Washington Math Olympiad

Proof Section

May 30, 2024

Welcome to the proof section of the Washington Math Olympiad (WAMO), a contest dedicated to serving and strengthening the Washington math community.

Instructions:

- You will have **4.5 hours** to solve **5 proof problems** worth **7 points each**.
- The only permitted aids are writing utensils, erasers, rulers, compasses, and blank scratch paper.
- Graph paper, calculators, and any other computing devices are **not** allowed.

1. **[7]** In equilateral triangle ABC, let D be a point on ray BA past A, and E a point on ray BC past C. Given that $CD = DE$, prove that $AD = BE$.

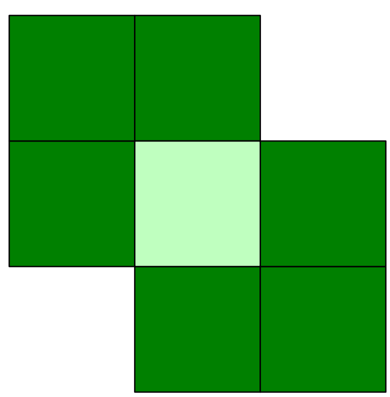
2. **[7]** Determine all ordered triples of positive integers (a, b, c) such that $2^a 3^b + 1 = c^2$.

3. **[7]** For his friend's 10^{th} birthday, Wesley bakes a perfectly circular cake measuring 10 inches in diameter. He puts 10 candles in the cake. Prove that there exist 2 of these candles which are at most 4 inches apart.

4. **[7]** Determine, with proof, the largest constant c such that the following inequality holds for all distinct positive integers x, y, z:

$$\frac{x+y+z}{3} \geq \sqrt[3]{xyz + c}.$$

5. **[7]** Kyle is coloring squares on an empty infinite grid. For each square he colors, his "score" is incremented by how many of its 8 neighbors were previously colored. For example, coloring the center square below would increment his score by 6.



Determine his largest possible score after coloring 2024 squares.