(extended) Generalized linear model

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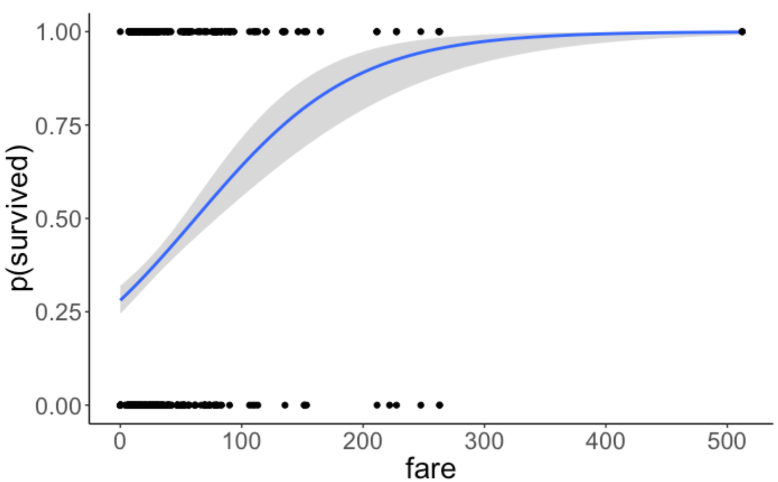
Reference: Statistical Rethinking by Richard McElreath
http://xcelab.net/rm/statistical-rethinking/



$$\ln\left(\frac{\mathrm{p(survived)}_i}{1-\mathrm{p(survived)}_i}\right) = b_0 + b_1 \cdot \mathrm{fare}_i + e_i$$

```
fit.glm = glm(formula = survived ~ 1 + fare,
              family = "binomial",
              data = df.titanic)

fit.glm %>% summary()
```

GLM in R

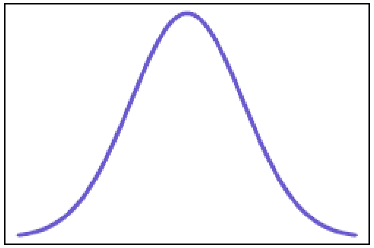
```
glm(formula, family = gaussian, data, weights, subset,
    na.action, start = NULL, etastart, mustart, offset,
    control = list(...), model = TRUE, method = "glm.fit",
    x = FALSE, y = TRUE, singular.ok = TRUE, contrasts = NULL, ...)
```

Usage

```
family(object, ...)

binomial(link = "logit")
gaussian(link = "identity")
Gamma(link = "inverse")
inverse.gaussian(link = "1/mu^2")
poisson(link = "log")
quasi(link = "identity", variance = "constant")
quasibinomial(link = "logit")
quasipoisson(link = "log")
```

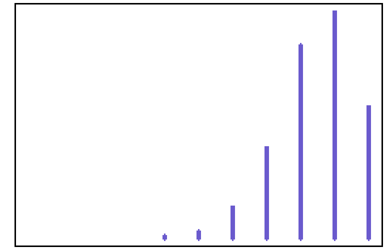
$y \sim \text{Normal}(\mu, \sigma)$



dnorm

Linear regression

$y \sim \text{Binomial}(n, p)$



dbinom

When n = 1: Bernoulli

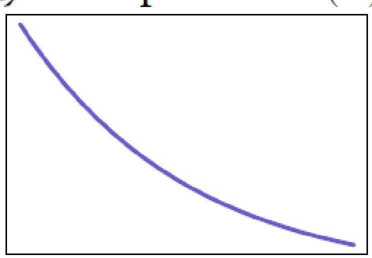
↓

Logistic regression

Exponential family

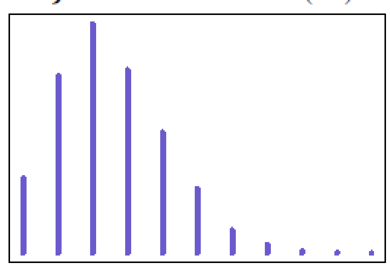
$$p(y; \eta) = b(y) \exp(\eta^T T(y) - a(\eta))$$

