CHAPTER 1: COORDINATES, GRAPHS AND LINES

CHAPTER OUTLINE:

1. Real Number System
    - Classification of Numbers
    - The real number lines
2. Inequalities
    - Properties of Inequalities
    - Solving inequalities (linear, quadratic and rational)
3. Absolute Value
    - Definition of Absolute Value
    - Equation involving Absolute Value
    - Inequalities involving Absolute Value
4. Complex Number
    - Definition of Complex Number & Geometrical Interpretation
    - Algebraic Operation
    - Quadratic Equation involving complex root
5. Coordinate Plane and Graph
    - The Cartesian coordinate System
    - Basic Graphs
    - Piecewise Function
6. Plane Analytic Geometry
    - Line
    - Circle
    - Parabola

1. REAL NUMBER SYSTEM

- The real number is the set of numbers containing all of the rational numbers and all of the irrational numbers.
- The real numbers are "all the numbers" on the number line.

(Ref: Varsity Tutor)

1.1 Classification of Numbers

There are 5 classification of numbers: Natural, Whole, Integers, Rational, Irrational.

Natural Number (N)

- Also called the counting number

- All positive number we use to count the object (not including 0)
- The set of Natural numbers {1, 2, 3, 4, …} is sometimes represented by N

Whole Number (W)

- The whole numbers are the numbers 0, 1, 2, 3, and so on (the natural numbers and 0)
- All natural numbers are whole numbers but not all whole numbers are natural numbers since zero is a whole number but not a natural number
- Denoted by W

Integers (Z)

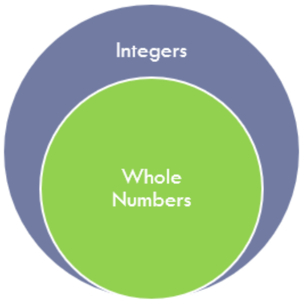
- Positive and Negative whole numbers and denoted by Z
- …, -3, -2, -1, 0, 1, 2, 3, …
- Includes even numbers, odd numbers, prime numbers, and composite numbers

WHOLE NUMBER = INTEGERS

All whole numbers are integers because integers include both positive and negative whole numbers

But not all integers are whole numbers or natural numbers

(eg: **- 5** is an integer but not a whole number or a natural number)



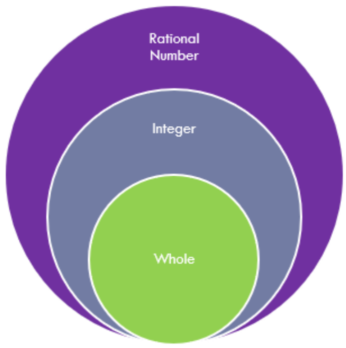
Rational Number (Q)

- Numbers that can be written as a fraction a/b
  (where a & b are integers and b ≠ 0).
- Numbers that have terminating decimals (end)
  2.5 = 25/10 = 2 ½
  0.75 = 3/4
- Numbers that have repeating decimals (repetition of digits)
  0.3333…. = 3/9 = 1/3
  1.454545….
  0.025025….

RATIONAL NUMBER = INTEGERS = WHOLE NUMBERS

All rational numbers are integers and whole numbers because you can make them into a ratio (or fraction) by putting a 1 under it.

24/1, -8/1, 567/1, -76/1, 24/3, -64/8

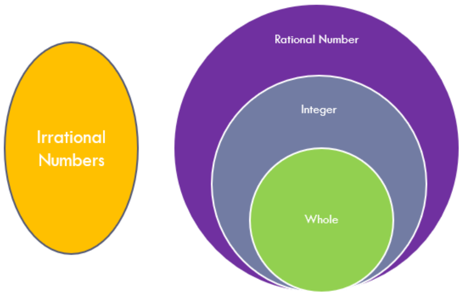


Irrational Number (I)

