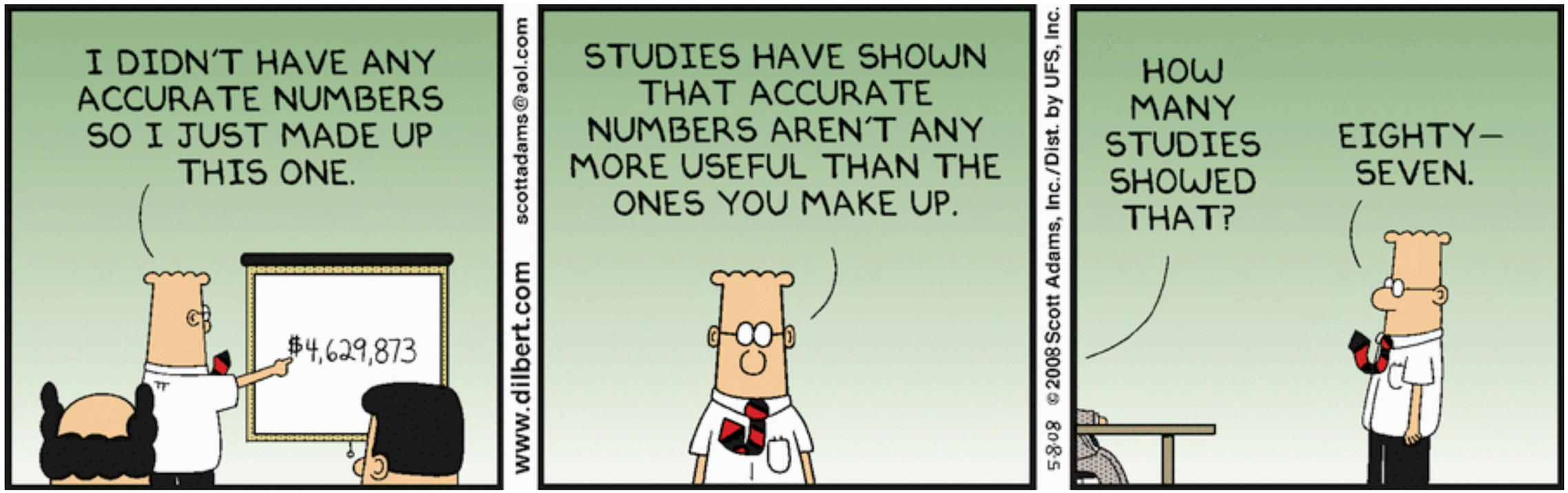
Linear model 2

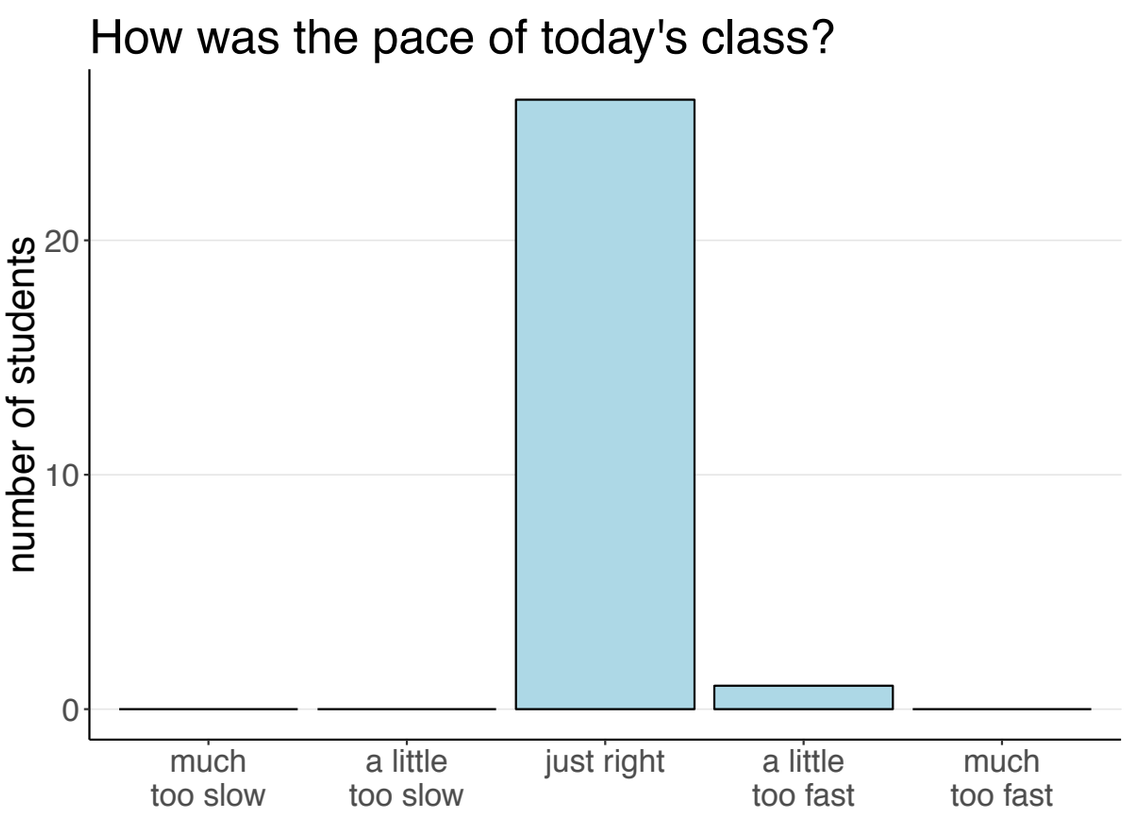


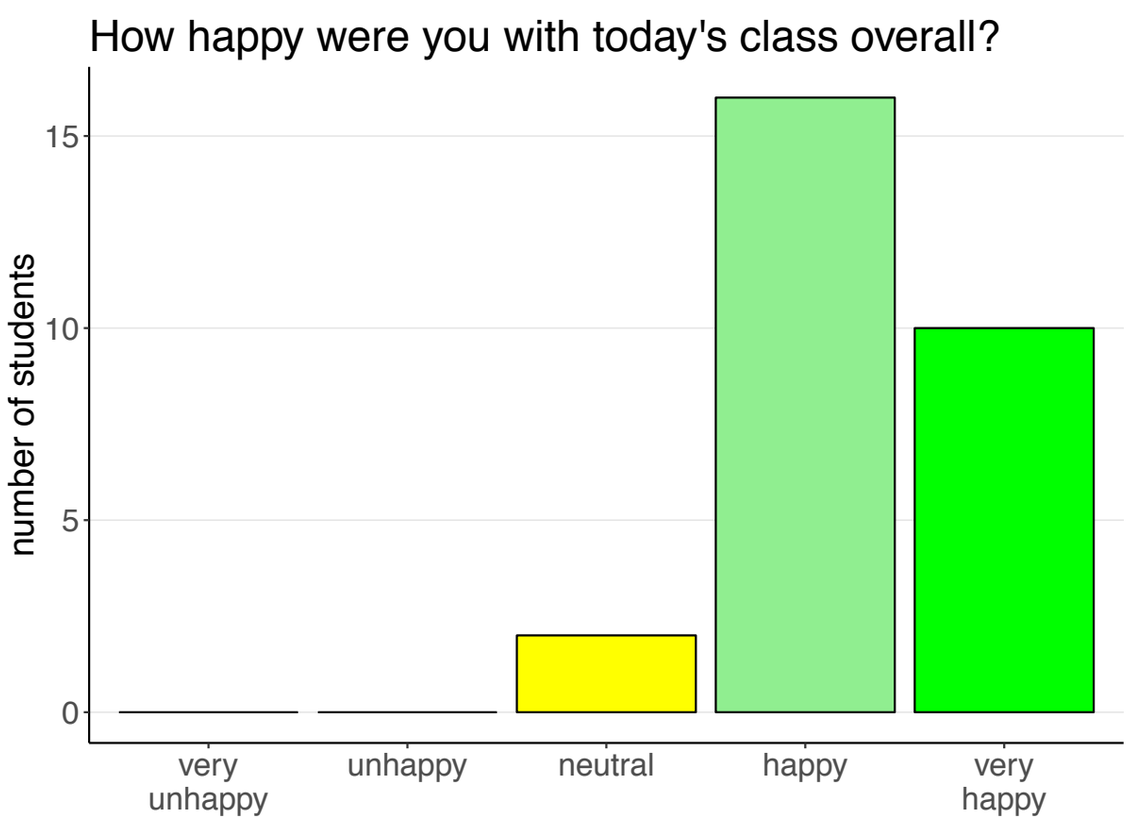
02/01/2019

Logistics

Your feedback

Your feedback





Your feedback

I finally understand all the stats I've been doing!! Well, most of it. What exactly does an ANOVA do?

we'll get there next week

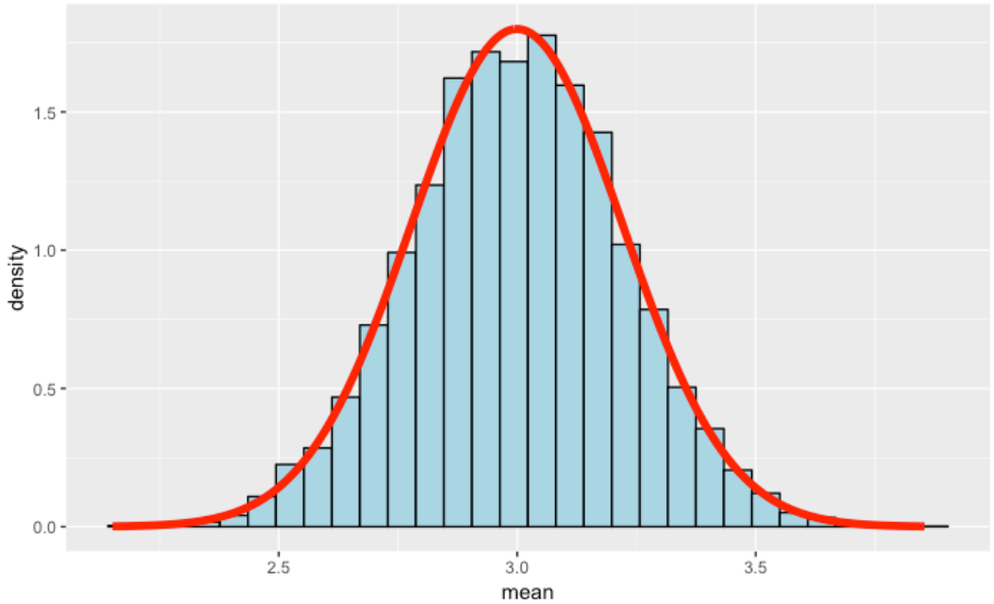
Your feedback

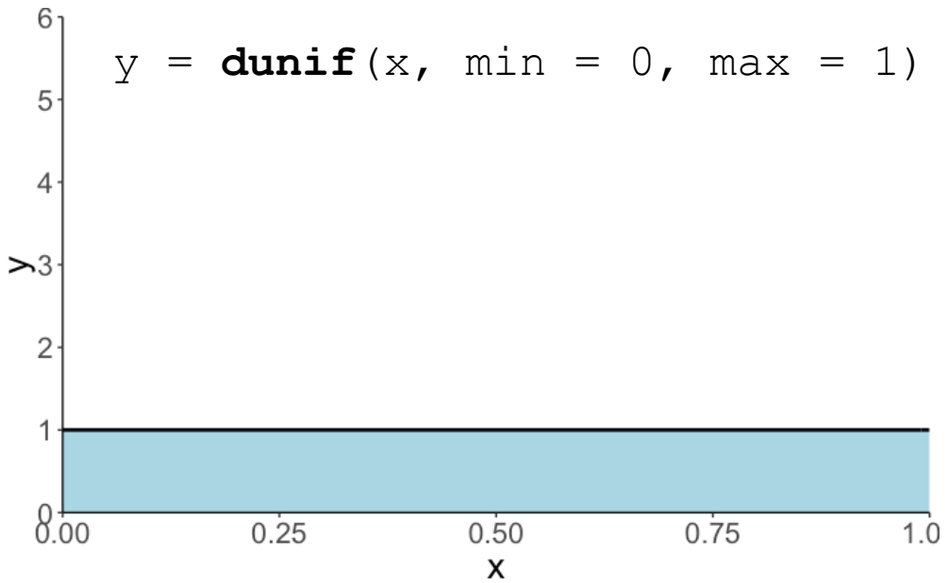
Does OLS stand for "Ordinary" Least Square instead?

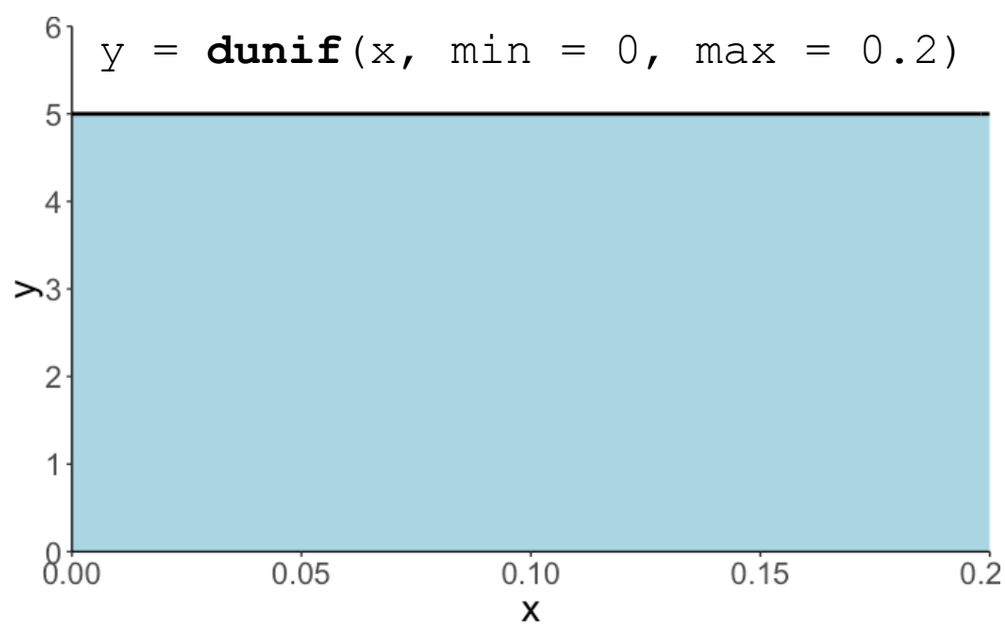
it does!

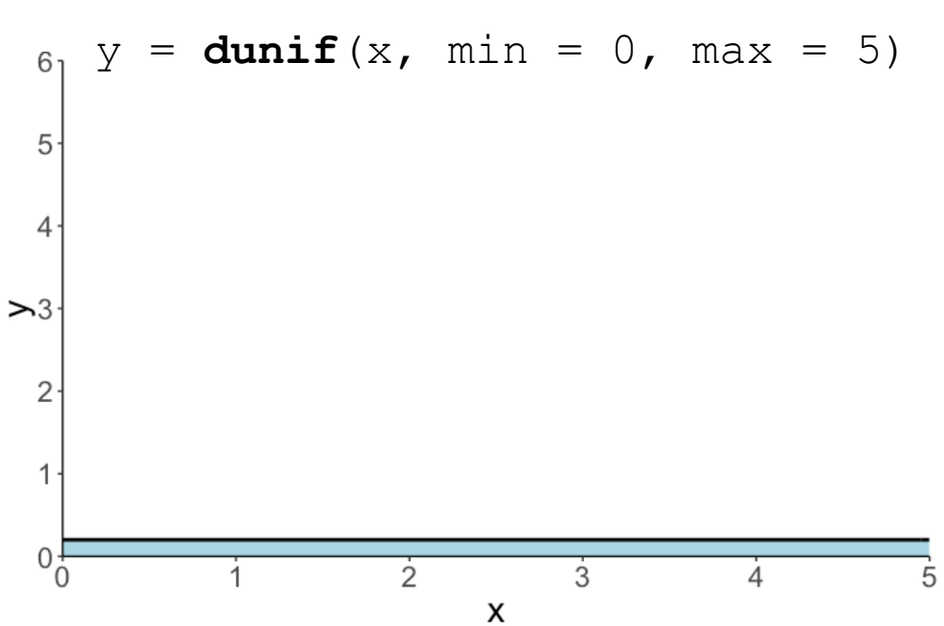
Piazza

My understanding is that the area under the density curve should add up to 1, or 100%. I'm very confused when I plot the density of the rating variable and see that the y-axis has labels that are higher than 1. How can density take on a value higher than 1? Thank you!









