Linear Regression

Linear Regression

- The first machine learning algorithm we will explore is also one of the oldest!
- Let’s have a quick overview of what is covered in this section of the course.

Linear Regression

- Linear Regression
  - Theory of Linear Regression
  - Simple Implementation with Python
  - Scikit-Learn Overview
  - Linear Regression with Scikit-learn
  - Polynomial Regression
  - Regularization
  - Overview of Project Dataset

Linear Regression

- Linear Regression
  - Theory of Linear Regression
  - Simple Implementation with Python
  - Scikit-Learn Overview
  - Linear Regression with Scikit-learn
  - Polynomial Regression
  - Regularization
  - Overview of Project Dataset

Linear Regression

- Unlike future ML Algorithm sections, the exercise project for linear regression will be spread over many sections, since we will first discuss feature engineering and cross-validation before tackling the full project exercise.

Introduction to Linear Regression

Algorithm Theory - Part One
History and Motivation

Linear Regression

- Before we do any coding, we will have a deep dive into building out an intuition of the theory and motivation behind Linear Regression.

رواد مصر الرقمية

Linear Regression

وزارة الاتصـــــــالات
وتكنولوجيا المعلومات

- This will include understanding:
  - Brief History
  - Linear Relationships
  - Ordinary Least Squares
  - Cost Functions
  - Gradient Descent
  - Vectorization

Linear Regression

- Relevant Reading in ISLR
  - Section 3 : Linear Regression
    - 3.1 Simple Linear Regression

Linear Regression

- The history of the “invention” of linear regression is a bit muddled.
- The linear regression methods based on least squares grew out of a need for mathematically improving navigation methods based on astronomy during the Age of Exploration in the 1700s.

Linear Regression

- 1722 - Roger Cotes discovers combining different observations yields better estimates of the true value.
- 1750 - Tobias Mayer explores averaging different results under similar conditions in studying librations of the moon.

Linear Regression

- 1757 - Roger Joseph Boscovich further develops combining observations studying the shape of the Earth.
- 1788 - Pierre-Simon LaPlace develops similar averaging theories in explaining the differences of motion between Jupiter and Saturn.

Linear Regression

- 1805 - First public exposition on Linear Regression with least squares method published by Adrien-Marie Legendre - *Nouvelles Méthodes pour la Détermination des Orbites des Comètes*

Linear Regression

- 2005 - Side portrait of Adrien-Marie Legendre is actually discovered to be Louis Legendre!
- Only one known sketch of Adrien-Marie Legendre...

Linear Regression

- Only one known watercolor caricature sketch of Adrien-Marie Legendre...

Linear Regression

- 1809 - Carl Friedrich Gauss publishes his methods of calculating orbits of celestial bodies.
- Claiming to have invented least-squares back in 1795!

Linear Regression

- Carl Friedrich Gauss was born in 1777, which would make him 18 years old at his proclaimed time of discovery!

Linear Regression

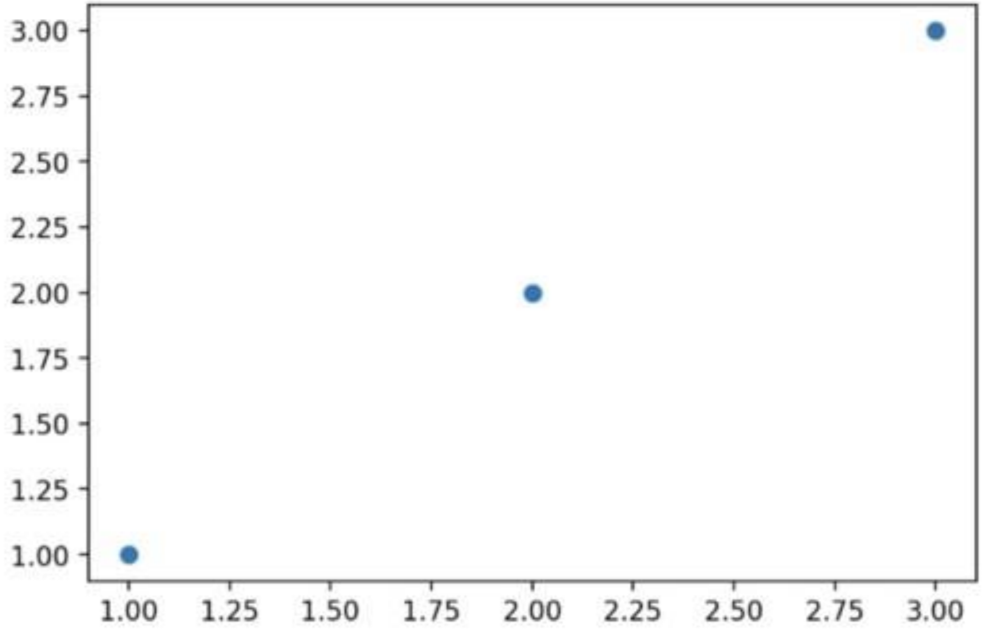
- 1808 - Robert Adrain published his formulation of least squares (a year before publication by Gauss).

Linear Regression

- Whatever the case of invention may be, let’s build an intuitive understanding of linear regression!

