$.\bar{9} = 1$
A Short Proof For Polymathematics

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I found this problem at a blog called Polymathematics (`http://polymathematics.typepad.com/polymath/2006/06/the_saga_contin.html`) and in the comments there were many people disputing the fact that $.\bar{9} = 1$. On the site and in the comments there were no proofs, although the author did a wonderful job appealing to logic and reasoning. In the end though, some people still refuse to believe it is true. Arguments often centered around the fact that $.\bar{9}$ is not an integer and therefore $.\bar{9} \neq 1$. The following is a short proof that in fact, $.\bar{9} = 1$. If you find any flaws, please let me know.

Proof. The repeating decimal $.\bar{9}$ can be represented as an infinite sum of fractions

$$.\bar{9} = \frac{9}{10} + \frac{9}{100} + \frac{9}{100} \dots \tag{1}$$

Which can be rewritten as

$$.\bar{9} = \sum_{n=1}^{\infty} \left(\frac{9}{10}\right)\left(\frac{1}{10}\right)^{n-1} \tag{2}$$

Which is a geometric series of the form

$$\sum_{n=1}^{\infty} ar^{n-1} \tag{3}$$

The series is geometric and since $|r| = \frac{1}{10}$ which is < 1, the series converges and its sum is given as $\frac{a}{1-r}$ (see Stewart's Calculus book). In our example $a = \frac{9}{10}$ and $r = \frac{1}{10}$, therefore

$$.\bar{9} = \sum_{n=1}^{\infty} \left(\frac{9}{10}\right)\left(\frac{1}{10}\right)^{n-1} = \frac{\frac{9}{10}}{1-\frac{1}{10}} = \frac{\frac{9}{10}}{\frac{9}{10}} = \frac{90}{90} = 1 \tag{4}$$

$\square$