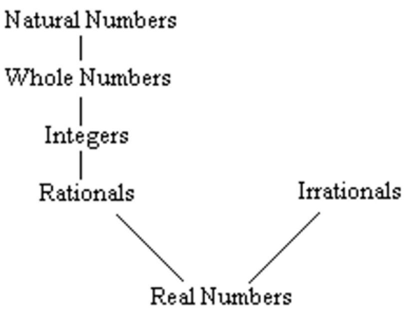
- Numbers that cannot be made into a simple fraction; they have a decimal that keeps going and going.
  π , √2 , 4.23233…. , -√8

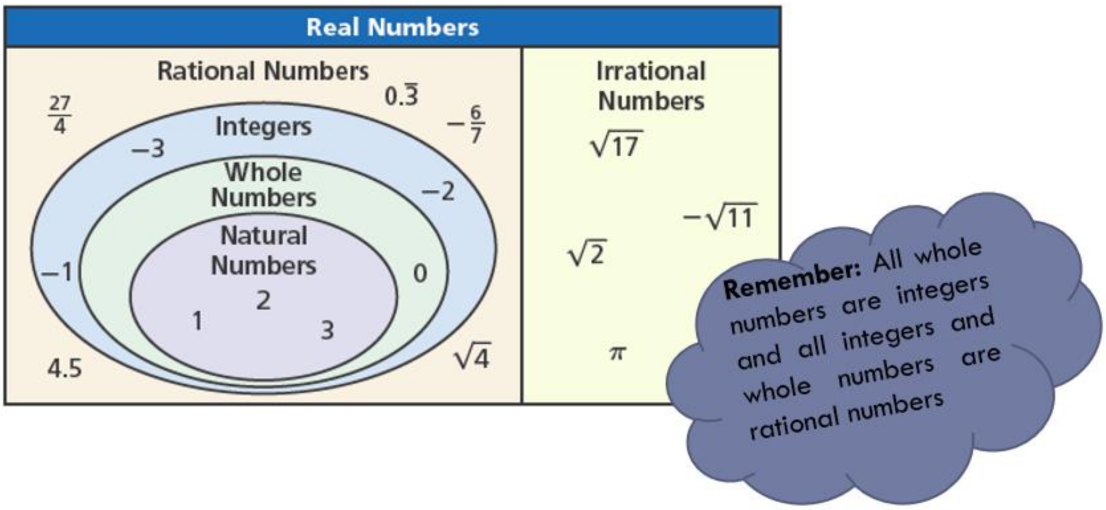
IRRATIONAL NUMBERS RELATED?

Irrational Numbers are by themselves because they cannot be made into fractions (rational numbers) or cannot be a positive or negative whole number since there is no decimal



CONCLUSION: All the rational numbers and all the irrational numbers together form the real numbers. R = Q + I





Let's Practice!

Type of Number

- 0
- 47
- 24/8
- 56/1
- 279

Type of Number

- 2.454554555….
- $\sqrt{6}$
- π
- -3.4224222….
- $-\sqrt{10}$

Type of Number

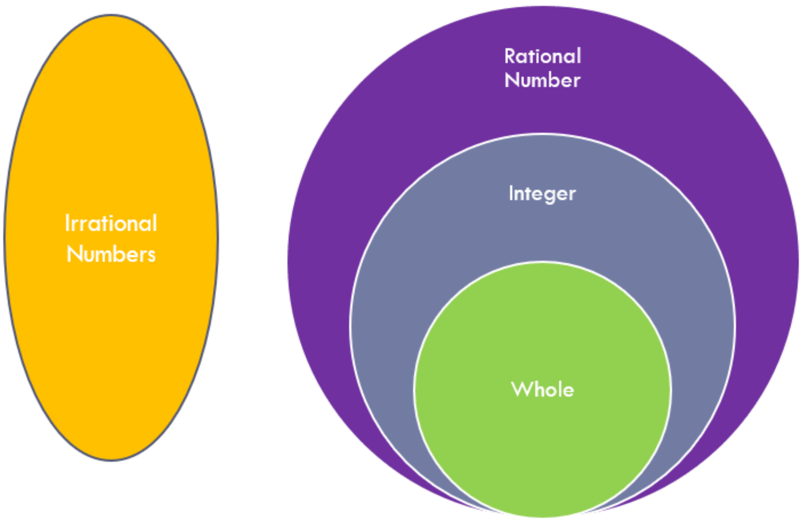
- -4
- $-\sqrt{100}$
- -12
- -81/9
- $-\sqrt{25}$

Type of Number

- 2.45
- -0.6060606
- 34 ½
- ¾
- -7.5

Place these numbers into correct category on the chart to prove your understanding!

