Research Commentary

Observations on Citation Practices in Mathematics Education Research

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In this research commentary, I argue that the field of mathematics education as a whole can and should improve its citation practices. I begin by discussing 4 forms of citation practice and considering how they vary with respect to transparency of voice. I then discuss several ways our citation practices may misrepresent cited authors' ideas, providing examples to illustrate the errors. I conclude by suggesting ways that we as writers (but also as reviewers and as graduate faculty) might jointly work toward improving our citation practices.

Key words: Citation practices; Researcher development

One critical aspect of conducting and reporting research in mathematics education is building on the work of others. We cite others' work for multiple reasons, among them to support rationales for the research problems we explore; to establish what we currently know and do not know in order to situate the relevance of our work; to help us describe the theoretical frameworks that guide our inquiry; and to demonstrate similarities and differences between perspectives, methods, and results. Indeed, "the embedding of arguments in networks of references not only suggests a cumulative and linear progression, but reminds us that statements are invariably a response to previous statements and are themselves available for further statements by others" (Hyland, 1999, p. 343). I believe we often take these citation practices for granted, paying insufficient attention to these practices in the refinement of our own writing and failing to explicitly articulate these practices as we give feedback to others, particularly novices. Along with Henige (2006), I posit that "the citation per se is below the radar in both scientific thought and scientific discourse" (p. 106).

The purposes of this research commentary are (a) to describe the forms and purposes of our citation practices, (b) to argue that the field of mathematics education as a whole can and should improve these practices, and (c) to suggest some ways that we might jointly work toward this improvement. Throughout the commentary, I use examples drawn from an analysis (Leatham & Winiecke, 2014)

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of a substantial set of references to the *Case of Benny* (Erlwanger, 1973).[1] That analysis revealed a large number of errors, causing me to reflect on the nature of those errors and the ways that, as a mathematics education research community, our citation practices might be improved. Taken as a whole, this commentary is intended to help all of us in our ongoing "efforts to accurately situate . . . [our] works in the context of the larger field of mathematics education" (Heid, Graysay, & Peters, 2011, p. 307).

Forms and Functions of Citation Practices

Any citation can be viewed as a claim—a claim that the cited reference supports the statement made by the citing author. Thus, one can examine citation practices by examining the ways those practices use citations to support claims. I subdivide citation practices into four main forms, each of which functions in a slightly different way to situate our work in the work of others: (a) stand-alone quotation, (b) integrated quotation, (c) paraphrasing citation, and (d) supporting citation.

When considering the validity of the claim made by each of these forms, I find it useful to consider how transparent each form is with respect to voice. Stand-alone quotations make transparent the voice of the cited author but leave fairly opaque the voice of the citing author, whereas supporting citations make transparent the voice of the citing author and leave opaque the voice of the cited author. Both integrated quotations and paraphrasing citations blur the voices.

Now, consider how the transparency of voice might be related to discerning the validity of a given claim. For example, suppose an integrated quotation accurately quotes the cited author but misrepresents the meaning of that quotation by placing it in a significantly different context. In this situation, the voices of the cited and citing authors are blurred. This citation instance would be inaccurate because the form implies that the citing and cited authors share the claim being made, but in reality, it is only the citing author who is making the claim. Or consider the ramifications of opaqueness in the cited author's voice. Given a sentence with no direct quotes that ends in a citation, the reader is left to discern whether the sentence is a paraphrasing or supporting citation. The citing author is making a claim about something the cited author said, but much is left to the reader to infer about the relationship between the voices of the citing and cited authors. I propose that these four citation forms can be better understood by analyzing them according to the transparency of voice. In what follows, I discuss each form and use examples to illustrate appropriate practice.

Stand-alone quotations are intended to share with the reader exactly what the cited author said and are thus quite transparent with respect to the cited author's voice. However, stand-alone quotations are also recontextualized by the citing

[1] Although Erlwanger (1973) is commonly referred to as the Case of Benny, the case is also presented along with five other cases in Erlwanger's dissertation (Erlwanger, 1974) and paired with the Case of Mat in Erlwanger (1975).

author from the original context to the new context. This recontextualization makes the citing author's voice fairly opaque. Consider the following example of a stand-alone quotation:

> Erlwanger [1973] concluded that the type of instruction Benny received "tends to develop in the pupil an inflexible rule-oriented attitude toward mathematics, in which rules that conflict with intuition are considered 'magical' and the quest for answers 'a wild goose chase'" (p. 25). (Yackel & Rasmussen, 2002, p. 328)

This direct quote from Erlwanger (1973) is quoted accurately—the words in quotes are exactly the same ones Erlwanger used. In addition, the beginning of the sentence is not in Erlwanger's exact words, but this setup is consistent with his meaning. Erlwanger did indeed make this statement in describing a conclusion of his study, and he did indeed make that claim with respect to "the type of instruction Benny received" (Yackel & Rasmussen, 2002, p. 328).

