Deep Learning Principles & Applications

Chapter 1 – Linear Algebra for Deep Learning

Sudarsan N.S. Acharya (sudarsan.acharya@manipal.edu)

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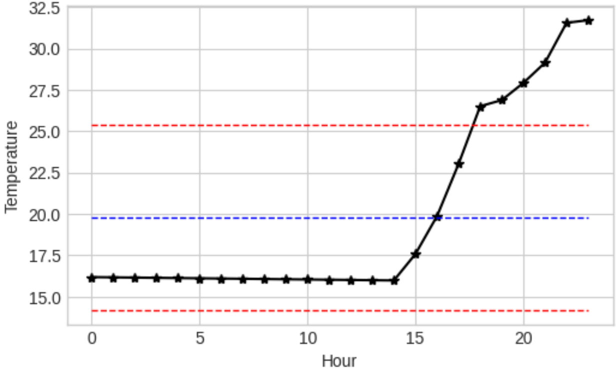

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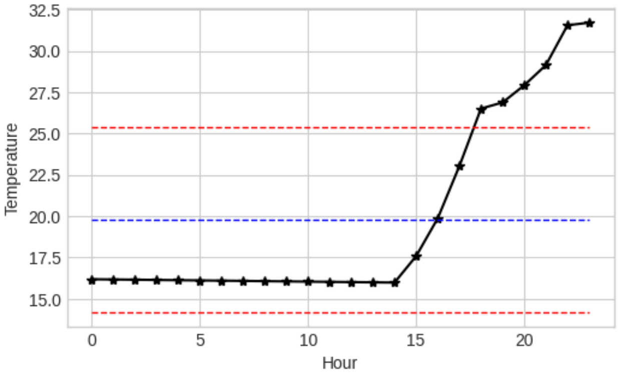

$$\mathbf{t} = \begin{bmatrix} 16.18 \\ 16.17 \\ 16.15 \\ 16.14 \\ 16.12 \\ 16.11 \\ 16.09 \\ 16.08 \\ 16.07 \\ 16.05 \\ 16.04 \\ 16.02 \\ 16.01 \\ 15.99 \\ 15.98 \\ 17.58 \\ 19.87 \\ 23.03 \\ 26.51 \\ 26.88 \\ 27.92 \\ 29.14 \\ 31.55 \\ 31.71 \end{bmatrix}$$

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$$\begin{array}{l} \text{word} \\ \text{in} \\ \text{number} \\ \text{horse} \\ \text{the} \\ \text{document} \end{array} \qquad \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \\ 4 \\ 2 \end{bmatrix}$$

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If $\mathbf{z}$ is a vector representing an audio signal (sound pressure), the scalar-vector product $2\mathbf{z}$ is the same audio signal perceived twice as loud.

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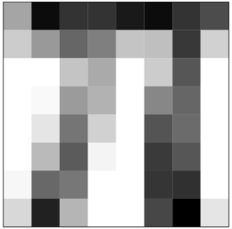

0.28 0.00 0.90

$$\begin{bmatrix} 0.65 & 0.05 & 0.2 & \cdots & \cdots & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & 0.28 & 0.0 & 0.9 \end{bmatrix}$$

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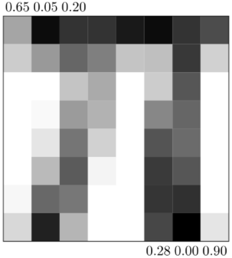

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The 4-row and 3-column patient data matrix $\mathbf{X}$ can be seen as a 3-vector (corresponding to each patient) repeating 4 times.

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$$\mathbf{X} = \begin{bmatrix} 76 & 126 & 38.0 \\ 74 & 120 & 38.0 \\ 72 & 118 & 37.5 \\ 78 & 136 & 37.0 \end{bmatrix} \Rightarrow \mathbf{X}^{\mathrm{T}} = \begin{bmatrix} 76 & 74 & 72 & 78 \\ 126 & 120 & 118 & 136 \\ 38.0 & 38.0 & 37.5 & 37.0 \end{bmatrix}$$

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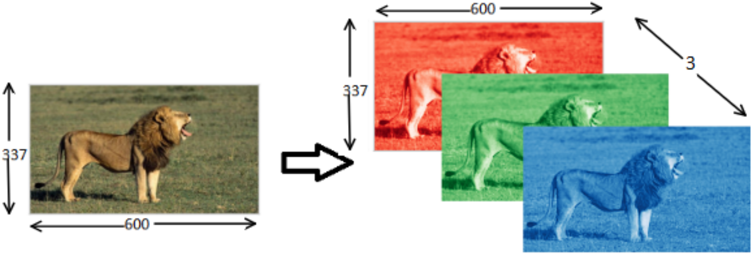