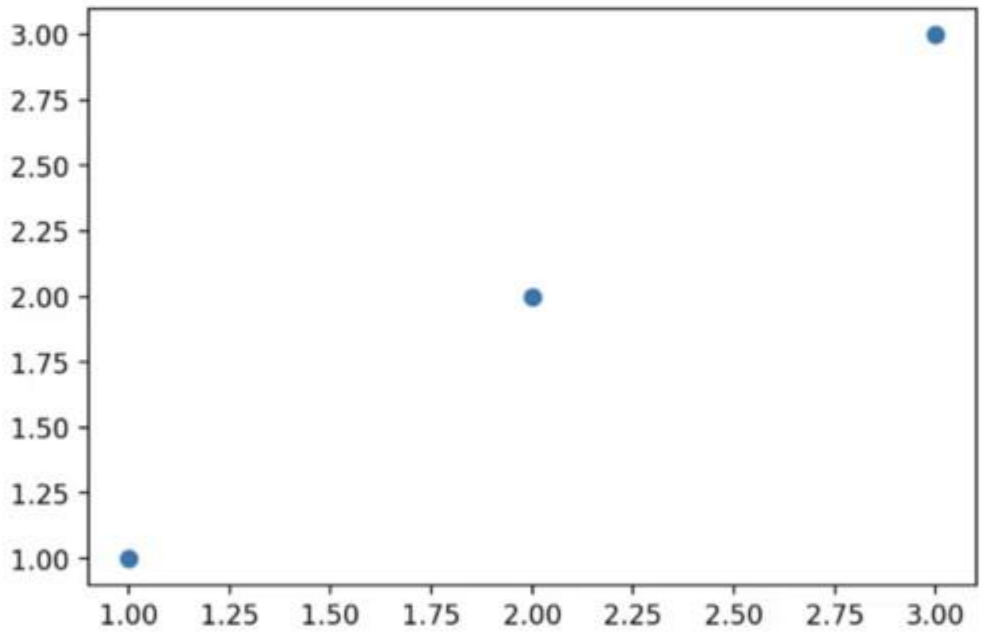
Linear Regression

- Put simply, a linear relationship implies some constant straight line relationship.
- The simplest possible being $y = x$.



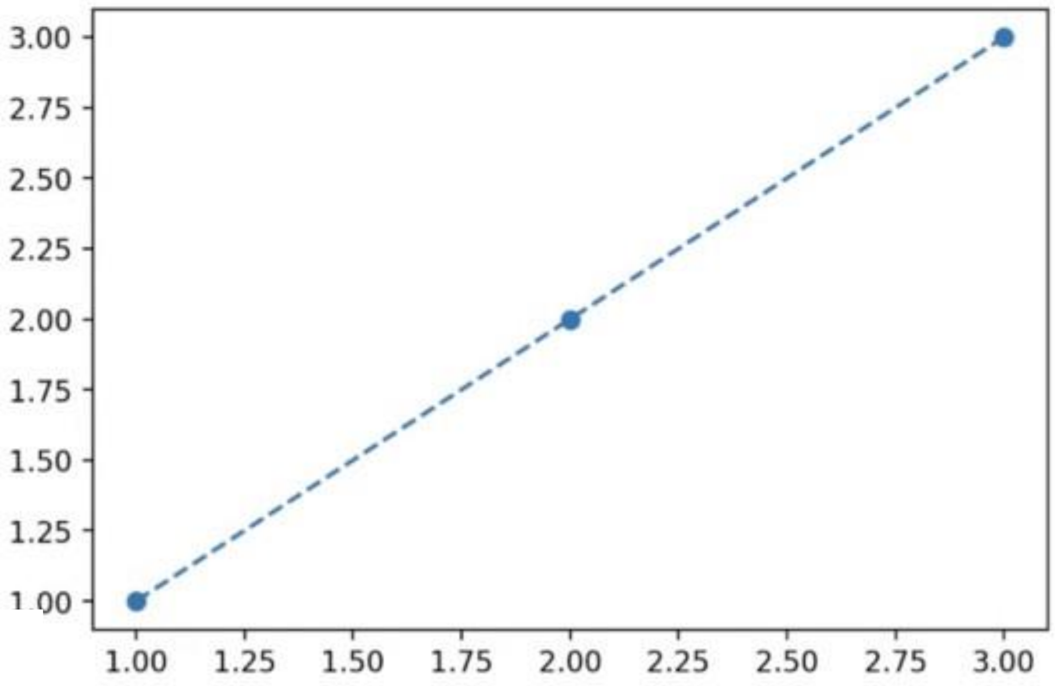
Linear Regression

- Here we see x = [1,2,3] and y = [1,2,3]



Linear Regression

- We could then (based on the three real data points) build out the relationship y=x as our “fitted” line.