$y \sim \text{Exponential}(\lambda)$



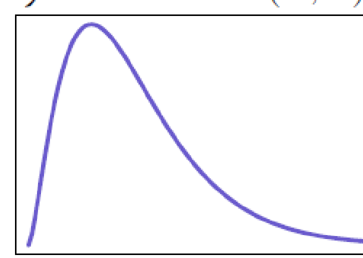
dexp

$y \sim \text{Poisson}(\lambda)$



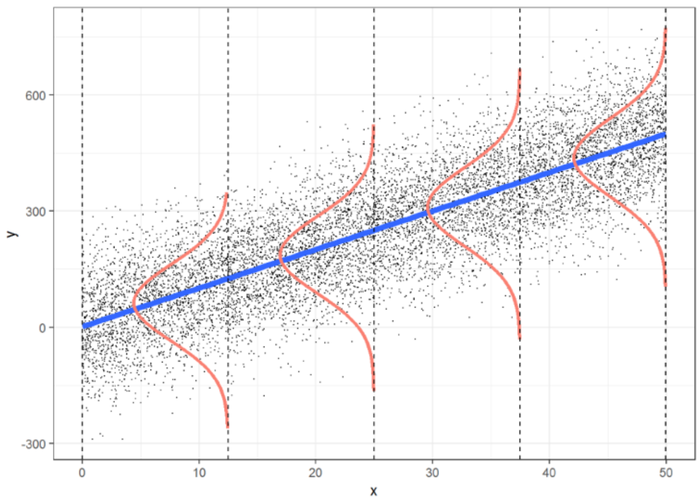
dpois

$y \sim \text{Gamma}(\lambda, k)$



dgamma

Linear regression



Data = Model + Error

$$Y_i = \underline{\beta_0 + \beta_1 X_i} + \underline{\epsilon_i}$$

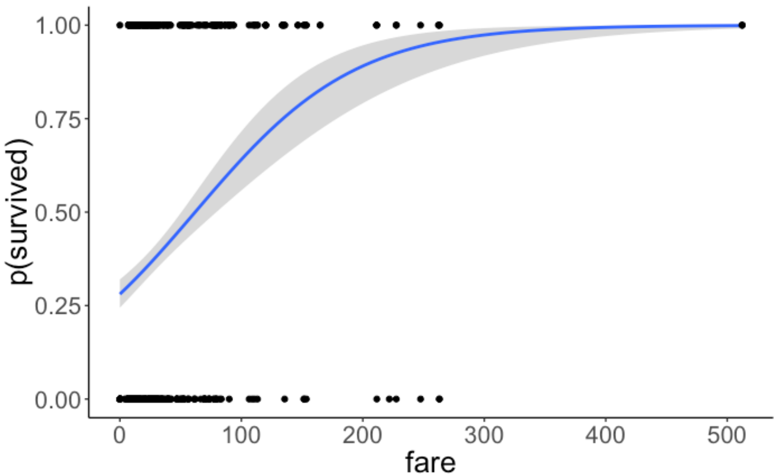
$$y_i \sim Normal(\mu_i, \sigma)$$
$$\mu_i = \beta_0 + \beta_1 x_i$$