-3, 27/3, π, 4.68, $\sqrt{13}$, $-\sqrt{49}$, 3.14144…, 8, ¼,
3.25, 61, 0.8 repeating, $\sqrt{144}$, -30/5, 244/2, 0



1.2 The Real Number Lines

- A real number line, or simply number line, allows us to visually display real numbers by associating them with unique points on a line.
- Can be arranged in horizontal line
- Positive and negative numbers

Black dot is used for $\leq$ or $\geq$ sign

White dot is used for $<$ or $>$ sign

2. INEQUALITIES

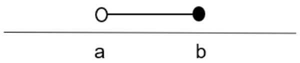
2.1 Properties of Inequalities

- Two quantities are sometimes not equal and cannot be equated using the equality sign "=“
- Unequal relationship: INEQUALITY
- Inequality signs: < , > , ≤ , ≥
- Types of Inequality:

  - Linear $2x+3<7$ or $3\geq 4-5x$
  - Quadratic $x^2+5x-7<0$ or $3x^2-5\geq 0$
  - Rational $\dfrac{x^2-1}{x-5}\geq 0$

- Three forms:

  - Inequality sign
    $$a<x\leq b$$
  - Interval notation
    $$[a,b)$$
  - Real number line



- Open and close bracket ( ) , [ ] , ( ] , [ )
- Black and white dot on the real number line
- Operations on Inequalities:

  - ADDITION
    $$9+4x>21$$
  - SUBTRACTION
    $$2x-8\geq 5$$
  - MULTIPLICATION
    $$3x(x+1)\leq x+5$$
  - DIVISION
    $$\frac{5-2x}{x-1}\leq 0$$

2.2 Solving Inequalities

2.2.1 a) Linear Inequality

A linear inequality in one variable can be solved by performing some or all of the four operations onto it to obtain its simplest form

Example 1: Solve the following inequality $-\frac{3}{4}x \le -3$ **(practice workbook: 2)**

Example 2: Find the value of x for i) $-3x < 15$ and ii) $3x > 15$

b) Compound Linear Inequality

Compound linear inequalities – each inequality is split into two simple linear inequalities and solved separately

Example 1: Solve the following inequality $2 + x < 3x - 4 \le 4 + 5x$
(practice workbook: 2)

2.2.2 Quadratic Inequality

Steps to solve:

- Simplify into its general form
- Change inequality sign into the equal sign
- Solve by factoring or completing the square or using quadratic formula
- Get the critical values (roots of the equation)
- Construct test table
- Pick the sign of the interval (s) that match the inequality sign of the quadratic inequality

Example: Solve the following inequality $3x(x+1) \le x + 5$

Example: Solve the following inequality $2x^2 + 5x - 12 \geq 0$ **(practice workbook: 3)**

2.2.3 Rational Inequality

Steps to solve:

- Rearrange it into the standard form
- Find the critical values by setting numerator and denominator equal to zero
- Construct test table
- Pick the sign of the interval (s) that match the inequality sign of the rational inequality

Example: Solve the following inequality $\dfrac{x^2 + 2x - 3}{x - 5} \geq 0$

Example: Solve the following inequality $\dfrac{5}{x-3} \geq \dfrac{3}{x-2}$ **(practice workbook: 7)**

