar and the streamfunction (or the pressure). Do the wave crests move relative to the scalar field, or do the waves carry the scalar with them as they propagate? Do the waves stir the fluid?

Repeat this calculation for a much larger value of the Rossby number (say $1/Ro = 1$). Now the nonlinear term will be quite large - how does the tracer transport change? You may have a difficult time diagnosing the wave nature of this flow since the nonlinear terms are so large. Try an intermediate value of the Rossby number ($1/Ro = 1/100$ seems to work well). Can you see a combination of wave and eddy-like characteristics?

Transport by nonlinear Rossby waves is extremely important in the ocean. To illustrate this, start with a tracer field with some structure in the $x$-direction (like a Gaussian). We can imagine that the tracer patch represents a patch of nutrient-rich fluid that has upwelled near a north/south coastline. Are your nonlinear Rossby waves capable of transporting this fluid long distances?

### B. Forced Rossby waves

Rossby waves can be generated in many ways - an important example is flow over topography. While our 2D code can't simulate flow over topography directly, we can add an artificial forcing to approximate the influence of a topographic feature. Set up a simulation with a small uniform velocity in the $x$-direction. Then add a region of localized forcing, opposing the initial flow, by adding a term to F1 in *user_rhs.m*, which will add a body force to the right hand side of the $x$-momentum equation. Here, we can think of the body force as an extra drag on the flow, which could be generated, for example, by flow over topography. To make sure that we can reach a steady-state, adjust the body force so that its mean vanishes. Run Diablo - eventually a standing wave pattern should develop. What does this say about the phase speed measured in a coordinate from moving with the mean flow?

Now, try making the forcing function time-dependent. By varying the frequency of the forcing, can you get waves to propagate both up and downstream? What scale do the waves have? Can you predict this from the dispersion relation?

## V. REFERENCES

In addition to the lecture notes, mathematical descriptions of Rossby waves can be found in nearly all Geophysical Fluid Dynamics textbook. A few examples are given below[1–3].

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[1] A. Gill, *Atmosphere-ocean dynamics*, vol. 30 (Academic Pr, 1982).

[2] J. Pedlosky, *Geophysical fluid dynamics* (Springer, 1987).

[3] R. Salmon, *Lectures on geophysical fluid dynamics* (Oxford University Press, USA, 1998).