Linear model 3

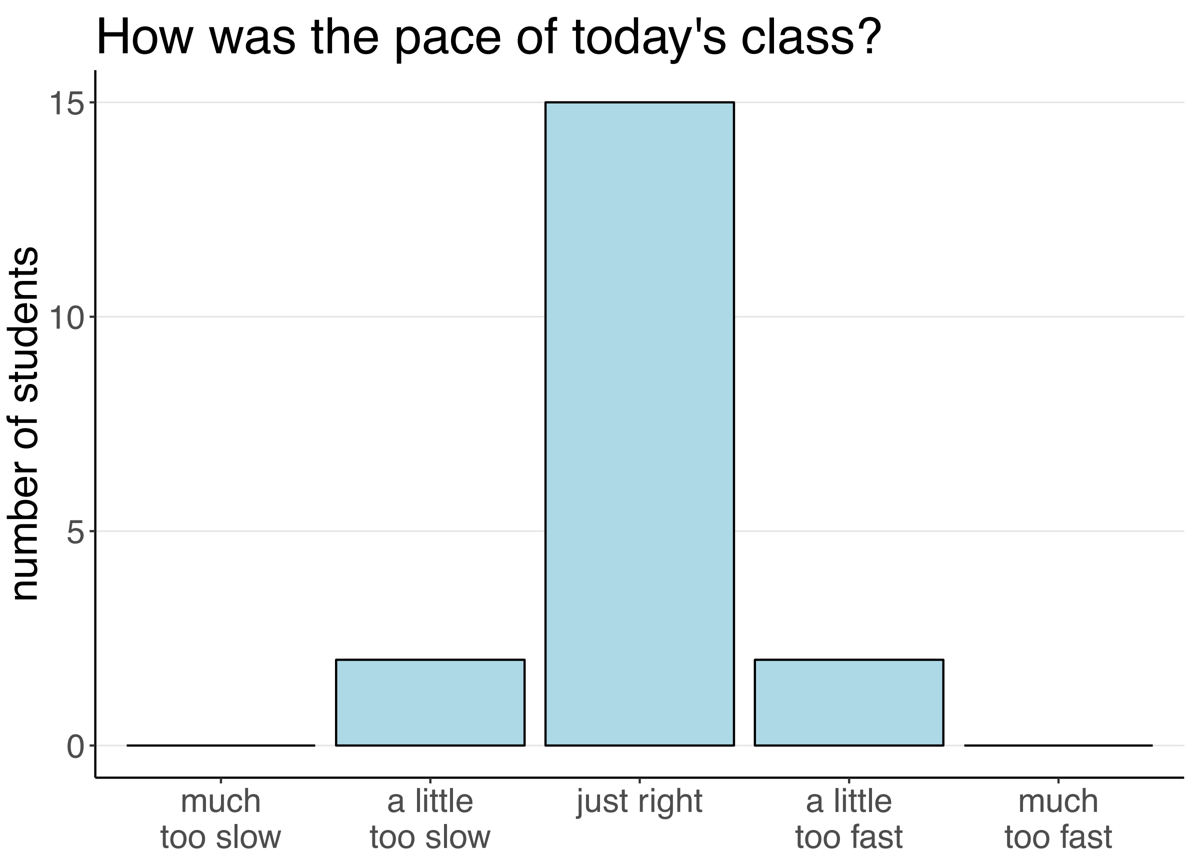
If she loves you more each and every day,
by linear regression she hated you before you met.

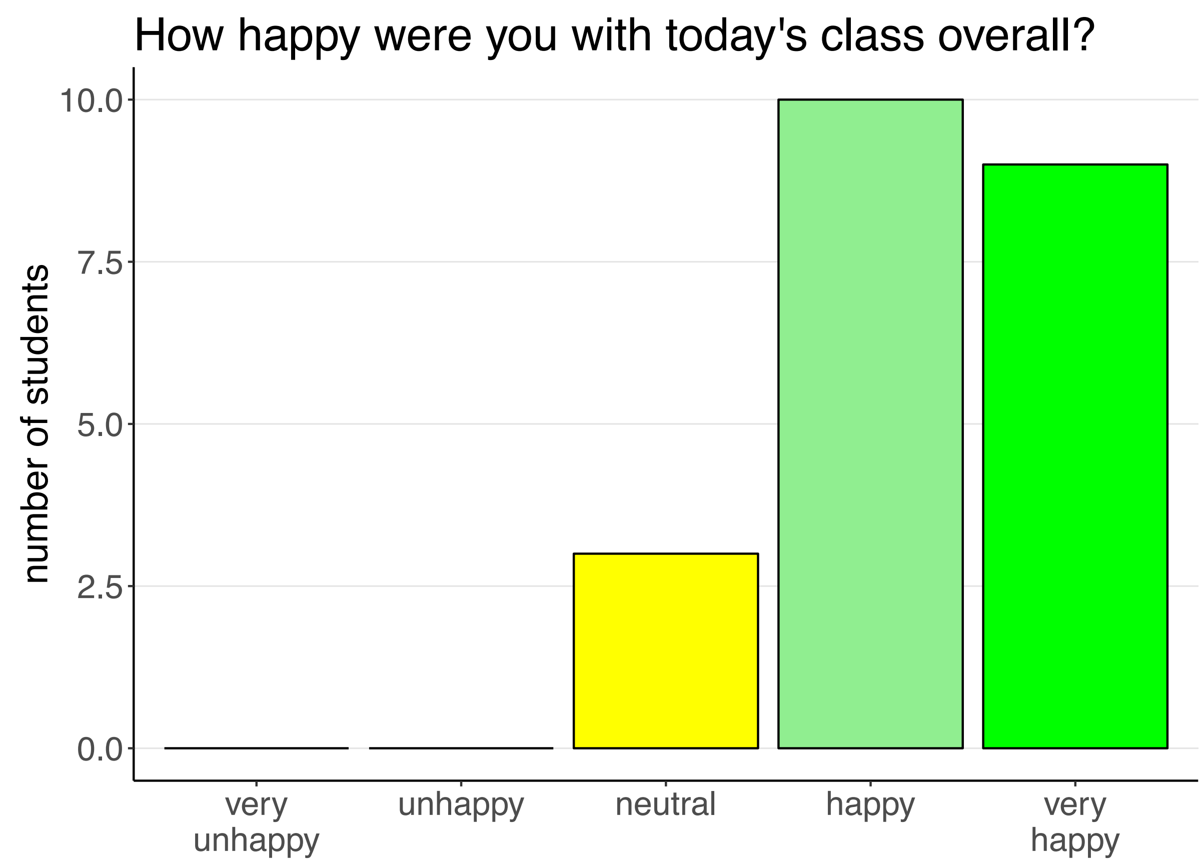
02/04/2019

Logistics

Your feedback

Your feedback





Your feedback

Class was good. Would appreciate a list of ways in which we might misinterpret importance of regression, as well as why we should care.

we'll talk about problem cases on Wednesday

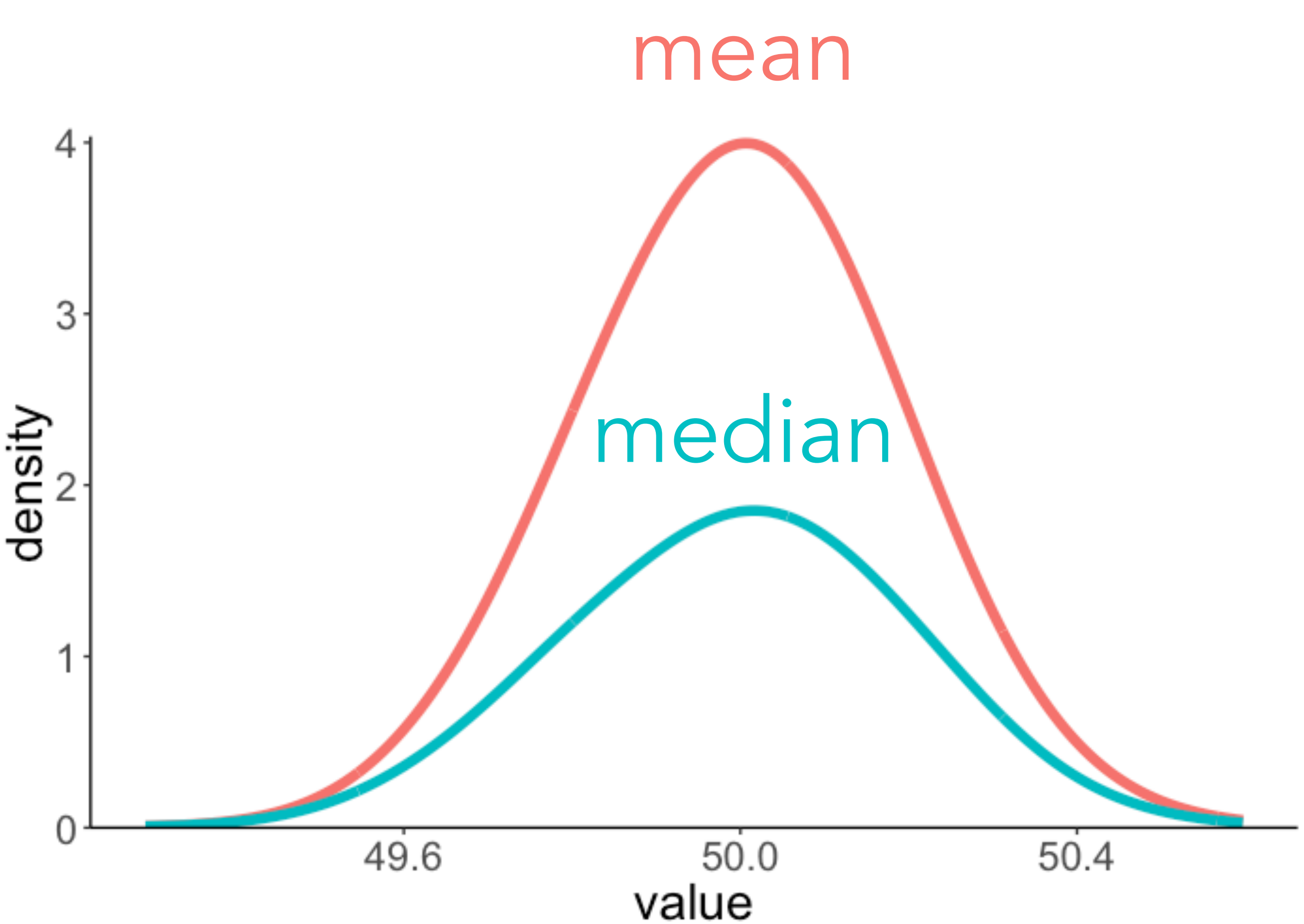
Homework 4

~~**Efficient estimator of the population variance**~~

~~Show that the sample variance is an efficient estimator of the population variance. Compare the sample variance $s^2 = \frac{\sum_{i=1}^{n}(Y_i - \overline{Y})^2}{n-1}$ with the *median absolute deviation* as an estimator of the population variance. Use the same procedure as above when you tried to figure out whether s^2 was an unbiased estimator.~~

population distribution

$$Y_i = 50 + \epsilon$$



But ...

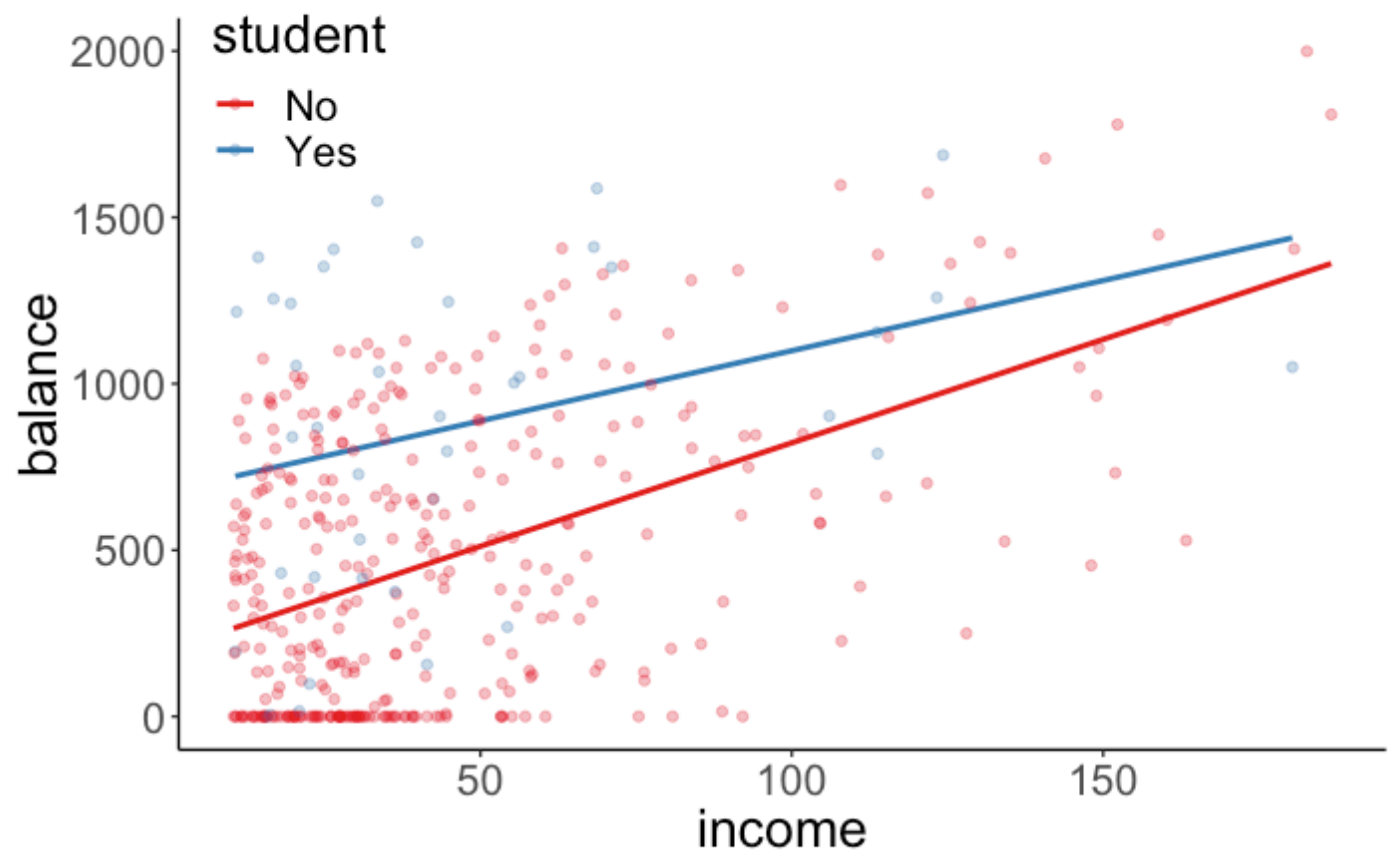
The *median absolute deviation* (MAD) is a biased estimator of the population standard deviation. It's even a biased estimator of the population MAD. So comparing efficiency doesn't make that much sense ...