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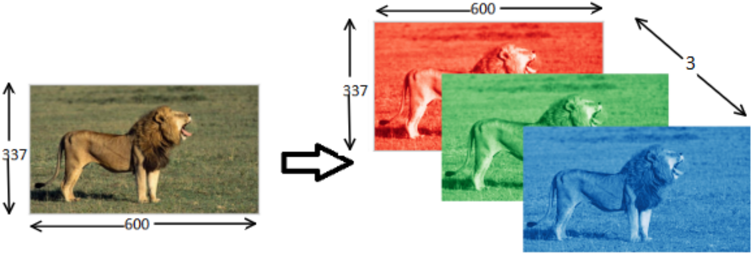

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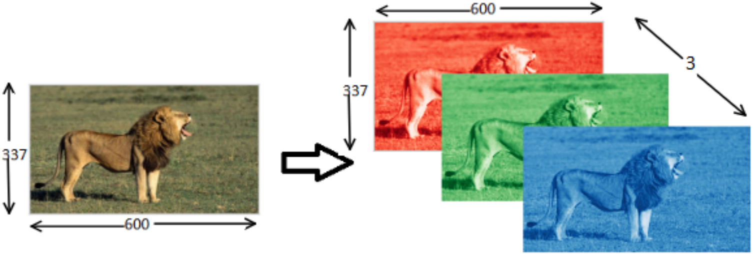

- The image tensor has the shape $337 \times 600 \times 3$, which can be seen as a 600×3-matrix (The R-G-B values for the horizontal 600 pixels from the top) repeating 337 times (for the vertical pixels).

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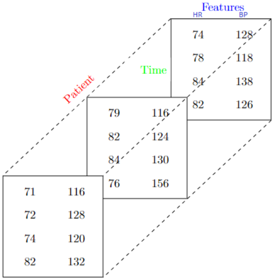

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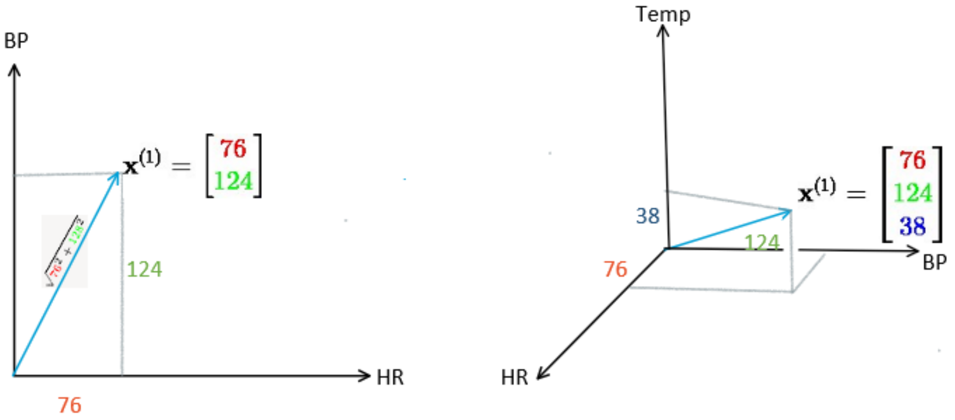

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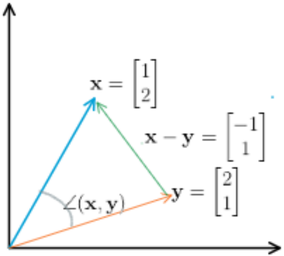

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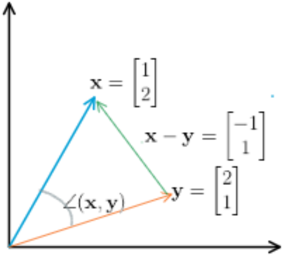

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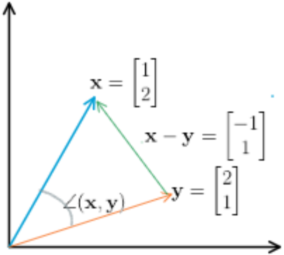

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- When calculating distance between sample vectors (such as 2 patients), units for the features have to be considered.

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- Standardizing the vectors (subtracting each feature from its mean and dividing by its standard deviation) prior to calculating distance dissimilarity will address this issue.

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A matrix-vector product can be seen as a sequence of dot products between the rows of matrix $\mathbf{A}$ (seen as vectors) and the vector $\mathbf{x}$.

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&= \begin{bmatrix} 4 \\ 8 \end{bmatrix} + \begin{bmatrix} 4 \\ -2 \end{bmatrix} + \begin{bmatrix} -8 \\ -6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.
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$$\Rightarrow \mathbf{AB} = \begin{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 2 \\ -2 \end{bmatrix} & \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix} \\ \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 2 \\ -2 \end{bmatrix} & \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix} \end{bmatrix}$$

Matrix-Matrix Product

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Matrix-Matrix Product–continued

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Hadamard Product

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The Hadamard product shows up in deep learning computations to simplify certain representations involving matrices and vectors.