Streaming Algorithms

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06/15/2018

Reminder

Final Project Presentation. Jun 22 10:10 - 12:40

Written Assignment # 3 Jun 24 23:55

The final grade will be announced on e3new by Jun 26. If you have any question regarding the grade, let me know no later than Jun 28.

Reduction-based Streaming Algorithms

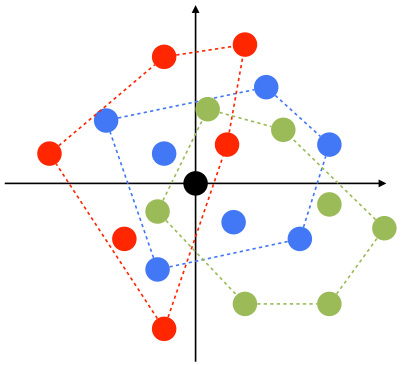
"Give me a place to stand and with a lever, I will move the whole world."

- Archimedes

There are many algorithmic results are reduction-based. To solve a new problem A, what we need is usually a reduction from A to some known problems.

Colorful Caratheodory Theorem

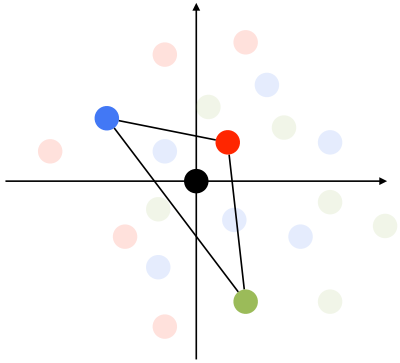
Given three point sets on a plane where the convex hull of each point set contains the origin.



Goal: find a colorful triangle that contains the origin.

Colorful Caratheodory Theorem

Given three point sets on a plane where the convex hull of each point set contains the origin.

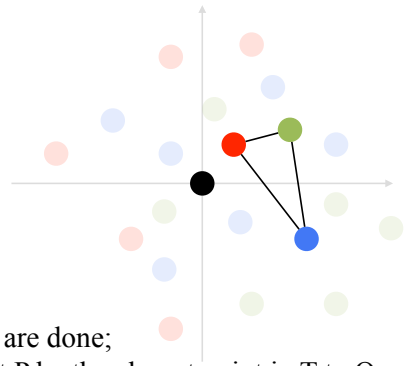


Goal: find a colorful triangle that contains the origin.

An Existence Proof

Colorful Caratheodory Theorem

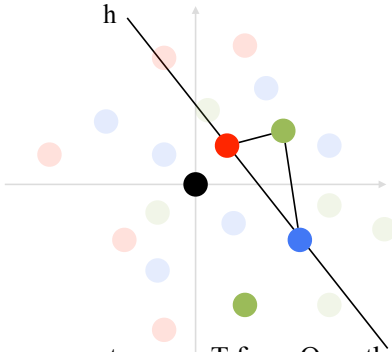
Starting from any colorful triangle T.



If O ∈ T, we are done;
otherwise, let P be the closest point in T to O, and h be the line orthorgonal to OP at the point P.

Colorful Caratheodory Theorem

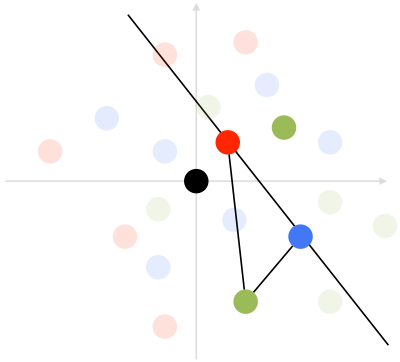
Starting from any colorful triangle T.



h separates a green vertex g on T from O ⇒ there exists a green point g' on the same side as O. Let T' = T-{g}∪{g'}. T' is closer to O than T.