Integrated quotations directly quote others, but these quotes are integrated into the fabric of the text at hand. Integrated quotations thus meld together the voices of the cited and citing authors, and their very form implies that the citing author's and cited author's meanings are the same. The cited author's ideas have been interpreted by the citing author as consistent with the latter's own intended meaning and as capable of being used to replace or supplement what the citing author is saying. Consider, for example, the following integrated quotation:

> The problem of applying rules without reason is that it can led [sic] to as many correct solutions as incorrect solutions, depending on the tasks provided, thus adding to the confusion and leading to the development of thinking that Benny exhibited (Erlwanger, 1973), namely, *but really they're the same, no matter what the key says* (p. 15). (Baturo, Cooper, & McRobbie, 1999, p. 79)

In this citation instance, Baturo, Cooper, and McRobbie (1999) are actually making their own claim, but using Benny's (Erlwanger, 1973) exact words to do so. In this case, their only claim about Benny is that he said this, which he did—Benny is quoted correctly. Their broader claim about "applying rules without reason" (Baturo et al., 1999, p. 79) leading to this kind of thinking is appropriately attributed to themselves, not to Erlwanger. Although the authors use italics rather than quotation marks to indicate the direct quote, Erlwanger's words are clearly and accurately attributed to him, as are his ideas.

Paraphrased citations remove the cited author's exact wording and replace it with the writer's interpretation written in their own words. A paraphrased citation is thus a claim about what the cited author said but in the voice of the citing author. Paraphrasing often involves summarizing the ideas of others. There is a great deal of interpretation involved in paraphrasing because the author is really sharing their understanding of the cited author's work. Consider the following example:

> Erlwanger (1973) found that Benny, a sixth grader, had developed a coherent rationale that accounted for his experiences in using Individually Prescribed Instruction (IPI) materials. (Cobb & Steffe, 1983, p. 84)

Here the authors accurately summarize a major theme of the Case of Benny—"that students are always trying to make sense of their environment, necessarily developing conceptions (more or less 'mathematical' depending on that environment) as they try to make sense of their experiences" (Leatham & Winiecke, 2014, p. 105). The details are correct in that Benny was indeed in sixth grade and was using the IPI materials, and the overall interpretation is consistent with Erlwanger's report of the consistency of Benny's conception and the circumstances in which he was learning mathematics. Furthermore, the citing authors use their own language and sentence structure to summarize Erlwanger's (1973) findings.

The final citation practice form—supporting citation—involves using citations alone to support the citing author's ideas. Such citing makes transparent the voice of the citing author but makes opaque that of the cited author, requiring the reader to draw inferences about how the cited author's work supports the citing author's claim. Consider, for example, the following supporting citation:

> A major problem with standard static tests featuring problems presented in canonical form is that getting the right answer does not necessarily indicate that a child knows what he or she is doing (Erlwanger, 1973). (Campione, Brown, & Connell, 1988, p. 107)

In this citation instance, the citing authors are making their own claim about the problematic nature of certain forms of assessment. They use Erlwanger (1973) to support this claim, and appropriately so given that Erlwanger makes a similar conclusion, claiming that "Benny's case indicates that a 'mastery of content and skill' does not imply understanding" (p. 12). That support, however, is implied because Erlwanger's voice is opaque. The citation instance does not indicate the nature of Erlwanger's work and thus leaves the reader to supply and judge the validity of the connection based on their own understanding of Erlwanger's article.

Types of Citation Errors

I now turn to discussing some types of errors that are associated with these various forms of citation practices. In a later section, I explicitly address ways that we can improve our citation practices, but I provide some suggestions along the way when these suggestions relate specifically to avoiding a particular type of error. The errors I discuss here are the ones that were most common in the Erlwanger (1973) citation instances, and these error types account for the vast majority of the identified errors.

Attribution Errors

The forms of stand-alone and integrated quotations give rise to particular issues related to the appropriate attribution of words. When appealing to the voice of others, we must accurately present their voice by using their exact words, and we must be explicit whenever we are doing so. To be certain, the ideas being discussed here are related to plagiarism, but I suspect that plagiarism—in the classic sense

of intentionally passing off the words or ideas of others as our own words or ideas—is rare in our field. The attribution errors I discuss here are much more subtle and, as such, are likely to be unintentional. If we are to improve our citation practices, however, I believe we must all become more aware of proper and improper ways to attribute ideas and words to the originators of those ideas and words.