3. ABSOLUTE VALUE

3.1 Definition of Absolute Value

- In practice "absolute value" means to remove any negative sign in front of a number, and to think of all numbers as positive (or zero).
- To show that we want the absolute value of something, we put "|" marks either side (they are called "bars" and are found on the right side of a keyboard), like these examples:

$|-5| = 5$ $\qquad\qquad$ $|7| = 7$

- Sometimes absolute value is also written as "abs ()", so **abs (−1) = 1** is the same as **|−1| = 1**

(Ref: Math is Fun web)

3.2 Equation involving Absolute Value

- An absolute value equation consisting of an absolute term or an absolute expression
- **Hint:** Separate the original absolute value equation into 2 parts (positive and negative components):

$$|a| = a \quad \text{if } a \geq 0$$
$$|a| = -a \quad \text{if } a \leq 0$$

- **Theorem:** $|a| = \sqrt{a^2}$

Example: Solve the following absolute value equations:

a) $|12x - 5| = 3x$ **(practice workbook: 8)**

b) $4|2x - 1| - 2 = 34$

c) $\left|\frac{2x-1}{x+1}\right| = 3$

d) $|2x+1| = \sqrt{1-x}$ **(practice workbook: 9)**

e) $|7x-3| = |3x+7|$

3.3 Inequality involving Absolute Value

- Involving the sign of < , > , ≤ , ≥
- **Theorem:**

$$\text{if } |x| < a \quad \text{then } -a < x < a$$
$$\text{if } |x| > a \quad \text{then } x > a \cup x < -a$$

Example: Solve the following absolute value inequalities:

a) $|2x + 3| \geq 5$

b) $\frac{|4x + 2|}{2} \geq 1$ **(practice workbook: 10)**

c) $2|x+3|-1<10$ **(practice workbook: 10)**

d) $\dfrac{1}{4|1-2x|}+3>6$

e) $|x+3|<\sqrt{3-x}$

4. COMPLEX NUMBER

4.1 Definition of Complex Number & Geometrical Interpretation

A **complex number** is a **number** that can be expressed in the form a + bi, where a and b are real **numbers**, and i is a symbol called the imaginary unit, and satisfying the equation $i^2 = -1$. It is usually denoted by the letter z → **z = a + bi**

(Ref: Foundation Mathematics textbook)

Plot the following complex numbers in the complex plane

a) $Z_1 = 3+i$

b) $Z_2 = -1-3i$

c) $Z_3 = -2+4i$

d) $Z_4 = 3-2i$

4.2 Algebraic Operation

In Mathematics, algebraic operations are similar to the basic arithmetic operations which include **addition**, **subtraction**, **multiplication**, and **division**.

Addition and Subtraction

$$(a+bi)+(c+di)=(a+c)+(b+d)i$$
$$(a+bi)-(c+di)=(a-c)+(b-d)i$$

Multiplication

$$(a+bi)(c+di)=(ac-cd)+(ad+bc)i$$

Product of Conjugate

$$z\bar{z}=(a+bi)(a-bi)=a^2+b^2$$

Scalar Multiplication

$kz = k(a+bi) = ka+kbi$, where k is a scalar

Quotient ($z_2 \neq 0$)

$$\frac{z_1}{z_2}=\frac{a+bi}{c+di}=\frac{(a+bi)(c-di)}{(c+di)(c-di)}=\frac{(ac+bd)(bc-ad)}{c^2+d^2}\,i$$

Examples:

1. $(5+3i)+(2-5i)$
2. $(6+2i)-(7-4i)$
3. $2i(6+4i)$
4. $(-1+4i)(3+2i)$

5. $\dfrac{3-5i}{2+3i}$

6. $\dfrac{3-i}{5+\sqrt{3}i}+\dfrac{2-i}{5-\sqrt{3}i}$

The length or magnitude of $z=a+bi$ is called the modulus of z and is written as $|z|$, that is the distance between point A (a, b) and the origin.

$$|z|=|a+bi|=\sqrt{a^2+b^2}$$

Examples:

1. $|2+5i|$
2. $|3i|$
3. $|4-4i|$

The conjugate of a complex number is given by:

$$Z=a+bi \quad \Longrightarrow \quad \bar{Z}=a-bi$$

Examples:

1. $z=3+2i$
2. $z=-4-i$
3. $z=10i$
4. $z=-\dfrac{2}{3}-\dfrac{1}{2}i$

Theorem of Conjugate:

$$z\bar{z}=a^2+b^2$$
$$\bar{\bar{z}}=z$$
$$\overline{z+w}=\bar{z}+\bar{w}$$
$$\overline{z\bullet w}=\bar{z}\bullet\bar{w}$$

i Index: to define a complex number we have to create a new variable. This new variable is **“i”**

$$i=\sqrt{-1}$$

Note: i is the <u>representation</u> of $\sqrt{-1}$

$i=\sqrt{-1}$	$i^5=i^4\cdot i=i$	$i^9=i^8\cdot i=i$
$i^2=-1$	$i^6=i^4\cdot i^2=-1$	$i^{10}=i^8\cdot i^2=-1$
$i^3=-i$	$i^7=i^4\cdot i^3=-i$	$i^{11}=i^8\cdot i^3=-i$
$i^4=1$	$i^8=1^4\cdot i^4=1$	$i^{12}=1^8\cdot i^4=1$

Example: Given $z = 4 + 3i$. Solve the following in the standard form of a + bi.
(practice workbook: 15)

1. $z + 3i^{40}(4i^{12} - 1)$
2. $\dfrac{z(i^{19} - 2)}{1 - 3i^{10}}$

4.3 Quadratic Equation involving complex root

In relation to quadratic equations, imaginary numbers (and complex roots) occur when the value under the radical portion of the quadratic formula is negative. When this occurs, the equation has no roots (or zeros) in the set of real numbers. The roots belong to the set of complex numbers, and will be called "complex roots" (or "imaginary roots"). These complex roots will be expressed in the form *a* ± *bi*.

(Ref: MathBitsNotebook.com)

Examples:

1. $x^2 + 16 = 0$
2. $x^2 - 2x + 10 = 0$

Example: Given $z_1 = 9 + i$ and $z_2 = 1 - 2i$. Express $\dfrac{(z_1)^2}{3 - z_2}$ in standard form of a + bi

(practice workbook: 13)

Exercise: Given $x = 2 + 3i$ and $y = 5 - i$. Express the following in the standard form of a + bi

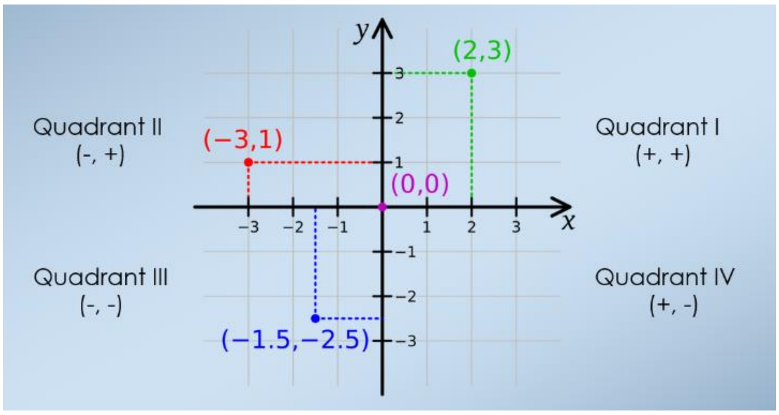
1. $5x - 2y$
2. $2ix - y$
3. $xy + 1$
4. $x^2 + |x - y|$

5. COORDINATE PLANE AND GRAPH

5.1 The Cartesian coordinate System

- The Cartesian plane which is also known as the rectangular coordinate plane, were introduce by Rene Descartes (1596 – 1650)
- Horizontal line (x-axis) and vertical line (y-axis)
- The intersection point of the two perpendicular axes is called the origin (0,0)
- Divided into 4 subsets or quadrants (Quadrant I, Quadrant II, Quadrant III, Quadrant IV)
- Any point on the plane can be represented by an ordered pair of numbers (x , y)