Thanks to Mark Roman Miller!!

Things that came up in class

How to report results of interaction

There is no significant difference in the relationship between income and balance for students versus non-students, $F(1, 396) = 1.33$, $p = 0.25$.



For *students*, an increase in $1000 income is associated with an increase in $4.21 of average credit card balance.

For *non-students*, an increase in $1000 income is associated with an increase in $6.22 of average credit card balance.

lm() output

```r
lm(formula = balance ~ income + student + income:student, data = df.credit) %>%
  summary()
```

```
Call:
lm(formula = balance ~ income + student + income:student,
data = df.credit)

Residuals:
    Min      1Q  Median      3Q     Max 
-773.39 -325.70  -41.13  321.65  814.04 

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)    
(Intercept)       200.6232    33.6984   5.953 5.79e-09 ***
income              6.2182     0.5921  10.502  < 2e-16 ***
studentYes        476.6758   104.3512   4.568 6.59e-06 ***
income:studentYes  -1.9992     1.7313  -1.155    0.249    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 
1

Residual standard error: 391.6 on 396 degrees of freedom
Multiple R-squared:  0.2799, Adjusted R-squared:  0.2744 
F-statistic:  51.3 on 3 and 396 DF,  p-value: < 2.2e-16
```

```r
fit_c = lm(formula = balance ~ student + income:student, data = df.credit)
fit_a = lm(formula = balance ~ income + student + income:student, data = df.credit)

anova(fit_c, fit_a)
```

```r
fit_c = lm(formula = balance ~ income + student, data = df.credit)
fit_a = lm(formula = balance ~ income + student + income:student, data = df.credit)

anova(fit_c, fit_a)
```

lm() output

```
lm(formula = balance ~ income + student + income:student, data = df.credit) %>%
  summary()
```

```
Call:
lm(formula = balance ~ income + student + income:student,
data = df.credit)

Residuals:
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-773.39 -325.70  -41.13  321.65  814.04

Coefficients:
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(Intercept)       200.6232    33.6984   5.953 5.79e-09 ***
income              6.2182     0.5921  10.502  < 2e-16 ***
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income:studentYes  -1.9992     1.7313  -1.155    0.249
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
1

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F-statistic:  51.3 on 3 and 396 DF,  p-value: < 2.2e-16
```

```
fit_c = lm(formula = balance ~ 1, data = df.credit)
fit_a = lm(formula = balance ~ income + student + income:student, data = df.credit)

anova(fit_c, fit_a)
```

`lm()` output

```
lm(formula = balance ~ income + student + income:student, data = df.credit) %>%
  summary()
```

```
Call:
lm(formula = balance ~ income + student + income:student, data =
df.credit)

Residuals:
    Min      1Q  Median      3Q     Max
-773.39 -325.70  -41.13  321.65  814.04

Coefficients:
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(Intercept)       200.6232    33.6984   5.953 5.79e-09 ***
income              6.2182     0.5921  10.502  < 2e-16 ***
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income:studentYes  -1.9992     1.7313  -1.155    0.249
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Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 391.6 on 396 degrees of freedom
Multiple R-squared:  0.2799,	Adjusted R-squared:  0.2744
F-statistic:  51.3 on 3 and 396 DF,  p-value: < 2.2e-16
```

- runs many hypothesis tests at the same time
- increases the danger of making a type-I error (incorrectly rejecting the H_0)
- will not give us p-values for mixed effects models ...

The model comparison approach

- allows to formulate hypotheses as specific comparisons between candidate models
- is more flexible: we could test a model with 2 predictors vs. one with 4 predictors
- gives us insight into the underlying statistical procedure

Another clarification

```
fit1 = lm(formula = balance ~ income + student + income:student, data = df.credit)
```

Explicitly encode the interaction

```
df.credit %>%
  mutate(student_dummy = ifelse(student == "No", 0, 1)) %>%
  mutate(income_student = income * student_dummy) %>%
  select(balance, income, student, student_dummy, income_student)
```

balance	income	student	student_dummy	income_student
333	14.89	No	0	0.00
903	106.03	Yes	1	106.03
580	104.59	No	0	0.00
964	148.92	No	0	0.00
331	55.88	No	0	0.00
1151	80.18	No	0	0.00
203	21.00	No	0	0.00
872	71.41	No	0	0.00
279	15.12	No	0	0.00
1350	71.06	Yes	1	71.06

```
fit2 = lm(formula = balance ~ income + student + income_student, data = df.credit)
```

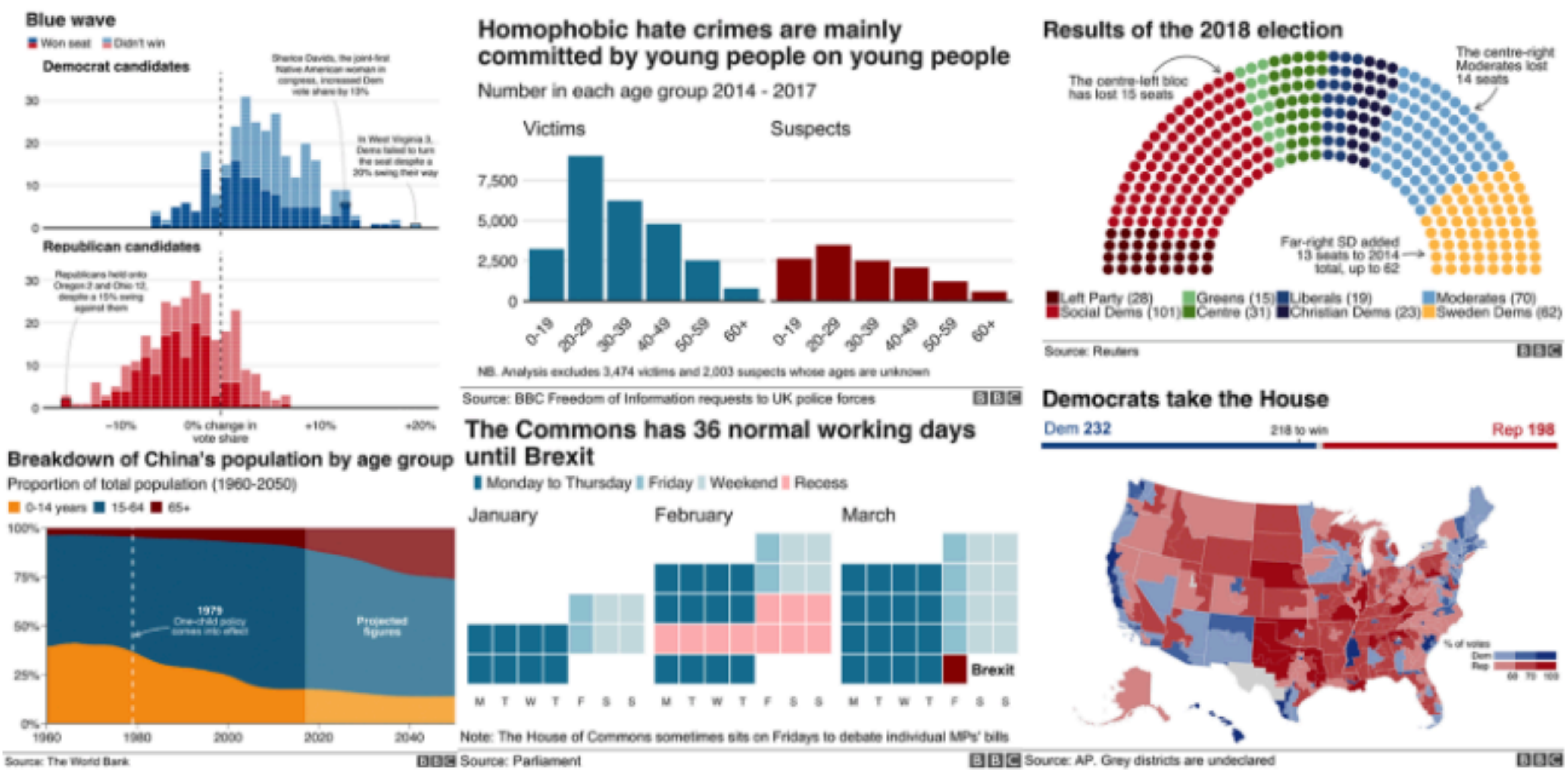
`fit1` and `fit2` are identical!

And now for something completely different …

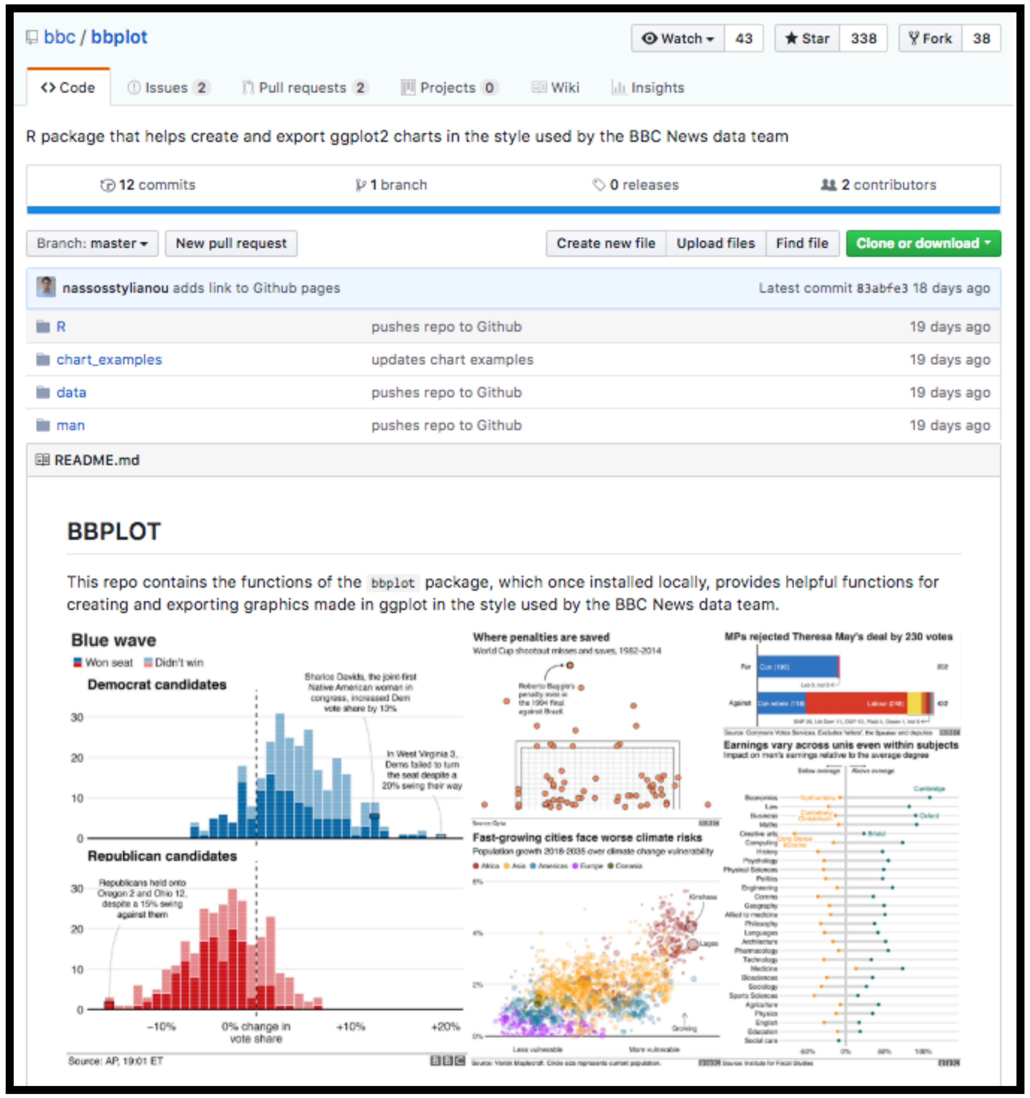
How the BBC Visual and Data Journalism team works with graphics in R