Plan for today

- Multiple regression
  - Model assumptions: multi-collinearity
- Several continuous predictors
  - Hypothesis tests
  - Interpreting parameters
  - Reporting results
- One categorical predictor
- Both continuous and categorical predictors
- Interactions

Multiple regression

Linear model

$$\texttt{Data} = \texttt{Model} + \texttt{Error}$$

Simple regression

$$Y_i = \beta_0 + \beta_1 X_{1i} + \epsilon_i$$

one predictor

Multiple regression

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \ldots + \beta_p X_{pi} + \epsilon_i$$

many predictors

Advertising data set

- Combine several predictors to explain an outcome variable of interest

money spent on different media (x $1000)

sales (x1000)

index	tv	radio	newspaper	sales
1	230.1	37.8	69.2	22.1
2	44.5	39.3	45.1	10.4
3	17.2	45.9	69.3	9.3
4	151.5	41.3	58.5	18.5
5	180.8	10.8	58.4	12.9
6	8.7	48.9	75.0	7.2
7	57.5	32.8	23.5	11.8
8	120.2	19.6	11.6	13.2
9	8.6	2.1	1.0	4.8
10	199.8	2.6	21.2	10.6

Model C

Simple regression

$$\text{sales}_i = b_0 + b_1 \cdot \text{tv}_i + e_i$$

Model A

Multiple regression

$$\text{sales}_i = b_0 + b_1 \cdot \text{tv}_i + b_2 \cdot \text{radio}_i + e_i$$

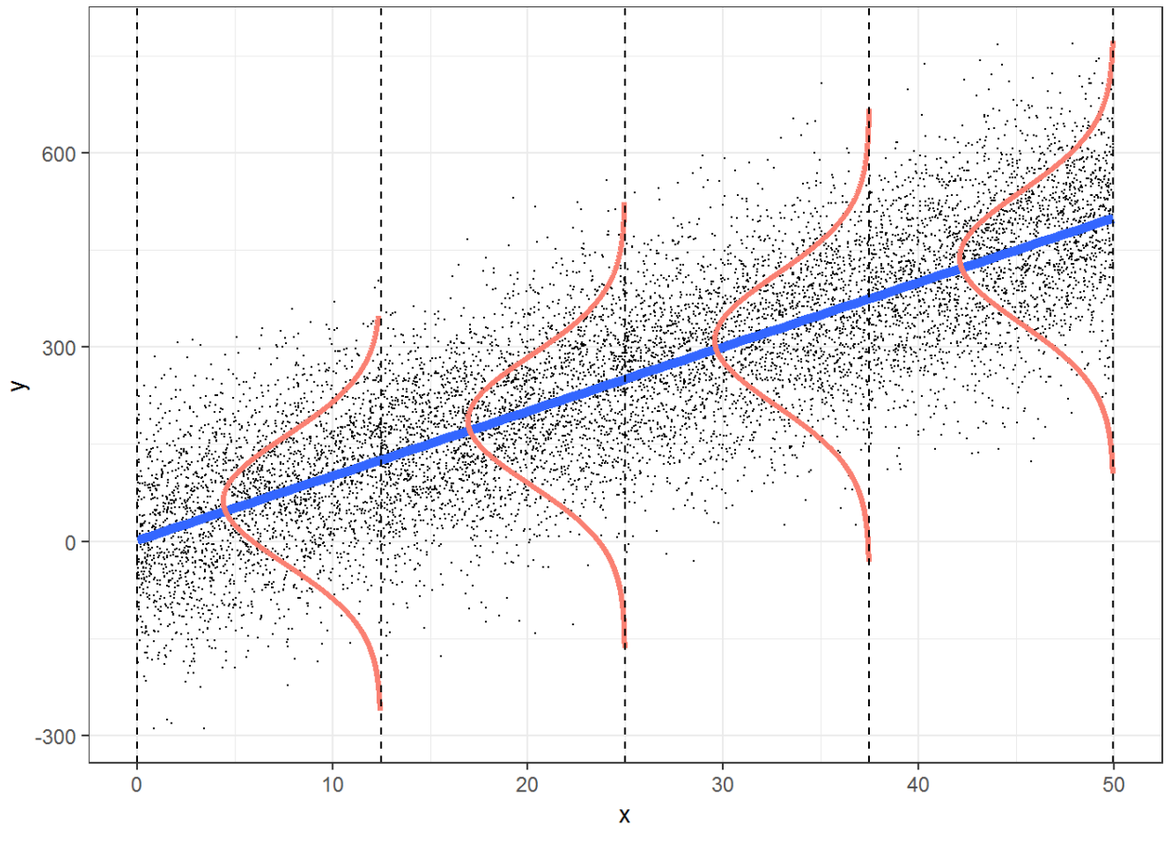
Can we predict sales better when we consider radio ads in addition to TV ads?

"Controlling" for TV ads, do radio ads explain any of the variance in sales?

Assumptions of multiple regression

- independent observations
- *Y* is continuous
- errors are normally distributed
- errors have constant variance
- error terms are uncorrelated
- **no multicollinearity**

predictors in the model should not be highly correlated with each other



Visualizing correlations

Visualizing correlations

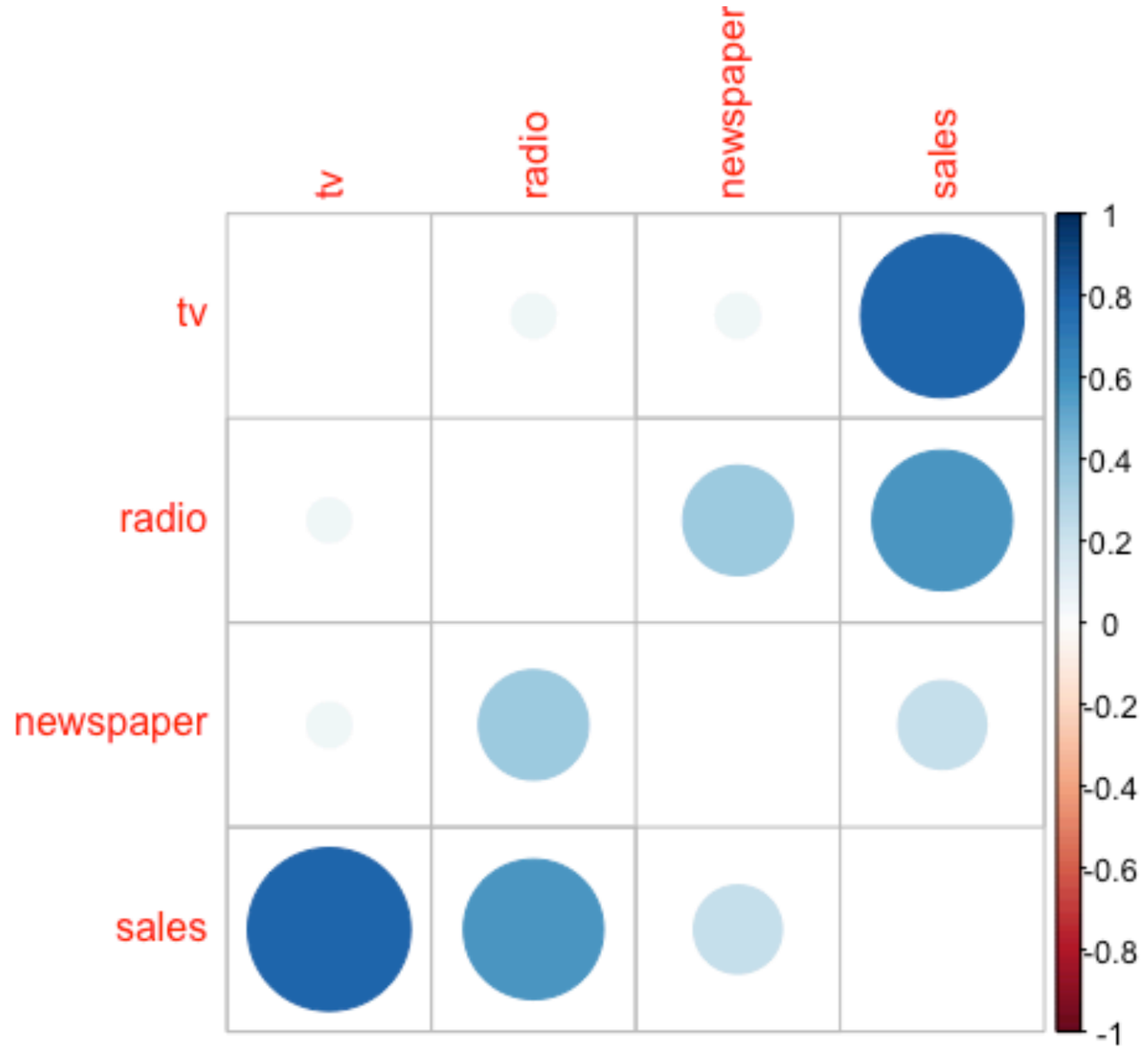
```
library("corrr")
```


```
df.credit %>%
  select_if(is.numeric) %>%
  correlate() %>%
  rearrange() %>%
  shave() %>%
  fashion()
```

rowname	index	newspaper	radio	sales	tv
index					
newspaper	-0.15				
radio	-0.11	0.35			
sales	-0.05	0.23	0.58		
tv	0.02	0.06	0.05	0.78	

Visualizing correlations

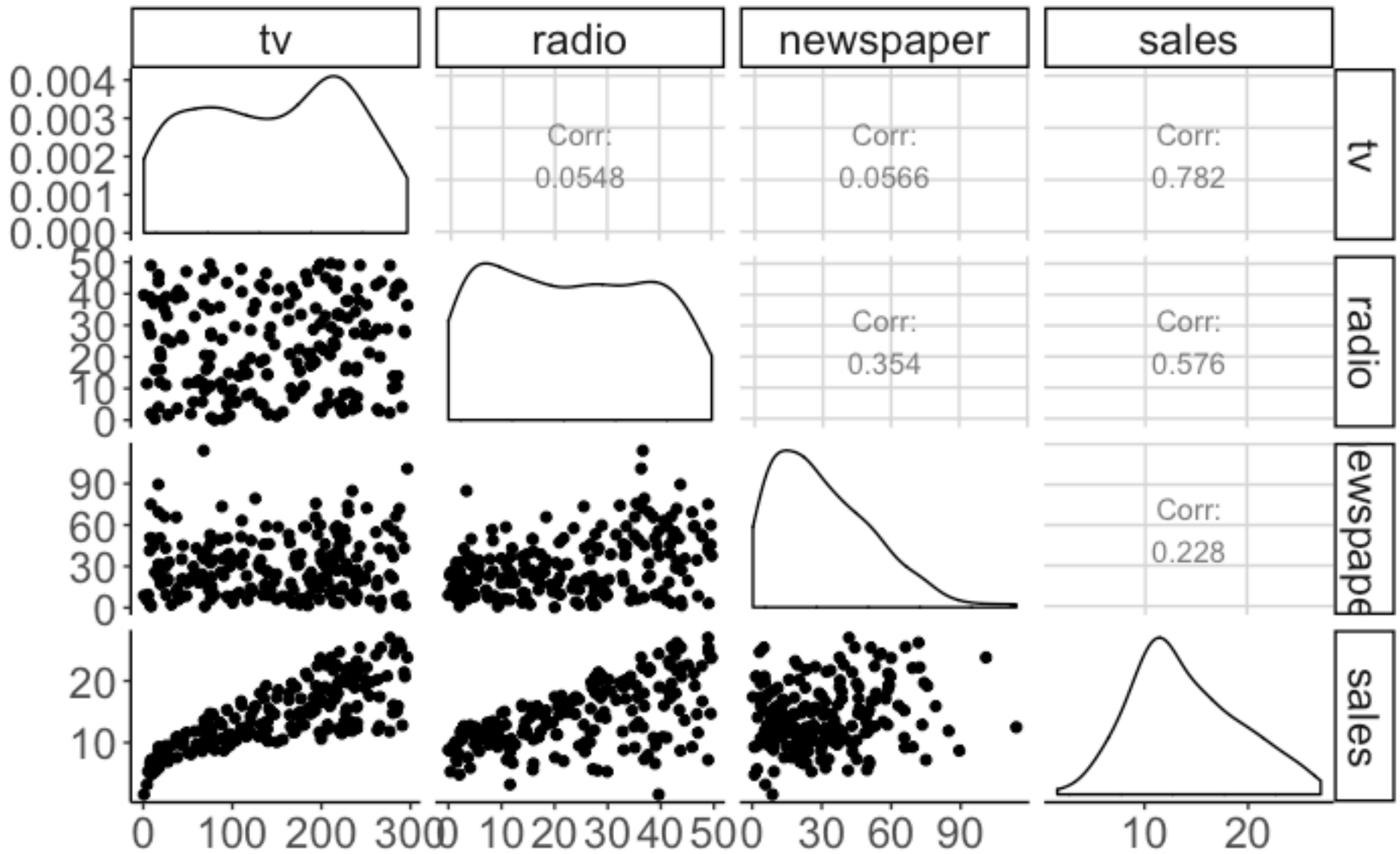
```
df.ads %>%
  select(-index) %>%
  correlate() %>%
  column_to_rownames() %>%
  as.matrix() %>%
  corrplot(diag = F)
```



Visualizing correlations

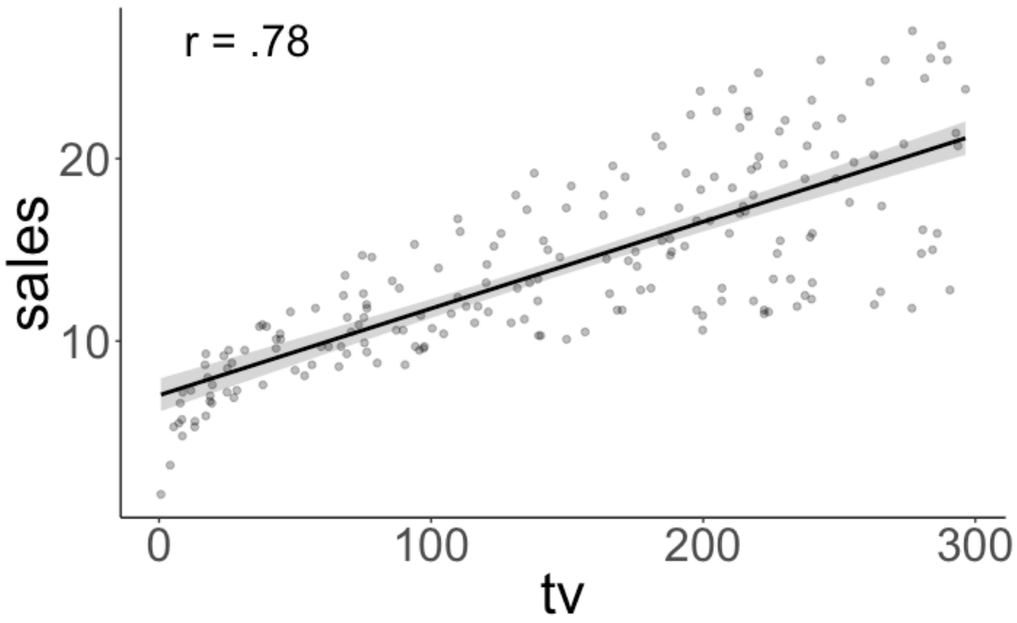
```
library("GGally")
```

```
1 df.ads %>%
2    select(-index) %>%
3    ggpairs()
```

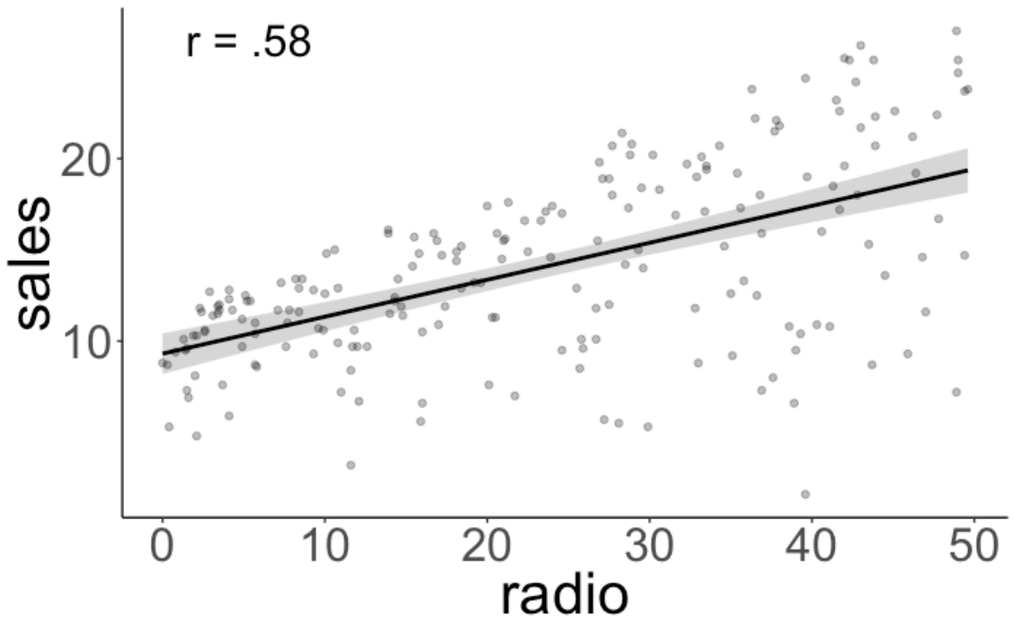


$$\text{sales}_i = b_0 + b_1 \cdot \text{tv}_i + b_2 \cdot \text{radio}_i + e_i$$

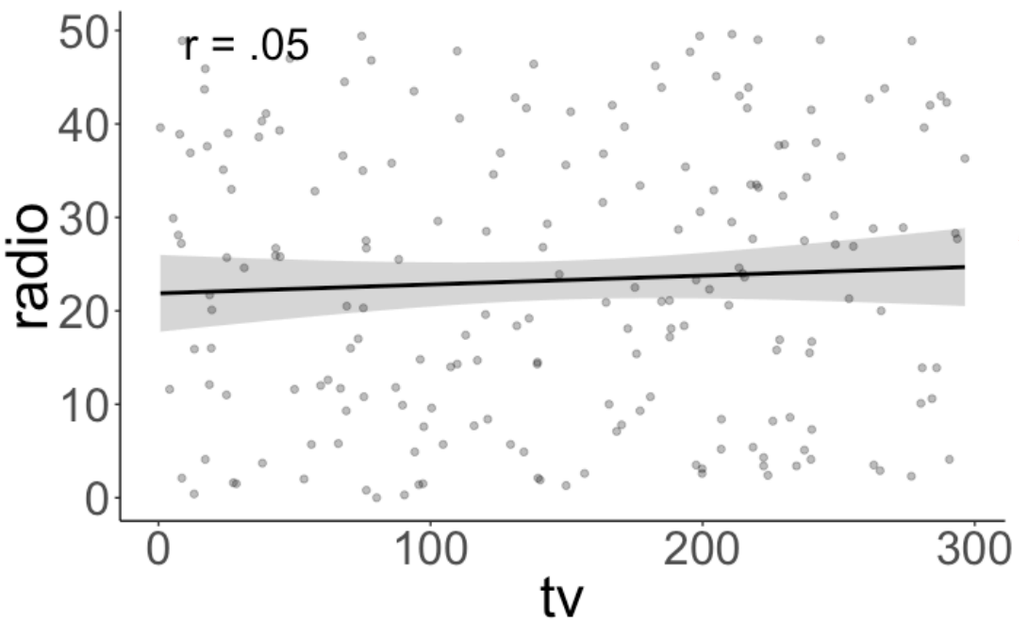
Relationship between TV ads and sales



Relationship between radio ads and sales



Relationship between TV ads and radio ads



predictors are not correlated, yay!