A fairly common attribution error (at least based on my analysis of the Case of Benny citation instances) is to attribute a direct quote to the cited author when some aspect of that direct quote is not actually correct. In order to build in productive ways on the work of others, we must be careful not to attribute words to someone that are not theirs. I found numerous instances in which authors simply changed quotes without indicating they had changed the quote. For example, consider the following citation instance:

> "In fractions, we have one-hundred different kinds of rules for solving one hundred different kinds of problems," declared 11-year old Benny after solving a variety of problems involving factions (Erlwanger 1973, p. 10).[2]

Now, beyond the fact that Benny was actually 12, compare this quote to the statement as actually reported: "In fractions we have 100 different kinds of rules . . ." (Erlwanger, 1973, p. 10). The added phrase "for solving one hundred different kinds of problems" should not be attributed to Benny, but the idea is supported by and should be attributed to Erlwanger (1973), who later in his report stated, "He believes that there are rules for every type of problem: ('In fractions, we have 100 different kinds of rules.')" (p. 17).

Consider another citation instance (a paraphrasing citation) with an attribution error of a slightly different nature but similarly related to Benny's rules:

> Benny's ideas about how to compute with fractions illustrate his nonstandard interpretation of mathematics. Benny understood 1.5 to be the same as 1/5.

One error here is that neither Erlwanger (1973), which is cited in connection with this citation instance, nor any other published report of the Case of Benny, reported Benny working with the fraction 1/5. The opaqueness of the cited author's voice along with the way in which this paraphrasing citation is written make it sound as though this example is drawn directly from Benny, which it is not. Although a number of other citation instances falsely attributed examples to Benny, in general these false examples were nevertheless consistent with his rules. This citation instance, however, makes a second error in that it misinterprets his rule as well: Had Benny converted 1/5 to a decimal number, he would have arrived at .6, not 1.5 because his rule for conversion here was "$a/b = .(a + b)$" (Erlwanger, 1973, p. 19). Furthermore, we are told that Benny actually did arrive at 1.5, but it was when

[2] I purposely did not provide the citation information for the citation instances I used as examples of errors throughout this commentary. The intent of this article is to help us improve as a community, not to call out any one of us for any particular error. I thus embrace the irony of these citation omissions.

he was asked to write 5/10 as a decimal and this time Benny used the rule "$b/ac = a.(b + c)$" (Erlwanger, 1973, p. 9).

Another kind of subtle attribution error occurs when citing authors use portions of the cited author's words or sentence structure but do not use direct quotes. In these situations, citing authors are attempting to use their own voice but do not sufficiently distinguish it from that of the cited author. At times, these errors come about simply because the citing authors leave out a few words of direct quotations (e.g., that Benny "could not have progressed" as opposed to "that Benny could not have progressed," Erlwanger, 1973, p. 7). In other instances, however, the citing authors fail to use direct quotes for much longer phrases and even for entire sentences, thus falling into the *paraphrase trap* wherein citing authors, in an attempt to use their own voice, merely replace a few words in a quote with synonyms, leaving the overall idea and sentence structure intact. For example, consider the following citation instance:

> Erlwanger (1973) used problem-based interviews to examine a sixth-grade student's (Benny's) understanding of decimals and fractions and his beliefs about the nature of rules, relationships, and answers in mathematics.

This citing author is attempting to briefly summarize the main focus of the Case of Benny. But the form of the citation (paraphrase) leaves quite opaque the issue of voice. In this form, the overall sentence structure, as well as the majority of word choices, should belong to the citing author. Compare, however, this citation instance to the relevant excerpt from Erlwanger:

> Subsequent discussions and interviews with Benny led me to an understanding of his concept of decimals and fractions, and his views about rules, relationships, and answers in mathematics. (Erlwanger, 1973, p. 7)

In this case, the citing author pulled too much of Erlwanger's own words, phrases, and basic sentence structure to legitimately claim that this is their voice. Actually, this is basically Erlwanger's voice with a few minor modifications. The citing author made it clear that they were referring to Erlwanger's ideas, but they did not appropriately attribute the words or sentence structure to Erlwanger. Given the desire to share the same basic summary that Erlwanger provided, in this case, the citing author could easily have chosen to use a direct quote for the latter half of their statement and thus avoided this attribution error.