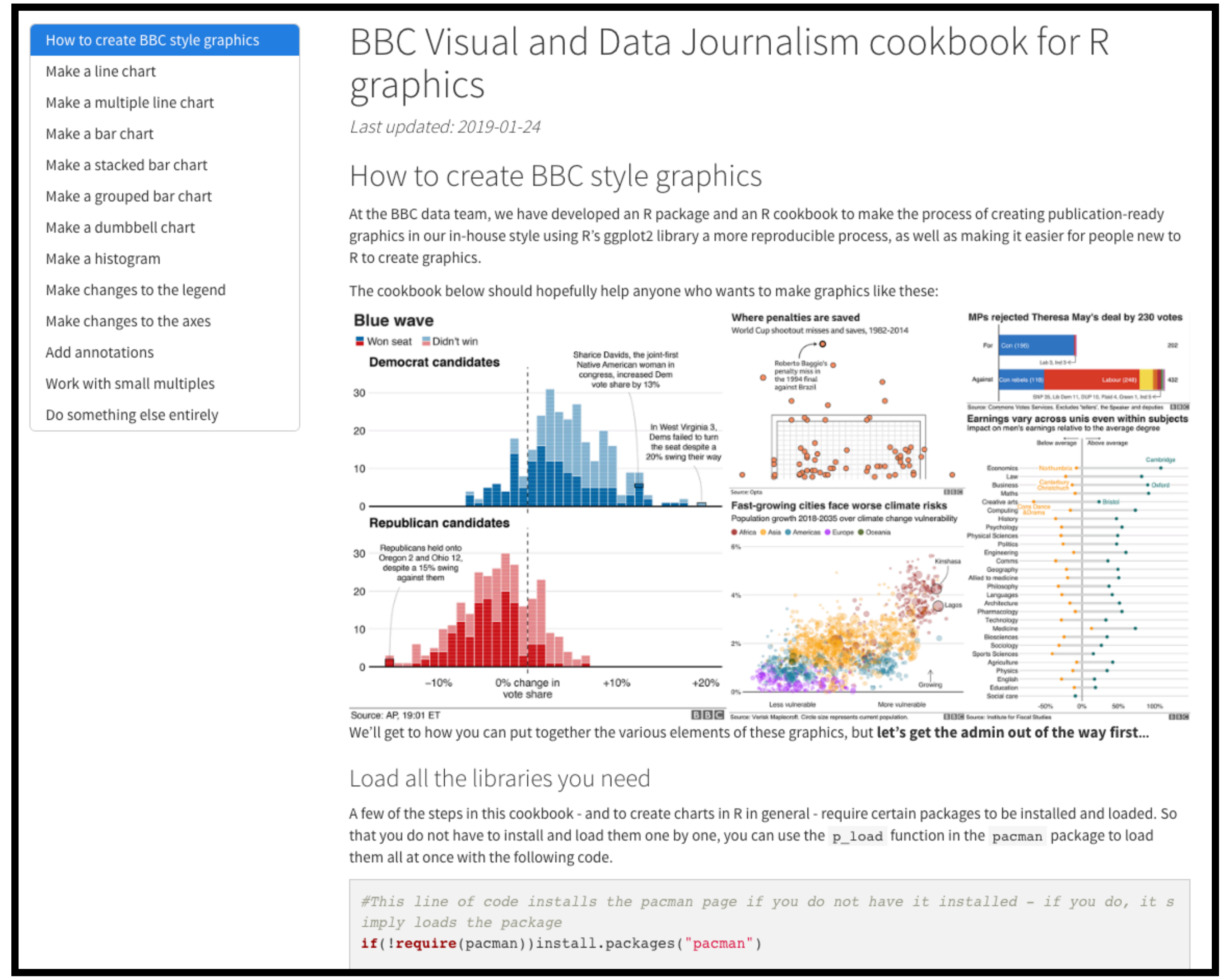
the BBC uses `ggplot2`

BBC Visual and Data Journalism Follow
Feb 1 · 8 min read



R Markdown doc!





https://github.com/bbc/bbplot

https://bbc.github.io/rcookbook/

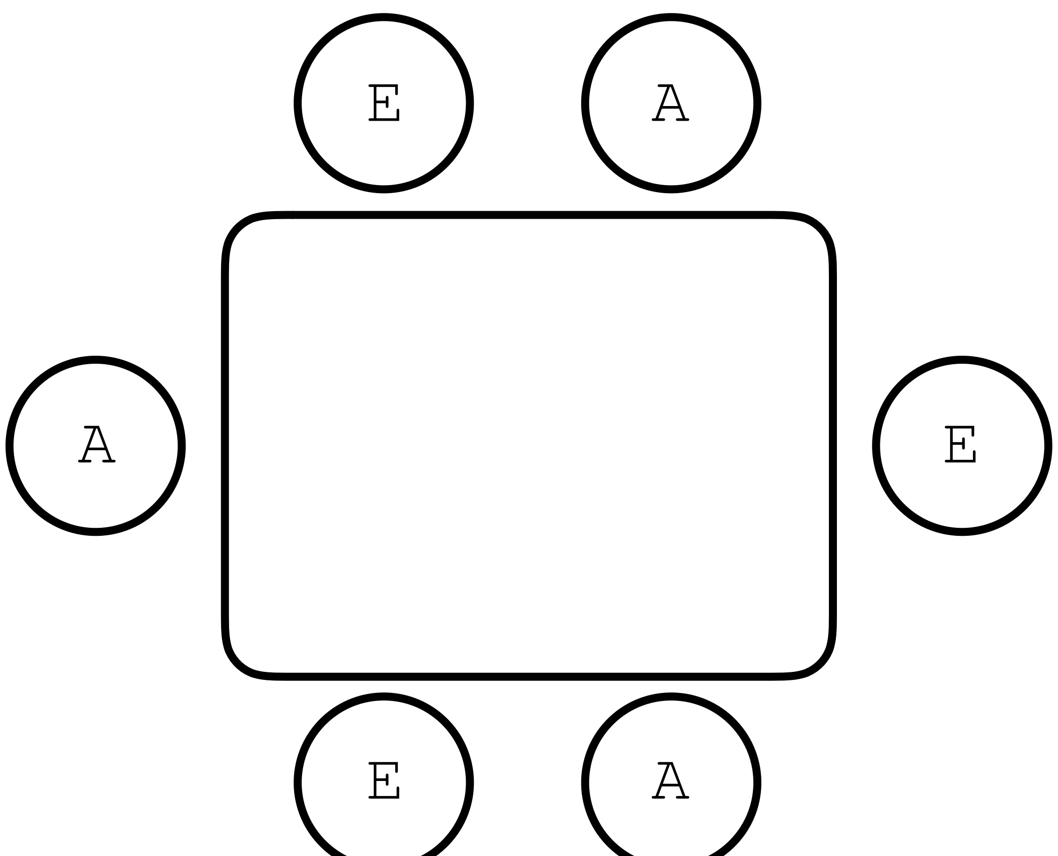
Plan for today

- Linear model with ...
  - categorical predictor that has more than two levels (One-way ANOVA)
  - multiple categorical predictors (N-way ANOVA)
    - dummy coding vs. effect coding
    - planned comparisons

What's the role of skill vs. chance in poker?

Abstract

Adopting a quasi-experimental approach, the present study examined the extent to which the influence of poker playing skill was more important than card distribution. Three average players and three experts sat down at a six-player table and played **60 computer-based** hands of the poker variant ''Texas Hold'em'' for money. In each hand, one of the average players and one expert received (a) better-than-average cards (winner's box), (b) average cards (neutral box) and (c) worse-than-average cards (loser's box). The standardized manipulation of the card distribution controlled the factor of chance to determine differences in performance between the average and expert groups. Overall, 150 individuals participated in a ''fixed- limit'' game variant, and 150 individuals participated in a ''no-limit'' game variant.



During the game, one expert player and one average player received

(a) the winning hand 15 times and the losing hand 5 times (winner's box condition)

(b) the winning hand 10 times and the losing hand 10 times (neutral box condition)

(c) the winning hand 5 times and the losing hand 15 times (loser's box condition)

Data set for today

participant	skill	hand	limit	balance
1	expert	bad	fixed	4.00
2	expert	bad	fixed	5.55
26	expert	bad	none	5.52
27	expert	bad	none	8.28
51	expert	neutral	fixed	11.74
52	expert	neutral	fixed	10.04
76	expert	neutral	none	21.55
77	expert	neutral	none	3.12
101	expert	good	fixed	10.86
102	expert	good	fixed	8.68

skill = expert/average

hand = bad/neutral/good

limit = fixed/none

balance = final balance in Euros

2 (skill) x 3 (hand) x 2 (limit) design

25 participants per condition

n = 300

Meyer, G., von Meduna, M., Brosowski, T., & Hayer, T. (2012). Is poker a game of skill or chance? A quasi-experimental study. *Journal of Gambling Studies*

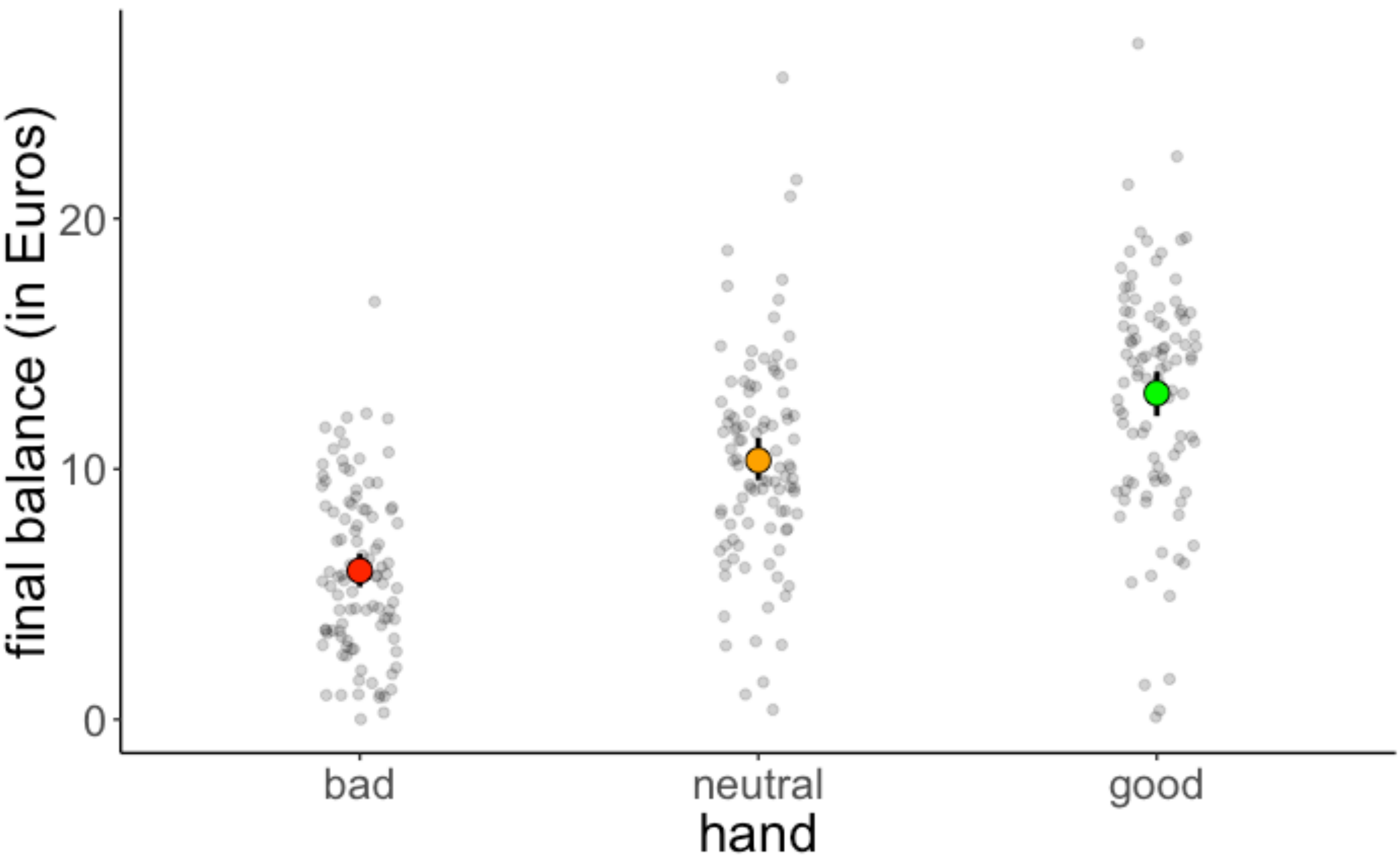
Categorical predictor with more than two levels

Do better hands win more money?

participant	skill	hand	limit	balance
1	expert	bad	fixed	4.00
2	expert	bad	fixed	5.55
26	expert	bad	none	5.52
27	expert	bad	none	8.28
51	expert	neutral	fixed	11.74
52	expert	neutral	fixed	10.04
76	expert	neutral	none	21.55
77	expert	neutral	none	3.12
101	expert	good	fixed	10.86
102	expert	good	fixed	8.68

hand = {bad, neutral, good}

Visualize the data first



H$_0$: Card quality does not affect the final balance.

H$_1$: Card quality affects the final balance.

