# Combining the power spectra

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Our goal is to combine spectra from different detector array and different season into a set of 'averaged' cross frequency spectra, the data model is the following

$$C^{Xs_1\alpha,Ys_2\beta}_{b,\nu_1\nu_2} = P^{XY}_{b,\nu_1\nu_2} + \epsilon^{Xs_1\alpha,Ys_2\beta}_{b,\nu_1\nu_2} \tag{1}$$

With $X, Y \in \{T, E\}$, $\nu_1, \nu_2$ index frequencies, $\alpha, \beta$ the detector arrays and $s_1, s_2$ the season of observation. $P^{XY}_{\ell,\nu_1\nu_2}$ is what we would like to estimate. Assuming gaussiannity we can write a likelihood

$$-2\ln\mathcal{L} \propto \sum_{\nu_1\nu_2\nu_3\nu_4}\sum_{X,Y,W,Z}\sum_{s_1s_2s_3s_4}\sum_{\alpha\beta\gamma\mu}\sum_{bb'}$$

$$\left(C^{Xs_1\alpha,Ys_2\beta}_{b,\nu_1\nu_2} - \sum_{A,B,b_a,\nu_a\nu_b}\mathcal{A}^{Xs_1\alpha,Ys_2\beta;AB}_{b,\nu_1\nu_2;b_a\nu_a\nu_b}P^{AB}_{b_a,\nu_a\nu_b}\right)^T \Xi^{-1}\left(C^{Ws_3\gamma,Zs_4\mu}_{b',\nu_3\nu_4} - \sum_{E,Fb_{a'},\nu_e\nu_f}\mathcal{A}^{Ws_3\gamma,Zs_4\mu;EF}_{b',\nu_3\nu_4;b_{a'}\nu_e\nu_f}P^{EF}_{b_{a'},\nu_e\nu_f}\right) \tag{2}$$

This looks daunting only because we decided to write all index explicitly, $\mathcal{A}$ associate the 'averaged' cross frequency spectra to the corresponding $C_b$ element, it's a rectangular matrix with expression

$$\mathcal{A}^{Xs_1\alpha,Ys_2\beta;AB}_{b,\nu_1\nu_2;b_a\nu_a\nu_b} = \delta_{b,b_a}\delta_{\nu_1,\nu_a}\delta_{\nu_2,\nu_b}\delta^{X,A}\delta^{Y,B} \tag{3}$$

and $\Xi$ is the usual covariance matrix. A maximum likelihood estimation of the 'averaged' cross frequency spectra is given by $\frac{\partial \ln\mathcal{L}}{\partial \boldsymbol{P}}|_{\hat{\boldsymbol{P}}} = 0$ We have

$$-2\mathcal{A}^T\Xi^{-1}\left(\boldsymbol{C} - \mathcal{A}\boldsymbol{P}\right) = 0 \tag{4}$$

$$\boldsymbol{P} = (\mathcal{A}^T\Xi^{-1}\mathcal{A})^{-1}\mathcal{A}^T\Xi^{-1}\boldsymbol{C} \tag{5}$$