I mention one other type of attribution error here—that of self-plagiarism. Although such a term seems oxymoronic, it is used to refer "to the practice of presenting one's own previously published work as though it were new" (American Psychological Association, 2010, p. 170). The issue with self-plagiarism is not that individuals use the exact same wording as they have in other publications but that they do not properly acknowledge that they have done so. Statements such as "as I have discussed elsewhere" or "as previously described" along with the appropriate citation are sufficient to alert the reader to the existence of the other work and to avoid the appearance of purposeful deception.

Erroneous Details

One way citation practices misrepresent work is when they report erroneous details about that work. Such errors typically occur in the paraphrase form. Almost one in four of the Case of Benny citation instances that I analyzed reported at least one faulty detail about the case. Accuracy with respect to the details of a study matter in general because we are trying to build on the work of others—to turn stories into cases and cases into knowledge. However, important cases like the Case of Benny are cases of something, and faulty details both misrepresent and fragment that something. Taken individually, erroneous details may seem of little consequence. Taken together, however, they paint a confusing picture.

To illustrate this issue, I quote the first paragraph from Erlwanger (1973). All bracketed statements are my own additions to the quote that present faulty details I found in the literature referencing the Case of Benny. I have used underlining to highlight the particular details that have been misrepresented:

> This study arose from visits made to a sixth [neither "fourth" nor "fifth"] grade class using Individually [neither "Individualize" nor "Individualized"] Prescribed [not "Programmed"] Instruction [not "Instructional"] (IPI) [not "PIP"] Mathematics in order to assist pupils who required remedial instruction and discover the nature of their trouble. In these terms, a twelve [not "eleven"] year old boy named Benny [not "Bennie"] did not seem a likely subject for the study. He was making much better than average [neither "medium" nor "superior"] progress through the IPI program, and his teacher regarded him as one of her best [neither "slightly above average" nor "the best"] pupils in mathematics [not "the class"]. In a structured program like IPI, it was expected by the teacher that Benny could not have progressed so far without an adequate understanding and mastery of previous work. (Erlwanger, 1973, p. 7)

Another striking collection of faulty details found in these citation instances was related to the nature of the IPI curriculum Benny was using. One of the most common reasons for citing the Case of Benny is to make a claim related to the impact of mathematics curricula on student learning (Leatham & Winiecke, 2014). In some citation instances, however, IPI was erroneously referred to as a computerized curriculum, and its assessments were sometimes erroneously assumed to be all multiple choice. In addition, several citation instances referred to the setting for the Case of Benny as one that was representative of "traditional" instruction. Erlwanger (1973) reported that "a basic goal of IPI is pupil independence, self-direction, and self-study" (p. 13), which was accomplished with "a minimum of direct teacher help to pupils" (Lindvall & Cox, 1970, p. 49, as cited in Erlwanger, 1973, p. 13). This emphasis on individualization and on the minimization of direct teacher instruction—what Erlwanger (1973) referred to as "a valuable and promising experiment in education," which, nevertheless, seemed to have "inherent weaknesses" (p. 25)—is not at all the picture one typically draws upon when conceptualizing traditional U.S. mathematics classrooms, well characterized by this oft-quoted description from the same era as Erlwanger's study:

> First, answers were given for the previous day's assignment. The more difficult problems were worked by the teacher or a student at the chalkboard. A brief explanation,

> sometimes none at all, was given of the new material, and problems were assigned for the next day. The remainder of the class was devoted to working on the homework while the teacher moved about the room answering questions. (Welch, 1978, p. 5-6)[3]

The IPI-intended environment and traditional instruction do share certain common traits (e.g., strong reliance on prescription through testing and little attention to students' mathematical thinking), but IPI was not traditional in nature, and Erlwanger's critiques of IPI are not direct critiques of traditional mathematics instruction.

The mathematics education community values the Case of Benny (Carpenter, Dossey, & Koehler, 2004), particularly as a story that provided "compelling evidence about the educational process" (Cooney, 1999, p. 1), but case studies are bounded, contextually rich studies of small numbers of participants in natural settings (VanWynsberghe & Khan, 2007). In order to successfully build on these cases, we must not compromise the contextual details that define them. Such erroneous details can easily creep into our writing, particularly when we use paraphrasing citations to present others' ideas in our own words.

Errors Involving Supporting Citations

Two types of errors seem to occur primarily when one uses the supporting citation form: attribution ambiguity and overgeneralization. Each of these types of errors seems to err in the placement and form of the citation itself more than it does in the citing authors' choice of words.