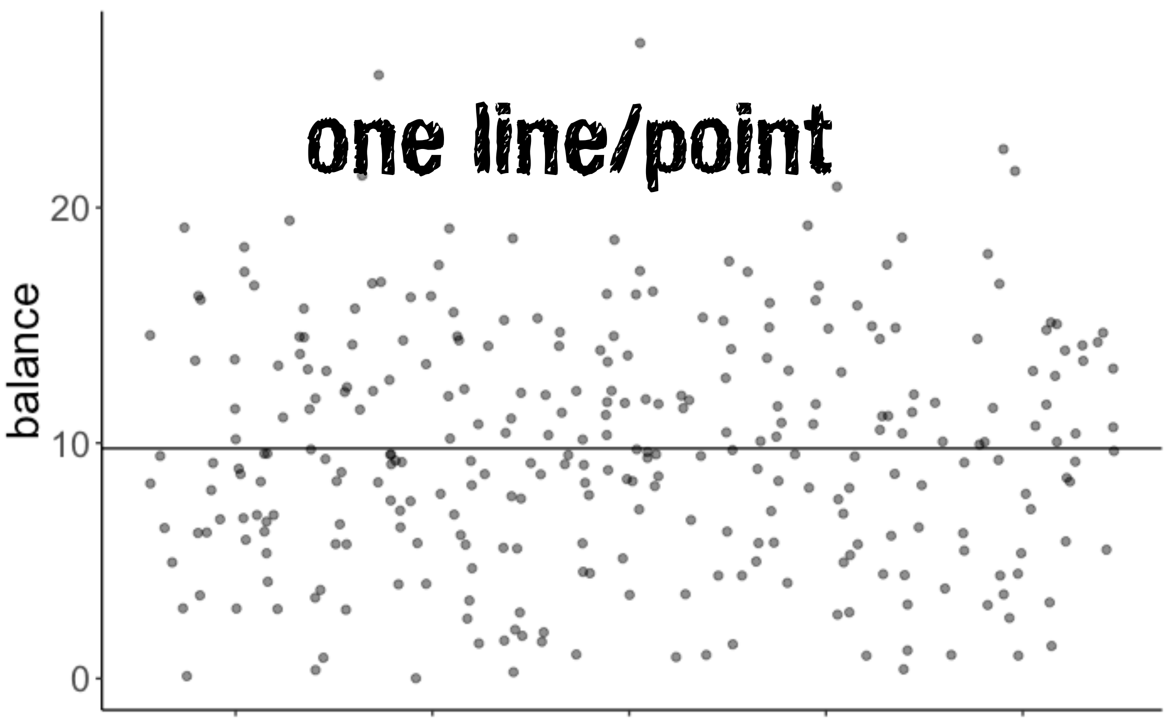
Model C

$$\text{balance}_i = \beta_0 + \epsilon_i$$

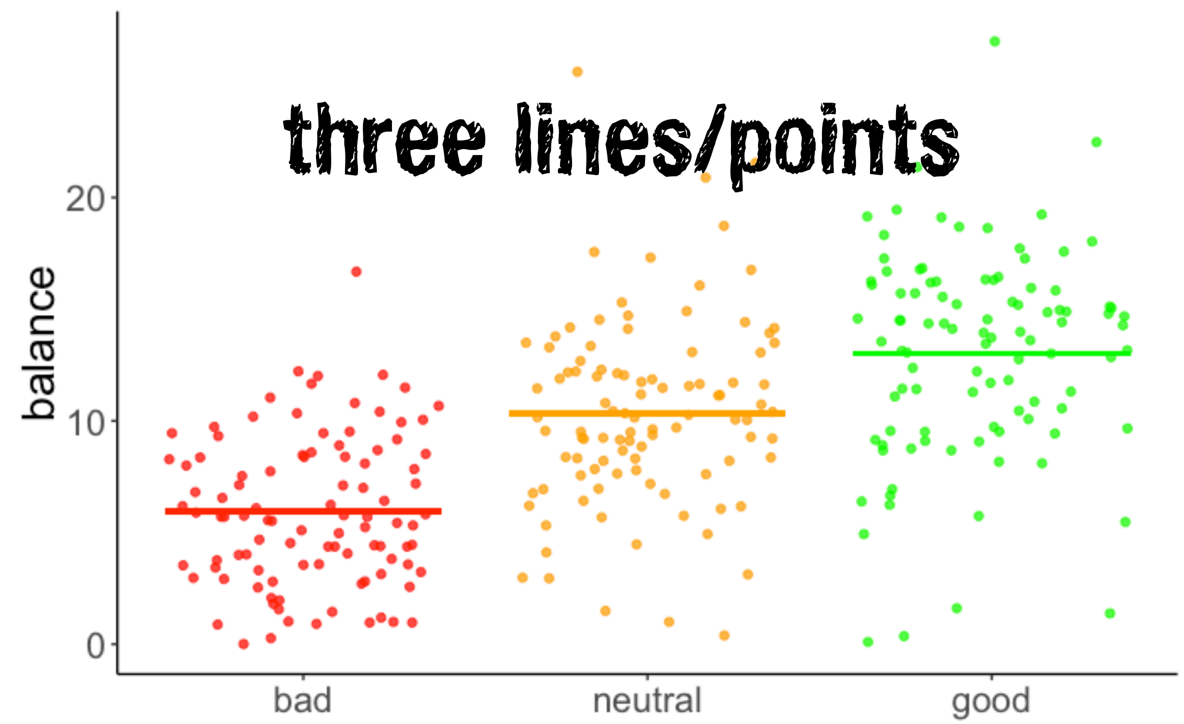
Model A

$$\text{balance}_i = \beta_0 + \beta_1 \text{hand\_neutral}_i + \beta_2 \text{hand\_good}_i + \epsilon$$

Model prediction



Model prediction



Fitted model

$$\widehat{\text{balance}}_i = 9.77$$

Fitted model

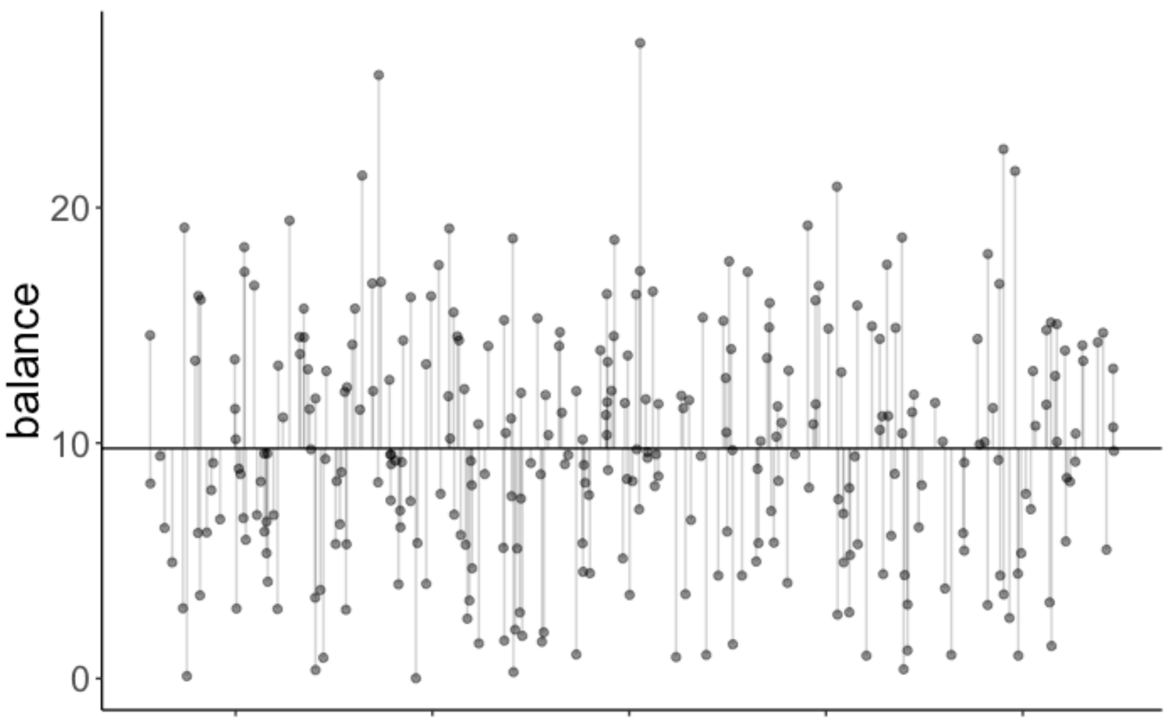
$$\widehat{\text{balance}}_i = 5.94 + 4.41 \cdot \text{hand\_neutral}_i + 7.08 \cdot \text{hand\_good}_i$$

H_0: Card quality does not affect the final balance.

Model C

$$\text{balance}_i = \beta_0 + \epsilon_i$$

Model prediction



SSE(C) = 7580

Fitted model

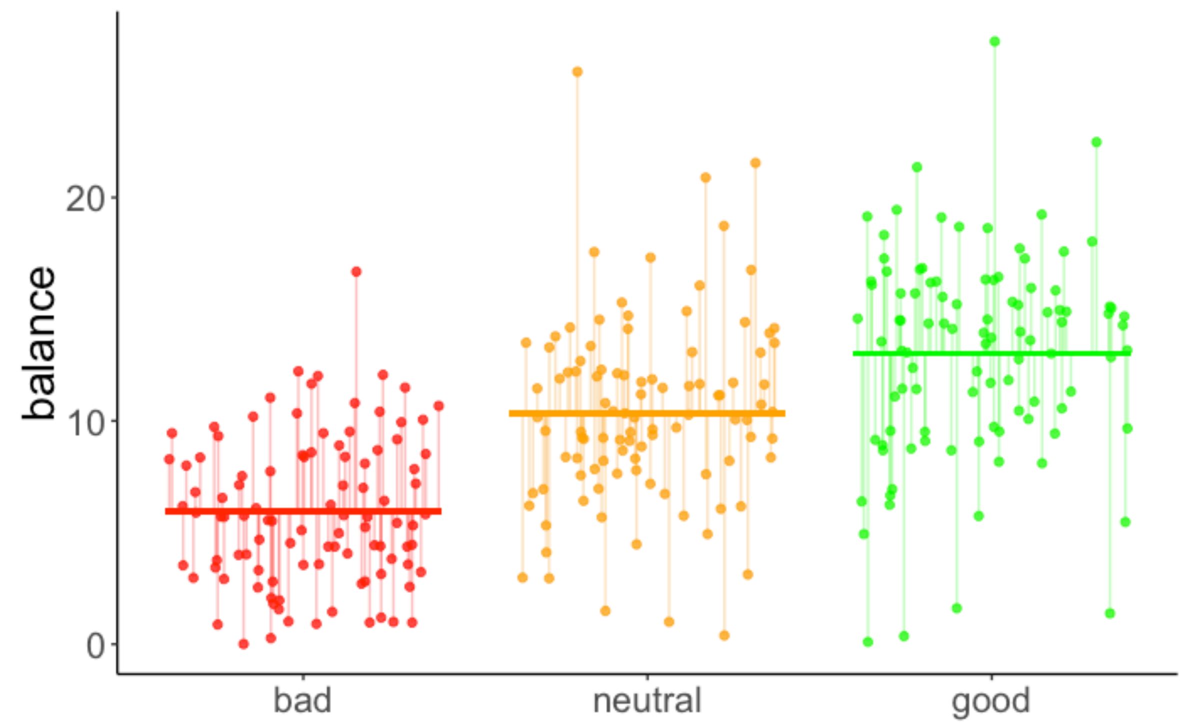
$$\widehat{\text{balance}}_i = 9.77$$

H_1: Card quality affects the final balance.

Model A

$$\text{balance}_i = \beta_0 + \beta_1 \text{hand\_neutral}_i + \beta_2 \text{hand\_good}_i + \epsilon$$

Model prediction



SSE(A) = 5021

Fitted model

$$\widehat{\text{balance}}_i = 5.94 + 4.41 \cdot \text{hand\_neutral}_i + 7.08 \cdot \text{hand\_good}_i$$

Does card quality affect the final balance?

SSE(C) = 7580

SSE(A) = 5021

$$\text{PRE} = 1 - \frac{\text{SSE}(A)}{\text{SSE}(C)}$$

worth it?

$$= 1 - \frac{5021}{7580} \approx 0.34$$

```r
# fit the models
fit_c = lm(formula = balance ~ 1, data = df.poker)
fit_a = lm(formula = balance ~ hand, data = df.poker)

# compare via F-test
anova(fit_c, fit_a)
```

```
Analysis of Variance Table

Model 1: balance ~ 1
Model 2: balance ~ hand
  Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1    299 7580.0
2    297 5020.6  2    2559.4 75.703 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Interpreting the results

```
lm(formula = balance ~ 1 + hand, data = df.poker)
```

```
Call:
lm(formula = balance ~ hand, data = df.poker)

Residuals:
     Min       1Q   Median       3Q      Max 
-12.9264  -2.5902  -0.0115   2.6573  15.2834 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   5.9415     0.4111  14.451  < 2e-16 ***
handneutral   4.4051     0.5815   7.576 4.55e-13 ***
handgood      7.0849     0.5815  12.185  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 4.111 on 297 degrees of freedom
Multiple R-squared:  0.3377, Adjusted R-squared:  0.3332 
F-statistic:  75.7 on 2 and 297 DF,  p-value: < 2.2e-16
```

Dummy coding

```
df.poker %>%
  mutate(hand_neutral = ifelse(hand == "neutral", 1, 0),
         hand_good = ifelse(hand == "good", 1, 0))
```

participant	hand	hand_neutral	hand_good	balance
31	bad	0	0	12.22
46	bad	0	0	12.06
50	bad	0	0	16.68
76	neutral	1	0	21.55
87	neutral	1	0	20.89
89	neutral	1	0	25.63
127	good	0	1	26.99
129	good	0	1	21.36
283	good	0	1	22.48

for a variable with k levels, we need k-1 dummy variables for encoding

same same, but different