Can we predict sales better when we consider radio in addition to TV ads?

1. Define H_0 as Model C (compact) and H_1 as Model A (augmented)
2. Fit model parameters to the data
3. Calculate the proportional reduction of error (PRE) in our sample
4. Decide whether the augmented model is **worth it** by comparing the observed PRE in our sample to the sampling distribution of PRE (assuming that H_0 is true)

H_0: Radio ads and sales are not related once we control for TV ads.

Model C

$$\text{sales}_i = b_0 + b_1 \cdot \text{tv}_i + e_i$$

H_1: Radio ads and sales are related even when we control for TV ads.

Model A

$$\text{sales}_i = b_0 + b_1 \cdot \text{tv}_i + b_2 \cdot \text{radio}_i + e_i$$

```
# fit the models
fit_c = lm(sales ~ 1 + tv, data = df.ads)
fit_a = lm(sales ~ 1 + tv + radio, data = df.ads)

# do the F test
anova(fit_c, fit_a)
```

we reject the H_0

```
Analysis of Variance Table

Model 1: sales ~ 1 + tv
Model 2: sales ~ 1 + tv + radio
  Res.Df     RSS Df Sum of Sq      F    Pr(>F)
1    198 2102.53
2    197  556.91  1    1545.6 546.74 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Evaluating the model: Model fit

```
fit_a %>%
  glance()
```

r.squared	adj.r.squared	sigma	statistic	p.value	df	logLik	AIC	BIC	deviance	df.residual
0.897	0.896	1.681	859.618	0	3	-386.197	780.394	793.587	556.914	197

`r.squared`	The percent of variance explained by the model
`adj.r.squared`	r.squared adjusted based on the degrees of freedom
`sigma`	The square root of the estimated residual variance
`statistic`	F-statistic
`p.value`	p-value from the F test, describing whether the full regression is significant
`df`	Degrees of freedom used by the coefficients
`logLik`	the data's log-likelihood under the model
`AIC`	the Akaike Information Criterion
`BIC`	the Bayesian Information Criterion
`deviance`	deviance
`df.residual`	residual degrees of freedom

Evaluating the model: Model fit

```
fit_a %>%
  glance()
```

r.squared	adj.r.squared	sigma	statistic	p.value	df	logLik	AIC	BIC	deviance	df.residual
0.897	0.896	1.681	859.618	0	3	-386.197	780.394	793.587	556.914	197

Compact Model

$$\text{sales}_i = b_0 + e_i$$

The augmented model reduces 89.7% of the error compared to a compact model that just predicts the mean.

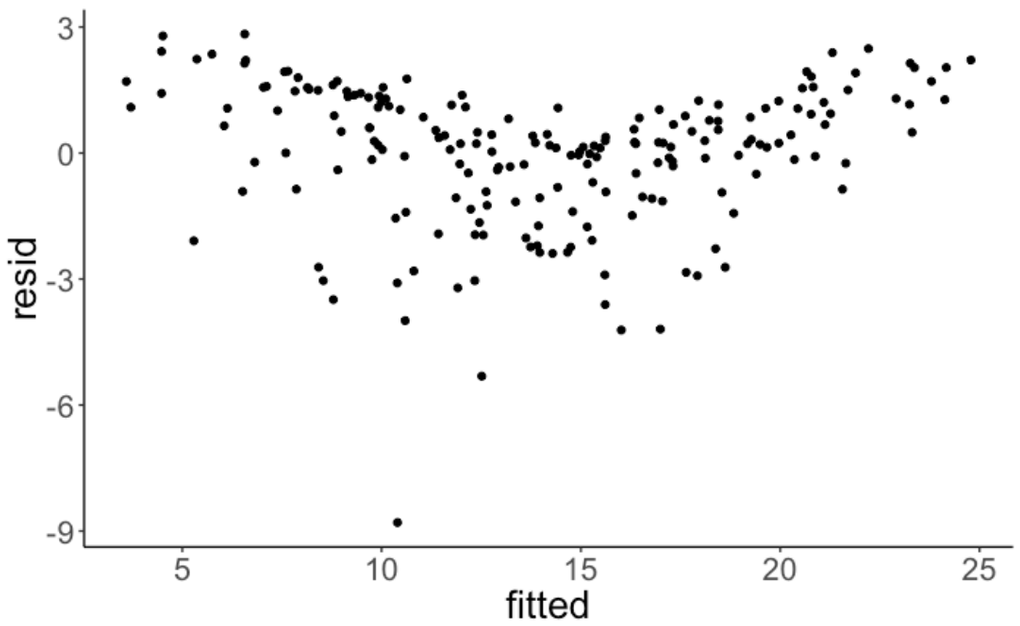
Augmented Model

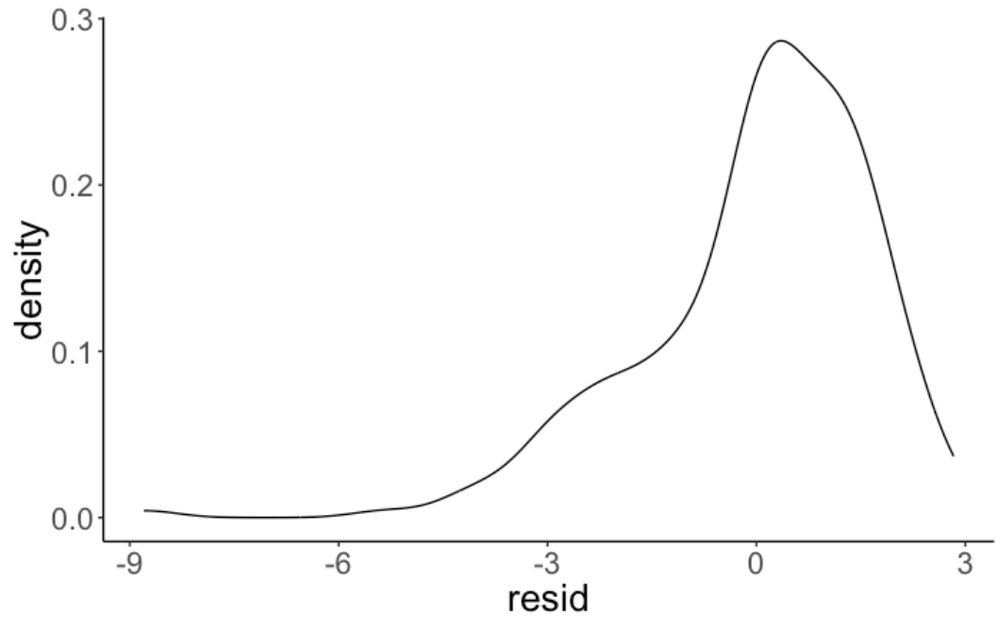
$$\text{sales}_i = b_0 + b_1 \cdot \text{tv}_i + b_2 \cdot \text{radio}_i + e_i$$

$$\text{PRE} = 1 - \frac{\text{SSE}(A)}{\text{SSE}(C)} = R^2$$

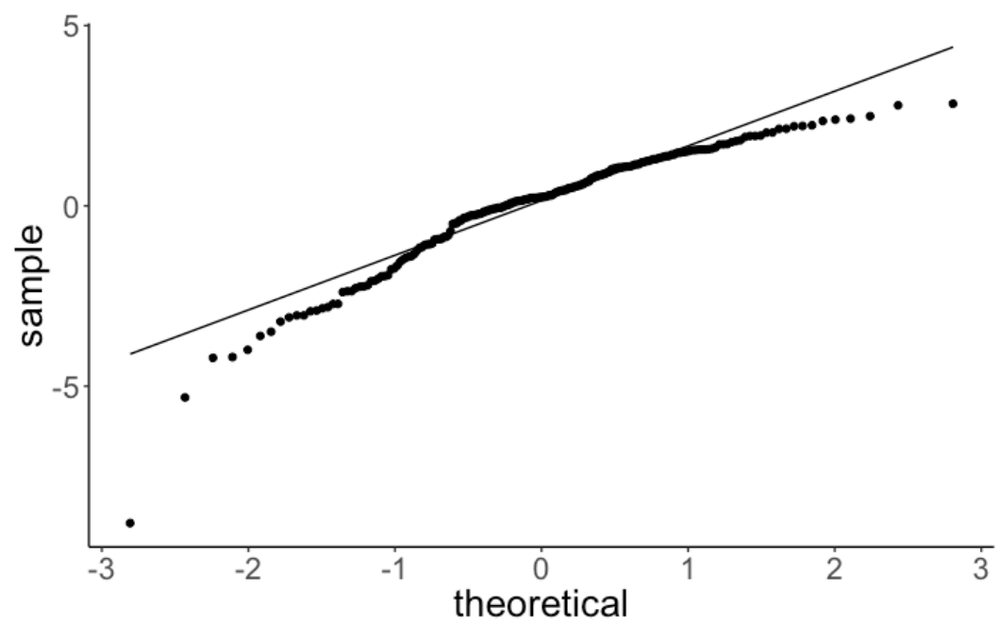
Evaluating the model: Residual plots

```
resid = sales - fitted
```





OKish overall



Interpreting the results

```
fit_a %>%
  tidy(conf.int = T)
```

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	2.92	0.29	9.92	0	2.34	3.50
tv	0.05	0.00	32.91	0	0.04	0.05
radio	0.19	0.01	23.38	0	0.17	0.20

$$\text{sales}_i = b_0 + b_1 \cdot \text{tv}_i + b_2 \cdot \text{radio}_i + e_i$$

$$\widehat{\text{sales}}_i = 2.92 + 0.05 \cdot \text{tv}_i + 0.19 \cdot \text{radio}_i$$

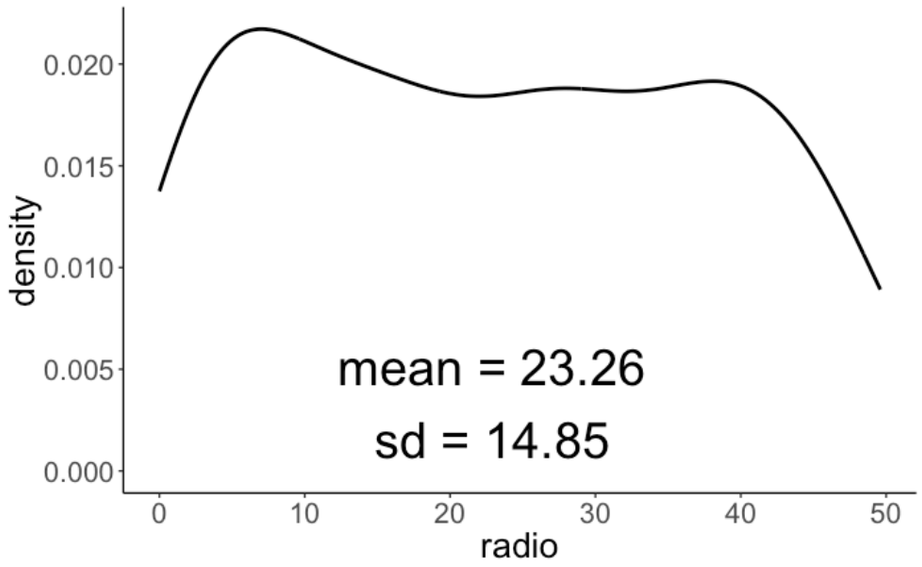
For a given amount of TV advertising, an additional $1000 on radio advertising leads to an increase in sales by 190 units.

anything surprising?

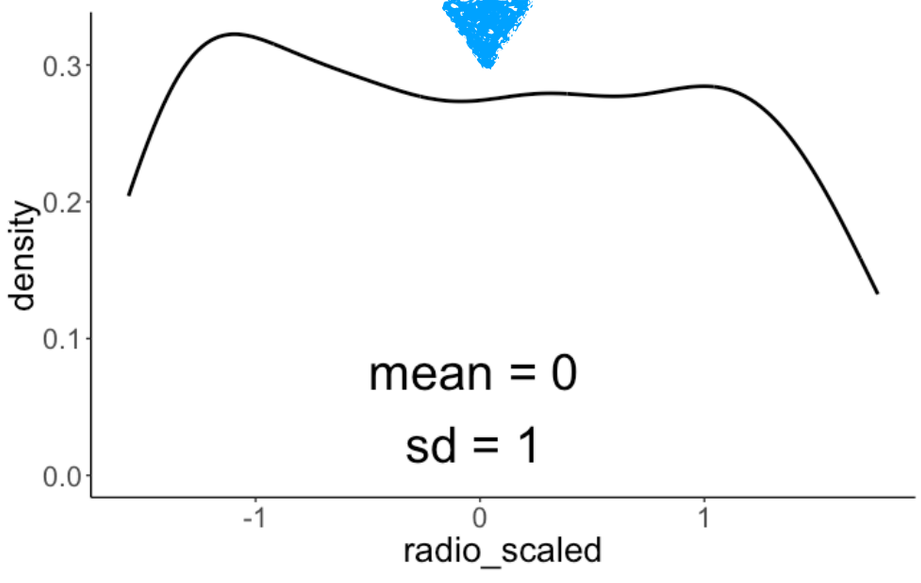
Standardizing predictors

$$z_i = \frac{Y_i - \bar{Y}}{s}$$

index	tv	radio	sales	tv_scaled	radio_scaled
1	230.1	37.8	22.1	0.97	0.98
2	44.5	39.3	10.4	-1.19	1.08
3	17.2	45.9	9.3	-1.51	1.52
4	151.5	41.3	18.5	0.05	1.21
5	180.8	10.8	12.9	0.39	-0.84
6	8.7	48.9	7.2	-1.61	1.73
7	57.5	32.8	11.8	-1.04	0.64
8	120.2	19.6	13.2	-0.31	-0.25
9	8.6	2.1	4.8	-1.61	-1.43
10	199.8	2.6	10.6	0.61	-1.39



$$z_i = \frac{Y_i - \bar{Y}}{s}$$



Interpreting the results

```
fit_a %>%
  tidy(conf.int = T)
```

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	14.02	0.12	117.94	0	13.79	14.26
tv_scaled	3.93	0.12	32.91	0	3.69	4.16
radio_scaled	2.79	0.12	23.38	0	2.56	3.03

$$\widehat{\text{sales}}_i = 14.02 + 3.93 \cdot \text{tv\_z}_i + 2.79 \cdot \text{radio\_z}_i$$

For a given amount of TV advertising, increasing radio advertising by one standard deviation (23.26 x $1000) leads to an increase in sales by 2790 units.

Interpreting the results

