Assignment 7

Exercise 31 (Checking that the risk-neutral probability is a probability)**.** Consider a one-period binomial model and suppose the no-arbitrage condition $d < 1 + r < u$ holds and suppose that $0 < d < u$, define

$$\tilde{p} = \frac{1 + r - d}{u - d}.$$

Show that $\tilde{p} \in (0, 1)$ and $1 - \tilde{p} = \frac{u-1-r}{u-d}$.

Exercise 32 (Sufficient condition for no-arbitrage)**.** Consider a one-period binomial model with $0 < d < u$ and $r > -1$ and assume that

$$d < 1 + r < u$$

holds. Show that there is no arbitrage in the model.

Exercise 33 (Pricing a European put option)**.**

1. Consider a one-period binomial model with model parameters $S_0 = 4$, $u = 2$, $d = \frac{1}{2}$, $r = \frac{1}{4}$. Compute the time-0 price of a European put option with maturity $T = 1$ and strike price $K = 5$ in this model (recall that its payoff at time 1 is $(K - S_1)^+$).

2. Write a function in Python that computes the price of a European put option in the one-period binomial model. (Test that it works by using the analytical price derived in 1.)