```
lm(formula = balance ~ 1 + hand_neutral + hand_good + data = df.poker)

lm(formula = balance ~ 1 + hand, data = df.poker)
```

Interpreting the results

regression coefficients encode differences between group means

term	estimate	std.error	statistic	p.value
(Intercept)	5.941	0.411	14.451	0
handneutral	4.405	0.581	7.576	0
handgood	7.085	0.581	12.185	0

$$\widehat{\text{balance}}_i = 5.94 + 4.41 \cdot \text{hand\_neutral}_i + 7.08 \cdot \text{hand\_good}_i$$

participant	hand	hand_neutral	hand_good	balance
31	bad	0	0	12.22
46	bad	0	0	12.06
50	bad	0	0	16.68
76	neutral	1	0	21.55
87	neutral	1	0	20.89
89	neutral	1	0	25.63
127	good	0	1	26.99
129	good	0	1	21.36
283	good	0	1	22.48

if hand == "bad":

$$\widehat{\text{balance}}_i = 5.94$$

if hand == "neutral":

$$\widehat{\text{balance}}_i = 5.94 + 4.41 = 10.35$$

if hand == "good":

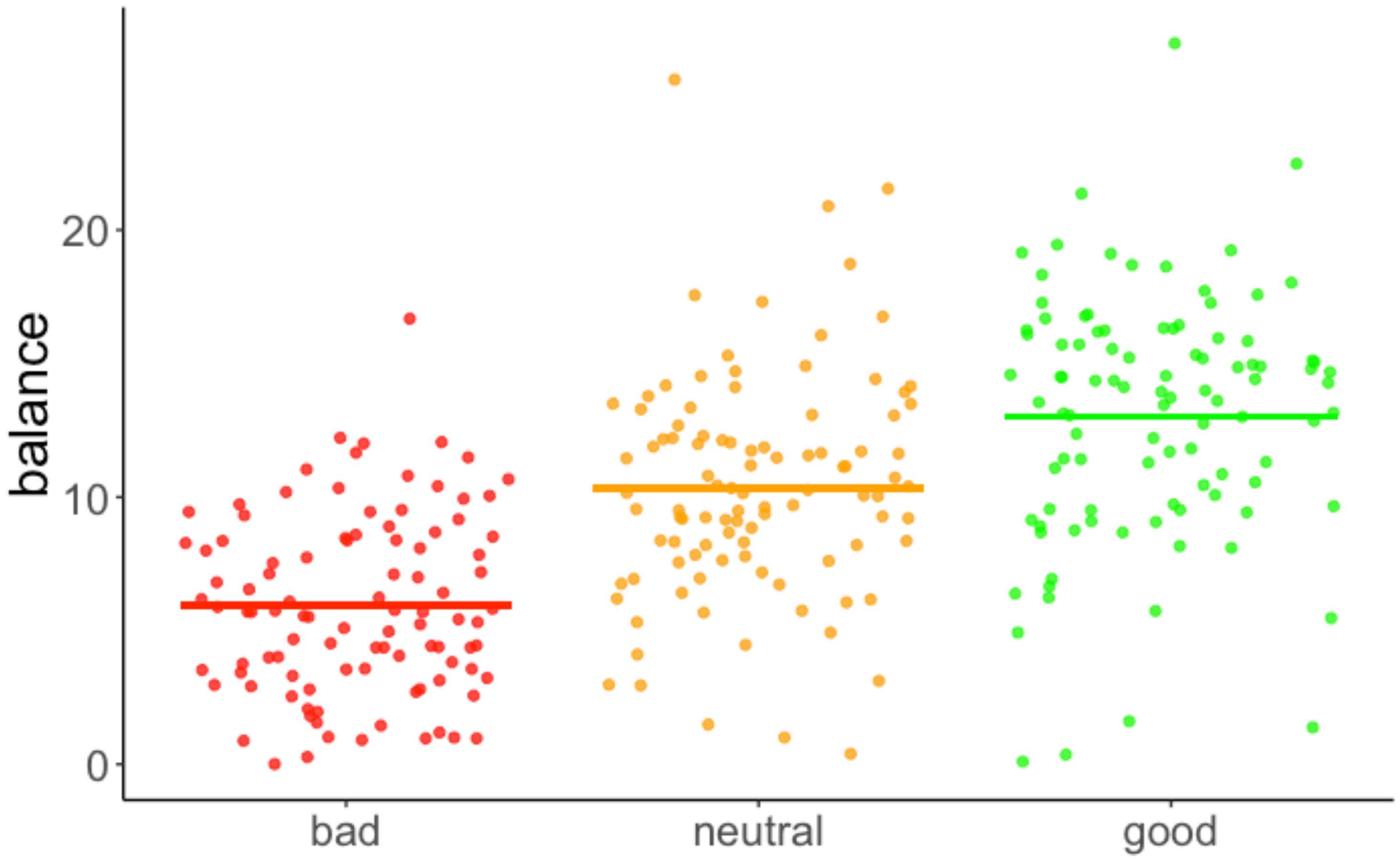
$$\widehat{\text{balance}}_i = 5.94 + 7.08 = 13.02$$

Interpreting the results

regression coefficients encode differences between group means

term	estimate	std.error	statistic	p.value
(Intercept)	5.941	0.411	14.451	0
handneutral	4.405	0.581	7.576	0
handgood	7.085	0.581	12.185	0

$$\widehat{\text{balance}}_i = 5.94 + 4.41 \cdot \text{hand\_neutral}_i + 7.08 \cdot \text{hand\_good}_i$$



if hand == "bad":

$$\widehat{\text{balance}}_i = 5.94$$

if hand == "neutral":

$$\widehat{\text{balance}}_i = 5.94 + 4.41 = 10.35$$

if hand == "good":

$$\widehat{\text{balance}}_i = 5.94 + 7.08 = 13.02$$

Follow-up tests

Asking more specific questions

Is there a difference in the final balance between bad hands and neutral hands?

```r
df.poker %>%
  filter(hand %in% c("bad", "neutral")) %>%
  lm(formula = balance ~ hand,
     data = .) %>%
  summary()
```

```
Call:
lm(formula = balance ~ hand, data = .)

Residuals:
    Min      1Q  Median      3Q     Max 
-9.9566 -2.5078 -0.2365  2.4410 15.2834 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   5.9415     0.3816  15.570  < 2e-16 ***
handneutral   4.4051     0.5397   8.163 3.76e-14 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.816 on 198 degrees of freedom
Multiple R-squared:  0.2518,	Adjusted R-squared:  0.248 
F-statistic: 66.63 on 1 and 198 DF,  p-value: 3.758e-14
```

Interpreting the results

```r
lm(formula = balance ~ hand, data = df.poker)
```

```
Call:
lm(formula = balance ~ hand, data = df.poker)

Residuals:
     Min       1Q   Median       3Q      Max 
-12.9264  -2.5902  -0.0115   2.6573  15.2834 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   5.9415     0.4111  14.451  < 2e-16 ***
handneutral   4.4051     0.5815   7.576 4.55e-13 ***
handgood      7.0849     0.5815  12.185  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.111 on 297 degrees of freedom
Multiple R-squared:  0.3377, Adjusted R-squared:  0.3332 
F-statistic:  75.7 on 2 and 297 DF,  p-value: < 2.2e-16
```

What does this summary not tell us?

Model comparison

Is there a difference in the final balance between neutral hands and good hands?

Model C

$$\text{balance}_i = \beta_0 + \epsilon_i$$

Model A

$$\text{balance}_i = \beta_0 + \beta_1 \cdot \text{good\_dummy}_i + \epsilon_i$$

(after having removed bad hands from the data set)

Asking more specific questions

Is there a difference in the final balance between neutral hands and good hands?

```r
df.poker %>%
  filter(hand %in% c("neutral", "good")) %>%
  lm(formula = balance ~ hand,
     data = .) %>%
  summary()
```

```
Call:
lm(formula = balance ~ hand, data = .)

Residuals:
     Min       1Q   Median       3Q      Max 
-12.9264  -2.7141   0.2585   2.7184  15.2834 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  10.3466     0.4448   23.26  < 2e-16 ***
handgood      2.6798     0.6291    4.26 3.16e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.448 on 198 degrees of freedom
Multiple R-squared:  0.08396,	Adjusted R-squared:  0.07933 
F-statistic: 18.15 on 1 and 198 DF,  p-value: 3.158e-05
```

Asking more specific questions

Is there a difference in the final balance between neutral hands and good hands?

```
df.poker %>%
  mutate(hand = fct_relevel(hand, "neutral")) %>%
  lm(formula = balance ~ hand,
     data = .) %>%
  summary()
```

same same, but different

```
Call:
lm(formula = balance ~ hand, data = .)

Residuals:
     Min       1Q   Median       3Q      Max 
-12.9264  -2.5902  -0.0115   2.6573  15.2834 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  10.3466     0.4111  25.165  < 2e-16 ***
handbad      -4.4051     0.5815  -7.576 4.55e-13 ***
handgood      2.6798     0.5815   4.609 6.02e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.111 on 297 degrees of freedom
Multiple R-squared:  0.3377,    Adjusted R-squared:  0.3332 
F-statistic:  75.7 on 2 and 297 DF,  p-value: < 2.2e-16
```

Is there a difference between bad hands vs. other hands?

```
df.poker %>%
  mutate(hand_other = ifelse(hand %in% c("neutral", "good"), 1, 0)) %>%
  lm(balance ~ 1 + hand_other,
     data = .) %>%
  summary()
```

```
Call:
lm(formula = balance ~ 1 + hand_other, data = .)