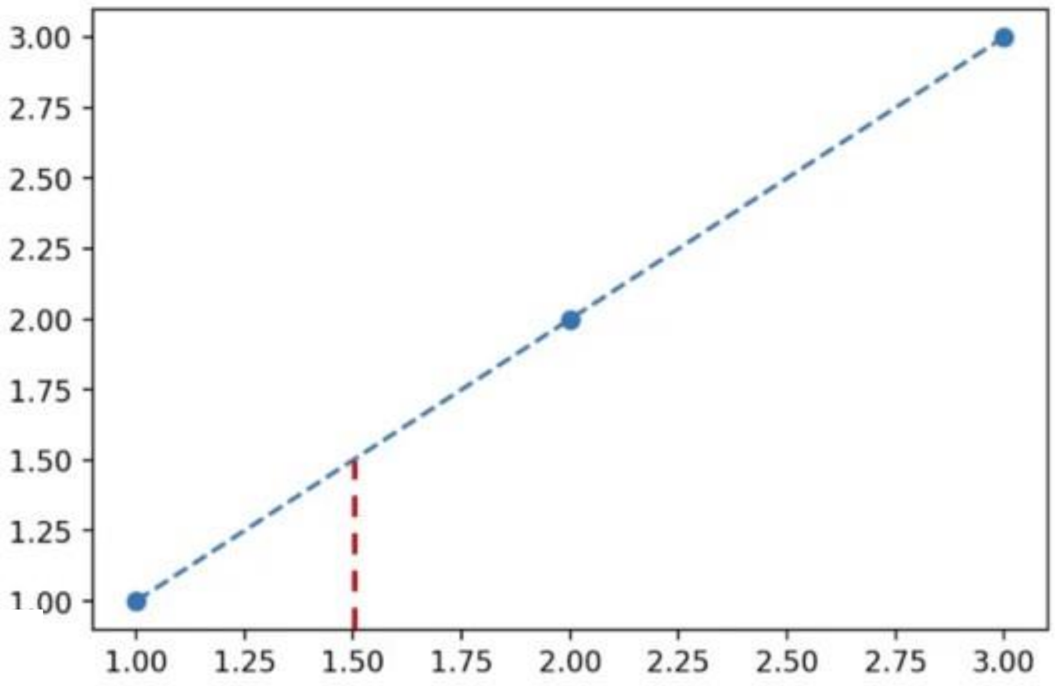


رواد مصر الرقمية

Linear Regression

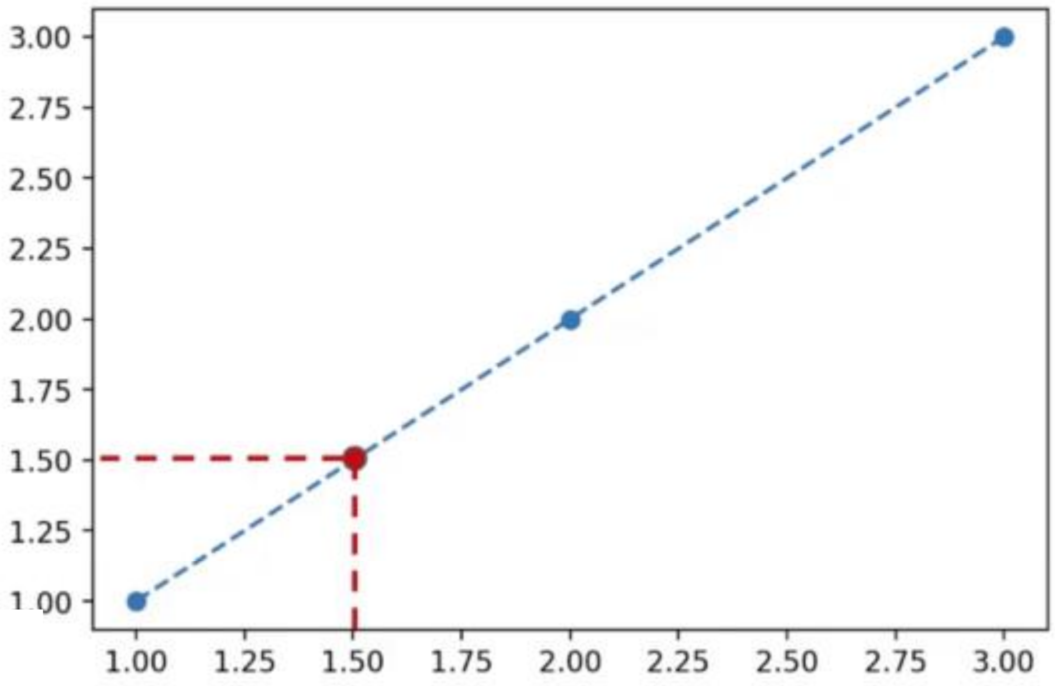
وزارة الاتصـــــالات
وتكنولوجيا المعلومات

- This implies for some new x value I can predict its related y.



Linear Regression

- This implies for some new x value I can predict its related y.

