Paper presentation: CONVEX OPTIMIZATION FOR FAST IMAGE DEHAZING

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Introduction - Image Dehazing

Haze occurs when dust and smoke particles accumulate in atmosphere.

The image dehazing problem is to apply post processing of hazed images to remove the haze effects.

In this paper, Haar wavelet transform is applied to derive a subband hazed image model with reduced dimension.

A convex optimization problem is formulate. It's solution is sufficient for reconstruction of the haze free image.

THE HAZED IMAGE MODEL

Let $J_c, I_c \in R_+^{M\times N}$ represent matrices of the haze free and hazed digital RGB color images, respectively.

$$\boldsymbol{I_c = J_c \odot t + a_c(1-t), c = 1, 2, 3} \quad (1)$$

a_c is the atmospheric light constant of the corresponding color channel.
$t \in R_{M\times N}$ is the transmission distribution representing the portion of the light illumination on camera sensors.
objective of image dehazing is to recover the haze free image
$J = J_1/J_2/J_3$ with a_c and t, from $\boldsymbol{I = I_1/I_2/I_3}$

Discrete Haar wavelet transform

In homogeneous atmosphere, t is characterized by $t = e^{-\beta d}$,
Where $d \in R^{M\times N}$ is the distance map from the objectives to the camera.
$\beta d \succcurlyeq 0$ implies $\boldsymbol{0 \prec t \preccurlyeq 1}$

Distance distribution d is piecewise constant for most images, Thus t is also piecewise constant.

t is 2-patch piecewise constant i.e. $t(2m+i, 2n+j) = t(2m, 2n)$, for $m \in [0, M/2)$, $n \in [0, N/2)$ and $i, j = 0, 1$

Discrete Haar wavelet transform

Let W be the well known discrete Haar wavelet transform (DHWT) matrix of appropriate dimension.

$$\hat{\mathbf{I}}_c = \mathbf{W}\mathbf{I}_c\mathbf{W}^T = \begin{bmatrix} \hat{\mathbf{I}}_c^a & \hat{\mathbf{I}}_c^h \\ \hat{\mathbf{I}}_c^v & \hat{\mathbf{I}}_c^d \end{bmatrix}, \hat{\mathbf{J}}_c = \mathbf{W}\mathbf{J}_c\mathbf{W}^T = \begin{bmatrix} \hat{\mathbf{J}}_c^a & \hat{\mathbf{J}}_c^h \\ \hat{\mathbf{J}}_c^v & \hat{\mathbf{J}}_c^d \end{bmatrix}.$$

It can be verified that, if $\mathbf{t}$ is 2-patch piecewise constant, its DHWT matrix has four identical subband blocks, i.e.,

$$\hat{\mathbf{t}} = \mathbf{W}\mathbf{t}\mathbf{W}^T = \begin{bmatrix} \hat{\mathbf{t}}^a & \hat{\mathbf{t}}^h \\ \hat{\mathbf{t}}^v & \hat{\mathbf{t}}^d \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{t}}^a & \hat{\mathbf{t}}^a \\ \hat{\mathbf{t}}^a & \hat{\mathbf{t}}^a \end{bmatrix}, 0 \prec \hat{\mathbf{t}}^a \preceq 1.$$

It can be further verified that the DHWT of the haze model (1) results in

$$\begin{bmatrix} \hat{\mathbf{I}}_c^a & \hat{\mathbf{I}}_c^h \\ \hat{\mathbf{I}}_c^v & \hat{\mathbf{I}}_c^d \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{J}}_c^a \odot \hat{\mathbf{t}}^a + 2a_c(1-\hat{\mathbf{t}}^a) & \hat{\mathbf{J}}_c^h \odot \hat{\mathbf{t}}^a \\ \hat{\mathbf{J}}_c^v \odot \hat{\mathbf{t}}^a & \hat{\mathbf{J}}_c^d \odot \hat{\mathbf{t}}^a \end{bmatrix}. \quad (2)$$

$$\hat{\boldsymbol{I}}_c^a = \hat{\boldsymbol{J}}_c^a \odot \hat{\boldsymbol{t}}^a + \hat{\boldsymbol{a}}_c(1-\hat{\boldsymbol{t}}^a),\ \hat{\boldsymbol{a}}_c = 2\boldsymbol{a}_c,\ c = 1,2,3 \quad (3)$$

THE CONVEX OPTIMIZATION

$\hat{I}_c^a$ and $\hat{a}_c$ are known variables.

$\hat{J}_c^a$ and $\hat{t}^a$ are unknown variables.

It is a nonconvex problem with the coupling term of $\hat{J}_c^a \odot \hat{t}^a$ in (3).