Residuals:
     Min       1Q   Median       3Q      Max 
-11.5865  -2.6203  -0.1815   2.8285  15.3035 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   5.9415     0.4249   13.98   <2e-16 ***
hand_other    5.7450     0.5204   11.04   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.249 on 298 degrees of freedom
Multiple R-squared:  0.2903,	Adjusted R-squared:  0.2879 
F-statistic: 121.9 on 1 and 298 DF,  p-value: < 2.2e-16
```

df.poker

participant	hand	hand_other	balance
31	bad	0	12.22
46	bad	0	12.06
50	bad	0	16.68
76	neutral	1	21.55
87	neutral	1	20.89
89	neutral	1	25.63
127	good	1	26.99
129	good	1	21.36
283	good	1	22.48

$$\widehat{\text{balance}}_i = b_0 + b_1 \cdot \text{hand\_other}_i$$

if hand == bad: $\widehat{\text{balance}}_i = b_0 = 5.94$

if hand != bad: $\widehat{\text{balance}}_i = b_0 + b_1 = 5.94 + 5.75 = 11.69$

group means

bad	neutral	good
5.94	10.35	13.03

Multiple categorical predictors

Do skill level and quality of cards affect the final balance?

participant	skill	hand	limit	balance
1	expert	bad	fixed	4.00
2	expert	bad	fixed	5.55
26	expert	bad	none	5.52
27	expert	bad	none	8.28
51	expert	neutral	fixed	11.74
52	expert	neutral	fixed	10.04
76	expert	neutral	none	21.55
77	expert	neutral	none	3.12
101	expert	good	fixed	10.86
102	expert	good	fixed	8.68

Why not just fit separate models?

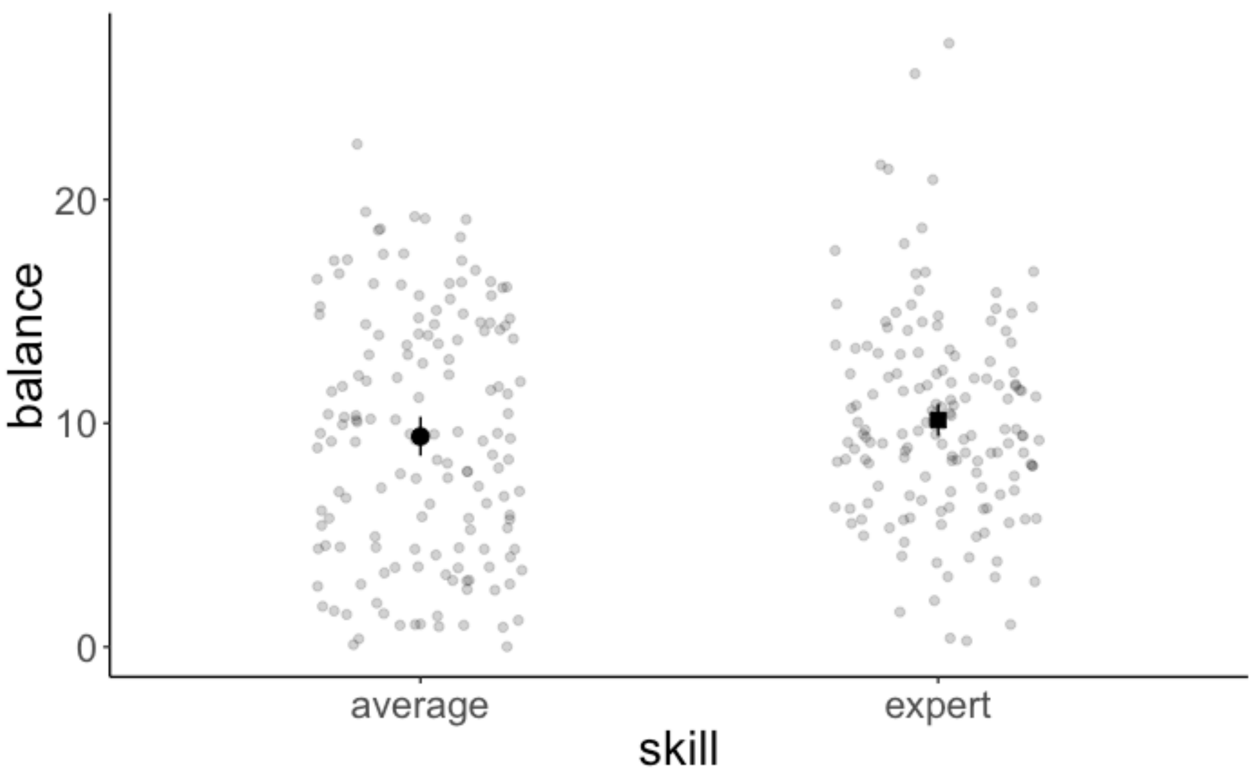
One testing whether skill level affects the final balance,
and one testing whether quality of cards affects the final balance?

Interested in interactions!

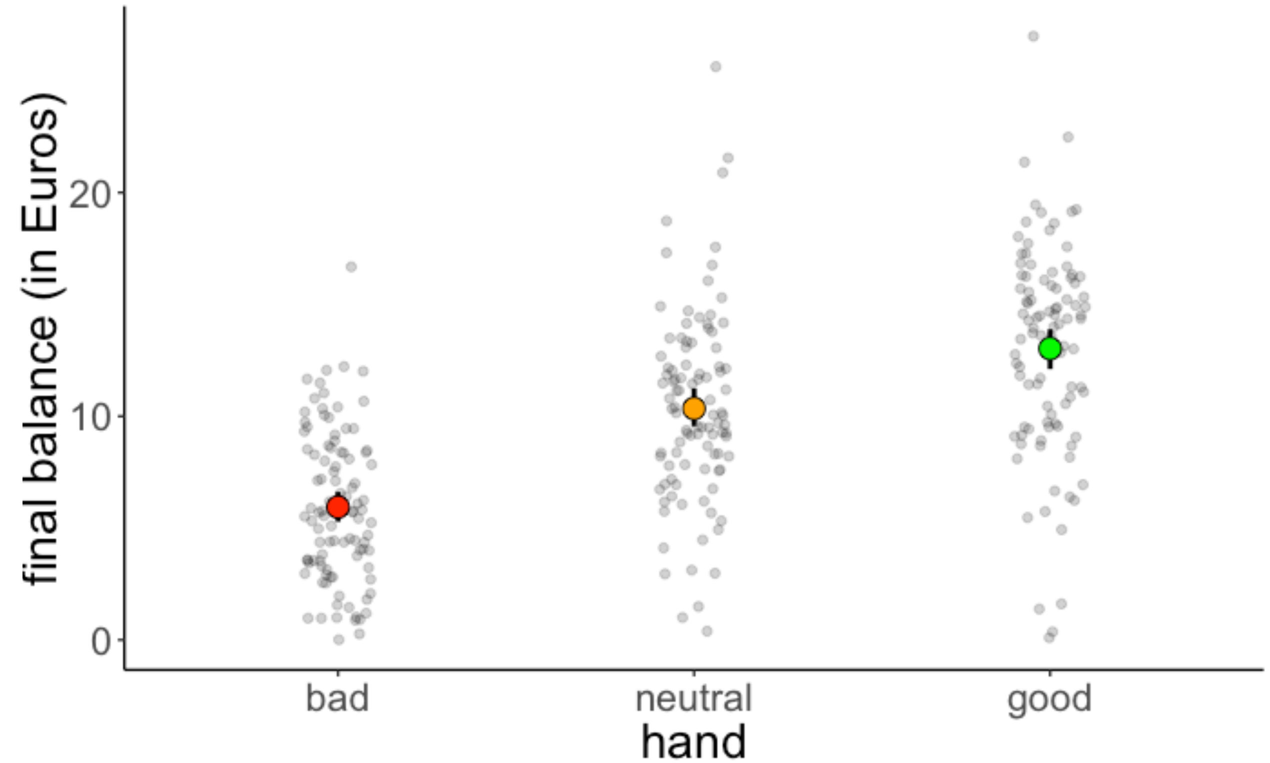
Does the effect of one variable depend on the other?

Visualize the data

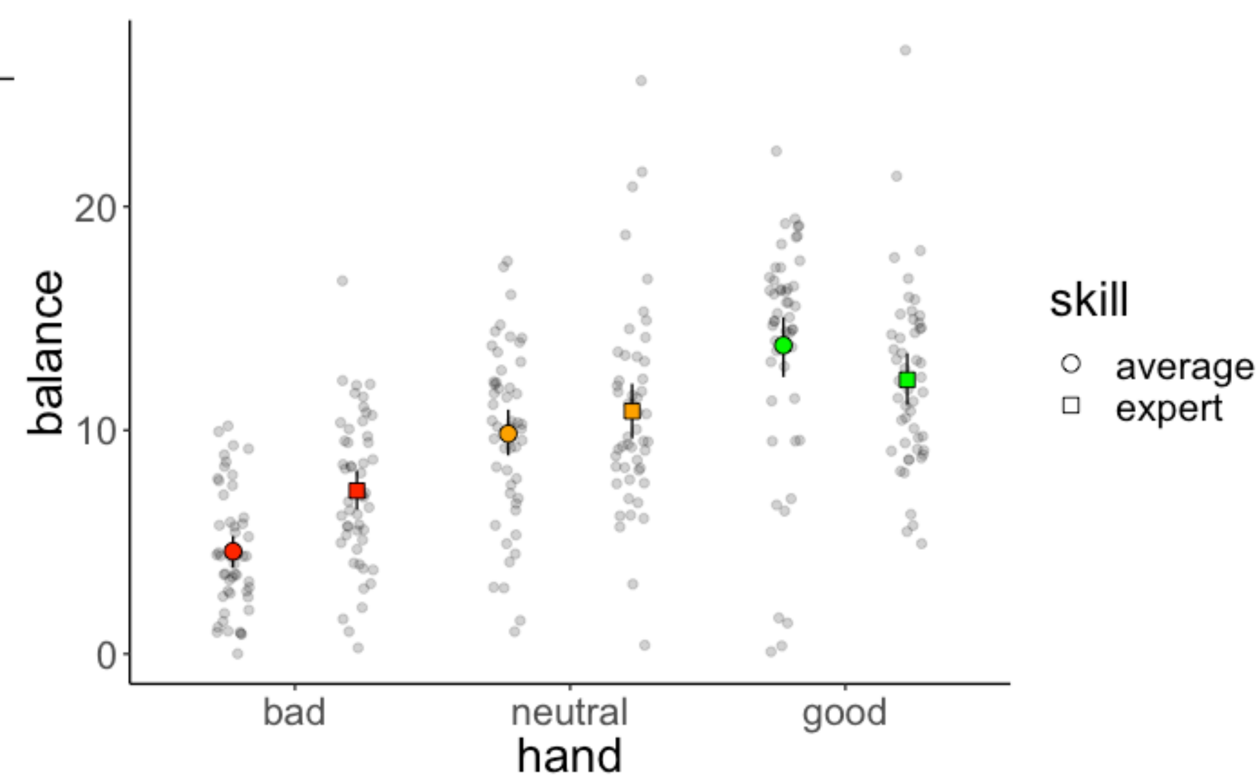
Skill



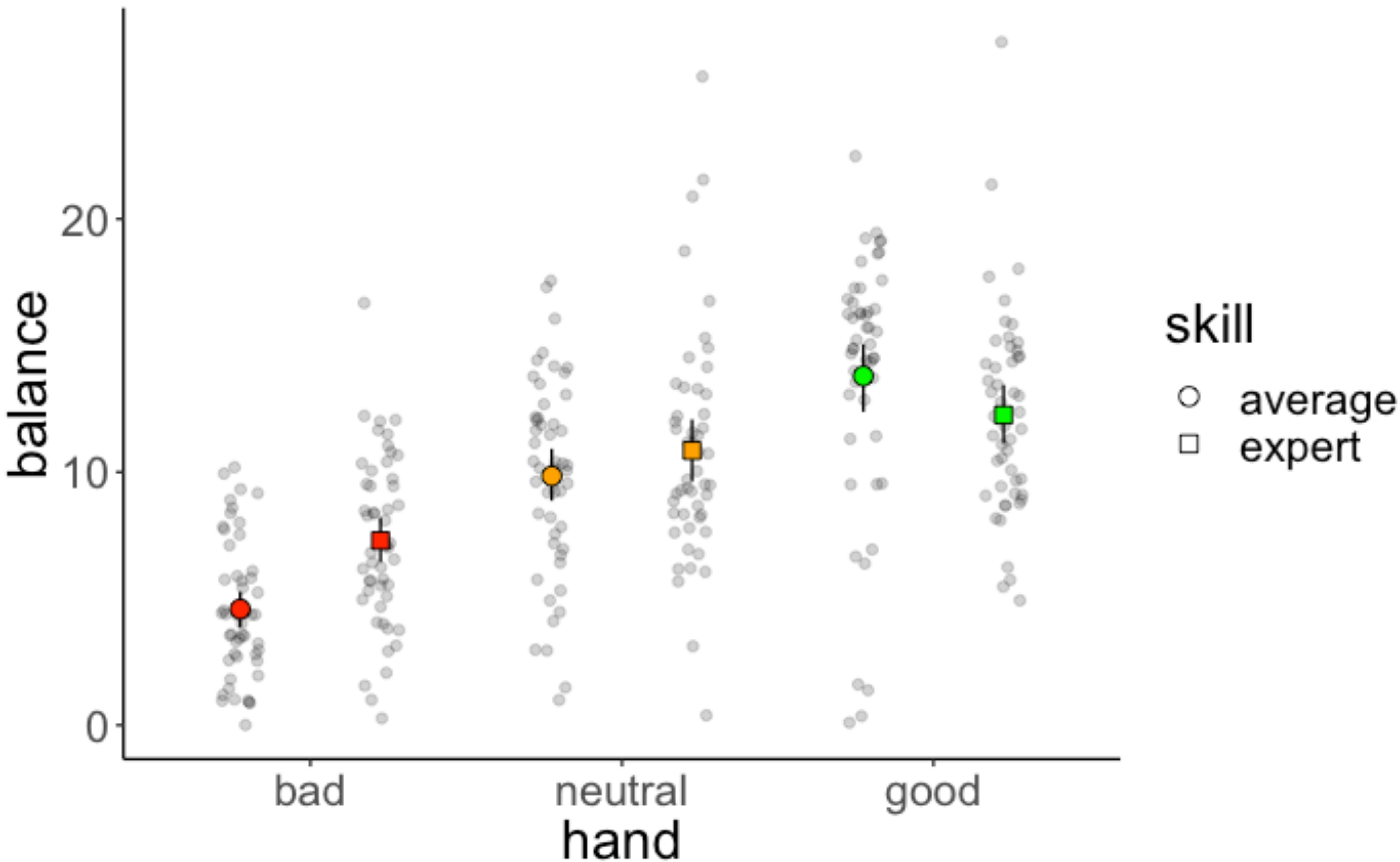
Quality of hand



Quality of hand & Skill



Visualize the data



Fit a model

```
lm(formula = balance ~ hand * skill, data = df.poker) %>%
  summary()
```

```
Call:
lm(formula = balance ~ hand * skill, data = df.poker)

Residuals:
     Min       1Q   Median       3Q      Max 
-13.6976  -2.4740   0.0348   2.4644  14.7806 

Coefficients:
                        Estimate Std. Error t value Pr(>|t|)    
(Intercept)               4.5866     0.5686   8.067 1.85e-14 ***
handneutral               5.2572     0.8041   6.538 2.75e-10 ***
handgood                  9.2110     0.8041  11.455  < 2e-16 ***
skillexpert               2.7098     0.8041   3.370 0.000852 ***
handneutral:skillexpert  -1.7042     1.1372  -1.499 0.135038    
handgood:skillexpert     -4.2522     1.1372  -3.739 0.000222 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 4.02 on 294 degrees of freedom
Multiple R-squared:  0.3731,Adjusted R-squared:  0.3624 
F-statistic: 34.99 on 5 and 294 DF,  p-value: < 2.2e-16
```

?

Interpretation

```
Coefficients:
                        Estimate Std. Error t value Pr(>|t|)
(Intercept)               4.5866     0.5686   8.067 1.85e-14 ***
handneutral               5.2572     0.8041   6.538 2.75e-10 ***
handgood                  9.2110     0.8041  11.455  < 2e-16 ***
skillexpert               2.7098     0.8041   3.370 0.000852 ***
handneutral:skillexpert  -1.7042     1.1372  -1.499 0.135038
handgood:skillexpert     -4.2522     1.1372  -3.739 0.000222 ***
```

group means

skill	bad	neutral	good
average	4.59	9.84	13.80
expert	7.30	10.85	12.26

$$\widehat{\text{balance}}_i = b_0 + b_1 \cdot \text{hand\_neutral}_i + b_2 \cdot \text{hand\_good}_i + b_3 \cdot \text{skill\_expert}_i + \\ b_4 \cdot \text{hand\_neutral:skill\_expert}_i + b_5 \cdot \text{hand\_good:skill\_expert}_i$$

hand = bad, skill = average

$$\widehat{\text{balance}}_i = b_0 = 4.59$$

hand = neutral, skill = average

$$\widehat{\text{balance}}_i = b_0 + b_1 \cdot \text{hand\_neutral}_i = 4.59 + 5.26 = 9.85$$

hand = good, skill = expert

$$\widehat{\text{balance}}_i = b_0 + b_2 \cdot \text{hand\_good}_i + b_3 \cdot \text{skill\_expert}_i + b_5 \cdot \text{hand\_good:skill\_expert}_i \\ = 12.26$$

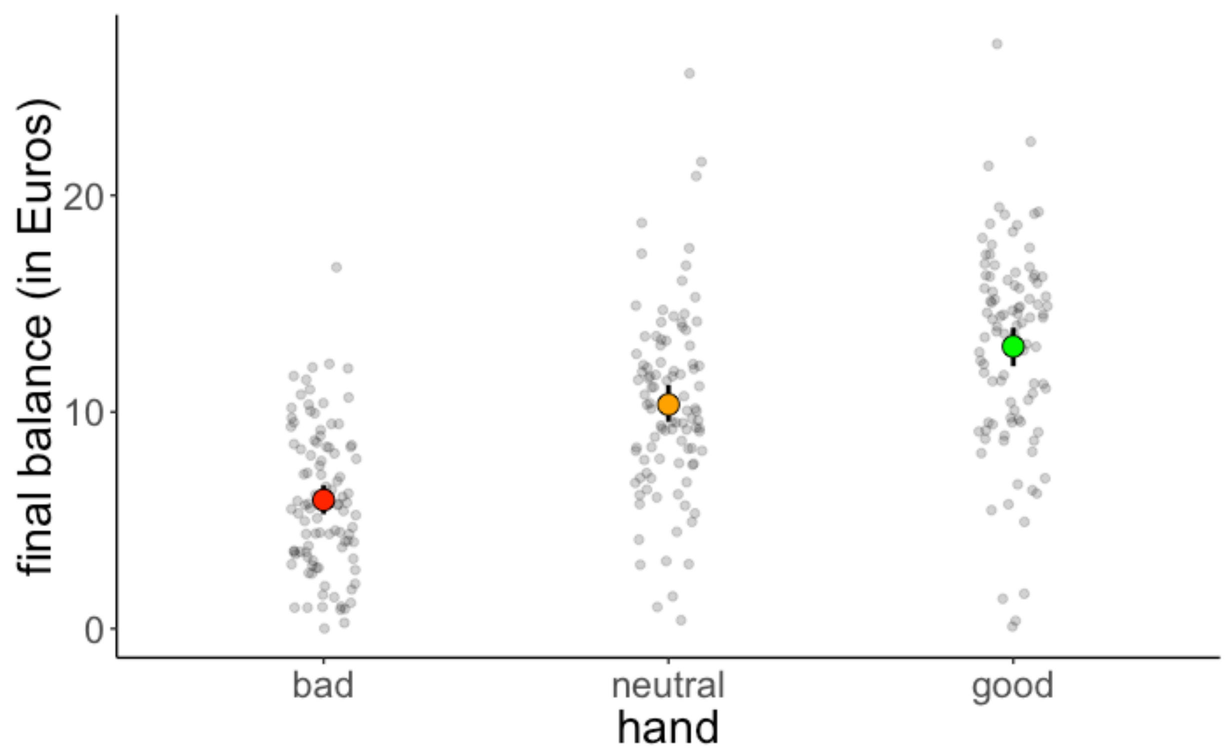
Analysis of variance

