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AE740 HW4 akshatdy

```
clc;
clear;
close all;
clear variables
format shortG;
```

1 Quadractic programming solver

1.a Standard dual projected gradient algorithm

Equation in the question is

$$3x_1^2 + x_2^2 + 2x_1x_2 + x_1 + 6x_2 + 2$$

the 2 can be ignored from the optimization problem as it is a constant

the resulting matrices that produce this equation are

```
H = [6 2;
     2 2];
q = [1; 6];
```

constraints are

$$2x_1 + 3x_2 \geq 4$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$

have to reverse the sign of the inequality to make it $\leq$

```
A = [-2 -3;
     -1 0;
```

```
     0 -1];
b = [-4; 0; 0];

% run with MATLAB's quadprog
[x, fval, exitflag, output, lam] = quadprog(H, q, A, b);
disp('1.a Standard dual projected gradient algorithm')
disp('MATLAB quadprog:');
disp('Solution =');
disp(x);
disp('Lagrange multipliers =');
disp(lam.ineqlin);

[x_my, lam_my] = myQP(H, q, A, b, zeros(size(A, 1), 1));
disp('myQP:');
disp('Solution =');
disp(x_my);
disp('Lagrange multipliers =');
disp(lam_my);
```

```
Minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

1.a Standard dual projected gradient algorithm
MATLAB quadprog:
Solution =
          0.5
            1

Lagrange multipliers =
            3
   6.8148e-10
   3.6335e-12
```

The solution from myQP is the same as MATLAB's quadprog, and the Lagrange multipliers are also the same.

Solutions:

- myQP: [0.5 1]

- MATLAB quadprog: [0.5 1]

Lagrange multipliers:

- myQP: [3 0 0]

- MATLAB quadprog: [3 6.8148e-10 3.6335e-12]

1.c Discretize the mass-spring-damper system

```
disp('1.c Discrete time model of the mass-spring system')
% have to re-declare A and B with consants for m and k
```

```
Ts = 0.1;
m = 1;
k = 1;
Ac = [0 1;
      -k / m 0];
Bc = [0;
      1 / m];

[Ad, Bd, Cd, Dd] = c2dm(Ac, Bc, [], [], Ts, 'zoh');
disp('Ad =');
disp(Ad);
disp('Bd =');
disp(Bd);
```

```
1.c Discrete time model of the mass-spring system
Ad =
        0.995     0.099833
    -0.099833        0.995

Bd =
    0.0049958
     0.099833
```

1.d Tracking MPC formulation

$\Delta x_{1,k+1}$ and $\Delta x_{2,k+1}$ can be represented as x_{k+1}

Then $\Delta x_{k+1} = x_{k+2} - x_{k+1}$

$= Ax_{k+1} + Bu_{k+1} - (Ax_k + Bu_k)$

$= A(x_{k+1} - x_k) + B(u_{k+1} - u_k)$

Using definitions of Δx_k and Δu_k

$\Delta x_{k+1} = A\Delta x_k + B\Delta u_k$

The error e can be calculated by:

$x_{k+1} = x_k + \Delta x_k$ -- (1)

$e_k = x_k - r$ -- (2)

$e_{k+1} = x_{k+1} - r$ -- (3)

Substituting (1) into (3)

$e_{k+1} = x_k + \Delta x_k - r$

Using (2) to reduce this equation

Thus: $e_{k+1} = e_k + \Delta x_k$

$u_{k+1} = u_k + \Delta u_k$, so it just uses the A_d to pick u and B_d to pick Δu.

$x_{1,k+1} = x_{1,k} + \Delta x_{1,k}$ by just rearranging the given equation for Δx_k

```
A = [Ad, zeros(2, 3);
     1, 0, 1, 0, 0;
     0, 0, 0, 1, 0;
     1, 0, 0, 0, 1];
B = [Bd; 0; 1; 0];
```