$[-\infty, +\infty]$

IID: independent and identically distributed

Y	X
y_1	x_1
y_2	x_2
y_3	x_3
y_4	x_4
…	…
y_{999}	x_{999}
y_{1000}	x_{1000}

Logistic regression



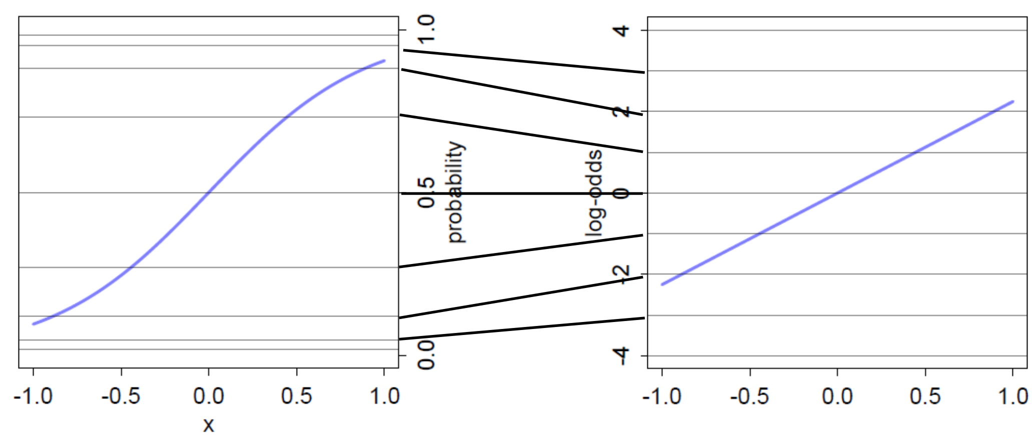
$$y_i \sim Binomial(1, p_i)$$ -- yes or no?

$$?\ p_i = \beta_0 + \beta_1 x_i$$ -- probability [0,1]

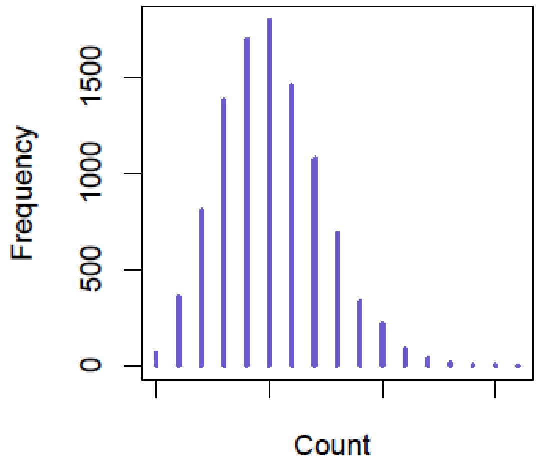
$$f(p_i) = \beta_0 + \beta_1 x_i$$

$$\log \frac{p_i}{1 - p_i} = \beta_0 + \beta_1 x_i$$

Y	X
0	x_1
1	x_2
0	x_3
0	x_4
…	…
1	x_{999}
1	x_{1000}



Poisson GLM



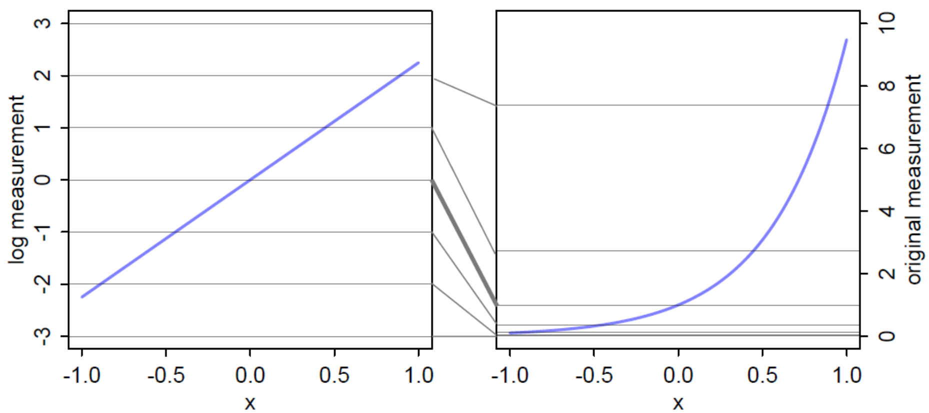
$$y \sim \text{Poisson}(\lambda), \lambda > 0$$

- Counts without upper limit, constant expected value
- Example: DNA mutations, soldiers killed by horses

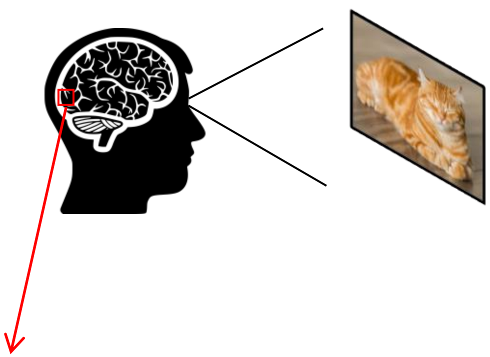
$$y_i \sim Poisson(\lambda_i)$$
$$?\, \lambda_i = \beta_0 + \beta_1 x_i$$