```
lm(formula = balance ~ hand * skill, data = df.poker) %>%
  anova()
```

```
Analysis of Variance Table

Response: balance
             Df Sum Sq Mean Sq F value    Pr(>F)    
hand          2 2559.4 1279.70 79.1692 < 2.2e-16 ***
skill         1   39.3   39.35  2.4344 0.1197776    
hand:skill    2  229.0  114.49  7.0830 0.0009901 ***
Residuals   294 4752.3   16.16                      
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

main effect of hand



Analysis of variance

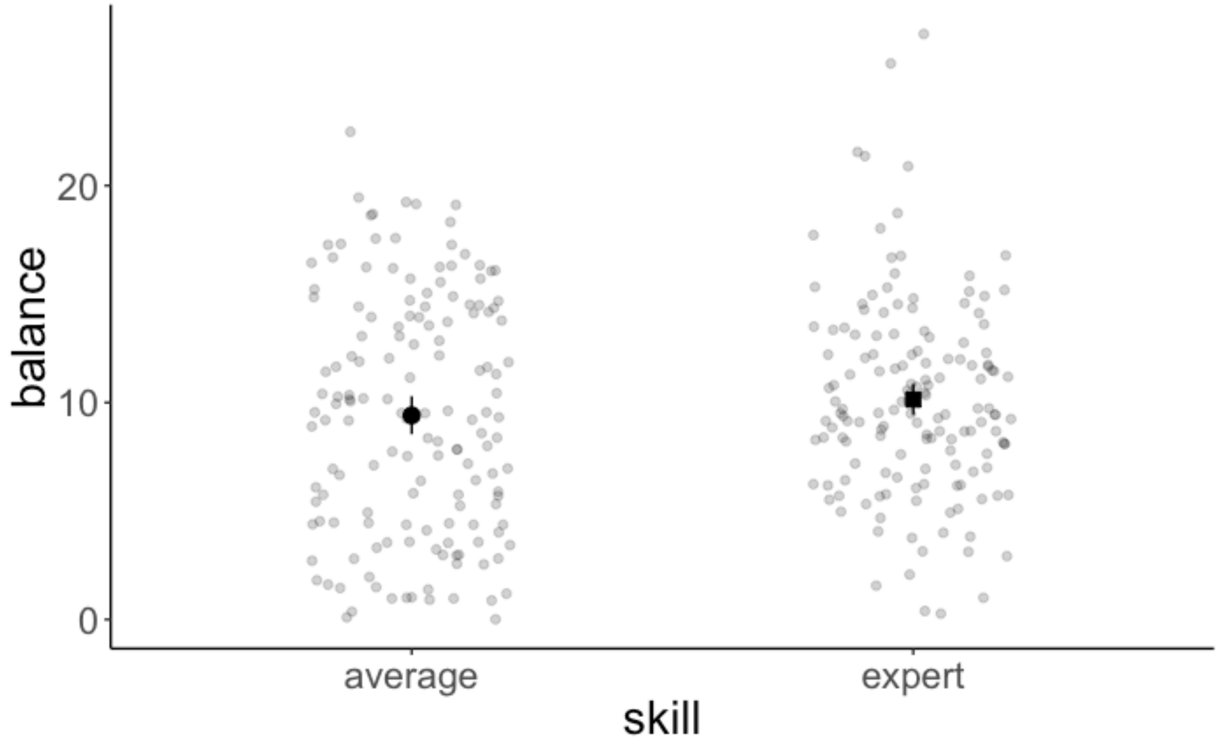
```
lm(formula = balance ~ hand * skill, data = df.poker) %>%
  anova()
```

```
Analysis of Variance Table

Response: balance
             Df Sum Sq Mean Sq F value    Pr(>F)    
hand          2 2559.4 1279.70 79.1692 < 2.2e-16 ***
skill         1   39.3   39.35  2.4344 0.1197776    
hand:skill    2  229.0  114.49  7.0830 0.0009901 ***
Residuals   294 4752.3   16.16                      
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

main effect of `hand`

no main effect of `skill`



Analysis of variance

```
lm(formula = balance ~ hand * skill, data = df.poker) %>%
  anova()
```

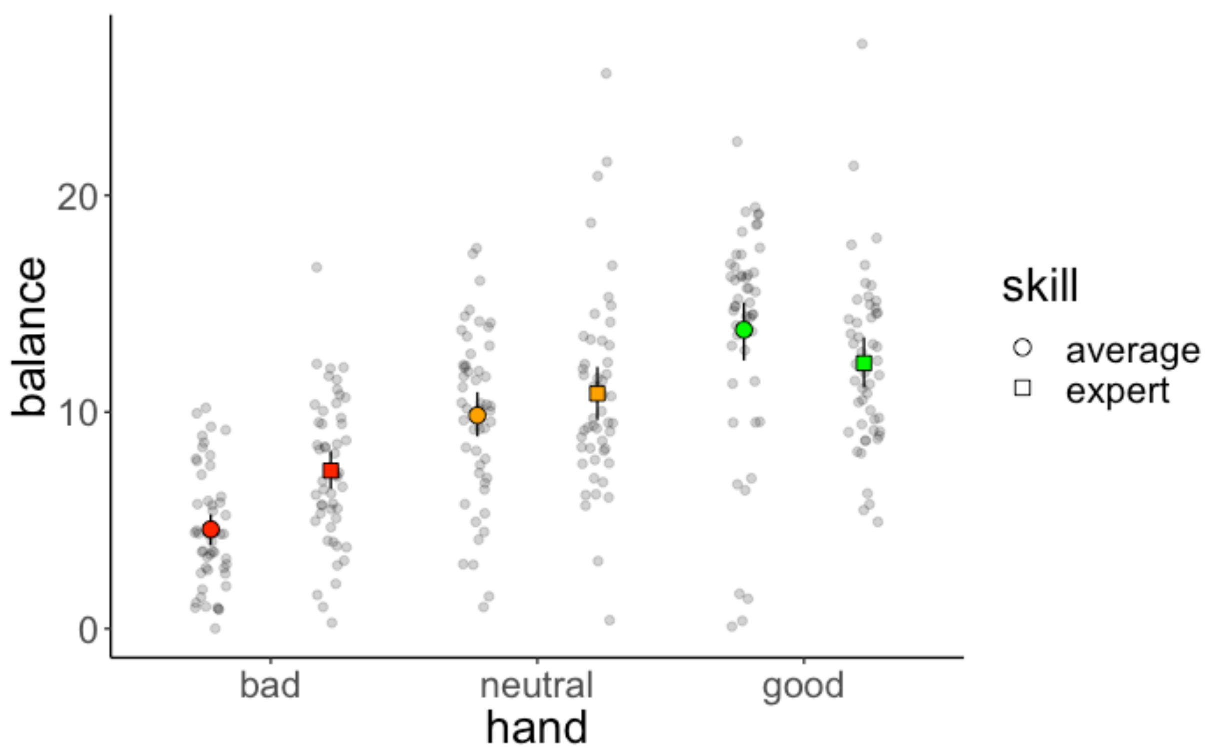
```
Analysis of Variance Table

Response: balance
            Df Sum Sq Mean Sq F value    Pr(>F)    
hand         2 2559.4 1279.70 79.1692 < 2.2e-16 ***
skill        1   39.3   39.35  2.4344 0.1197776    
hand:skill   2  229.0  114.49  7.0830 0.0009901 ***
Residuals  294 4752.3   16.16                      
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