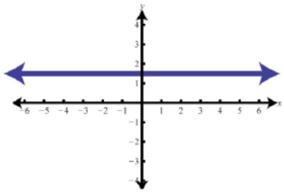
(Ref: Foundation Mathematics Textbook)



5.2 Basic Graphs

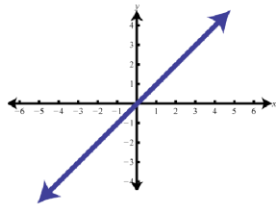
Constant Function

$f(x)=c$



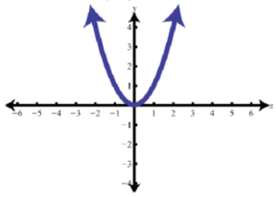
Identity Function

$f(x)=x$



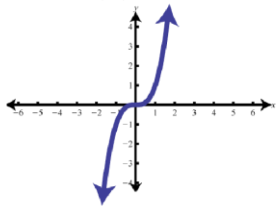
Squaring Function

$f(x)=x^2$



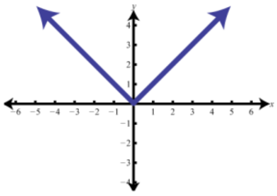
Cubing Function

$f(x)=x^3$



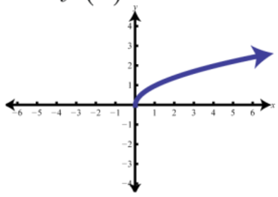
Absolute Value Function

$f(x)=|x|$



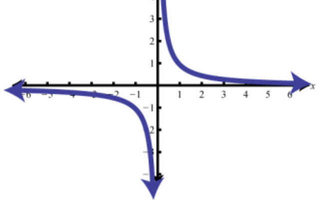
Square Root Function

$f(x)=\sqrt{x}$



Reciprocal Function

$f(x)=\dfrac{1}{x}$



5.3 Piecewise Function

- A **piecewise function** is a function where more than one formula is used to define the output over different pieces of the domain.
- **Example 1:**

$$f(x)=\begin{cases} 7x+3 & \text{if } x<0 \\ 7x+6 & \text{if } x\geq 0 \end{cases}$$

Evaluate:

1. $f(-1)$ 2. $f(0)$ 3. $f(2)$

Example 2:

Given $f(x)=\begin{cases} x^2 & \text{if } x\leq 1 \\ 3 & \text{if } 1<x\leq 2 \\ x & \text{if } x>2 \end{cases}$

Evaluate:

1. $f(-1)$ 2. $f(3)$ 3. $f(0)$ 4. $f(2)$

Try this!

Given $f(x)=\begin{cases} x+2 & \text{if } x<-1 \\ x^3+3 & \text{if } -1\le x<1 \\ 3 & \text{if } x\ge 1 \end{cases}$

Find:

1. $f(-1)$ 2. $f(4)$ 3. $f(5)$ 4. $f(-4)$ 5. $f(0)$

6. PLANE ANALYTICAL AND GEOMETRY

6.1 Straight Lines

6.1.1 Determine Slope (gradient), Midpoint, Distance, Equation of a Line

Midpoint is a middle point in a line segment. The formula is as follows:

$$AB=\left(\frac{x_2+x_1}{2},\frac{y_2+y_1}{2}\right)$$

Distance between 2 points: Distance is a numerical measurement of how far apart objects or points are. The distance formula is:

$$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Gradient/Slope is a measure of the steepness of the line and represented by the symbol *m*. The formula is:

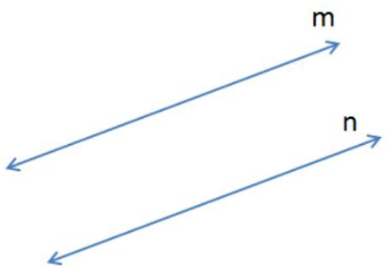
$$m=\frac{y_2-y_1}{x_2-x_1} \text{ where } x_2\neq x_1$$