1.e LQ-MPC Problem formulation

```
Q = diag([0, 0, 1, 0, 0]);
R = 1;
[K, Pdxu, E] = dlqr(A(1:3, 1:3), B(1:3), Q(1:3, 1:3), R);
P = blkdiag(Pdxu, zeros(2, 2));
disp('1.e LQ-MPC Problem formulation')
disp('P =');
disp(P);
```

```
1.e LQ-MPC Problem formulation
P =
       304.52       85.262       44.817            0            0
       85.262       40.051       9.9917            0            0
       44.817       9.9917       10.033            0            0
            0            0            0            0            0
            0            0            0            0            0
```

1.f Represent constraints

```
lN = 10;
% large number
XLIM.max = [lN, lN, lN, 0.2, 0.2]; % [dx1, dx2, e, u, x_1]
XLIM.min = -XLIM.max;
xlim20.max = [lN, lN, lN, 0.25, 0.2]; % [dx1, dx2, e, u, x_1]
xlim20.min = -xlim20.max;
% Note our "control" is "control increment", actual control is the fourth
state
ulim.max = [lN];
ulim.min = -ulim.max;
```

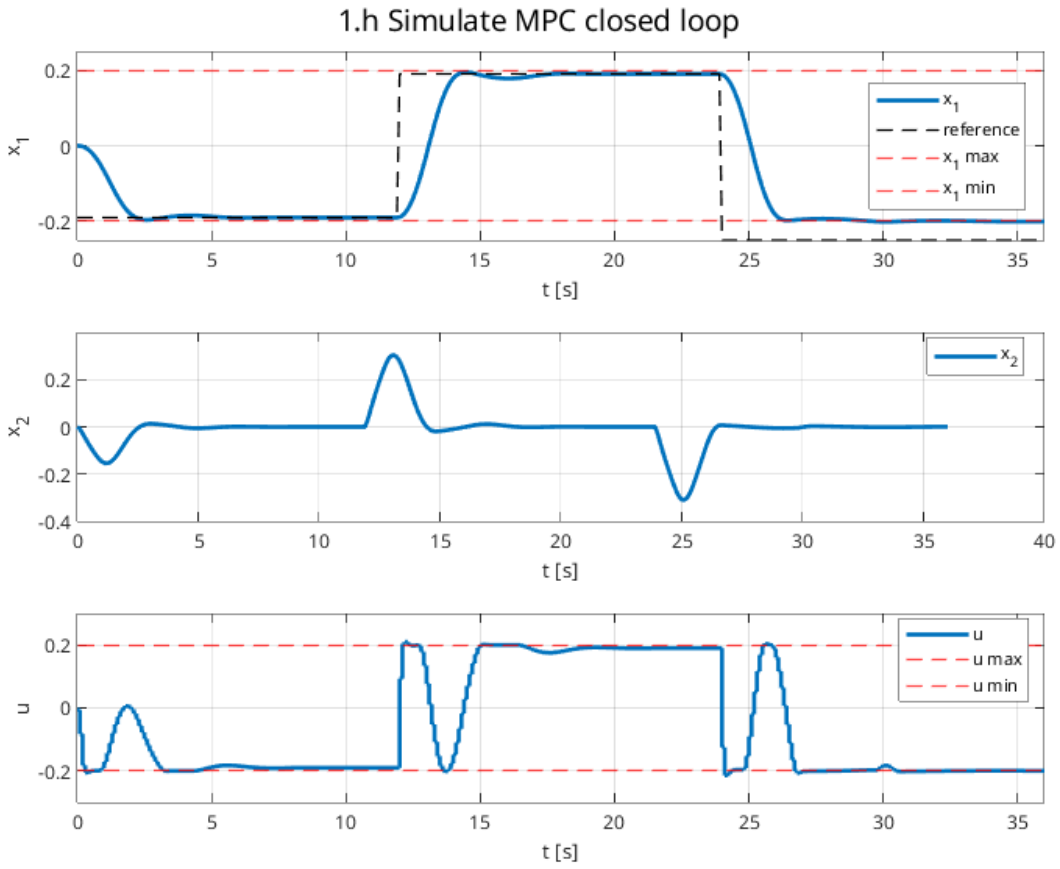
1.h/i Simulate MPC closed loop

```
% m = 1;
% k = 1;
N = 15;
cmds = [-0.19 0.19 -0.25];
cmd = cmds(1);
prev_cmd = cmd;
% start with everything as 0, so the error is now the step reference command
xk_msd = [0; 0];
```

```
xk_msd20 = [0; 0];
xk = [0; 0; -cmd; 0; 0];
xk20 = [0; 0; -cmd; 0; 0];
Tcmd = 12;
Tsim = Tcmd * length(cmds);
times = 0:Ts:Tsim;
fidelity = Ts / 10;
% form the matrices needed for QP
[H, L, G, W, T, IMPC] = formQPMatrices(A, B, Q, R, P, XLIM, ulim, N);
[H20, L20, G20, W20, T20, IMPC20] = formQPMatrices(A, B, Q, R, P, xlim20,
ulim, N);
lam = ones(size(G, 1), 1);
lam20 = ones(size(G20, 1), 1);

data.x_msd = zeros(2, Tsim / Ts);
data.x_msd20 = zeros(2, Tsim / Ts);
data.u = zeros(1, Tsim / Ts);
data.u20 = zeros(1, Tsim / Ts);
data.r = zeros(1, Tsim / Ts);
data_idx = 1;

for t = times
  % get the current command
  cmd = cmds(min(1 + floor(t / Tcmd), length(cmds)));

  if cmd ~= prev_cmd
    % update the error in the state
    xk(3) = xk(3) - cmd + prev_cmd;
    xk20(3) = xk20(3) - cmd + prev_cmd;
  end

  % solve the QP
  [U, lam] = myQP(H, L * xk, G, W + T * xk, lam);
  [U20, lam20] = myQP(H20, L20 * xk20, G20, W20 + T20 * xk20, lam20);
  % get the first control increment
  delta_uk = IMPC * U;
  delta_uk20 = IMPC20 * U20;