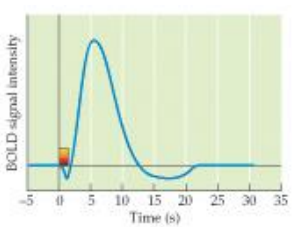
$$\log(\lambda_i) = \beta_0 + \beta_1 x_i$$

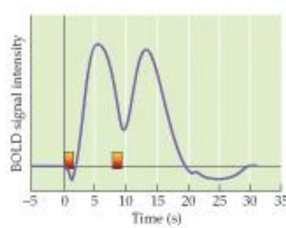
Y	X
3	x_1
5	x_2
4	x_3
5	x_4
…	…
2	x_{999}
6	x_{1000}



GLM for fMRI data

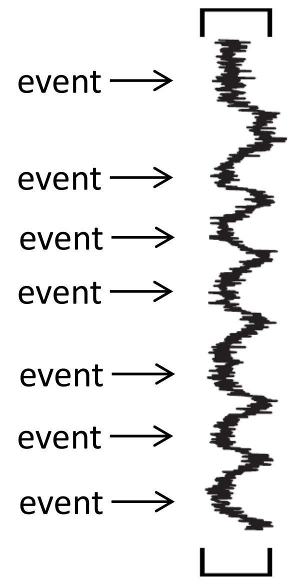






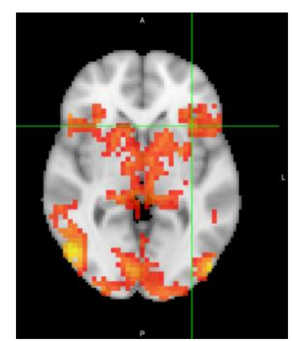
hemodynamic response functions (HRF)

t	event	signal
1	1	y_1
2	0	y_2
3	0	y_3
4	0	y_4
…	…	…
11	1	y_{11}
12	0	y_{12}
…	…	…