Attribution ambiguity. With attribution ambiguity, the placement of the citation leaves unclear whether the author intended to attribute the quote, idea, or focus of the sentence to the cited author. For example, consider the following claim:

> Effective content programming requires a very deliberate effort to incorporate sound behavioral systems and operant learning principles, and poor adherence to those principles should not lay blame to the principles themselves as too often occurs. (Erlwanger, 1973)

In this citation instance, it is unclear whether the author is claiming that Erlwanger made the mistake of blaming the principles themselves or whether he is claiming that Erlwanger is one who supports the claim that such mistaken blaming often occurs—a large swing in meaning.

Furthermore, the possibilities for attribution ambiguity increase drastically when we cite multiple references at once. When using more than one reference, we should consider whether it is true that all of the references support a single claim and whether that claim is clear. When making complex arguments involving multiple claims, we should strive to place supporting citations adjacent to the claims they are intended to support, not in a clearinghouse fashion at the end of

[3] Given the nature of this article, I feel that I should point out that this page number is indeed correct. The pages of this article are numbered 5-1 through 5-33, and this quote is found on page 5-6.

the sentence or paragraph. To illustrate the ambiguity that can result from such practice, consider the following (somewhat extreme) citation instance:

> "Understanding" takes many forms, among which *conceptual understanding*, learning from relevant *experience*, clarity of *goals*, appropriate *analyses* of ways to reach these goals, and the use of *heuristics* are especially important (Davis et al. 1982; Davis 1984; Zukav 1979, 7–8; Pólya 1957; Kantowski 1981; Papert 1980; Alderman et al. 1979; and Erlwanger 1973).

Although much could be discussed about what is and is not being said here, I make but two comments. First, all of these citations do not support all of these claims; this is simply demonstrated by the fact that Erlwanger (1973) says nothing about heuristics. The cited references could be much more accurately represented were they to accompany the specific pieces of this complex claim that they are intended to support. Second, we see here how the organization of the sentence, not so much the ideas themselves, can get in the way of contributing valuable insights. I suspect the author was aware of the various complex connections that could be made between these other works and their own treatise on understanding, but the misuse of citations here serves to obfuscate rather than to illuminate these ideas. Such citation practices leave me less rather than more convinced of the soundness of the citing author's ideas and of the support in the literature for those ideas.

Overgeneralization. Beyond creating attribution ambiguity, when using the supporting citation form the mere placement of the citation can unintentionally alter the nature of the claim. When analyzing the Case of Benny citation instances, I found numerous examples of citing authors making claims that students always or often think or act like Benny and then citing the Case of Benny to support those claims. Consider, for example, the following citation instance:

> Classroom research studies indicate that curricula emphasizing facts and skills really make sense to only a few students (e.g., Erlwanger, 1973).

Here the author claimed that the Case of Benny is an example of a study that has shown "that curricula emphasizing facts and skills really make sense to only a few students." But Erlwanger made no such claim, nor can we do so based on what we know about Benny. That said, Benny is indeed a student who was using a procedure-focused curriculum. Whether he made sense of that curriculum or not, his story simply does not support the claim that few students make sense of such curricula. As an aside, Erlwanger's very premise was that Benny actually *was* "making sense" of his curriculum—the problem was that he was not making the sense that was intended by the curriculum. This citation is one of several that mischaracterized Benny as being a student who was failing to make sense rather than one who was making the "wrong" sense.

Now, consider this pair of citations, which appear on consecutive pages of a single publication:

> Erlwanger (1975) has shown that children can invent a multitude of idiosyncratic rules in order to provide *the right answer.*
>
> It is well known that most errors made by students are not due to their lack of attention but to false rules which they have invented and which seem perfectly logical to them (Erlwanger, 1975; Ginsburg, 1977).

The author accurately represented Erlwanger's work in the first instance, claiming the cases of Benny and Mat demonstrate that students "can invent" erroneous rules that sometimes work. These cases certainly support such a claim because Erlwanger presented numerous examples of such rules. In the second instance, however, the author overgeneralized with regard to this same issue, now claiming that "most errors made by students" occur because of such "false rules." There are at least two fundamental overgeneralization problems with this claim. The first problem is in direct contrast to the correct usage in the first citation instance. Erlwanger (1975) presented data about Benny and Mat—just two students. Regardless of the number of students with whom Ginsburg (1977) worked (and there were indeed many), Erlwanger's contribution of two more simply does not provide sufficient evidence that most students do anything. Ginsburg (which, by the way, contains a lengthy citation instance referencing the Case of Benny) provided compelling evidence that many students behave in particular ways, but still not most. In addition, this citation instance makes the further false generalization that "most student errors" are caused by "false rules." Again, the work of Erlwanger, Ginsburg, and others (e.g., Clements, 1980, 1982) did indeed reveal many instances in which false rules were behind student errors. However, other common error types were also identified, including a large number of errors that could be attributed to a lack of reading comprehension and or to carelessness (Clements, 1980, 1982).