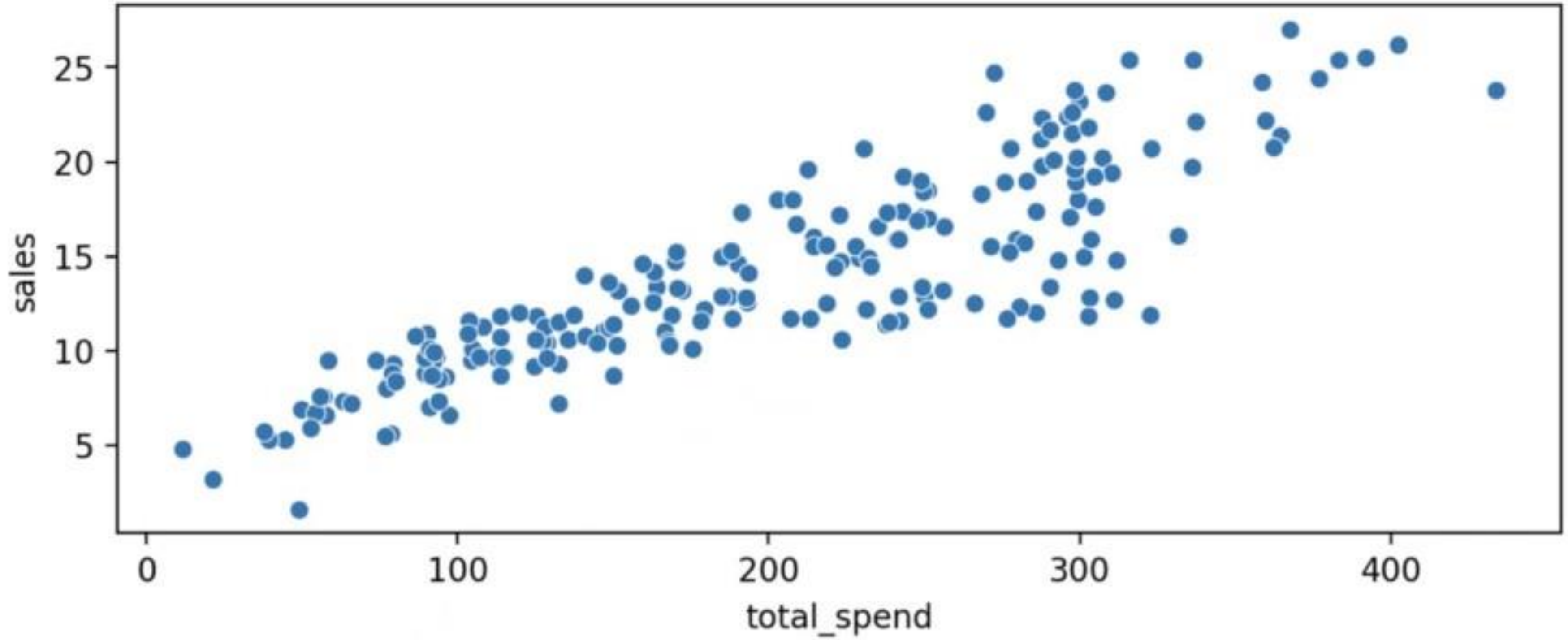


Linear Regression

وزارة الاتصـــــــالات
وتكنولوجيا المعلومات

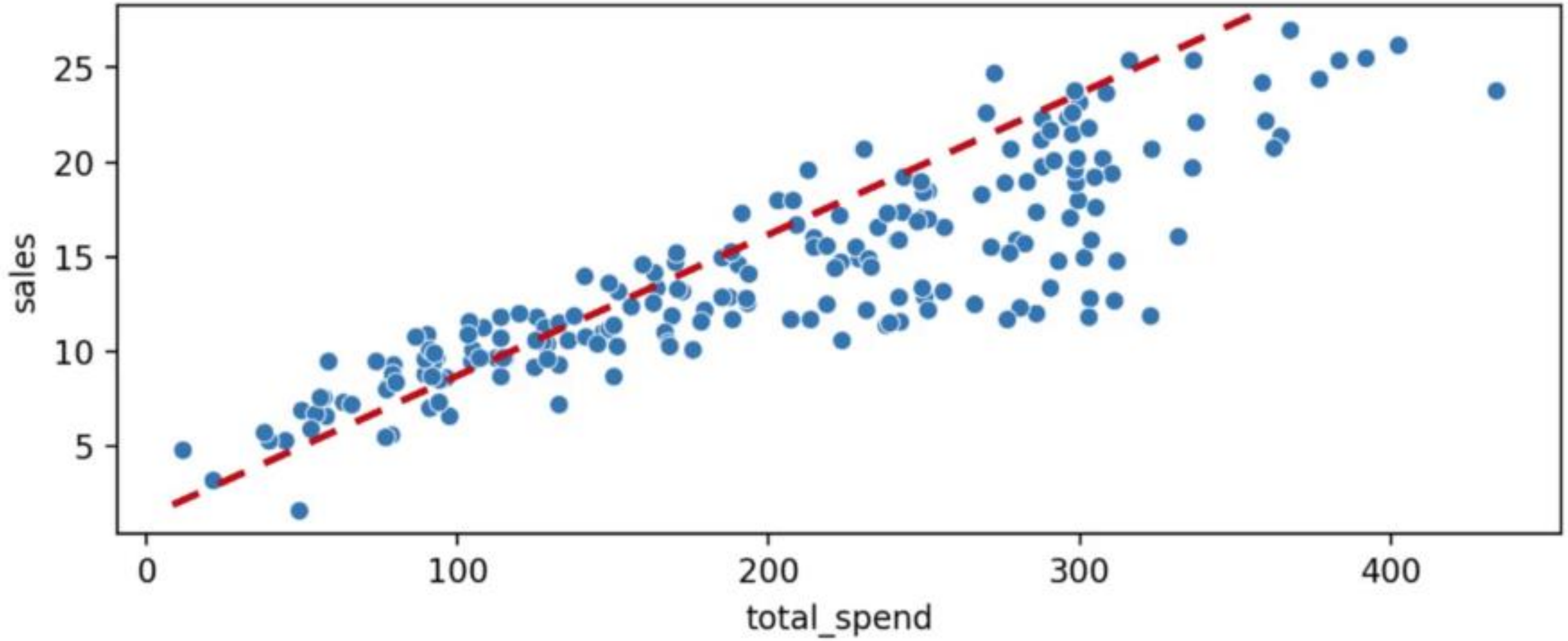
رواد مصر الرقمية

- But what happens with real data? Where do we draw this line?