```
fit_a %>%
  tidy(conf.int = T)
```

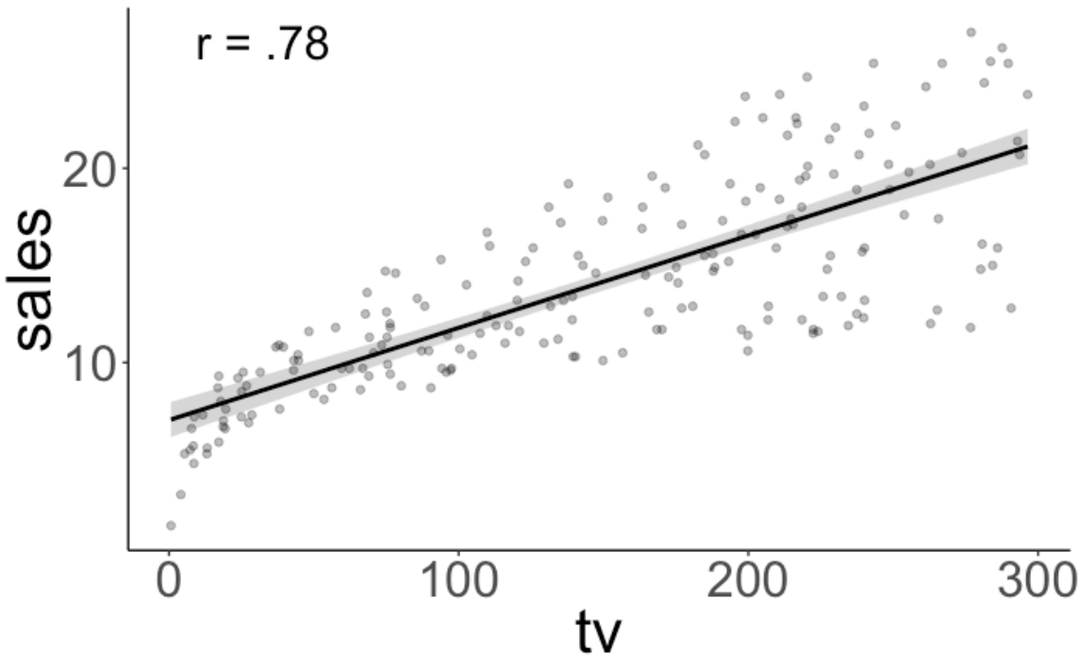
term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	14.02	0.12	117.94	0	13.79	14.26
tv_scaled	3.93	0.12	32.91	0	3.69	4.16
radio_scaled	2.79	0.12	23.38	0	2.56	3.03

$$\widehat{\text{sales}}_i = 14.02 + 3.93 \cdot \text{tv\_z}_i + 2.79 \cdot \text{radio\_z}_i$$

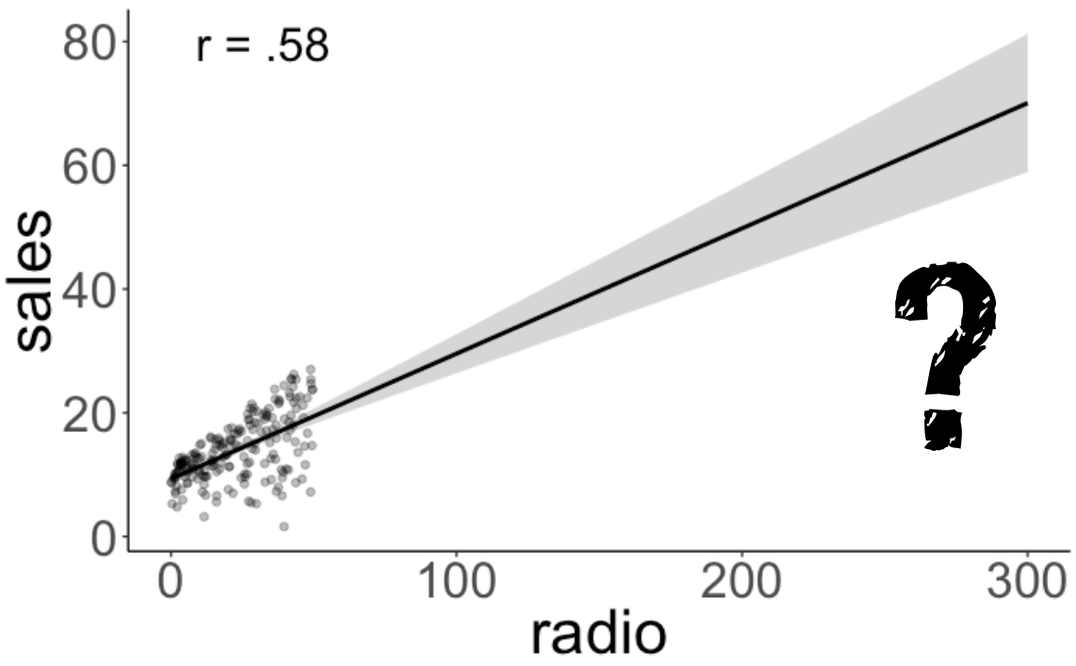
For a given amount of radio advertising, increasing TV advertising by one standard deviation (85.85 x $1000) leads to an increase in sales by approximately 3930 units.

On average, 14,020 units are being sold.

Question of extrapolation ...

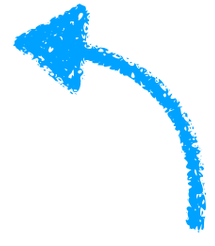


Radio ads give more bang for the buck but it's unclear whether this would extrapolate ...



Interpreting coefficients

- For multiple regression, the meaning of a coefficient is: How much is the outcome predicted to change for a unit increase in the predictor, holding all the other predictors fixed.
- Standardizing predictors can help with interpretation (particularly when comparing predictors with different units, ranges, ...).
- Standardizing coefficients means that you can compare the relative importance of each coefficient in a regression model.



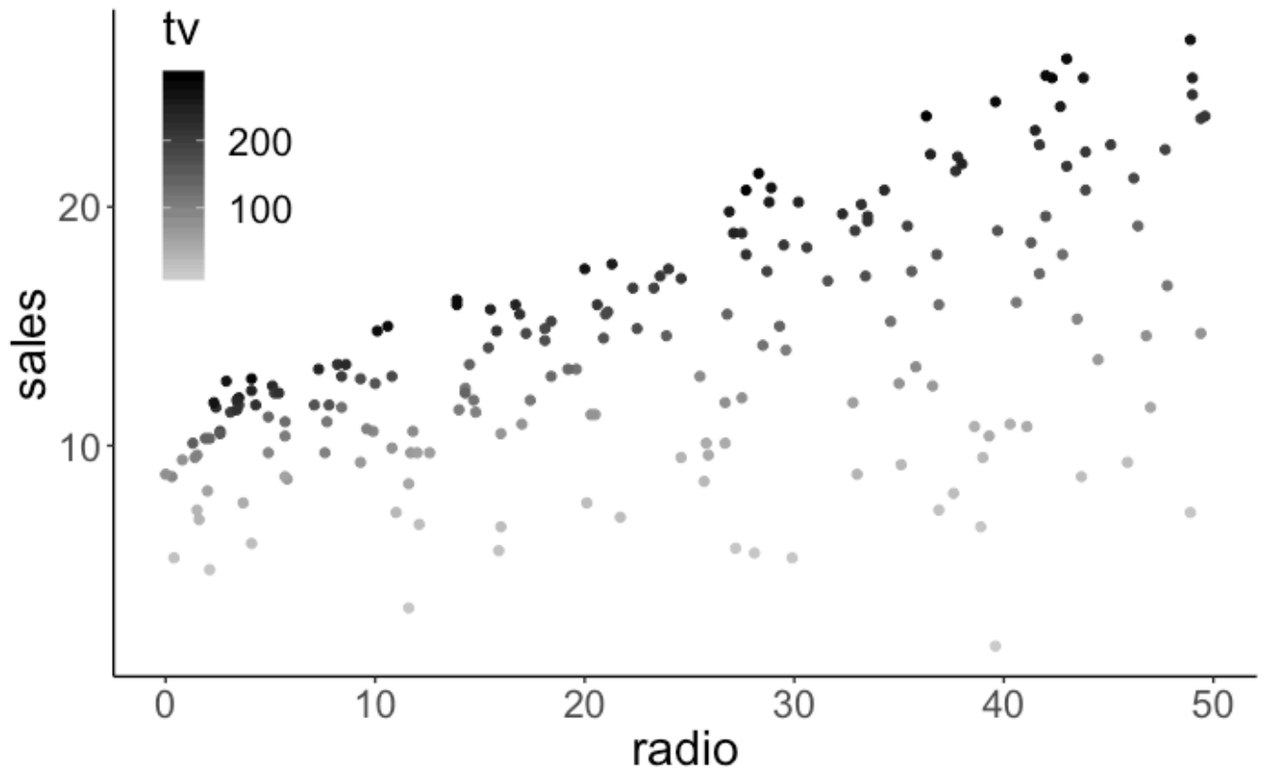
well, sort of ...

Interpreting coefficients

> "The fundamental problem is that we cannot frame a question about relative importance in terms of a Model A versus Model C comparison because both models would necessarily have the same predictors. Hence, our machinery for statistical inference cannot address the question of relative importance.
>
> Relative importance of predictor variables is a slippery concept. Any statement about relative importance must be accompanied by many caveats that it applies to this particular range of variables in this situation. As a result they are, in our opinion, seldom useful. Thus, although it is tempting to compare the importance of predictors in multiple regression, it is almost always best not to do so."

Judd, C. M., McClelland, G. H., & Ryan, C. S. (2011). Data analysis: A model comparison approach. Routledge.

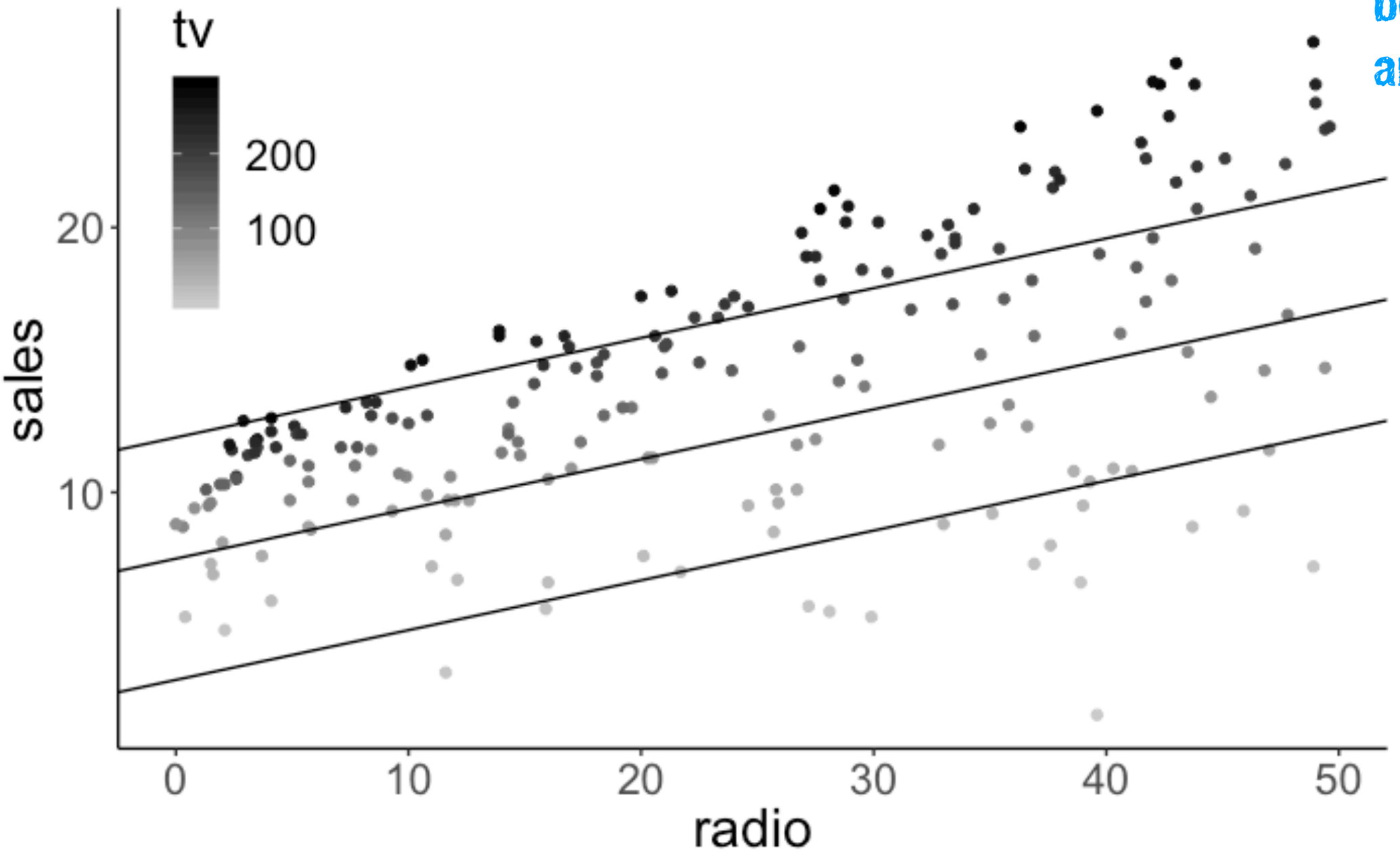
Reporting results



There is a significant relationship between sales and radio ads, controlling for TV ads $F(1, 197) = 546.74$, $p < .001$.

Holding TV ads fixed, an increase in $1000 on radio ads is predicted to increase sales by 190 units [170, 200] (95% confidence intervals).

Visualizing the results



predicted relationship between radio and sales when:

tv = 200

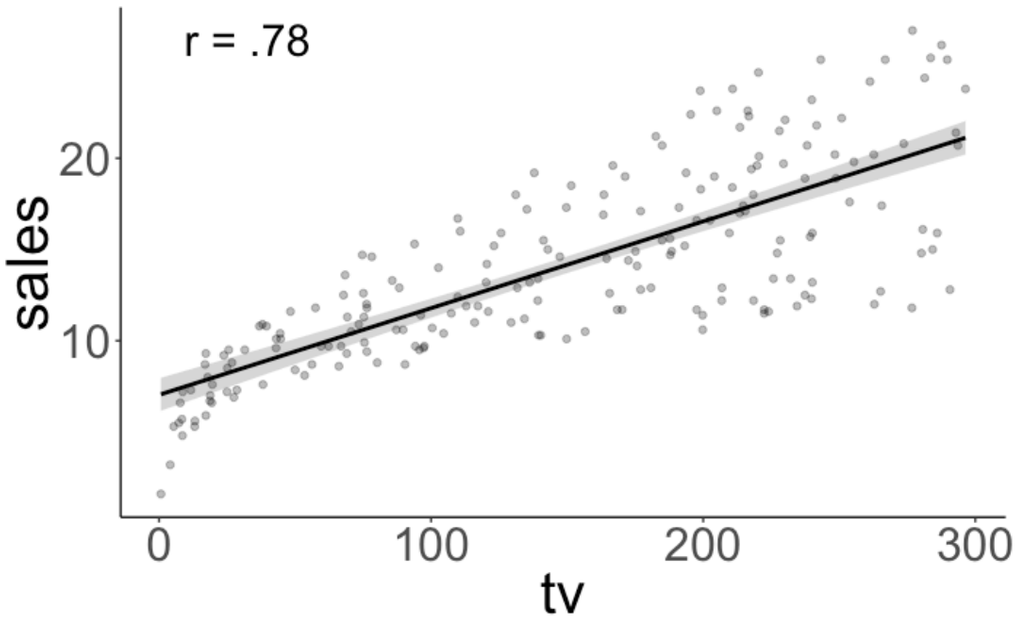
tv = 100

tv = 0

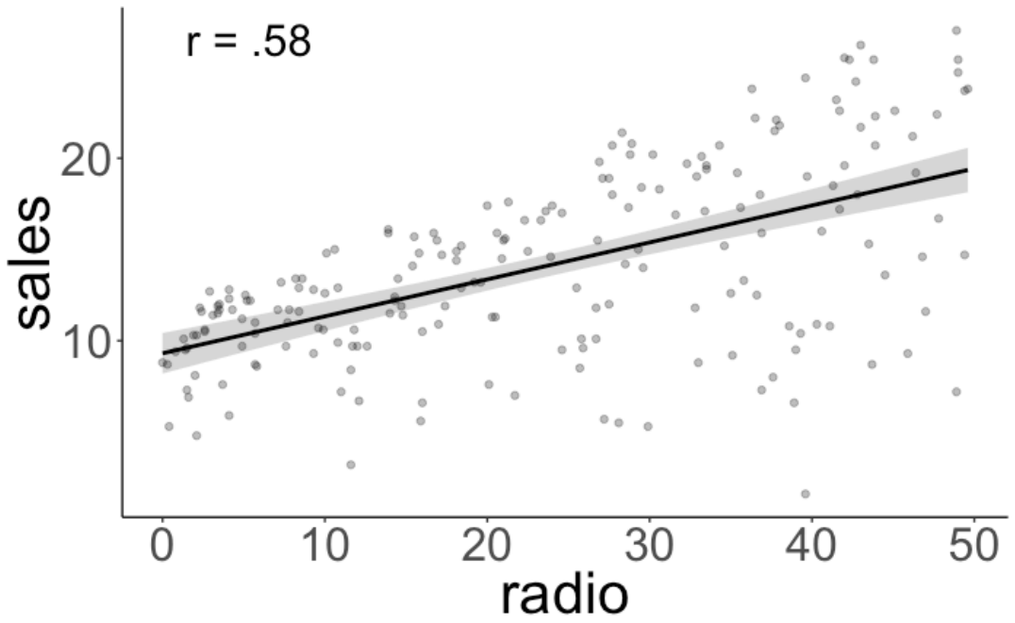
just imagine infinitely many more lines ...

Why can't I just run several simple regressions?

Relationship between TV ads and sales



Relationship between radio ads and sales



We found that both TV ads and radio ads were related to sales.

But did we need to run a multiple regression? Could we not just have looked at correlations?

Advertising data set

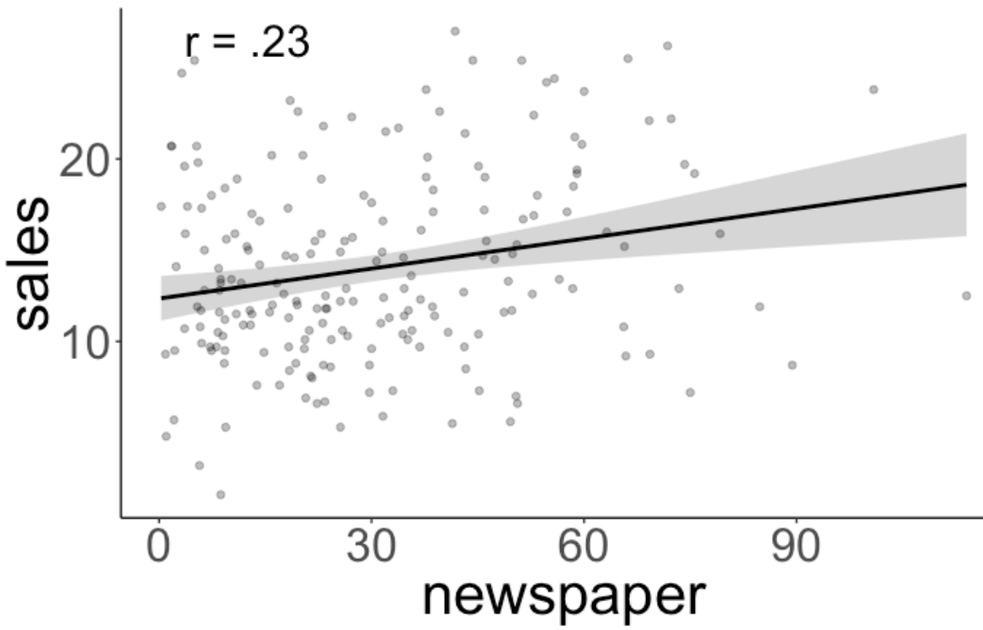
money spent on different media (x $1000)

sales (x1000)

index	tv	radio	newspaper	sales
1	230.1	37.8	69.2	22.1
2	44.5	39.3	45.1	10.4
3	17.2	45.9	69.3	9.3
4	151.5	41.3	58.5	18.5
5	180.8	10.8	58.4	12.9
6	8.7	48.9	75.0	7.2
7	57.5	32.8	23.5	11.8
8	120.2	19.6	11.6	13.2
9	8.6	2.1	1.0	4.8
10	199.8	2.6	21.2	10.6

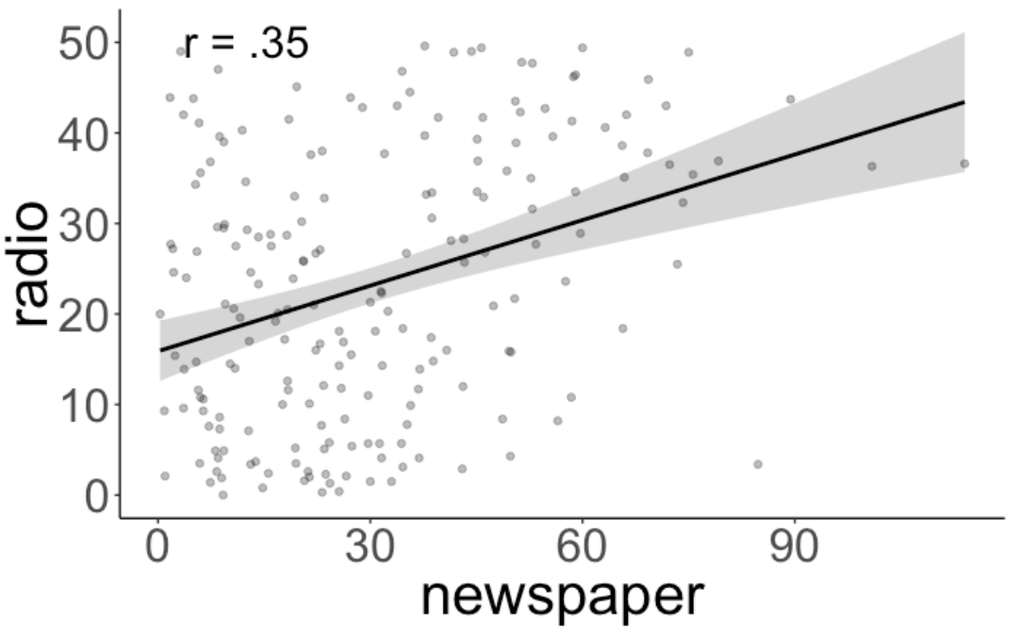
Are newspaper ads and sales related when controlling for radio ads and TV ads?

Relationship between newspaper ads and sales

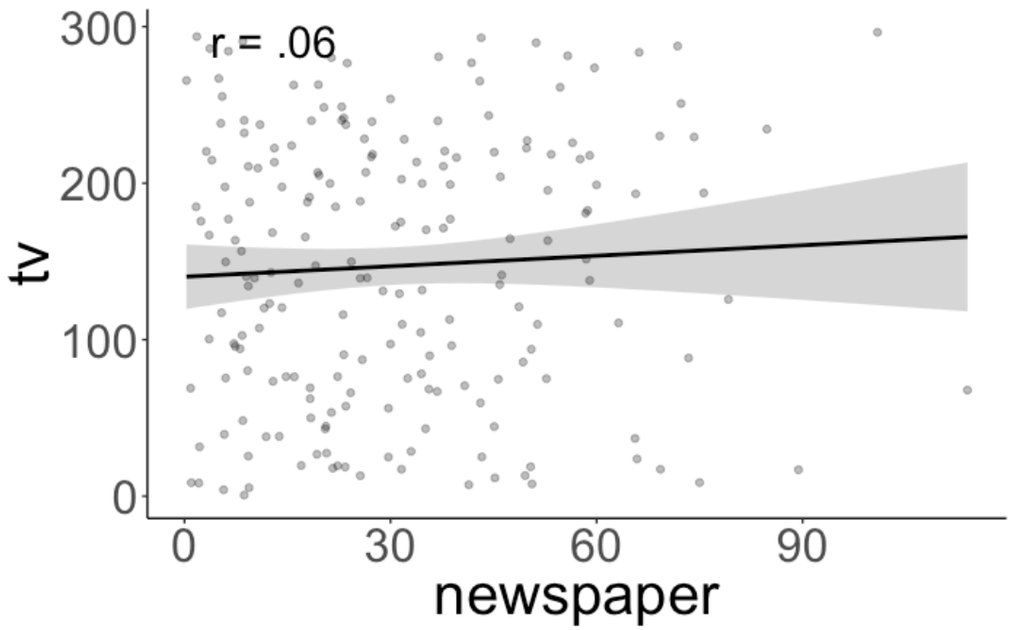


this is significant

Relationship between newspaper and radio ads



Relationship between newspaper and TV ads