As one final example, consider the following citation instance:

> What soon became evident in the Back to Basics movement was that there was no such a thing as a "teacher-proof" mathematics curriculum (Erlwanger, 1973), and the mathematical education community once again was faced with the challenge of developing a curriculum to bring effective mathematics instruction into the classroom.

This final example illustrates the strongest form of overgeneralizing—the claim that something never exists. In this citation instance, the author seems to be using the Case of Benny to support the claim that "teacher-proof" mathematics curricula cannot exist. Although the teacher still had an important role to play in the IPI curriculum, suppose one did assume that its goal was to be teacher-proof. One simply cannot infer that, because the IPI curriculum failed to be teacher-proof, no curriculum ever could.

These examples illustrate how errors in citation practices can cause us to overgeneralize the work of others when we use it to support our claims. The Case of Benny is an existence proof. It demonstrates but one student's experience, albeit in a poignant and important way. In fact, one of the primary reasons for citing

the Case of Benny is to illustrate quality qualitative research (Leatham & Winiecke, 2014). The Case of Benny is indeed a compelling case, but it is a case with what Schoenfeld (2007) would call "limited warranted generality" (p. 89). The case demonstrates what is possible, not what is prevalent or likely. Such a warning is of particular note given that "if anyone should understand existence proofs and how they relate to universal quantifiers, it should be mathematics people" (S. R. Williams, personal communication, October 23, 2013). The overgeneralization errors I identified typically occurred because citing authors used a phenomenon identified in the Case of Benny to support a claim that the given phenomenon *often occurs*. We should be careful when using adverbs of frequency (e.g., *never* is seldom true), particularly when speaking beyond our own opinions. It is perfectly acceptable to believe that there are many Bennys in our classrooms, but we should not attribute such commonality to the Case of Benny. We should hedge whenever we speak beyond the evidence provided by our data or reported in the literature. The tone and style of our writing, which includes how we use citations to support our work, should make it clear whether we are sharing our own opinion or the collective wisdom of the research community.

Improving Our Citation Practice

The responsibility to improve citation practices in mathematics education research rests with our entire community—with both graduate students and graduate faculty, with both novice and seasoned writers, and with all of us in our role as reviewers of each other's work. Here I suggest some ways that we as writers, reviewers, and graduate faculty might jointly work toward this improvement. Although reflection and feedback on our writing certainly should be focused on the big ideas, we also need to attend to whether we have properly articulated and supported those ideas. A good place to begin giving and receiving such explicit feedback is likely graduate school, but it should continue on throughout our careers as we reflect on our own work and give critical feedback to others, acting for each other as "insiders who can help us see our work whole" (Kilpatrick, 2013, p. 173). Side by side with discussions about quality research should be discussions of quality research writing. With such explicit attention in mind, I state here several suggestions for how we as individuals and as a community of mathematics education researchers can explicitly improve our citation practice.

Alter Quotes Legitimately

There are times when it is desirable to alter direct quotes, and we typically use ellipses, brackets, or italics to alert the reader to these changes. We risk misrepresenting the voice of the cited author in these situations, however, particularly when we quote transcript excerpts (i.e., when we quote quotes). Ellipses and brackets seldom occur outside of transcript, so we can usually infer they are the addition of the quoting author; the use of italics, however, is common across text types, so

we should always make note of such additions. Making any of these alterations to quoted quotes, however, can cause considerable confusion and risks misrepresenting the cited author. To illustrate the difficulties related to such alterations, consider the following citation instance:

> In another example Benny wrote 400/400 equals 8.00 because "the numbers are the same . . . say like 4,000 over 5,000. [Benny is referring to the number of digits in each case.] All you do is add them up; put the answer down; then put your decimal in the right place . . . in front of the [last] three numbers."