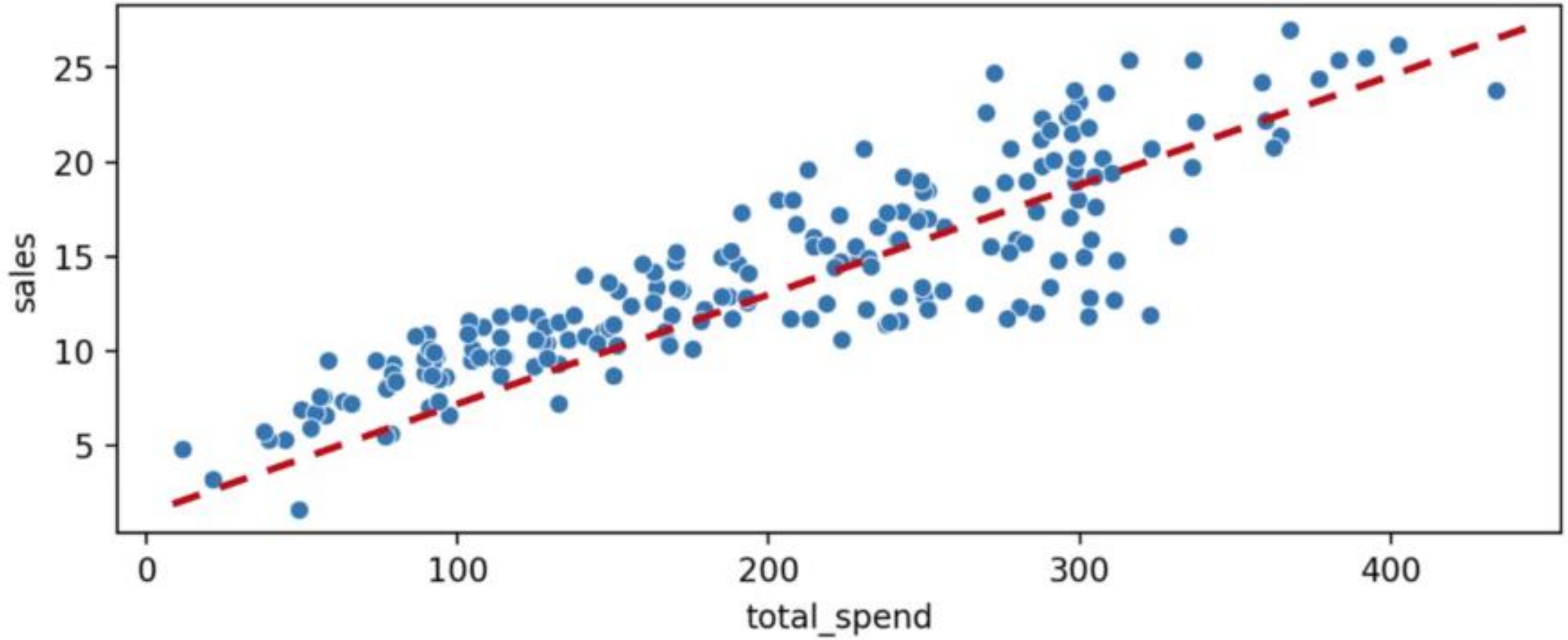
Linear Regression

- But what happens with real data? Where do we draw this line?



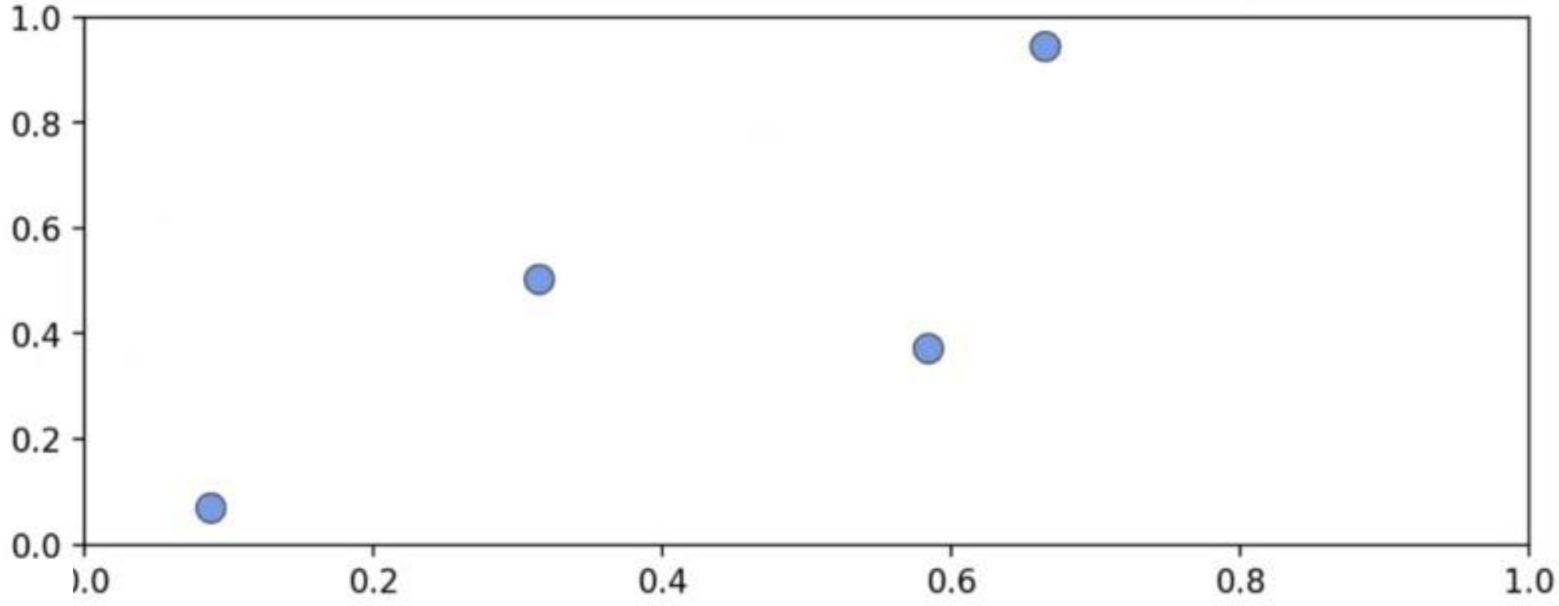
Linear Regression

- But what happens with real data? Where do we draw this line?