```r
# fit the models
fit_c = lm(sales ~ 1 + tv + radio, data = df.ads)
fit_a = lm(sales ~ 1 + tv + radio + newspaper, data = df.ads)

# do the F test
anova(fit_c, fit_a)
```

```
Analysis of Variance Table

Model 1: sales ~ 1 + tv + radio
Model 2: sales ~ 1 + tv + radio + newspaper
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1    197 556.91
2    196 556.83  1  0.088717 0.0312 0.8599
```

it's not worth it

`sales ~ 1 + tv`

`sales ~ 1 + tv + newspaper`

it's worth it

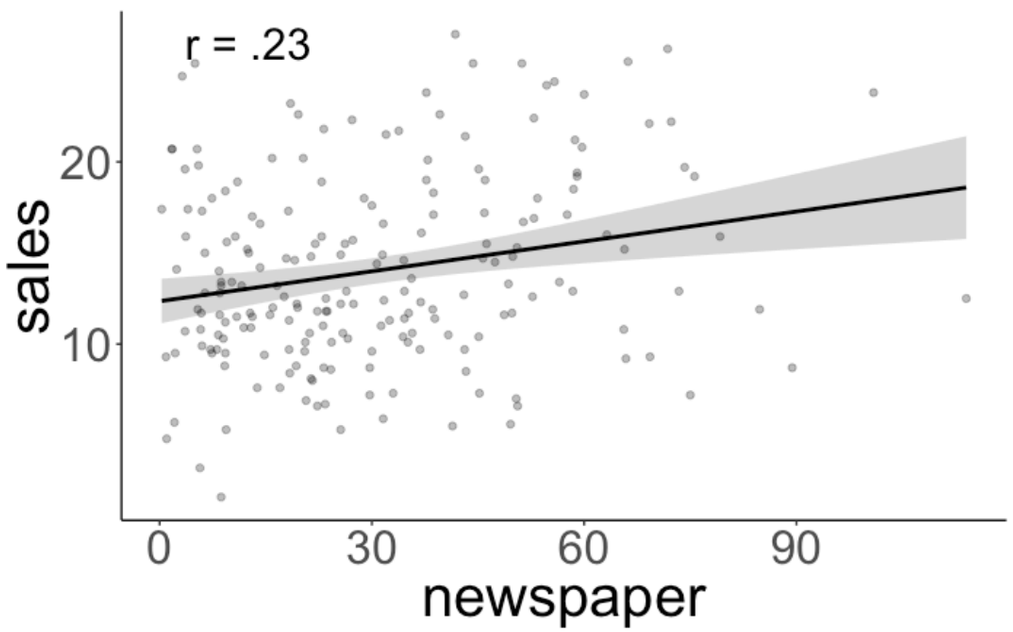
`sales ~ 1 + radio`

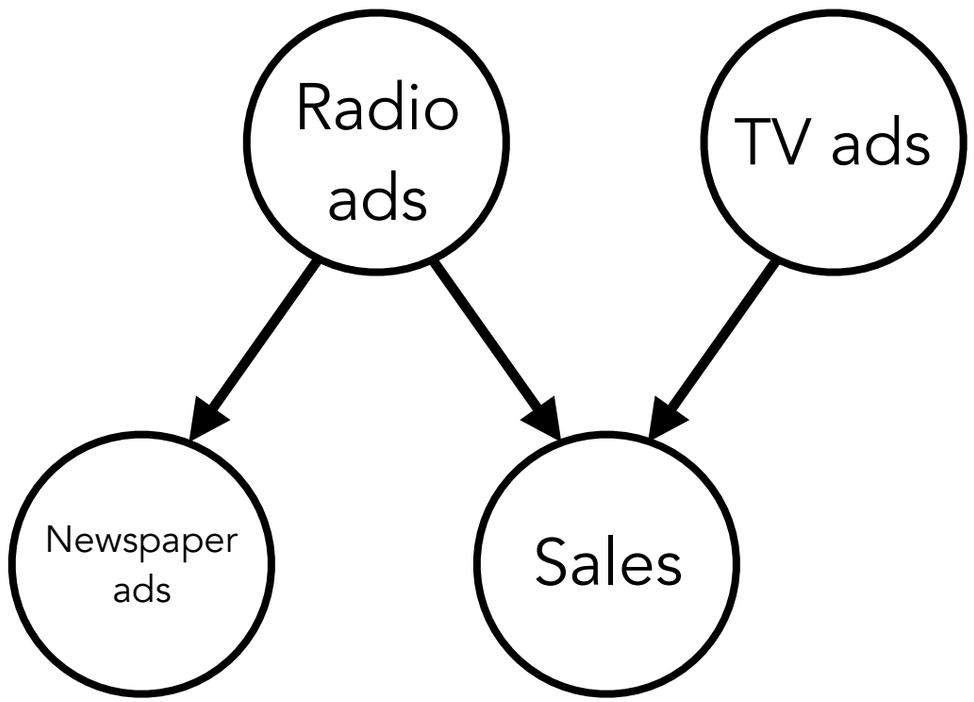
`sales ~ 1 + radio + newspaper`

it's not worth it

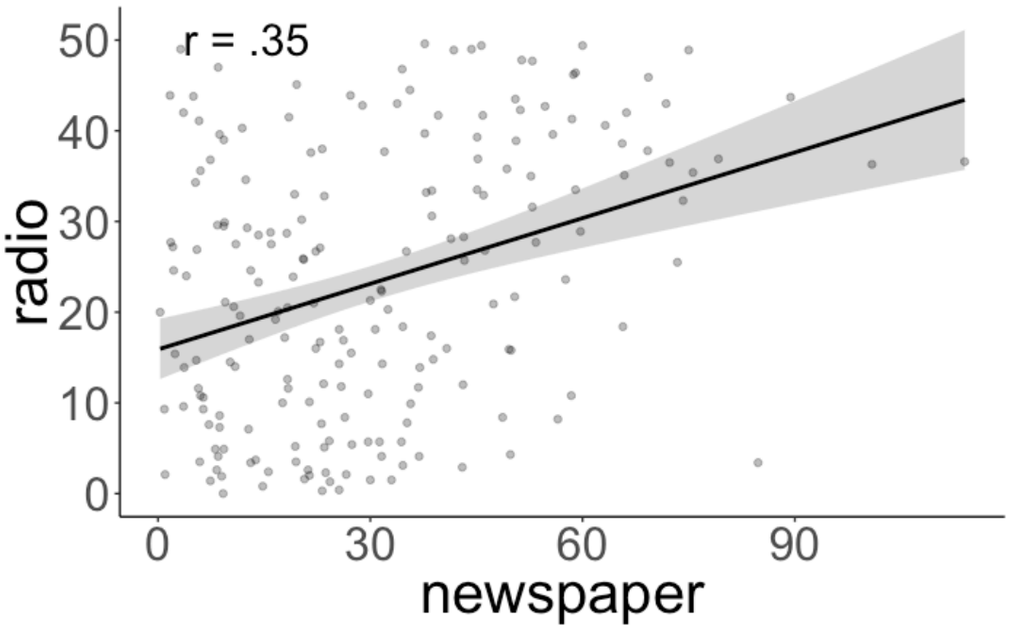
Are newspaper ads and sales related when controlling for radio ads and TV ads?

Relationship between newspaper ads and sales

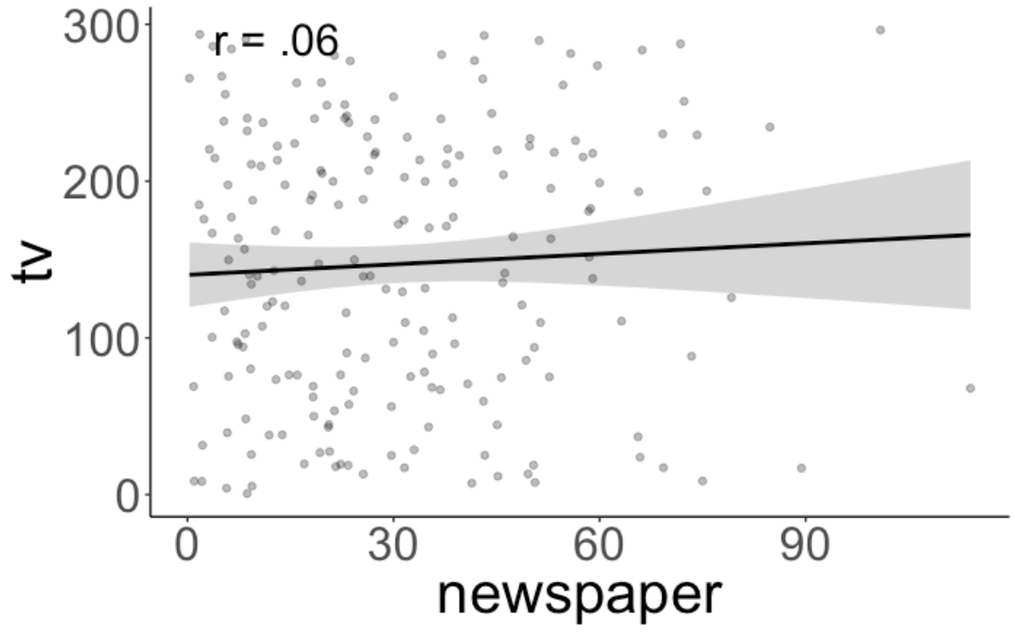




Relationship between newspaper and radio ads



Relationship between newspaper and TV ads



Categorical predictors

Credit data set

`df.credit`

index	income	limit	rating	cards	age	education	gender	student	married	ethnicity	balance
1	14.89	3606	283	2	34	11	Male	No	Yes	Caucasian	333
2	106.03	6645	483	3	82	15	Female	Yes	Yes	Asian	903
3	104.59	7075	514	4	71	11	Male	No	No	Asian	580
4	148.92	9504	681	3	36	11	Female	No	No	Asian	964
5	55.88	4897	357	2	68	16	Male	No	Yes	Caucasian	331
6	80.18	8047	569	4	77	10	Male	No	No	Caucasian	1151
7	21.00	3388	259	2	37	12	Female	No	No	African American	203
8	71.41	7114	512	2	87	9	Male	No	No	Asian	872
9	15.12	3300	266	5	66	13	Female	No	No	Caucasian	279
10	71.06	6819	491	3	41	19	Female	Yes	Yes	African American	1350

variable	description
income	in thousand dollars
limit	credit limit
rating	credit rating
cards	number of credit cards
age	in years
education	years of education
gender	male or female
student	student or not
married	married or not
ethnicity	African American, Asian, Caucasian
balance	average credit card debt in dollars

`nrow(df.credit) = 400`

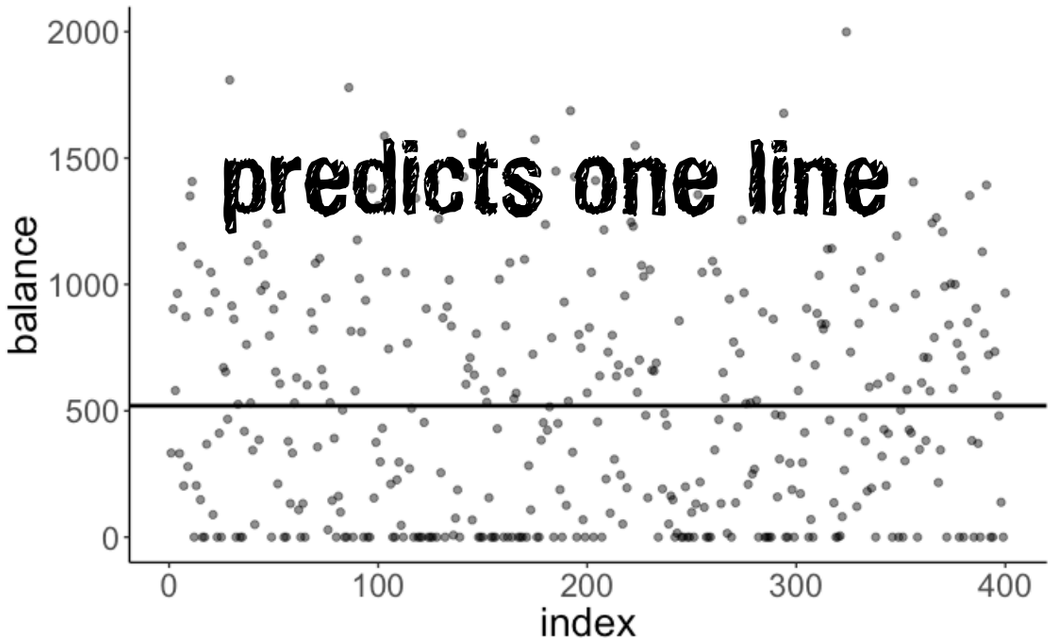
Do students have a different credit card balance from non-students?

H_0: Students and non-students have the same balance.

Model C

$$Y_i = \beta_0 + \epsilon_i$$

Model prediction



Fitted model

$$Y_i = 520.02 + e_i$$

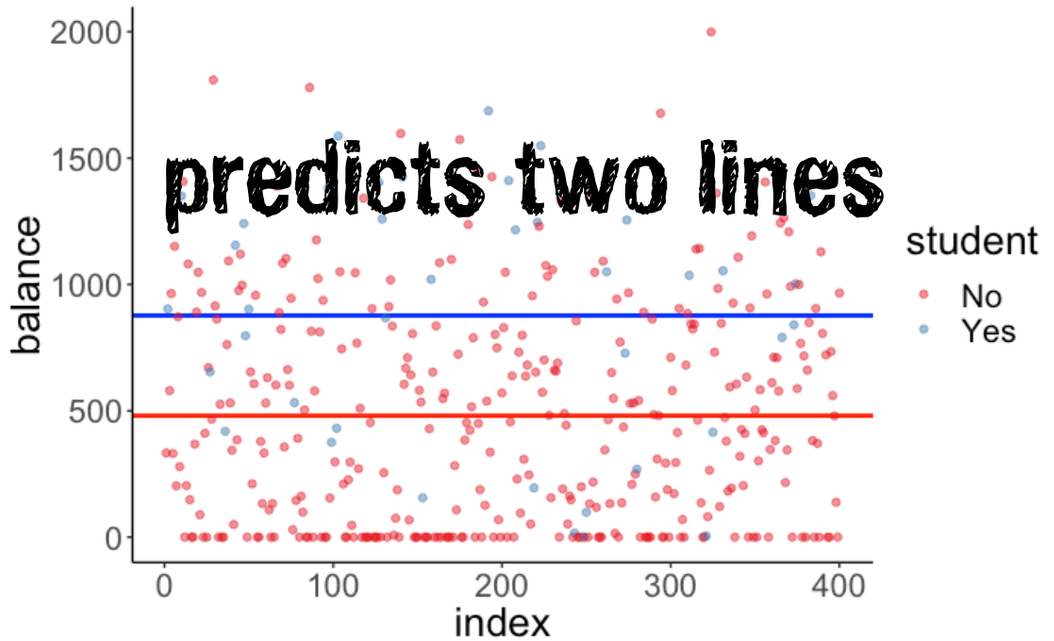
H_1: Students and non-students have different balances.

Model A

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

student

Model prediction



Fitted model

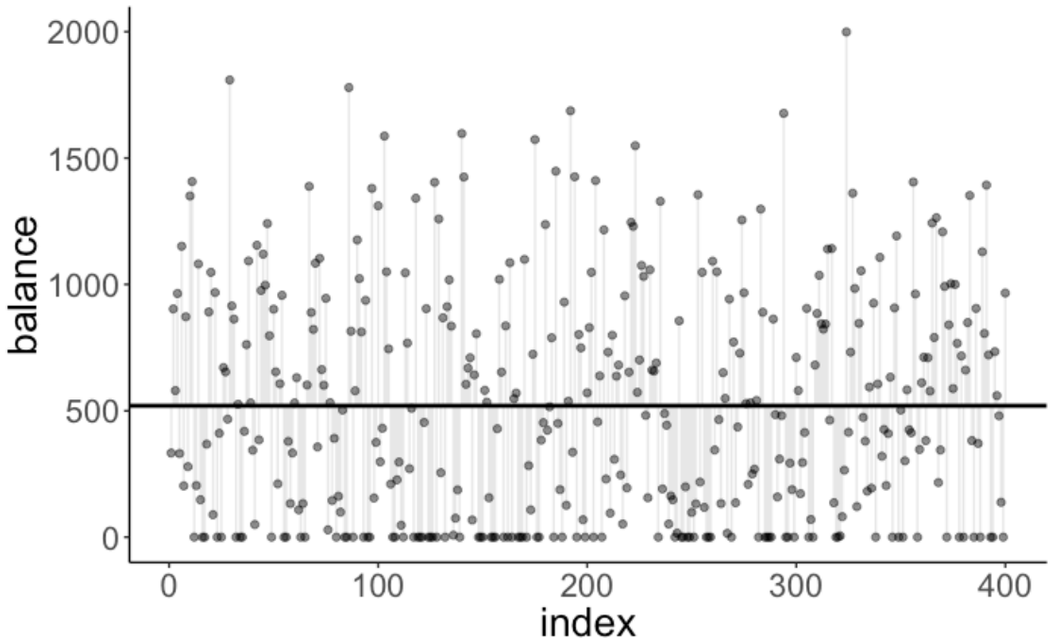
$$Y_i = 480.37 + 396.46 X_i + e_i$$

H_0: Students and non-students have the same balance.

Model C

$$Y_i = \beta_0 + \epsilon_i$$

Model prediction



Fitted model

$$Y_i = 520.02 + e_i$$

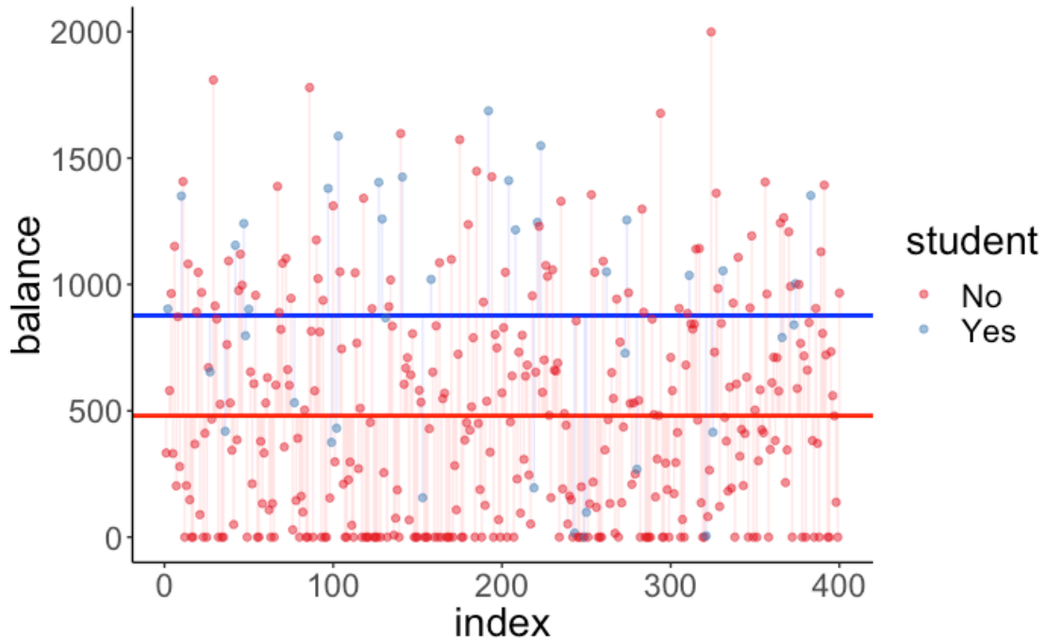
H_1: Students and non-students have different balances.

Model A

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

student

Model prediction



Fitted model

$$Y_i = 480.37 + 396.46 X_i + e_i$$

Worth it?