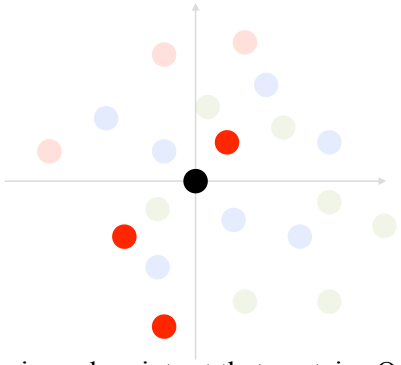
Colorful Caratheodory Theorem

Starting from any colorful triangle T.



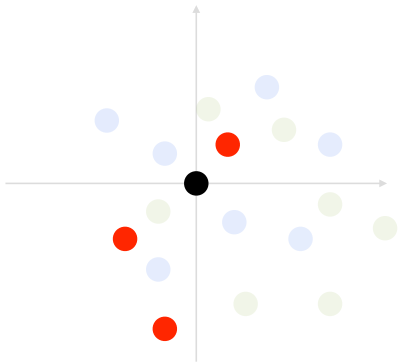
There are $O(n^3)$ different distances, so the number of iterations is $O(n^3)$. Can we do better?

Colorful Caratheodory Theorem



Find a triangle in each point set that contains O, and remove the rest of points in the set.

Colorful Caratheodory Theorem



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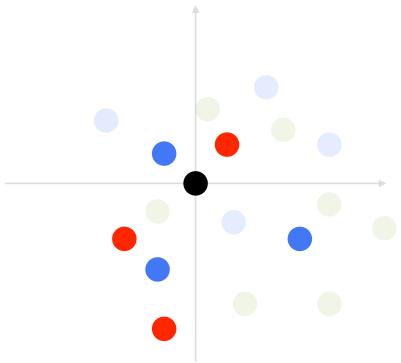

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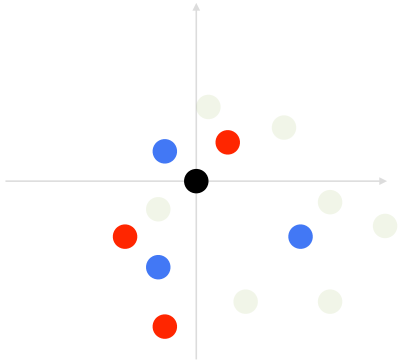

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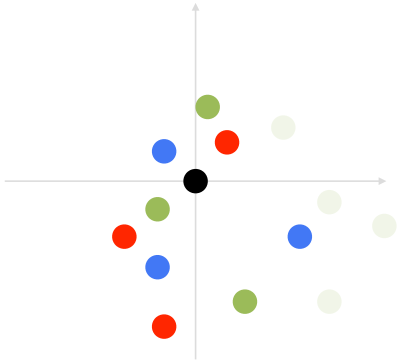
Colorful Caratheodory Theorem



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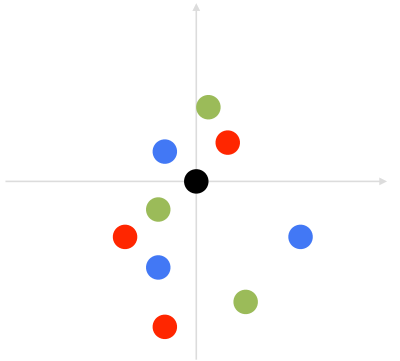
Colorful Caratheodory Theorem



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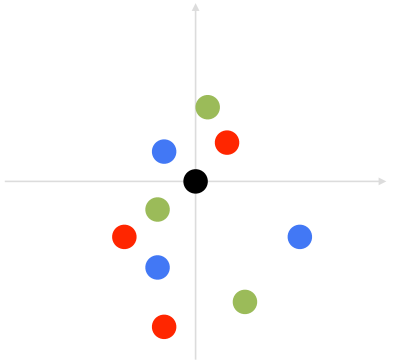
Colorful Caratheodory Theorem



The left 9 points has a colorful triangle that contains O. Conseqently, the running time is $O(n \log n + 9^3)$.