  uk = xk(4) + delta_uk; % nu is just 1 so this works
  uk20 = xk20(4) + delta_uk20; % nu is just 1 so this works
  % simulate the system
  [~, xk1_msd_ode] = ode45(@(t, x) msd(t, x, uk, 1, 1), [t +
fidelity:fidelity:t + Ts], xk_msd);
  [~, xk1_msd_ode20] = ode45(@(t, x) msd(t, x, uk20, 0.8, 1.2), [t +
fidelity:fidelity:t + Ts], xk_msd20);
  xk1_msd = xk1_msd_ode(end, :)';
  xk1_msd20 = xk1_msd_ode20(end, :)';
  % update the state
  delta_xk_msd = xk1_msd - xk_msd;
  xk1 = [delta_xk_msd; % delta x1 and x2
         xk(3) + delta_xk_msd(1); % e
         uk; % u
         xk1_msd(1)]; % x1

  delta_xk_msd20 = xk1_msd20 - xk_msd20;
```

```
  xk120 = [delta_xk_msd20; % delta x1 and x2
           xk20(3) + delta_xk_msd20(1); % e
           uk20; % u
           xk1_msd20(1)]; % x1
  % update
  xk_msd = xk1_msd;
  xk_msd20 = xk1_msd20;
  xk = xk1;
  xk20 = xk120;
  prev_cmd = cmd;
  % save data
  data.x_msd(:, data_idx) = xk_msd;
  data.x_msd20(:, data_idx) = xk_msd20;
  data.u(data_idx) = uk;
  data.u20(data_idx) = uk20;
  data.r(data_idx) = cmd;

  data_idx = data_idx + 1;
end

% plot the results
fig = figure();
fig.Position(3:4) = [800, 600];
sgtitle('1.h Simulate MPC closed loop');
subplot(3, 1, 1);
plot(times, data.x_msd(1, :), 'DisplayName', 'x_1', "LineWidth", 2);
grid on;
hold on;
plot(times, data.r, '--k', 'DisplayName', 'reference', "LineWidth", 1);
yline(0.2, '--r', 'DisplayName', 'x_1 max', "LineWidth", 1);
yline(-0.2, '--r', 'DisplayName', 'x_1 min', "LineWidth", 1);
xlabel('t [s]');
ylabel('x_1');
xlim tight;
ylim([-0.25, 0.25]);
legend("Location", "best");

subplot(3, 1, 2);
plot(times, data.x_msd(2, :), 'DisplayName', 'x_2', "LineWidth", 2);
grid on;
xlabel('t [s]');
ylabel('x_2');
ylim([-0.40, 0.40]);
legend("Location", "best");

subplot(3, 1, 3);
stairs(times, data.u, 'DisplayName', 'u', "LineWidth", 2);
grid on;
xlabel('t [s]');
ylabel('u');
yline(0.2, '--r', 'DisplayName', 'u max', "LineWidth", 1);
yline(-0.2, '--r', 'DisplayName', 'u min', "LineWidth", 1);
xlim tight;
ylim([-0.30, 0.30]);
```

```
legend("Location", "best");

disp('1.h Simulate MPC closed loop');
fprintf('x_1 limits = [%f  %f]\n', min(data.x_msd(1, :)),
max(data.x_msd(1, :)));
fprintf('u limits = [%f  %f]\n', min(data.u), max(data.u));
```

