

Assignment 10

Exercise 40 (Central Limit Theorem). Without looking at the lecture notes state the Central Limit Theorem.

Exercise 41 (Illustration of the Central Limit Theorem). Generate samples from the Beta distribution in Python using `numpy.random.beta` (read the numpy help page for this). Read the Wikipedia page for the Beta distribution to see what it looks like and what its expectation and variance are.

Investigate (using Python) what the empirical CDF of the standardised mean of the samples of the Beta distribution looks like (similarly to the examples discussed in the lecture) for different numbers of samples. From the Central Limit Theorem we know that for large sample size this should look like the CDF of the standard normal distribution.

In addition to the empirical CDF plot histograms of the standardised mean in Python as well.

Exercise 42 (Black-Scholes option pricing formula - European call option). Consider the Black-Scholes market. Compute analytically the time-0 price of a European call option with strike price K , i.e., show that

$$E \left[e^{-rT} (S_T - K)^+ \right] = S_0 \Phi(D_1) - K e^{-rT} \Phi(D_1 - \sigma \sqrt{T}),$$

where

$$D_1 = \frac{\log \left(\frac{S_0}{K} \right) + \left(r + \frac{\sigma^2}{2} \right) T}{\sigma \sqrt{T}}.$$