Streaming Algorithm for CCT

Run Convex Hull on each point set, and memoize the 3 points in each point set that contains O.

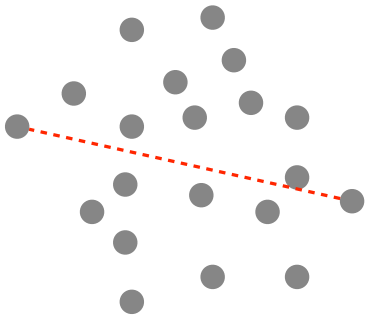
This give a streaming algorithm whose pass complexity is $O(1/\boldsymbol{\delta})$ and space complexity is $O(hn^{\boldsymbol{\delta}} \log n)$.

Diameter

Given a point set on a plane.

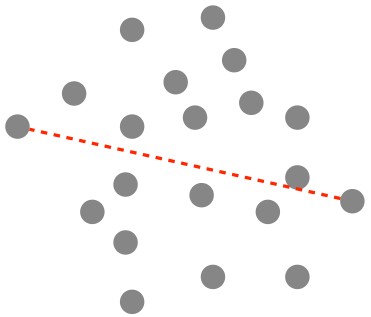
Goal: find the pair of points whose distance is maximum.

Diameter



Observation: the pair of point is an antipodal pair.

Diameter



Running rotating calipers along the hull edges, we can enumerate all antipodal pair and the farthest pair has the maximum distance.

Streaming Algorithm for Diameter

Simulate the rotating caliper by running two copies of convex hull algorithms.

This give a streaming algorithm whose pass complexity is $O(1/\delta^2)$ and space complexity is $O(hn^{\delta} \log n)$.

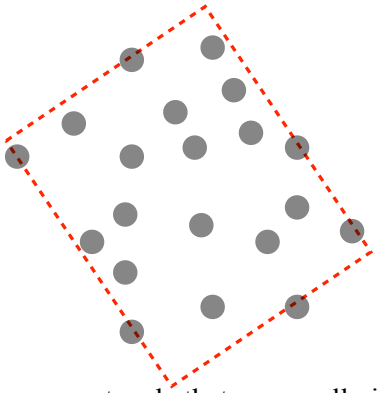
Minimum Enclosing Rectangle

Given a point set on a plane.

Goal: find a min-area rectangle that covers all given points.

Minimum Enclosing Rectangle

Given a point set on a plane.



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Minimum Enclosing Rectangle

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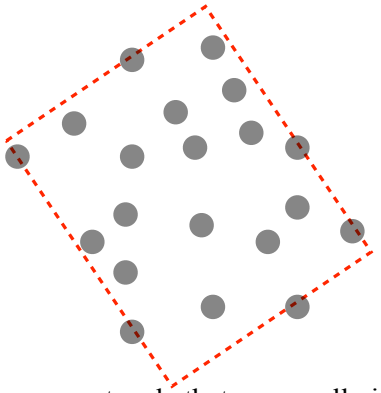
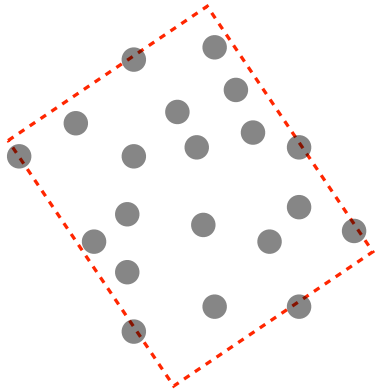


Goal: find a min-area rectangle that covers all given points.

Minimum Enclosing Rectangle

Lemma. The min-area enclosing rectangle has one side conlinear with one of the hull edges.



Streaming Algorithm for Diameter

Simulate the rotating caliper by running four copies of convex hull algorithms.

This give a streaming algorithm whose pass complexity is $O(1/\delta^4)$ and space complexity is $O(hn^{\delta} \log n)$.

Thanks. Any Questions?