Equation of a Straight Lines	Forms
1. Point Slope form	$y-y_1=m(x-x_1)$
2. Slope Intercept Form	$y=mx+c$

3. Double Intercept Form	$\frac{x}{a}+\frac{y}{b}=1$
4. Standard Form	$ax+by=c$
5. General Form	$ax+by+c=0$ where a and b cannot be zero

6.1.2 Parallel and Perpendicular Lines

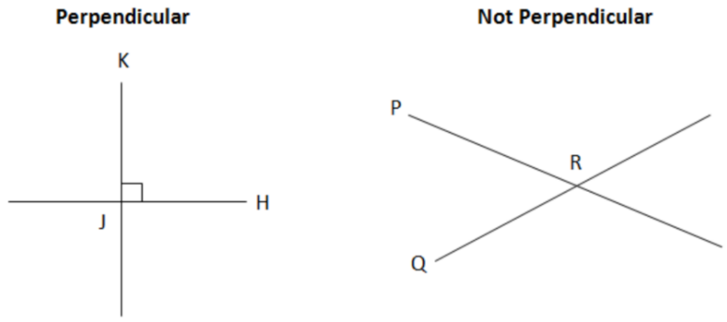
Parallel lines are **lines** in a plane which do not meet; that is, two straight **lines** in a plane that do not intersect at any point are said to be **parallel**.



They have the same slope/gradient: $m_1 = m_2$

For example: $y=3x+5$ and $3x-y+2=0$ are parallel lines because both of them have the same slope's value, that is, m = 3

Perpendicular Lines is the line that intersect and form 90° angle.



They have different slope/gradient and can be calculated by the formula: $m_1 \times m_2 = -1$

For example: $y=3x+2$ and $y=-\frac{1}{3}x+\frac{10}{3}$ are perpendicular because both of them have different value for the slope.

Example: Find an equation of the line that passes through the origin and is parallel to the line $y-3x+2=0$. **(practice workbook: 19)**

Example: Find an equation of the line that perpendicular to the line $y = -5x + 2$ and passes through (1 , 1). **(practice workbook: 19)**

Exercises:

1. Find an equation of the line that passes through point $A(1, -2)$ and $B(3, 2)$
2. Given a point $A(-4, 3)$ and slope, $m = \frac{1}{2}$. Find the equation of a straight line in the slope intercept form
3. Find the equation of the straight line passing through the point $A(1, 5)$ and parallel to the line $3y - 6x + 2 = 0$
4. Find the equation of the straight line passing through the point $A(1, 3)$ and parallel to the line $y = 3x + 2$

6.1.3 Intersection Between 2 Lines

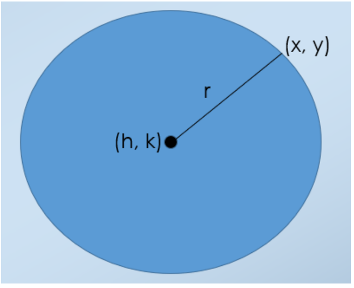
The point of intersection of two non-parallel lines can be found from the equations of the two lines.

For example:

Find the equation of the straight line that passes through the point of intersection of the two straight lines $2x - y - 1 = 0$ and $3x + y - 9 = 0$ and is parallel to the line $x + 4y = 10$ **(practice workbook: 21).**

6.2 Circle

- A circle is a set of points in a plane that are equidistant from a fixed point
- A fixed point is called the **centre** and the distance between the fixed point to any of the point is called the **radius** of the circle denoted by, r.