Which of these alterations to what Benny said should be attributed to Erlwanger, and which should be attributed to the author of the citation instance? Compare the instance to the direct quote from Erlwanger (1973):

> In another example, 400/400 = 8.00 because "The numbers are the same [number of digits] . . . say like 4000 over 5000. All you do is add them up; put the answer down; then put your decimal in the right place . . . in front of the [last] three numbers." (p. 8)

Both the citing and cited authors have made alterations using ellipses and brackets. The first use of ellipses in the citation instance is problematic because there were already ellipses in Erlwanger's direct quotation, indicating the removal of portions of what Benny said; the citing author has made an additional omission adjacent to the original one, this time omitting the cited author's brackets. It thus becomes impossible to decipher the meaning of these ellipses without referring to the original work. The reader would likely attribute the ellipses solely to the citing author, having no idea that it was actually the cited author who did the original redacting and who is currently being redacted. Furthermore, the first bracketed clarification that actually appears in the citation instance is not from Erlwanger, whereas the second one is. Again, this attribution is impossible to discern without referring to the original source. Although the citing author may have had legitimate reasons for wanting to make such alterations to the original work, the lack of proper acknowledgement of the changes misrepresented Erlwanger's work. In this situation, the following slight reorganization and rewording could make the attribution more clear:

> Elsewhere Benny concluded that 400/400 equals 8.00 "because the numbers are the same," referring to the number of digits in 400. He went on to explain, "say like 4,000 over 5,000. All you do is add them up; put the answer down; then put your decimal in the right place," which he indicated would be "in front of the [last] three numbers" (p. 8, brackets in original).

Loquere Latine

One way to avoid misrepresentation in our citation practice is to learn how to appropriately take advantage of Latin abbreviations commonly used in academic writing. In situations where we wish to cite literature as illustrative examples (most often using the supported citation form), it is useful to use "e.g.," which is short

for the Latin phrase "exempli gratia." This phrase literally means "for the sake of example" and can be read as "for example" or "for instance." Now, consider this citation instance:

> Moreover, teachers often focus only on whether students' answers *are* completed or "correct" rather than on the thinking used to obtain the answers, so that mistakes and misconceptions frequently go unnoticed and uncorrected (Bennett, Desforges, Cockburn, & Wilkinson, 1984; Erlwanger, 1975).

As it stands, this example misrepresents the work of Erlwanger (1975), whose cases of Benny and Mat involved but did not actually study two elementary teachers, and Bennett, Desforges, Cockburn, and Wilkinson (1984), who studied 16 elementary teachers, by inferring that the cases of these 18 elementary teachers imply, in general, that teachers often act in certain ways and that those actions frequently yield certain results. Adding "e.g." inside the citation would instead characterize these studies as examples of studies wherein the authors observed this phenomenon rather than as studies that establish its commonality.

Another useful abbreviation is "cf.," which is short for the Latin word "confer." This word literally means "bring together" and can be read as "compare" or as "for the sake of comparison, see." Proper use of this abbreviation can help writers to avoid the appearance of plagiarism. It seems reasonable to assume that in our work studying the learning and teaching of mathematics, many of us have had similar thoughts and made similar connections. Yes, we need to appropriately attribute others' words and ideas when we are basing our work on theirs, but it is also perfectly appropriate to claim original thoughts and ideas that are similar to what others have said. Although not always the case, such situations seem to occur when, in the midst of writing about a phenomenon, we become aware of related work through searching the literature. In cases such as these, the use of "cf." can help us to maintain the originality of our own ideas while at the same time acknowledging the comparable work of others.

One other useful Latin abbreviation is "i.e.," which is short for the Latin phrase "id est." This phrase literally means "that is" and can be read as "which means" or "in other words." This abbreviation is typically used to allow the writer to restate or clarify an idea and would not typically be used in conjunction with citations. I mention it here in part because of its common use and in part because "i.e." is sometimes incorrectly used in place of "e.g."

Avoid Secondary Sources

Following reference trails found within research publications is a great way to build an understanding of related literature, but there is danger in choosing to cite Author A based solely on Author B's interpretation of Author A's work. Relying on secondary sources in this way blurs the voices of the cited and citing authors even more. If using a secondary source seems sufficiently compelling to warrant the risk, one can preface the citation instance with phrases like "as noted by" or "commenting on Erlwanger's (1973) work." In other words, it is important to make clear when we are not the ones who made the connection between the ideas and the

secondary source. Alternatively (and preferably, in my opinion), one can locate the original source and develop a slightly different take on the significance of the work. (It would then be appropriate to provide a "cf." citation to the secondary source.)

To illustrate the dangers of secondary sources, consider the following citation instance:

> In mathematics learning, the intention to make sense is essential. (Erlwanger, 1973)

Now, to be clear, this citation instance is a direct quote. The problem is that it is not from Erlwanger (1973) but from Wheatley (1992, p. 533). Wheatley made this statement and used Erlwanger (1973) to support it. The author who mistook Wheatley's words for Erlwanger's clearly used Wheatley as a secondary source for Erlwanger—as evidenced by the exact wording of the quotation as well as by the fact that the author provided an extensive quote from Wheatley on the following page of the article.