```r
# fit the models
fit_c = lm(balance ~ 1, data = df.credit)
fit_a = lm(balance ~ student, data = df.credit)

# run the F test
anova(fit_c, fit_a)
```

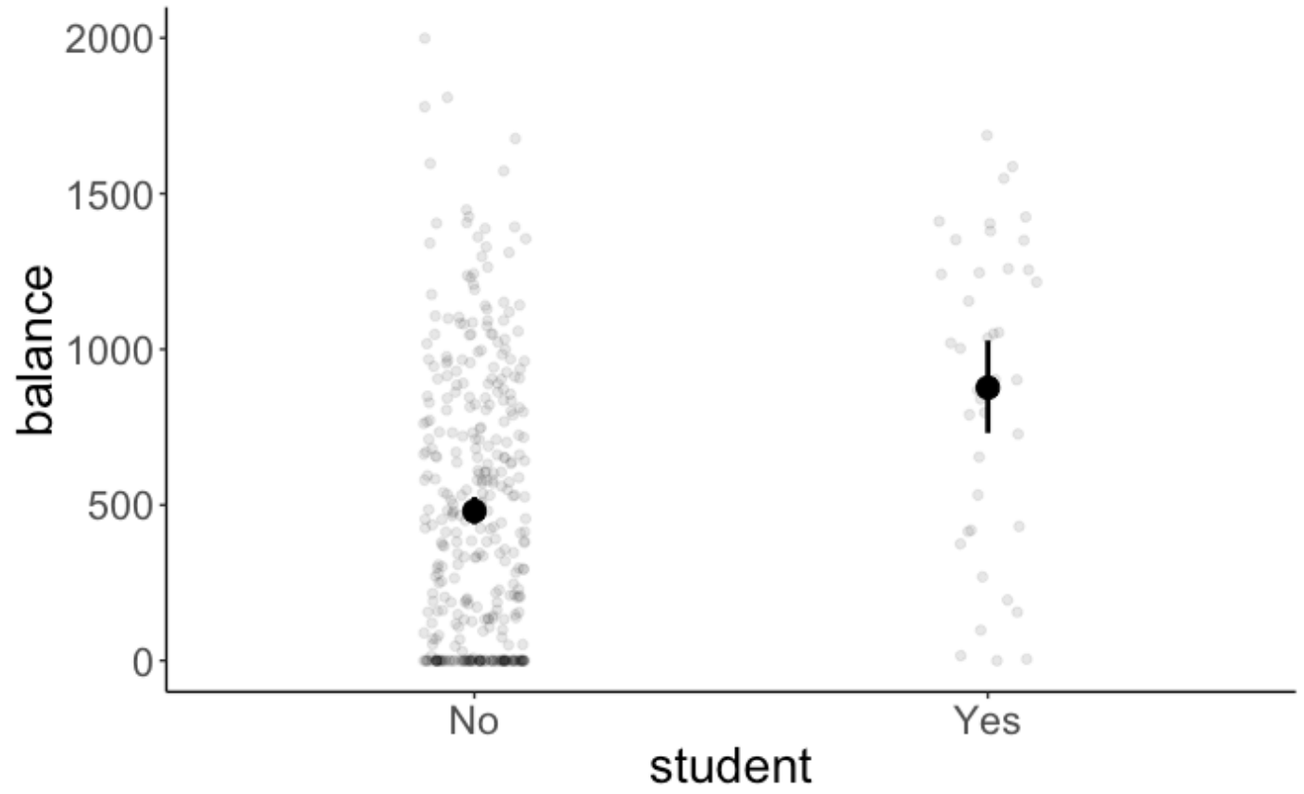
```
Analysis of Variance Table

Model 1: balance ~ 1
Model 2: balance ~ student
  Res.Df      RSS Df Sum of Sq      F    Pr(>F)
1    399 84339912
2    398 78681540  1   5658372 28.622 1.488e-07 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Worth it!

Two sample t-test (with independent groups)

Reporting the results



Students have a significantly higher average credit card balance (*Mean* = 876.83, *SD* = 490.00) than non-students (*Mean* = 480.37, *SD* = 439.41), $F(1, 398) = 28.622$, $p < .001$.

Interpreting the model

```
1 fit_a = lm(balance ~ student, data = df.credit)
2 fit_a %>%
3   summary()
```

```
Call:
lm(formula = balance ~ student, data = df.credit)

Residuals:
    Min      1Q  Median      3Q     Max
-876.82 -458.82  -40.87  341.88 1518.63

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   480.37      23.43   20.50  < 2e-16 ***
studentYes    396.46      74.10    5.35 1.49e-07 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 444.6 on 398 degrees of freedom
Multiple R-squared:  0.06709,   Adjusted R-squared:  0.06475
F-statistic: 28.62 on 1 and 398 DF,  p-value: 1.488e-07
```

Dummy coding

Dummy coding

$$\hat{Y}_i = 480.37 + 396.46 \cdot \text{student\_dummy}_i$$

if student = "No" $\hat{Y}_i = 480.37$

if student = "Yes" $\hat{Y}_i = 480.37 + 396.46 = 876.83$

student	student_dummy
No	0
Yes	1
No	0
No	0
No	0
No	0
No	0
No	0
No	0
No	0
No	0
Yes	1

- Reference category is coded as 0, the other category is coded as 1
- When thrown into an `lm()`, R automatically turns character columns into factors, and determines the reference category alphabetically

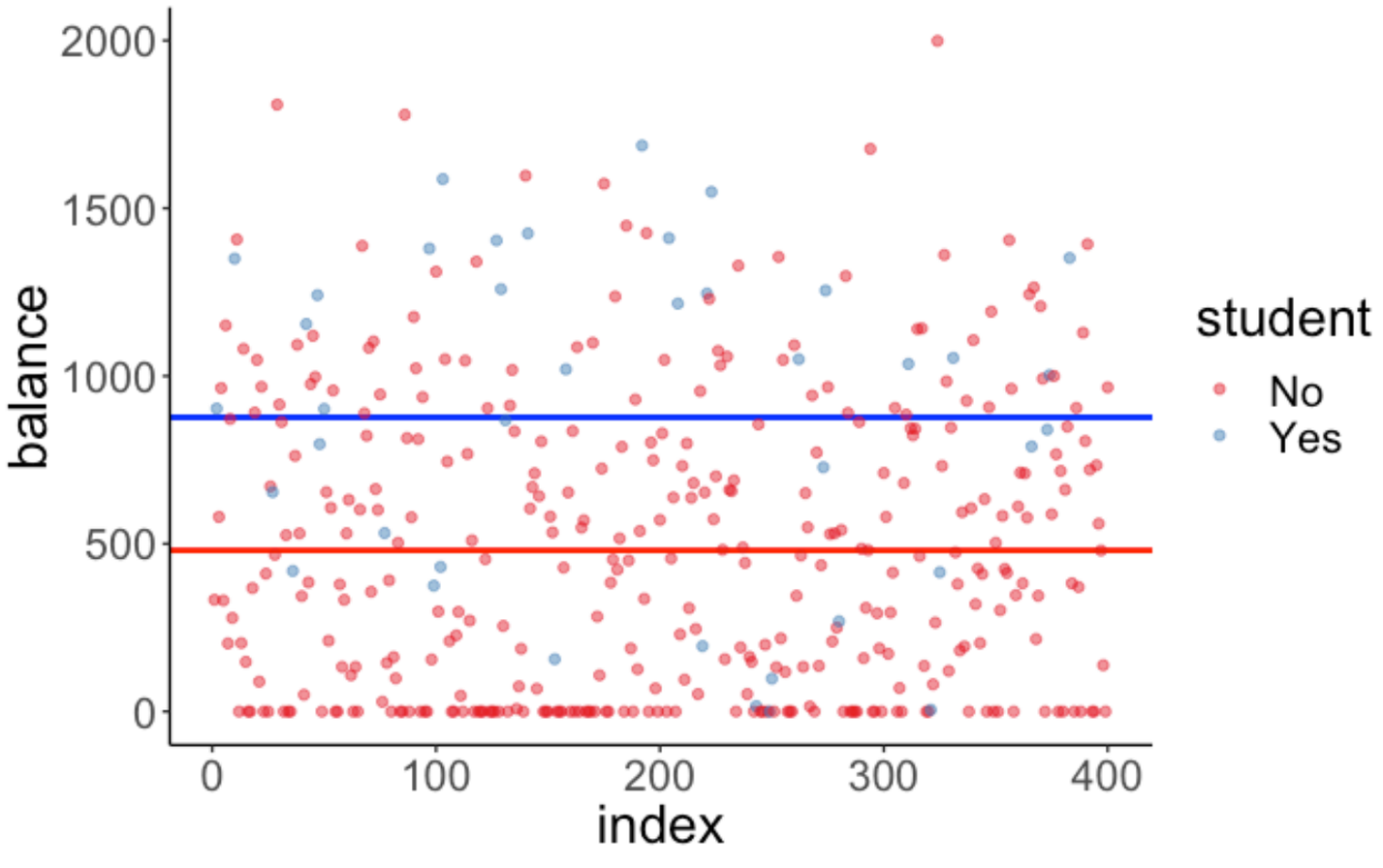
Dummy coding

$$\hat{Y}_i = 480.37 + 396.46 \cdot \text{student\_dummy}_i$$

if student = "No" $\hat{Y}_i = 480.37$

if student = "Yes" $\hat{Y}_i = 480.37 + 396.46 = 876.83$

student	student_dummy
No	0
Yes	1
No	0
No	0
No	0
No	0
No	0
No	0
No	0
No	0
No	0
Yes	1



Categorical and continuous predictor

Credit data set

`df.credit`

index	income	limit	rating	cards	age	education	gender	student	married	ethnicity	balance
1	14.89	3606	283	2	34	11	Male	No	Yes	Caucasian	333
2	106.03	6645	483	3	82	15	Female	Yes	Yes	Asian	903
3	104.59	7075	514	4	71	11	Male	No	No	Asian	580
4	148.92	9504	681	3	36	11	Female	No	No	Asian	964
5	55.88	4897	357	2	68	16	Male	No	Yes	Caucasian	331
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7	21.00	3388	259	2	37	12	Female	No	No	African American	203
8	71.41	7114	512	2	87	9	Male	No	No	Asian	872
9	15.12	3300	266	5	66	13	Female	No	No	Caucasian	279
10	71.06	6819	491	3	41	19	Female	Yes	Yes	African American	1350

variable	description
income	in thousand dollars
limit	credit limit
rating	credit rating
cards	number of credit cards
age	in years
education	years of education
gender	male or female
student	student or not
married	married or not
ethnicity	African American, Asian, Caucasian
balance	average credit card debt in dollars

`nrow(df.credit) = 400`

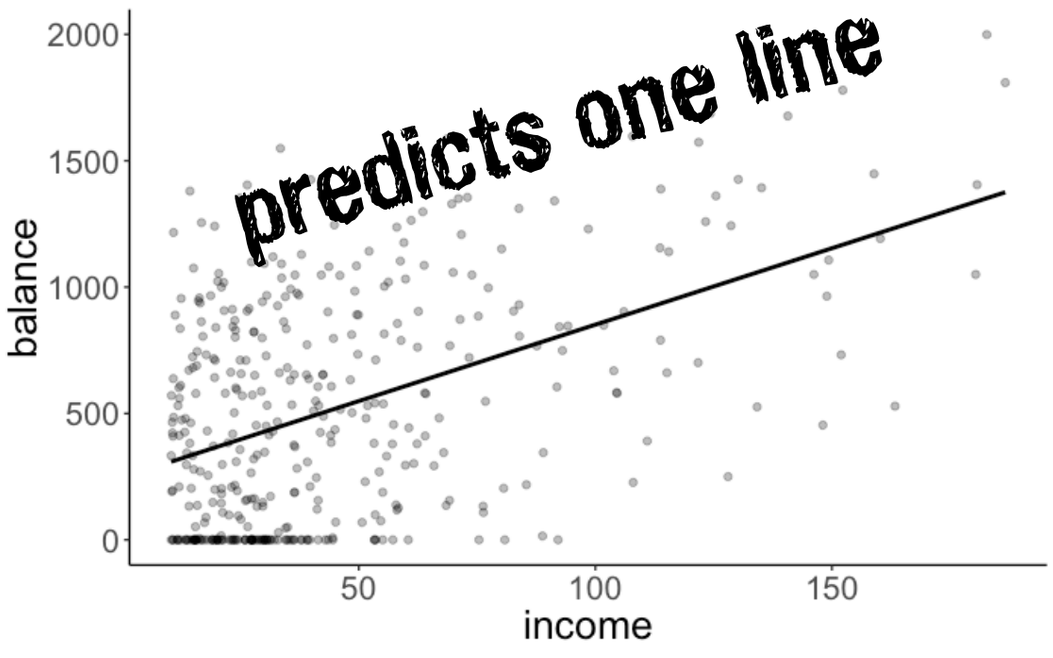
Do students have a different credit card balance from non-students, when controlling for income?