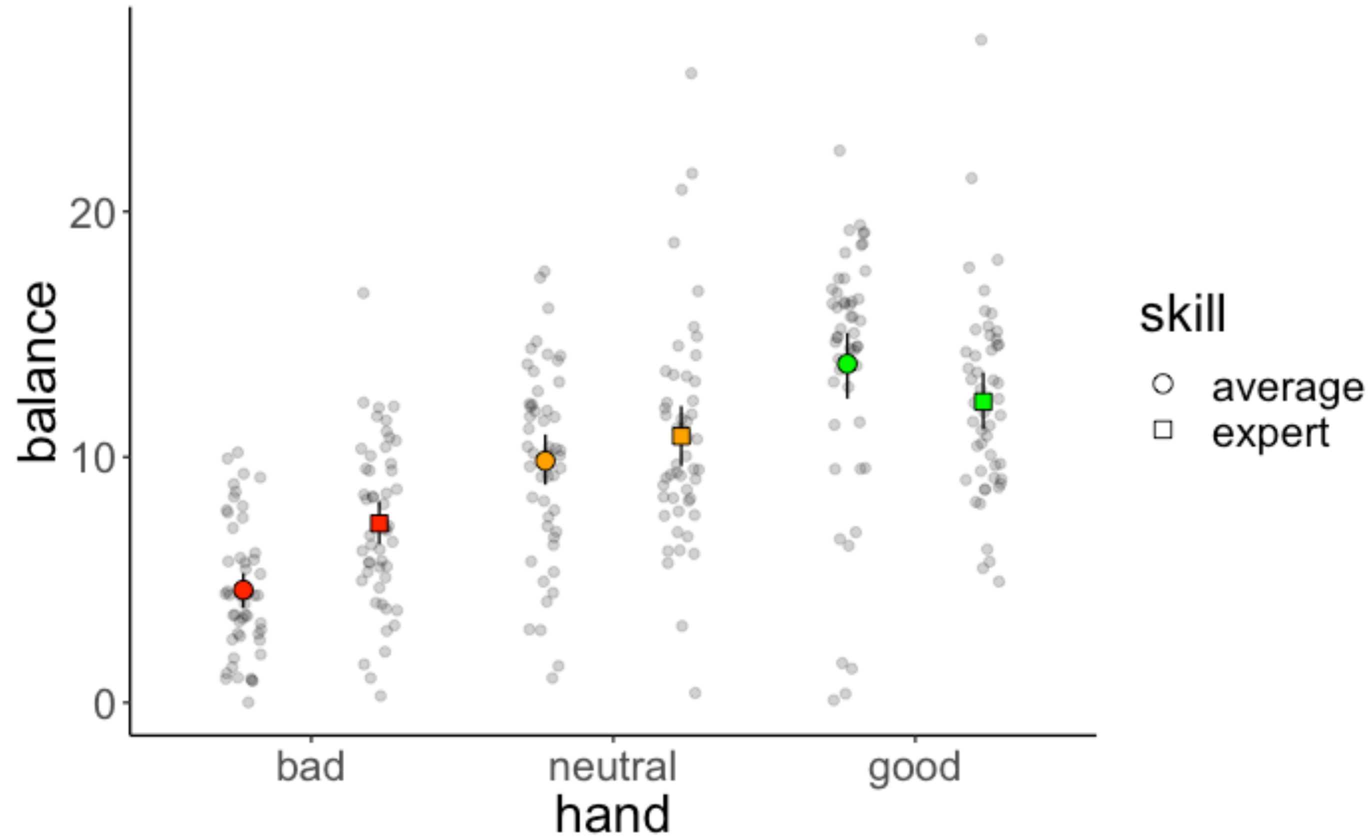
main effect of `hand`

no main effect of `skill`

interaction between `hand` and `skill`



Reporting the results



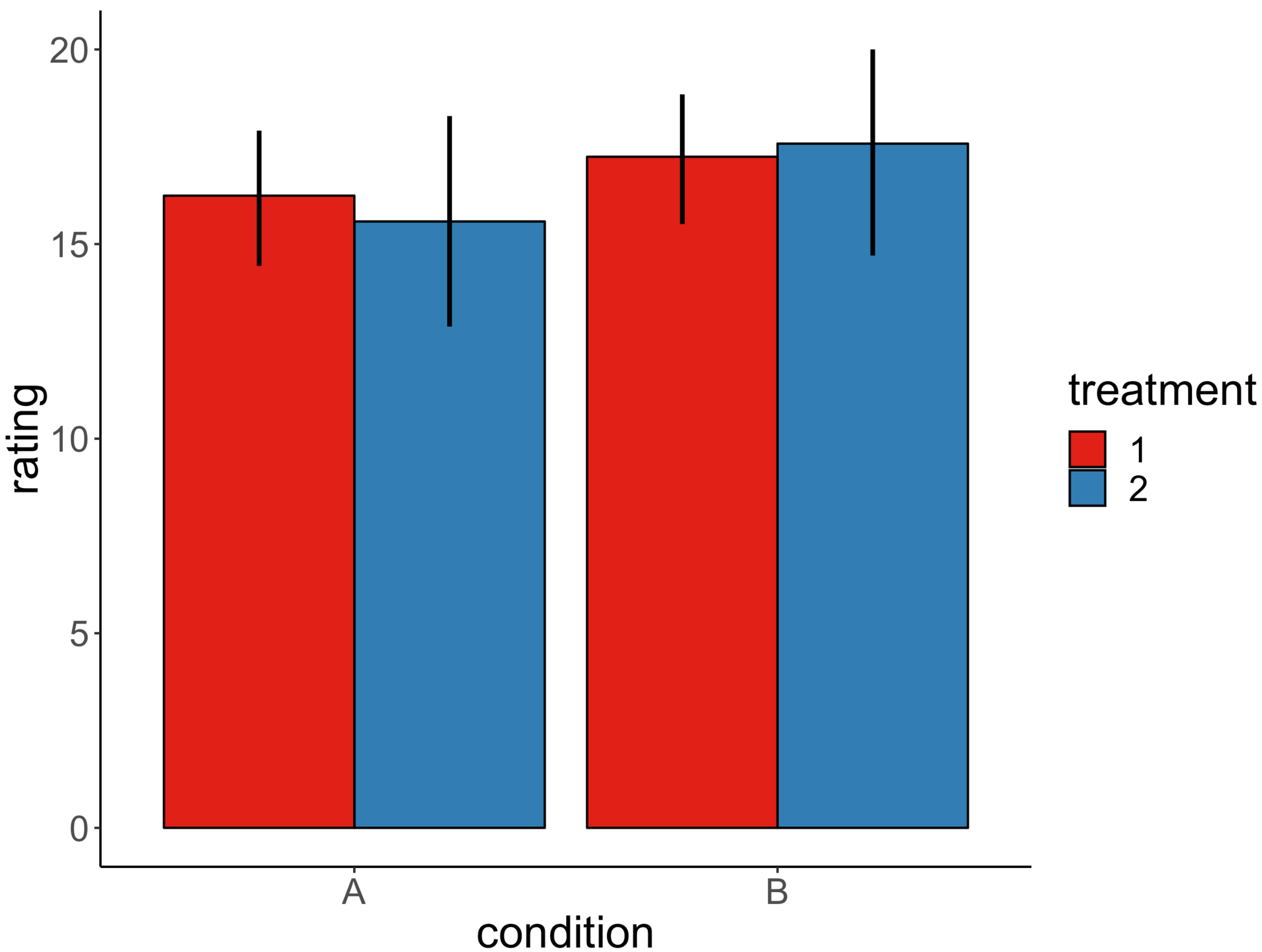
There was no main effect of skill *F*(1, 294) = 2.43, p = .12. The final balance of average (*M* = 9.41, *SD* = 5.51) and expert poker players (*M* = 10.13, *SD* = 4.50) did not differ significantly.

However, the quality of a player's hand significantly affected the final balance F(2, 294) = 79.17, p < .001. The final balance for good hands (M = 13.03, SD = 4.65) was significantly greater than for neutral hands (M = 10.35, SD = 4.24), and the balance for neutral hands was significantly higher than for bad hands (M = 5.94, SD = 3.34).

There was also a significant interaction between the quality of a player's hand and the player's skill level F(2, 294) = 7.08, p < .001. Whereas for bad hands, average players had a lower final balance than experts, for good hands, average players had a higher final balance than experts.

Who is the ANOVA champ?

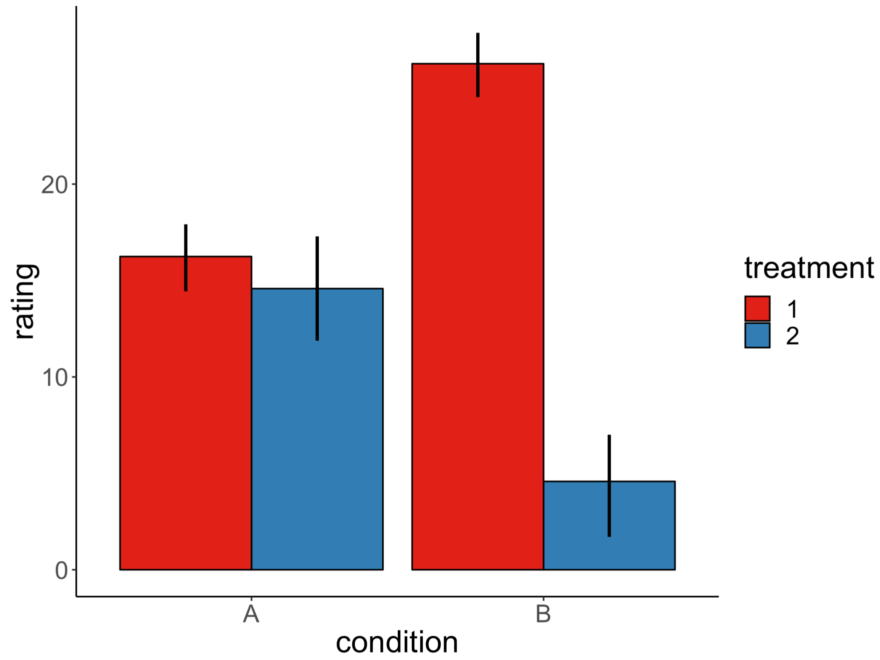
Which effects are significant?



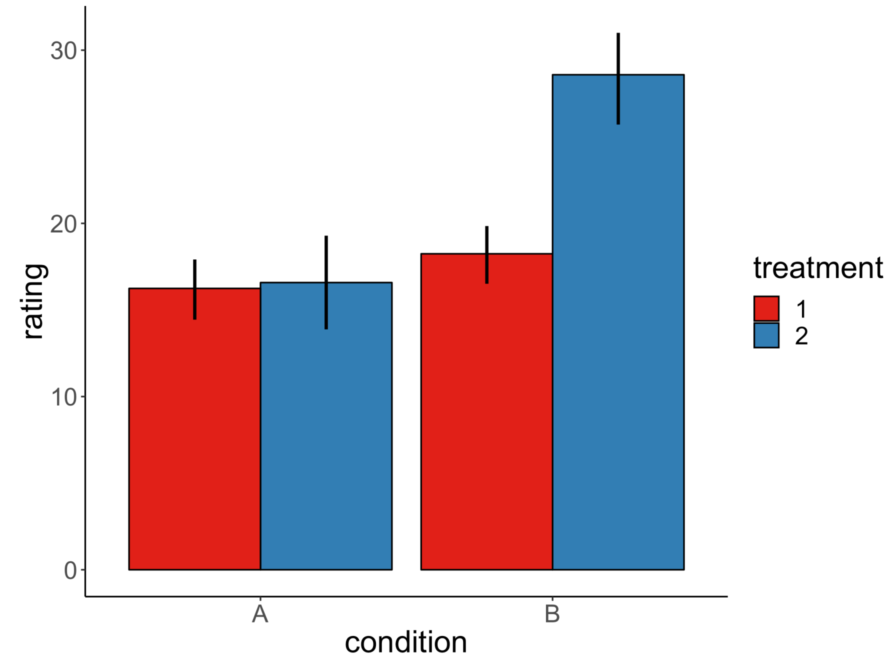
- Condition
- Treatment
- Condition x Treatment **interaction effect**
- Condition, Treatment **two main effects**
- Condition, Condition x Treatment
- Treatment, Condition x Treatment
- Condition, Treatment, Condition x Treatment

Solution

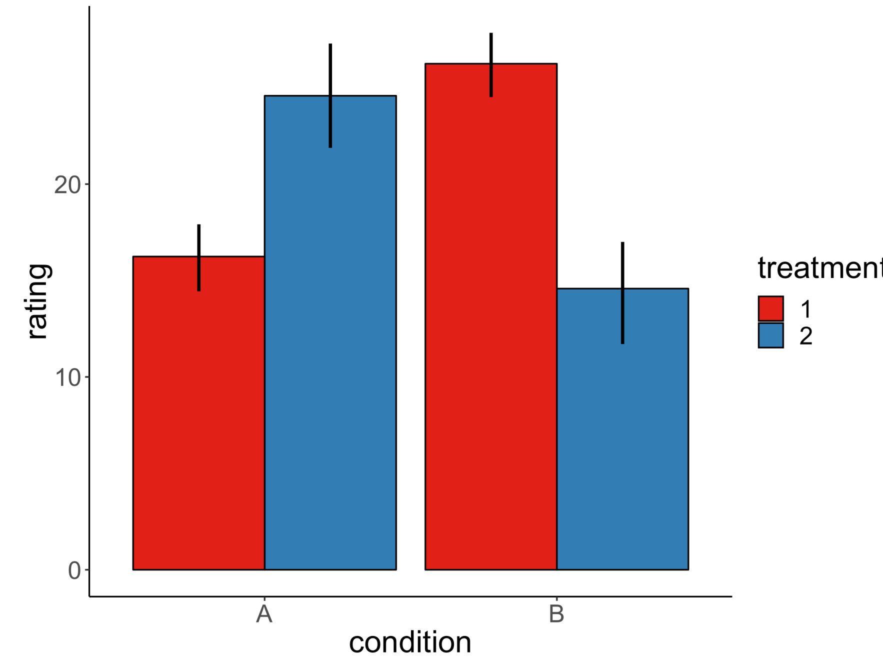
Treatment
Condition x Treatment



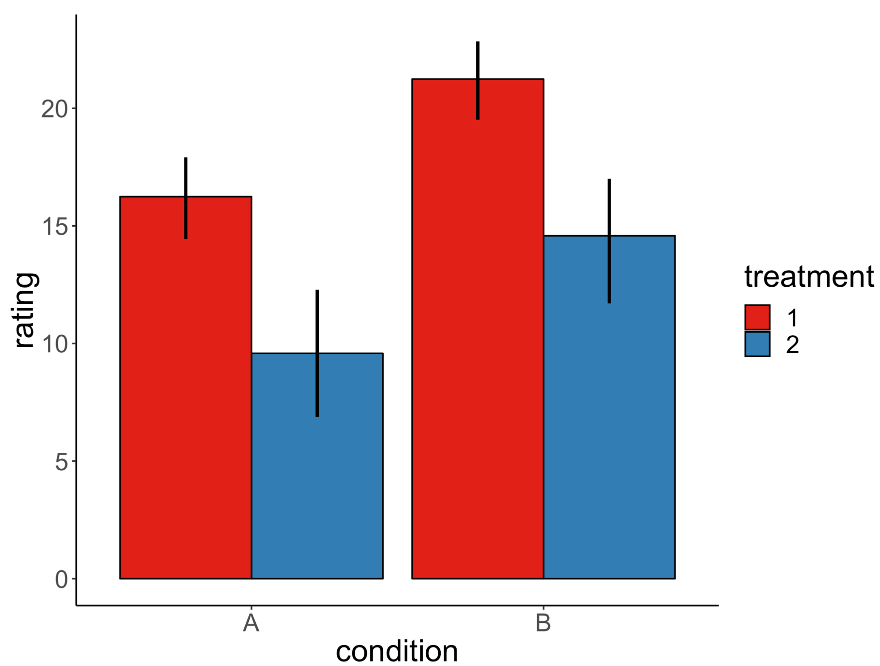
Condition,
Treatment,
Condition x Treatment



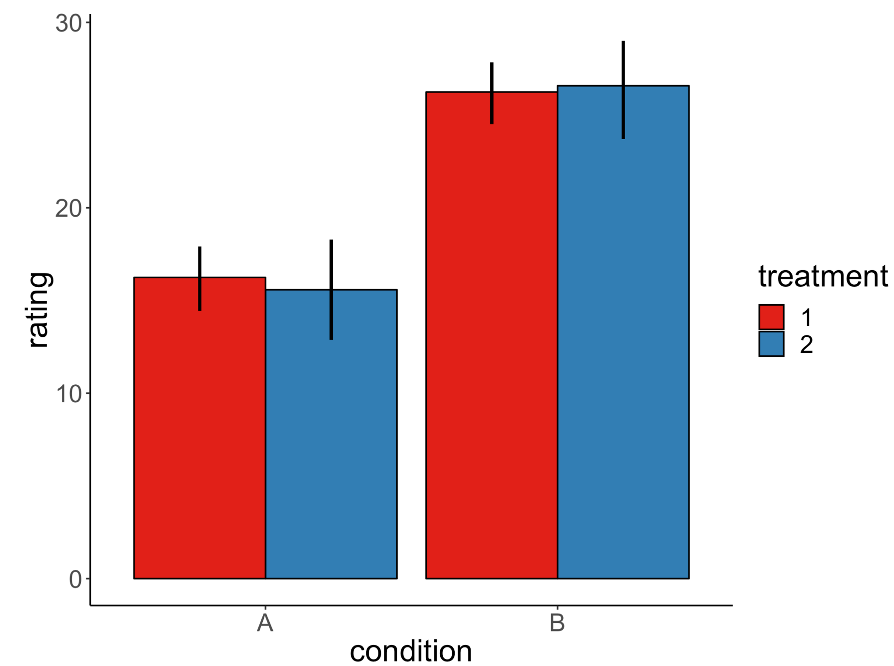
Condition x Treatment



Condition
Treatment



Condition



Feedback

How was the pace of today's class?

much too slow

a little too slow

just right

a little too fast

much too fast

How happy were you with today's class overall?

What did you like about today's class? What could be improved next time?

Thank you!