Fit Form to Function

I feel that my own understanding of citation practices has been improved by considering the various forms citations can take and the basic functions those forms accomplish. Being aware of these differences, and of the common errors that are often associated with the different forms, might allow us to make more wise choices about the citation form we choose in a given circumstance and to use those forms more appropriately. Direct quotes are useful when we want to take advantage not just of the cited author's ideas but of the way they articulated those ideas. When using direct quotes, we should seek to be explicit about our reasons for sharing the quote. This practice lets the reader know why we included the quote, but it also serves to separate our voice from that of the cited author. Integrated quotes are powerful ways of situating our work within the work of others, but in so doing, we risk attribution ambiguity. Here, as with paraphrased citations, we should be careful to avoid the appearance of plagiarism by attending not only to the words but also to the sentence structure of the cited author. Finally, the biggest danger with supporting citations is that the specifics of the connection we see between our work and the work of the cited author may be opaque to the reader and possibly to ourselves.

I suspect that it is rare for arbitrary citations (e.g., Asimov, 1959) to creep into our writing. When analyzing the collection of instances citing the Case of Benny, for example, I was able to discern connections between the Case of Benny and all of the citation instances I examined. Although it seemed as though some errors did occur because authors misinterpreted Erlwanger's work, many errors seemed to misrepresent that work because the authors were trying to say one thing but unintentionally said another. I suspect we could improve much in our citation practices if we developed the habit of reviewing our citations, regardless of the citation forms we are employing, with the following question in mind: "How does this source support the claim I am making?"

In addition to developing this self-interrogation skill in the midst of our writing, I believe this skill can be learned by and made more explicit in our work with graduate students. The design of the citation study (Leatham & Winiecke, 2014) that motivated the writing of this commentary was actually an expanded version of a "learning-to-write-research" activity I often do with my own graduate students. It is relatively easy to locate references to readings by using Google Scholar. Thus, for example, when asking students to read classic works such as Erlwanger (1973/2004) or Skemp (1976/2006), we could ask them to locate a small set of publications that reference the given reading and analyze the extent to which those references accurately represent the author's ideas. Such activities serve to highlight appropriate citation practices and also encourage students to engage with the given reading at a different level than they might do otherwise. We can employ a similar activity (but on a smaller scope) when we ask students to read a pair of publications, one of which draws or builds explicitly and extensively on the work of the other.

Conclusion

We all can play a role in improving citation practices in mathematics education. Not only as writers but also as reviewers, committee members, colleagues, and editors, we each have a responsibility to help our complex system self correct—"if we do not scrutinize each other's ideas, we are not in a scholarly field" (Kilpatrick, 2013, p. 175). As reviewers, I think we should pay closer attention to authors' citation practices. I am not suggesting that we seek out each reference to check for accuracy, although, as mentioned earlier, I do think that small exercises of this sort are an excellent learning activity and suggest that we all double-check the citation instances in our own writing.[4] Rather, it would be valuable to pay more attention to citations when we review others' work and to ask questions such as these: Do I recognize any of these citations? For the ones I do recognize, do the foregoing claims seem to be appropriately connected to the citation? Does anything seem amiss? For most journals, three to five reviewers and several members of the editorial team read every article. With a collective increase in attention to citations, I believe we could significantly decrease the misrepresentation of ideas and significantly increase citation integrity.

As we seek to build on the work of others, particularly when that work is qualitative in nature, we should pay greater attention to detail and be wary of how easy it is to overgeneralize. As readers, writers, reviewers, and editors, we must be concerned with citation accuracy as we seek to read, understand, and reference others' work. The power in building on the work of others is diluted when that work is misrepresented. Hopefully the arguments and examples put forth in this

[4] In preparing to submit this manuscript for review, I checked each of my own citations against the original sources. This review revealed two incorrect page numbers, one incorrect date, an incorrect author, two misspelled words, several missing words, an added word, and two instances in which I had failed to include the cited authors' italics.

commentary convince us all of the need for greater attention to accuracy and integrity as we seek to productively build on the work of others. "Credible citation practice is more than a matter of selective quotation, fluent paraphrase, accurate summary, avoidance of plagiarism, and precise punctuation. It is an act of building community, collaboratively constructing shared knowledge" (Rose, 1996, p. 45). As a community of researchers in mathematics education, we need to help each other increase the accuracy and integrity of our citation practices, thus enabling a jointly built, firm foundation for our field.

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