H_0: Students and non-students have the same balance, when controlling for income.

Model C

$$\text{balance}_i = \beta_0 + \beta_1 \text{income}_i + \epsilon_i$$

Model prediction



Fitted model

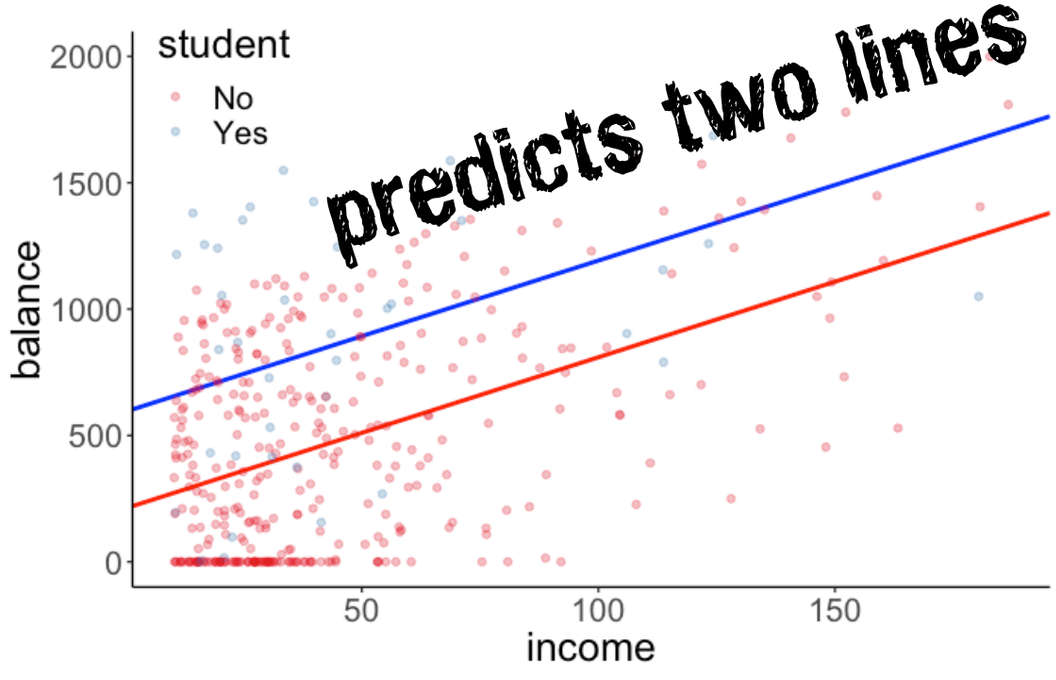
$$\widehat{\text{balance}}_i = 246.515 + 6.048 \cdot \text{income}_i$$

H_1: Students and non-students have different balances, when controlling for income.

Model A

$$\text{balance}_i = \beta_0 + \beta_1 \text{income}_i + \beta_2 \text{student}_i + \epsilon_i$$

Model prediction



Fitted model

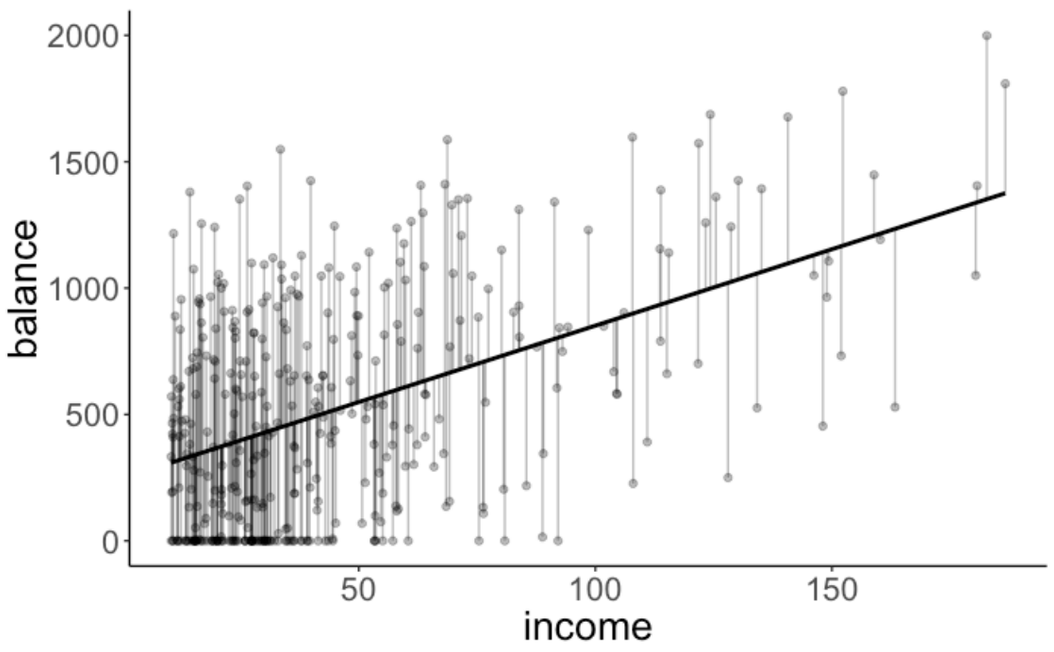
$$\widehat{\text{balance}}_i = 211.14 + 5.98 \cdot \text{income}_i + 382.67 \cdot \text{student}_i$$

H_0: Students and non-students have the same balance, when controlling for income.

Model C

$$\text{balance}_i = \beta_0 + \beta_1 \text{income}_i + \epsilon_i$$

Model prediction



Fitted model

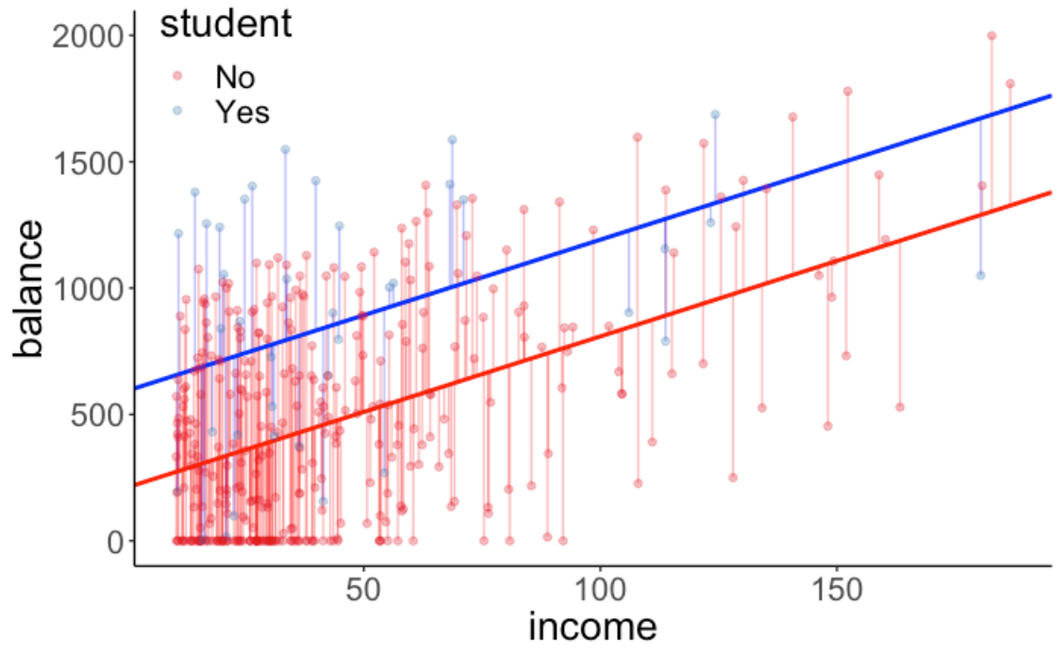
$$\widehat{\text{balance}}_i = 246.515 + 6.048 \cdot \text{income}_i$$

H_1: Students and non-students have different balances, when controlling for income.

Model A

$$\text{balance}_i = \beta_0 + \beta_1 \text{income}_i + \beta_2 \text{student}_i + \epsilon_i$$

Model prediction



Fitted model

$$\widehat{\text{balance}}_i = 211.14 + 5.98 \cdot \text{income}_i + 382.67 \cdot \text{student}_i$$