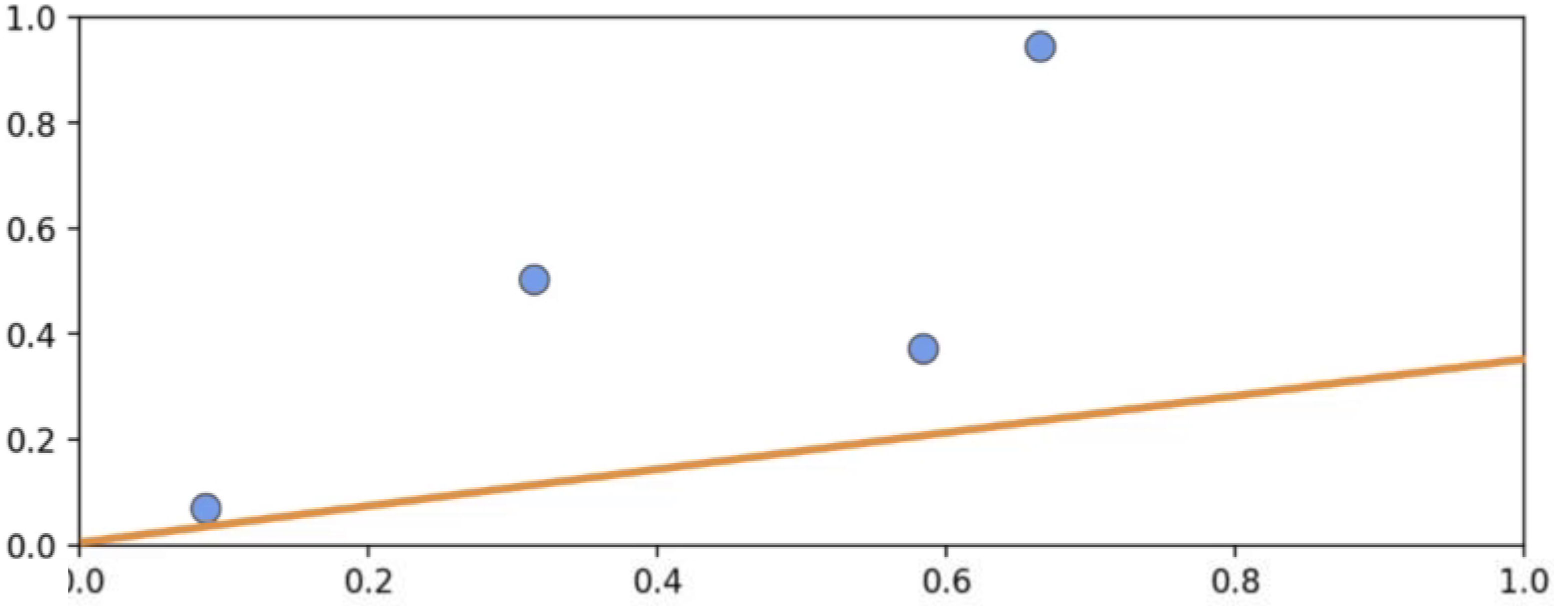
Linear Regression

- Fundamentally, we understand we want to minimize the overall distance from the points to the line.



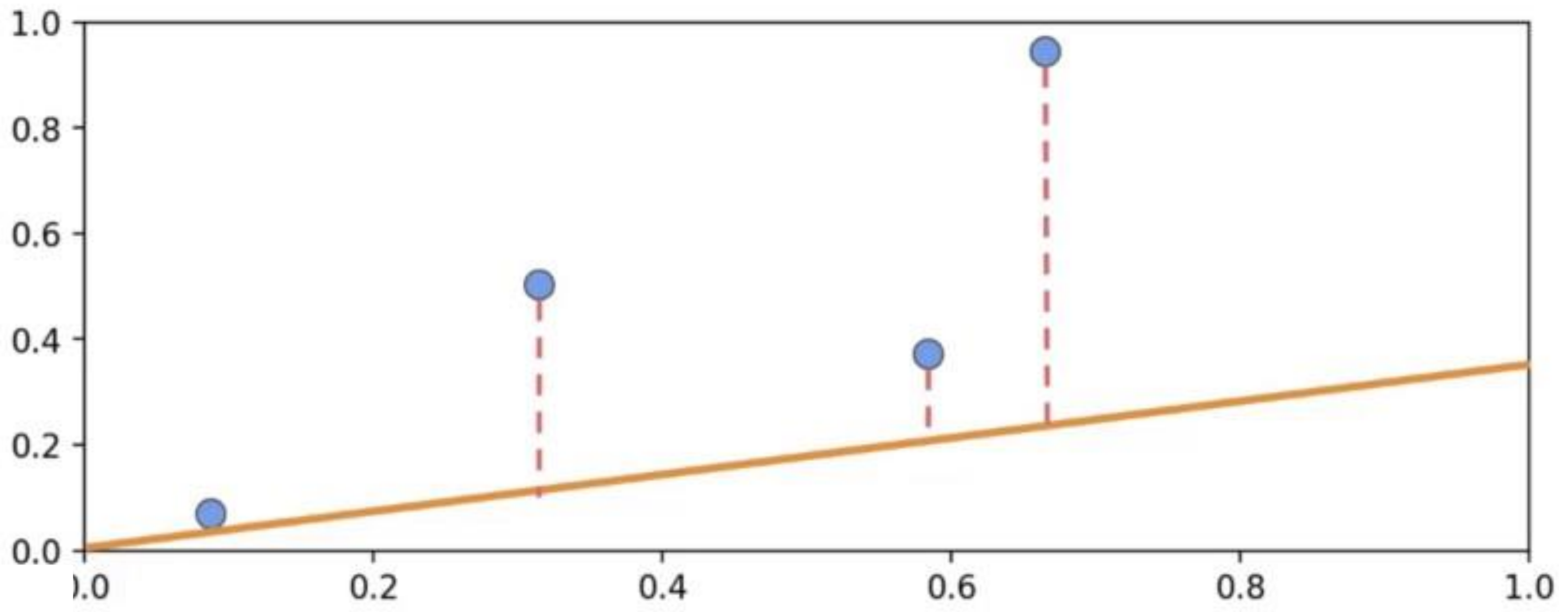
Linear Regression

- Fundamentally, we understand we want to minimize the overall distance from the points to the line.



