

# Regularization

# Regularization

- Regularization seeks to solve a few common model issues by:
  - Minimizing model complexity
  - Penalizing the loss function
  - Reducing model overfitting (add more bias to reduce model variance)

# Regularization

- In general, we can think of regularization as a way to reduce model overfitting and variance.
  - Requires some additional bias
  - Requires a search for optimal penalty hyperparameter.

# Regularization

- Three main types of Regularization:
  - L1 Regularization
    - LASSO Regression
  - L2 Regularization
    - Ridge Regression
  - Combining L1 and L2
    - Elastic Net

# Regularization

- L1 regularization adds a penalty equal to the **absolute value** of the magnitude of coefficients.
  - Limits the size of the coefficients.
  - Can yield sparse models where some coefficients can become zero.

# Regularization

- L1 regularization adds a penalty equal to the **absolute value** of the magnitude of coefficients.

$$\sum_{i=1}^n \left( y_i - \beta_0 - \sum_{j=1}^p \beta_j x_{ij} \right)^2 + \lambda \sum_{j=1}^p |\beta_j| = \text{RSS} + \lambda \sum_{j=1}^p |\beta_j|$$

# Regularization

- L2 regularization adds a penalty equal to the **square** of the magnitude of coefficients.
  - All coefficients are shrunk by the same factor.
  - Does not necessarily eliminate coefficients.



# Regularization

- L2 regularization adds a penalty equal to the **square** of the magnitude of coefficients.

$$\sum_{i=1}^n \left( y_i - \beta_0 - \sum_{j=1}^p \beta_j x_{ij} \right)^2 + \lambda \sum_{j=1}^p \beta_j^2 = \text{RSS} + \lambda \sum_{j=1}^p \beta_j^2$$



# Regularization

- Elastic net combines L1 and L2 with the addition of an alpha parameter deciding the ratio between them:

$$\frac{\sum_{i=1}^n (y_i - x_i^J \hat{\beta})^2}{2n} + \lambda \left( \frac{1 - \alpha}{2} \sum_{j=1}^m \hat{\beta}_j^2 + \alpha \sum_{j=1}^m |\hat{\beta}_j| \right)$$

# Regularization

- Elastic net combines L1 and L2 with the addition of an alpha parameter deciding the ratio between them:

$$\alpha = 0$$

$$\frac{\sum_{i=1}^n (y_i - x_i^J \hat{\beta})^2}{2n} + \lambda \left( \frac{1 - \alpha}{2} \sum_{j=1}^m \hat{\beta}_j^2 + \alpha \sum_{j=1}^m |\hat{\beta}_j| \right)$$

# Regularization

- Elastic net combines L1 and L2 with the addition of an alpha parameter deciding the ratio between them:

$$\alpha = 1$$

$$\frac{\sum_{i=1}^n (y_i - x_i^J \hat{\beta})^2}{2n} + \lambda \left( \frac{1 - \alpha}{2} \sum_{j=1}^m \hat{\beta}_j^2 + \alpha \sum_{j=1}^m |\hat{\beta}_j| \right)$$

# Regularization

- These regularization methods do have a cost:
  - Introduce an additional hyperparameter that needs to be tuned.
  - A multiplier to the penalty to decide the “strength” of the penalty.

# Regularization

- Later on, we will actually cover L2 regularization (Ridge Regression) first, due to the intuition behind the squared term being easier to understand.

# Regularization

- Before we dive straight into coding regularization with Scikit-Learn, we need to discuss a few more relevant topics:
  - Feature Scaling
  - Cross Validation

# Feature Scaling



# Feature Scaling

- Feature scaling provides many benefits to our machine learning process!
- Some machine learning models that rely on distance metrics (e.g. KNN) **require** scaling to perform well.
- Let's discuss the main ideas behind feature scaling...

# Feature Scaling

- Feature scaling improves the convergence of steepest descent algorithms, which do not possess the property of scale invariance.
- If features are on different scales, certain weights may update faster than others since the feature values  $x_j$  play a role in the weight updates.

# Feature Scaling

- Critical benefit of feature scaling related to gradient descent.
- There are some ML Algos where scaling won't have an effect (e.g. CART based methods).

# Feature Scaling

- Scaling the features so that their respective ranges are uniform is important in comparing measurements that have different units.
- Allows us directly compare model coefficients to each other.

# Feature Scaling

- Feature scaling caveats:
  - Must always scale new unseen data before feeding to model.
  - Effects direct interpretability of feature coefficients
    - Easier to compare coefficients to one another, harder to relate back to original unscaled feature.



# Feature Scaling

- Feature scaling benefits:
  - Can lead to great increases in performance.
  - Absolutely necessary for some models.
  - Virtually no “real” downside to scaling features.

# Feature Scaling

- Two main ways to scale features:
  - Standardization
    - Rescales data to have a mean ( $\mu$ ) of 0 and standard deviation ( $\sigma$ ) of 1.
  - Normalization
    - Rescales all data values to be between 0-1.



# Feature Scaling

- Standardization:
  - Rescales data to have a mean ( $\mu$ ) of 0 and standard deviation ( $\sigma$ ) of 1 (unit variance).

$$X_{changed} = \frac{X - \mu}{\sigma}$$

# Feature Scaling

- Standardization:
  - Namesake can be confusing since this is also referred to as “Z-score normalization”.

$$X_{changed} = \frac{X - \mu}{\sigma}$$

# Feature Scaling

- Normalization:
  - Scales all data values to be between 0 and 1.

$$X_{changed} = \frac{X - X_{min}}{X_{max} - X_{min}}$$

# Feature Scaling

- Normalization:
  - Simple and easy to understand.

$$X_{changed} = \frac{X - X_{min}}{X_{max} - X_{min}}$$

# Feature Scaling

- There are many more methods of scaling features and Scikit-Learn provides easy to use classes that “fit” and “transform” feature data for scaling.
- Let’s quickly discuss the fit and transform calls in more detail when it comes to scaling.

# Feature Scaling

- A `.fit()` method call simply calculates the necessary statistics ( $X_{min}$ ,  $X_{max}$ , mean, standard deviation).
- A `.transform()` call actually scales data and returns the new scaled version of data.
- Previously saw a similar process for polynomial feature conversion.

# Feature Scaling

- Very important consideration for fit and transform:
  - We only **fit** to training data.
  - Calculating statistical information should only come from training data.
  - Don't want to assume prior knowledge of the test set!



# Feature Scaling

- Using the full data set would cause **data leakage**:
  - Calculating statistics from full data leads to some information of the test set leaking into the training process upon `transform()` conversion.

# Feature Scaling

- Feature scaling process:
  - Perform train test split
  - Fit to training feature data
  - Transform training feature data
  - Transform test feature data

# Feature Scaling

- Do we need to scale the label?
  - In general it is not necessary nor advised.
  - Normalising the output distribution is altering the definition of the target.
  - Predicting a distribution that doesn't mirror your real-world target.

# Feature Scaling

- Do we need to scale the label?
  - Can negatively impact stochastic gradient descent.
- **[stats.stackexchange.com/questions/111467](https://stats.stackexchange.com/questions/111467)**

# Feature Scaling

- Now that we understand the benefits of feature scaling, let's move on to understanding the benefits of cross-validation!