So let

$\hat{Y}_c^a = \hat{I}_c^a - \hat{a}_c 1$ and $\hat{Q}_c^a = \hat{J}_c^a \odot \hat{t}^a$

$\hat{Y}_c^a = \hat{Q}_c^a - \hat{a}_c\hat{t}^a,\ c = 1,2,3 \quad (4)$

The reformulated model is linear in $\hat{Q}_c^a$ and $\hat{t}^a$, which enables the following convex optimization.

$$\min_{\hat{Q}_c^a, \hat{t}^a} \sum_{c=1,2,3} (\| \hat{Y}_c^a - \hat{Q}_c^a + \hat{a}_c\hat{t}^a \|_2^2) + R(\hat{Q}_c^a, \hat{t}^a)$$
$$s.t.\ 0 \prec t \preccurlyeq 1, 0 \preccurlyeq \hat{Q}_c^a, c = 1,2,3$$

$R(\hat{Q}_c^a, \hat{t}^a)$ is the regularization function.

Selection of the regularization function

Since the haze effect reduces the degree of contrast in images, the image dehazing process is to enhance the level of image contrast

Further, for the sake of smooth and low pass feature, $R(\hat{Q}_c^a, \hat{t}^a)$ is specified as

$$R(\hat{Q}_c^a, \hat{t}^a) = \lambda_1 \| \hat{t}^a \|_2^2 + \lambda_2 \| \hat{t}^a \|_{TV} + \lambda_3 \sum_{c=1,2,3} \| \hat{Q}_c^a \|_2^2$$

CO-DHWT

The convex optimization problem become

$$\min_{\hat{Q}_c^a, \hat{t}^a} \sum_{c=1,2,3} (\| \hat{Y}_c^a - \hat{Q}_c^a + \hat{a}_c\hat{t}^a \|_2^2) + \lambda_1 \| \hat{t}^a \|_2^2 + \lambda_2 \| \hat{t}^a \|_{TV} + \lambda_3 \sum_{c=1,2,3} \| \hat{Q}_c^a \|_2^2$$
$$s.t.\ 0 \prec t \preccurlyeq 1, 0 \preccurlyeq \hat{Q}_c^a, c = 1,2,3$$

($\hat{Q}_c^a$ and $\hat{t}^a$ are unknown variables.)

It is called CO-DHWT in this paper.

Remark 1: directly applicable to the original hazed image model(1)

Remark 2: directly extendable to multilevel subband DHWT of the hazed image model.

COMPUTATIONAL RESULTS

CO-DHWT(7) was implemented with the Split Bregman iteration algorithm and coded with Matlab.

regularization parameters were empirically adjusted and set as $\lambda_1 = 0.02$, $\lambda_2 = 0.002$, $\lambda_3 = 0.04$

Compared with state of the art image dehazing algorithms, including that of Fattal in [14], He et al. in [10], Tarel and Hautiere in [21], Meng et al. in [24], Wang and Fan in [18], and Zhu et al. in [25].

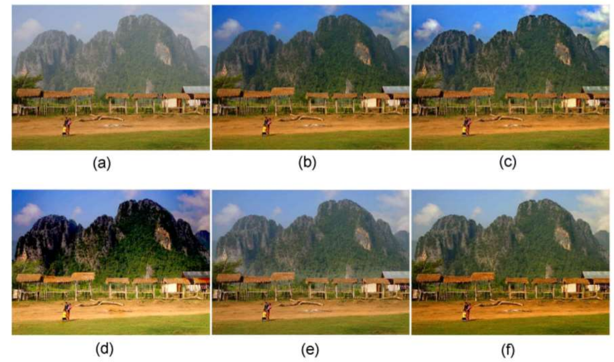

Fig. 1. Dehazing results of the "Mountain" image by different algorithms, (a) the hazed input image, (b) by He et al.'s algorithm [10], (c) by Meng et al.'s algorithm [24], (d) by Fattal's algorithm [14], (e) by Zhu et al.'s algorithm [25], (f) by CO-DHWT.

Fig. 2. Dehazing results of the "House" image by different algorithms, (a) the hazed input image, (b) by He et al.'s algorithm [10], (c) by Meng et al.'s algorithm [24], (d) by Wang and Fan's algorithm [18], (e) by Zhu et al.'s algorithm [25], (f) by CO-DHWT.

Computational time

Table 1. Comparison of computational time (second) of different algorithms for images of different sizes

Algorithm \ Size	600 × 450	1024 × 768	1536 × 1024
He et al. [9]	10.04	29.46	65.4
Tarel et al. [21]	8.81	68.90	335.79
Meng et al. [24]	4.782	5.60s	10.27
Zhu et al. [25]	3.68	4.08	8.08
CO	2.25	6.85	12.9
CO-DHWT	0.70	2.08	3.72

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CONCLUSION

By reformulating in the hazed image model, this paper has presented a convex optimization formulation for the image dehazing problem in the low pass subband of the Haar wavelet transform domain.

Its dehazing results outperform state of the art image dehazing algorithms.