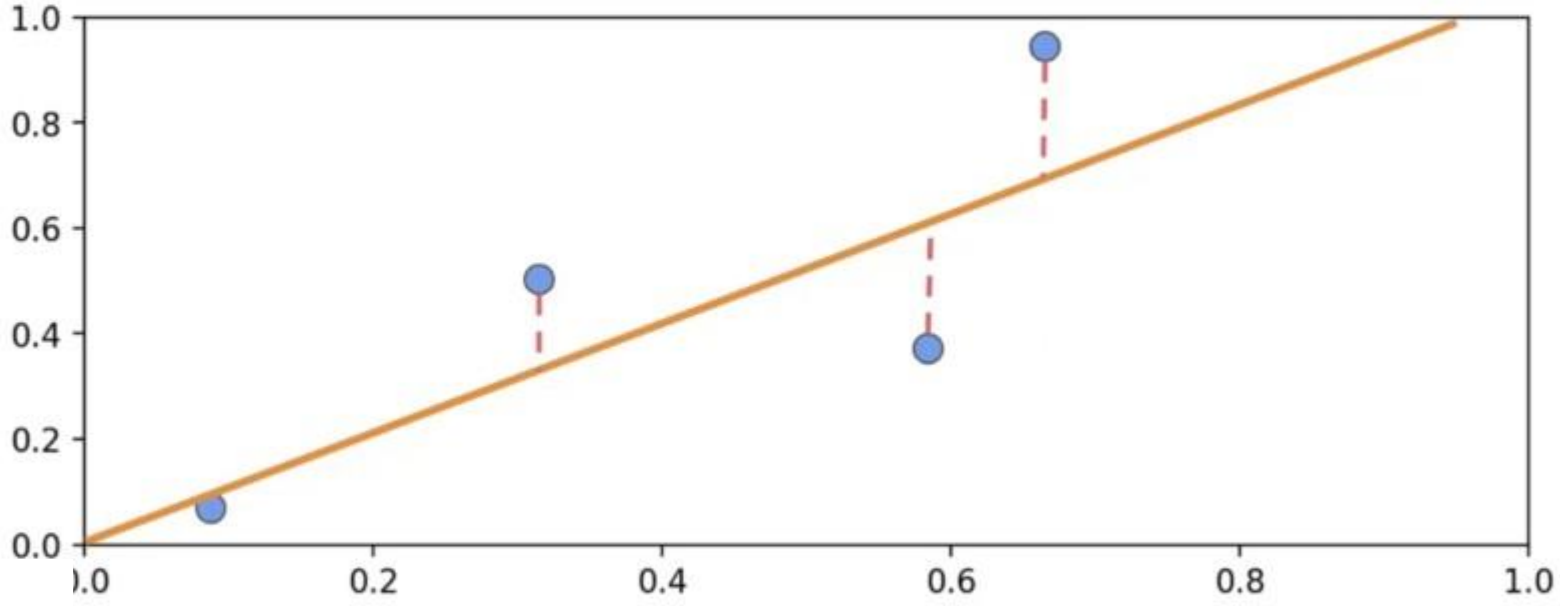
Linear Regression

- We also know we can measure this error from the real data points to the line, known as the **residual error.**



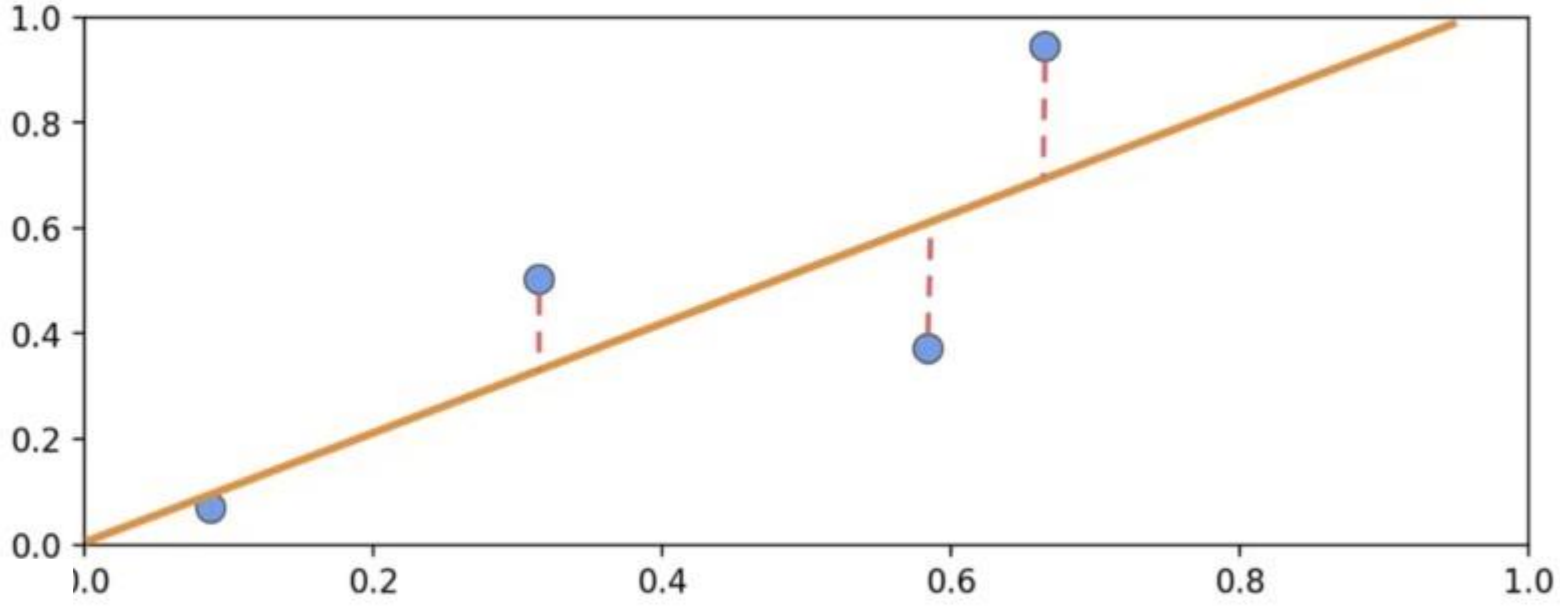
Linear Regression

- Some lines will clearly be better fits than others.



Linear Regression

- We can also see the **residuals** can be both positive and negative.