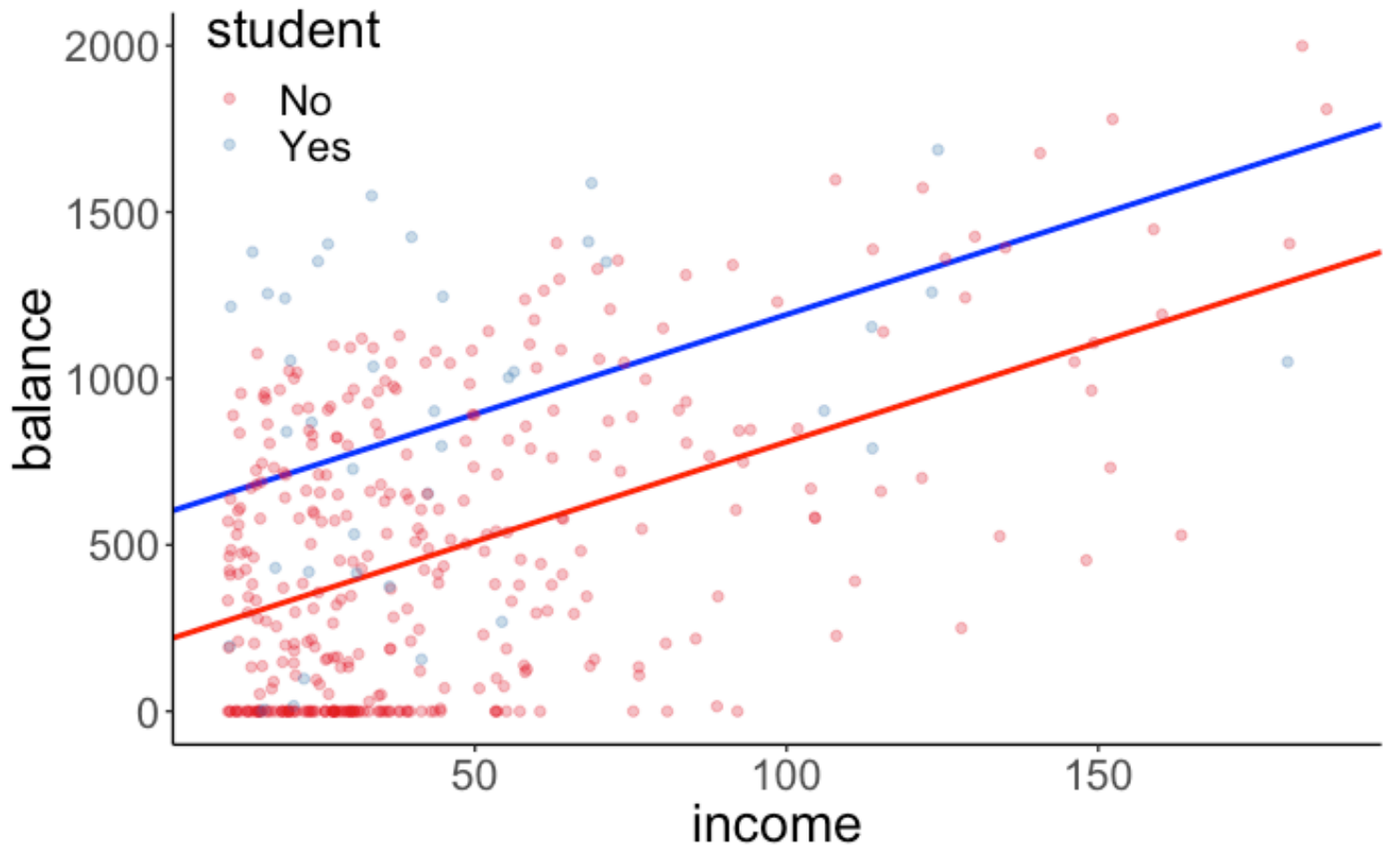
Worth it?

```r
# fit the models
fit_c = lm(balance ~ 1 + income, df.credit)
fit_a = lm(balance ~ 1 + income + student, df.credit)

# run the F test
anova(fit_c, fit_a)
```

```
Analysis of Variance Table

Model 1: balance ~ 1 + income
Model 2: balance ~ 1 + income + student
  Res.Df      RSS Df Sum of Sq      F    Pr(>F)
1    398 66208745
2    397 60939054  1   5269691 34.331 9.776e-09 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Worth it!

Interpretation

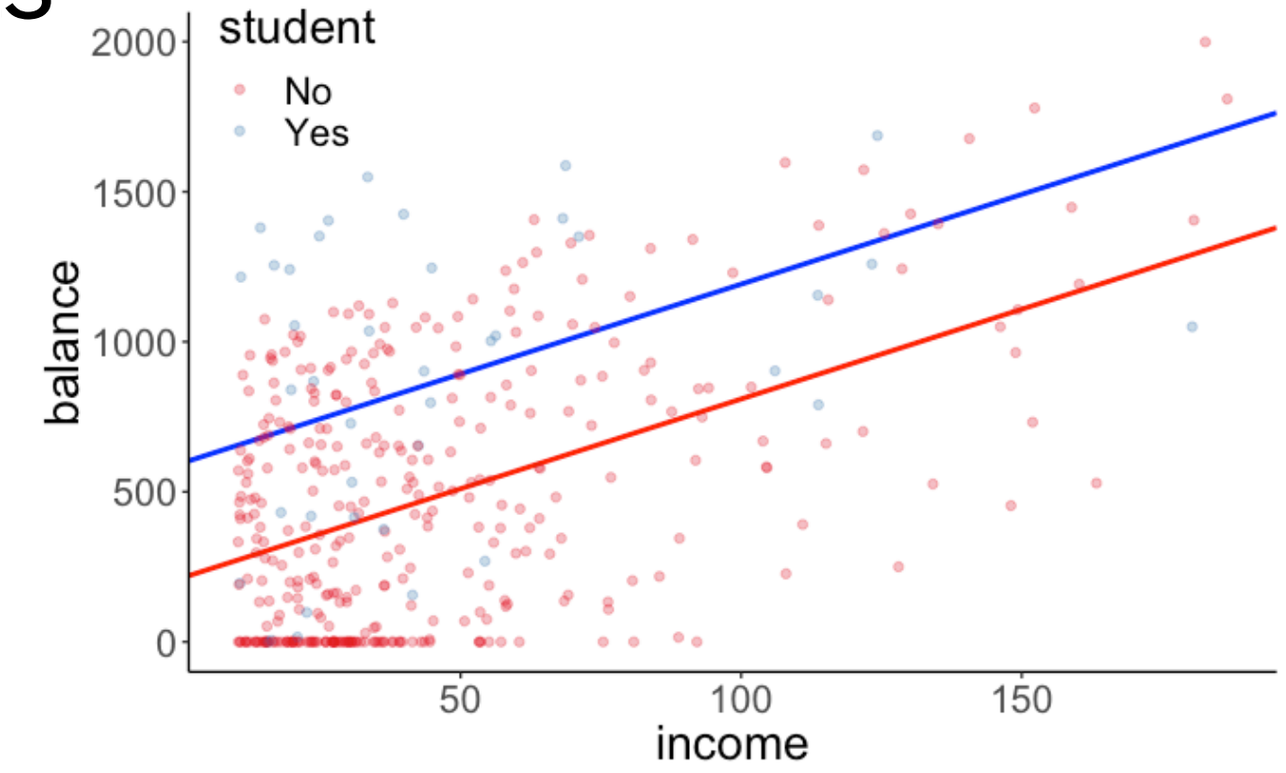


$$\widehat{\text{balance}}_i = 211.14 + 5.98 \cdot \text{income}_i + 382.67 \cdot \text{student}_i$$

if student = "No" $\widehat{\text{balance}}_i = 211.14 + 5.98 \cdot \text{income}_i$

if student = "Yes" $\begin{aligned} \widehat{\text{balance}}_i &= 211.14 + 5.98 \cdot \text{income}_i + 382.67 \\ &= 211.14 + 382.67 + 5.98 \cdot \text{income}_i \\ &= 593.81 + 5.98 \cdot \text{income}_i \end{aligned}$

Reporting the results



Controlling for income, students have a significantly higher average credit card balance (*Mean* = 876.83, *SD* = 490.00) than non-students (*Mean* = 480.37, *SD* = 439.41), $F(1, 397) = 34.331$, $p < .001$.

Interaction

Is the relationship between level of income and balance different for students than it is for non-students?

Compact Model

$$\widehat{\text{balance}}_i = b_0 + b_1\text{income}_i + b_2\text{student}_i$$

Augmented Model

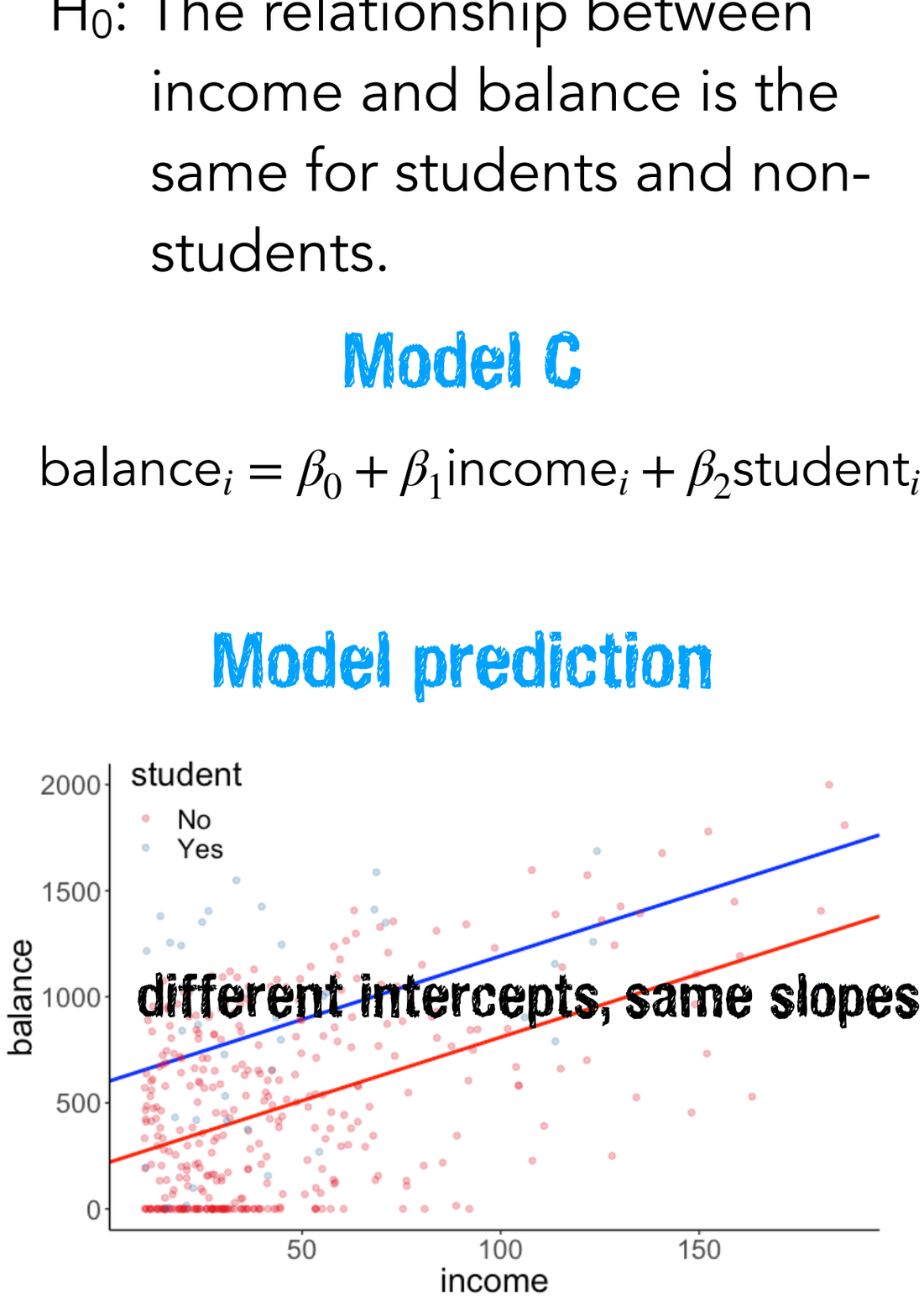
$$\widehat{\text{balance}}_i = b_0 + b_1\text{income}_i + b_2\text{student}_i + b_3(\text{income}_i \times \text{student}_i)$$

H_0: The relationship between income and balance is the same for students and non-students.

Model C

$$\text{balance}_i = \beta_0 + \beta_1 \text{income}_i + \beta_2 \text{student}_i + \epsilon_i$$

Model prediction



Fitted model

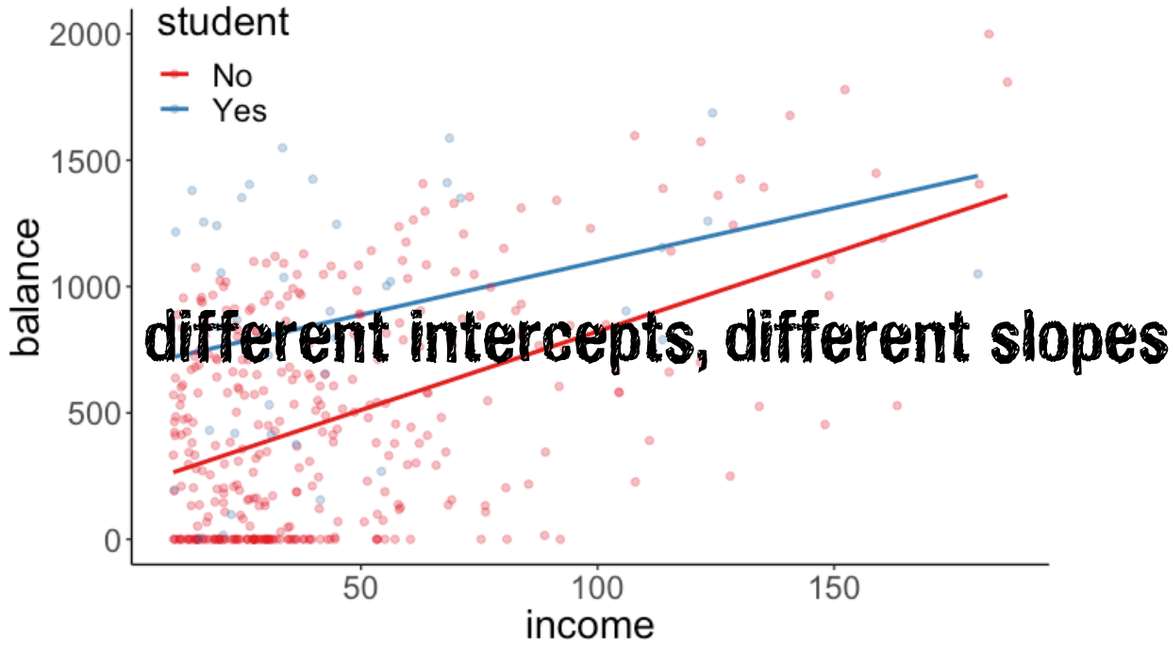
$$\widehat{\text{balance}}_i = 211.14 + 5.98 \cdot \text{income}_i + 382.67 \cdot \text{student}_i$$

H_1: The relationship between income and balance differs between students and non-students.

Model A

$$\widehat{\text{balance}}_i = b_0 + b_1 \text{income}_i + b_2 \text{student}_i + b_3(\text{income}_i \times \text{student}_i)$$

Model prediction



Fitted model

$$\widehat{\text{balance}}_i = 200.62 + 6.22 \cdot \text{income}_i + 476.68 \cdot \text{student}_i - 2.00 \cdot (\text{income}_i \times \text{student}_i)$$

Worth it?

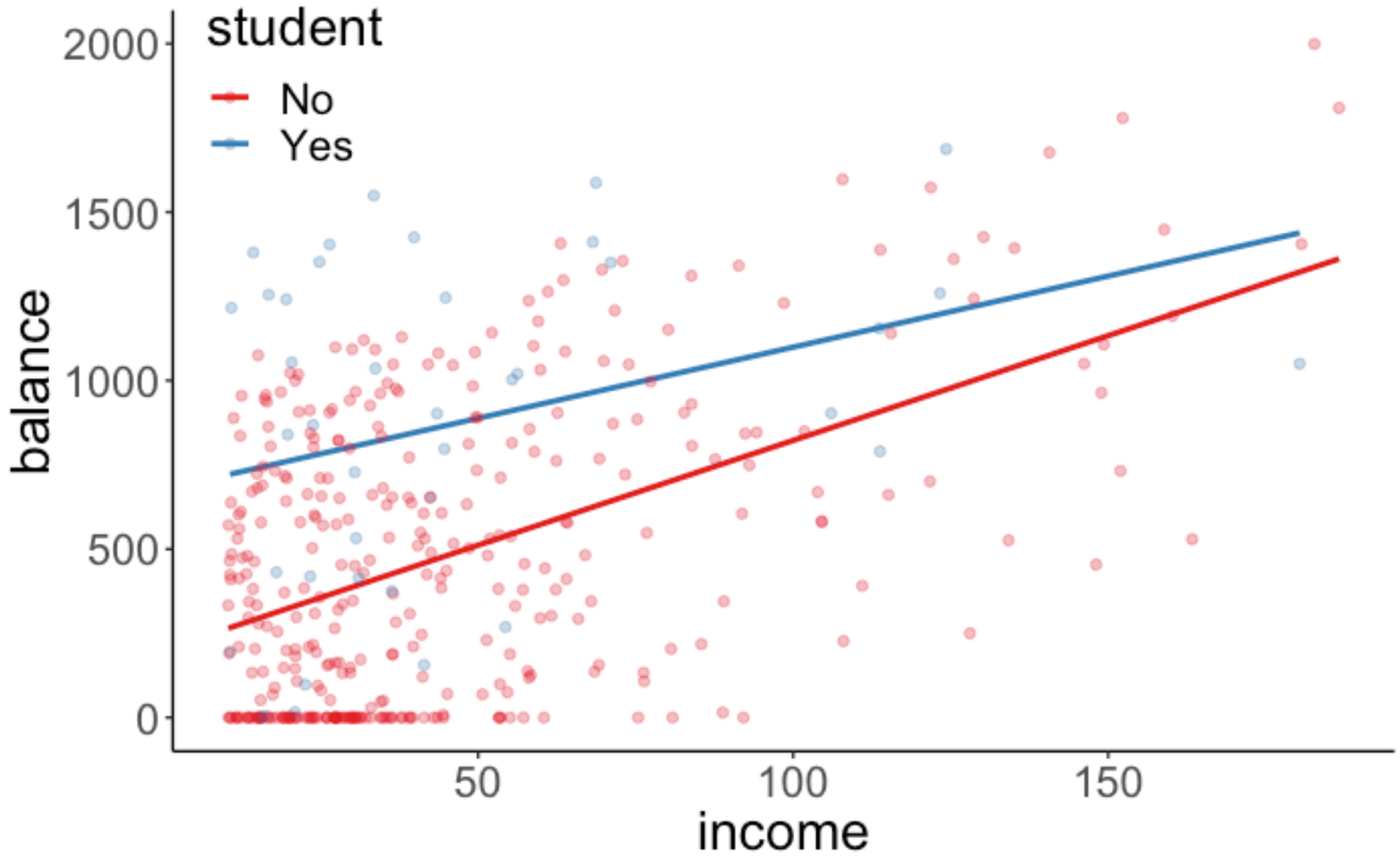
Is the relationship between level of income and balance different for students than it is for non-students?

```
1 # fit models
2 fit_c = lm(formula = balance ~ income + student, data = df.credit)
3 fit_a = lm(formula = balance ~ income * student, data = df.credit)
4
5 # F-test
6 anova(fit_c, fit_a)
```

```
Analysis of Variance Table

Model 1: balance ~ income + student
Model 2: balance ~ income * student
  Res.Df      RSS Df Sum of Sq      F Pr(>F)
1    397 60939054
2    396 60734545  1    204509 1.3334 0.2489
```

not worth it!

Interpretation



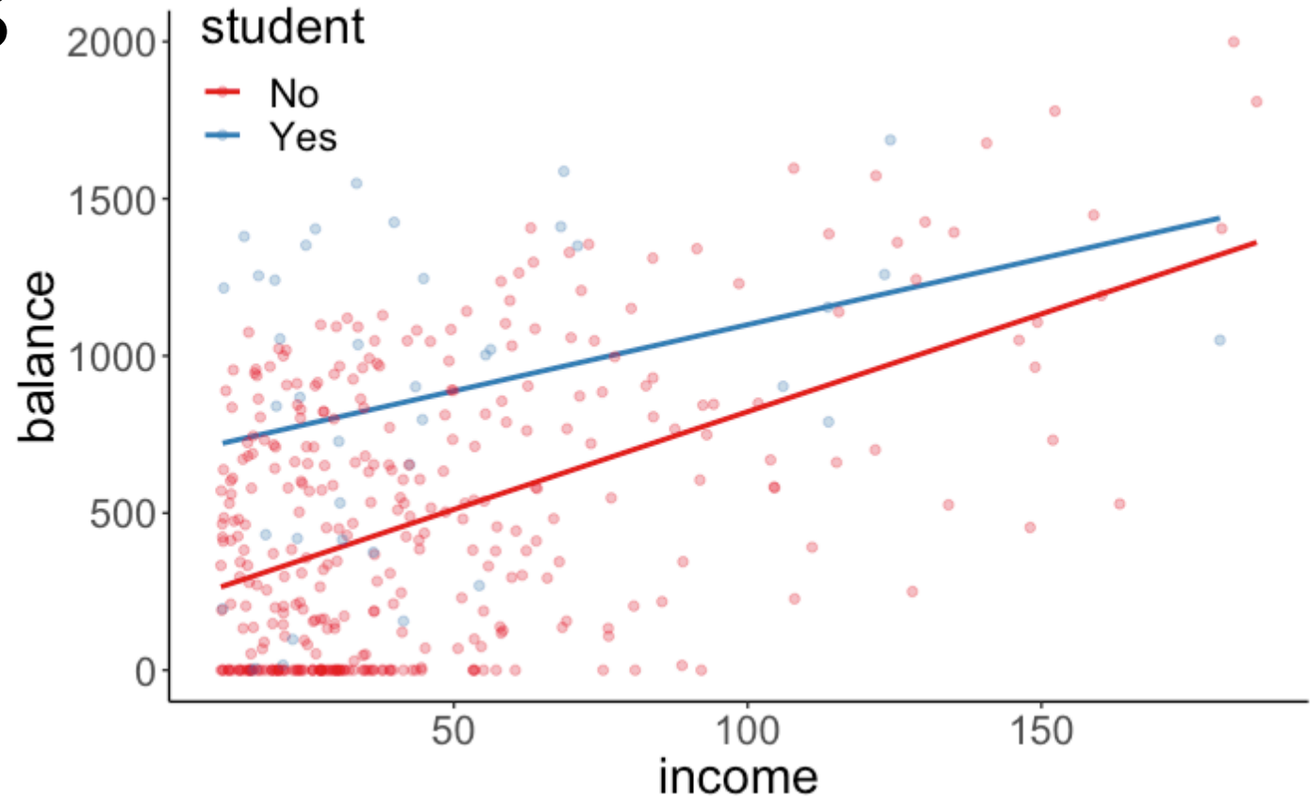
$$\widehat{\text{balance}}_i = 200.62 + 6.22 \cdot \text{income}_i + 476.68 \cdot \text{student}_i - 2.00 \cdot (\text{income}_i \times \text{student}_i)$$

if student = "No" $\widehat{\text{balance}}_i = 200.62 + 6.22 \cdot \text{income}_i$

if student = "Yes"

$$\begin{aligned}\widehat{\text{balance}}_i &= 200.62 + 6.22 \cdot \text{income}_i + 476.68 \cdot 1 - 2.00 \cdot (\text{income}_i \times 1) \\ &= 677.3 + 6.22 \cdot \text{income}_i - 2.00 \cdot \text{income}_i \\ &= 677.3 + 4.22 \cdot \text{income}_i\end{aligned}$$

Reporting the results



There is no significant difference in the relationship between income and balance for students versus non-students, $F(1, 396) = 1.33$, $p = 0.25$.

Summary

- Multiple regression
  - Model assumptions: multi-collinearity
- Several continuous predictors
  - Hypothesis tests
  - Interpreting parameters
  - Reporting results
- One categorical predictor
- Both continuous and categorical predictors
- Interactions

Thank you!