$$\text{signal} = \beta_0 + \beta_1 \cdot HRF(event) + \epsilon_i$$

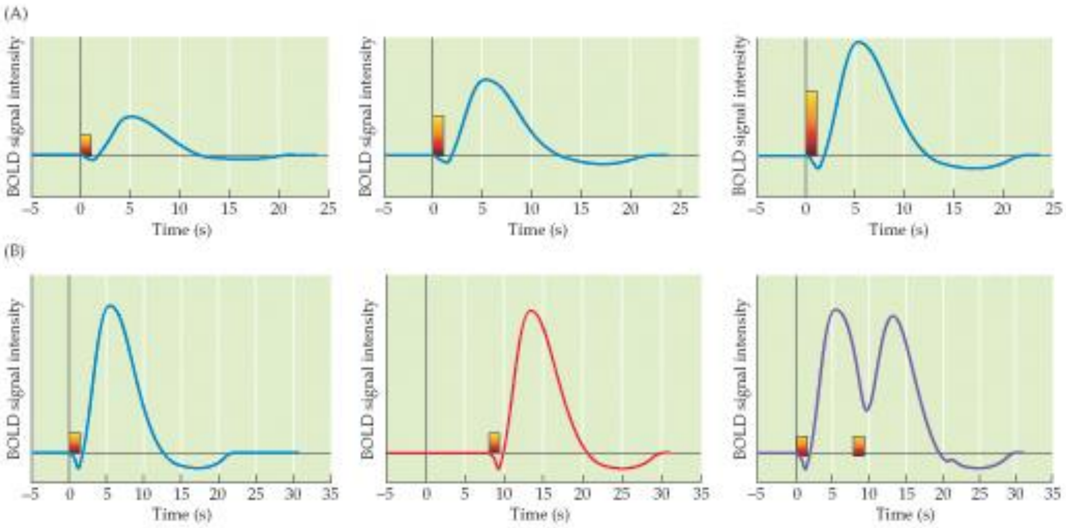
Brain map of β_1

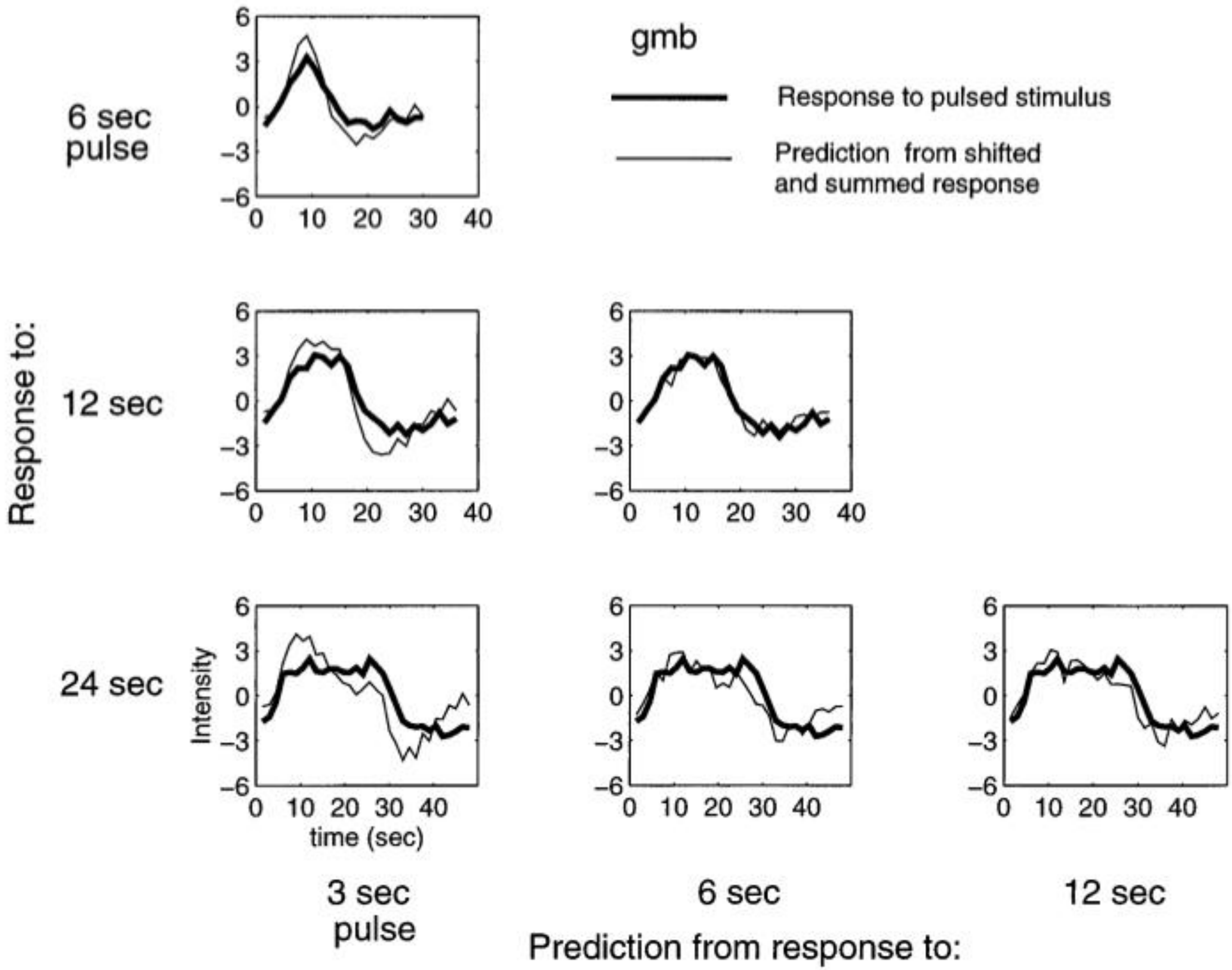


Thank you!

Supplementary slides

Brain response is (approximately) a linear system


(A)
(B)
BOLD signal intensity
Time (s)
–5 0 5 10 15 20 25
–5 0 5 10 15 20 25 30 35



Bonyton GM, Engel SA, Glover GH & Heeger DJ (1996) Journal of Neuroscience 16:4207-21