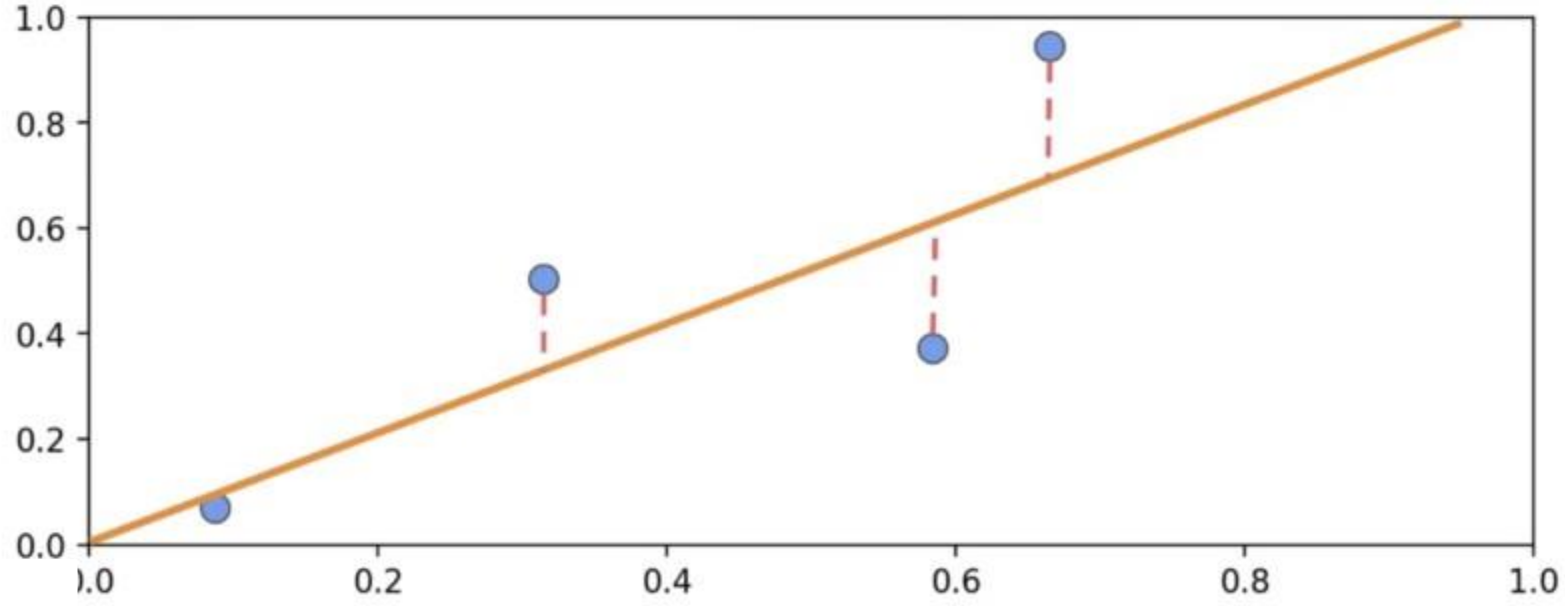


Linear Regression

- **Ordinary Least Squares** works by minimizing the sum of the squares of the differences between the observed dependent variable (values of the variable being observed) in the given dataset and those predicted by the linear function.

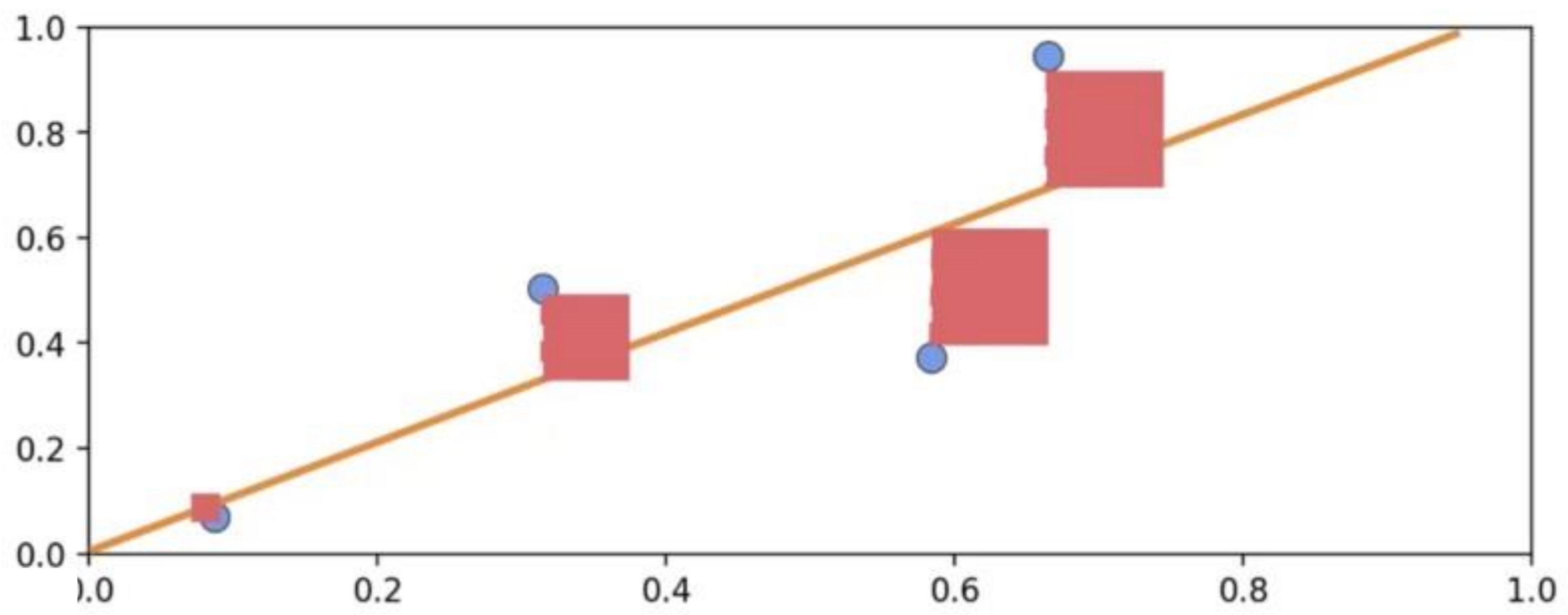
Linear Regression

- We can visualize squared error to minimize:



Linear Regression

- We can visualize squared error to minimize:



Linear Regression

- Having a squared error will help us simplify our calculations later on when setting up a derivative.
- Let's continue exploring OLS by converting a real data set into mathematical notation, then working to solve a linear relationship between features and a variable!