- If the point (h, k) is the centre of the circle, the radius can be calculated by the distance formula;

$$\sqrt{(x-h)^2+(y-k)^2}=r$$

- Squaring both sides of the equation yields the standard form of the equation of a circle;

$$(x-h)^2+(y-k)^2=r^2$$

- General form;

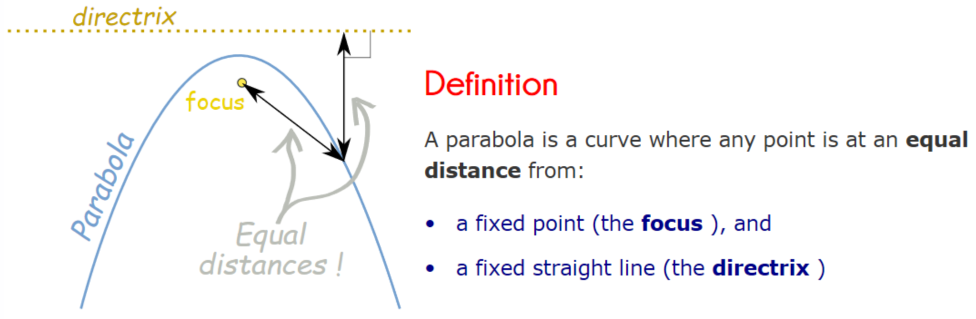
$$x^2+y^2+ax+by+c=0$$

Example: Given the equation of the circle $2x^2+6x-8y+2y^2+12=0$, write the equation in the standard form. Hence, find the radius and the center of the circle and sketch the graph. **(practice workbook: 23)**

Example: Find the equation of a circle which passes through the points A (3 , 4) and B (5 , 8) where AB is the diameter of the circle. **(practice workbook: 24)**

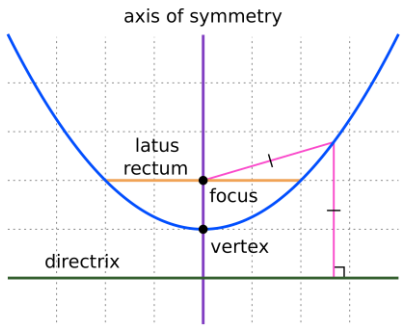
6.3 Parabola

- A parabola is a set of points in a plane that are equidistant from a fixed-point F called the focus and a fixed line called the directrix.

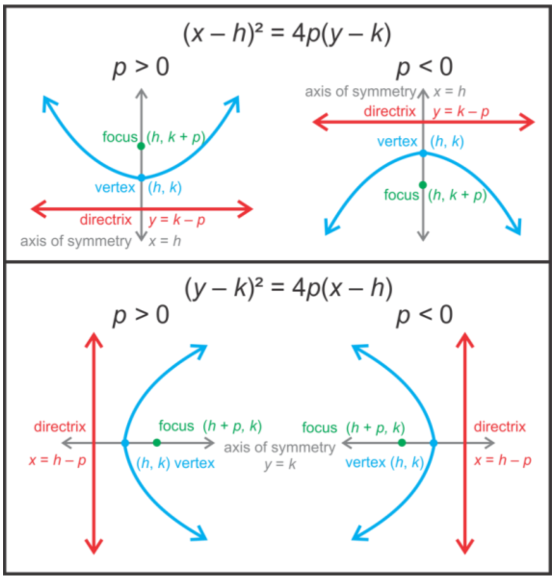


- The line passing through the focus and perpendicular to the directrix is called the **axis of symmetry**.
- The point which is midway on this axis and between the focus and the directrix is called **vertex**.
- The distance between the vertex and the focus is called the **focal length**, p
- The line segment passing through the focus of a parabola and perpendicular to the axis of symmetry is called the **latus rectum**

(Ref: Math is Fun!)



6.3.1 Equation of Parabola



Vertex = (h, k)

p = distance from vertex to focus and directrix

Example: Write down the equation of $y^2 + 2y + 4x - 8 = 0$ in the standard form. Hence, find the vertex, focus and directrix of the graph. Sketch the graph **(practice workbook: 26)**

Example: Find the equation of the parabola that has a minimum at (- 2 , 6) and passes through the point (2 , 8) **(practice workbook: 27)**