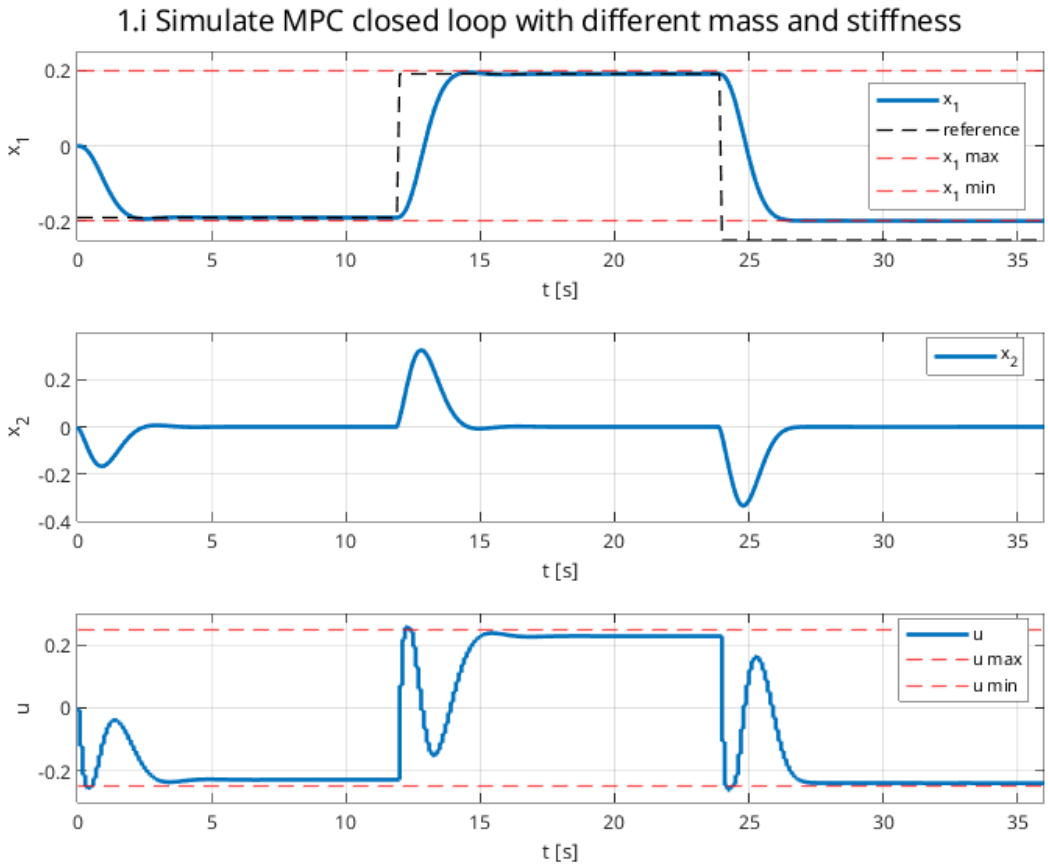


x_1 limits = [-0.201262 0.194408] u limits = [-0.213918 0.209788] There is a very slight constraint violation for both the state and the control input

1.i Simulate MPC closed loop and comment on differernces

```
fig = figure();
fig.Position(3:4) = [800, 600];
sgtitle('1.i Simulate MPC closed loop with different mass and stiffness');
subplot(3, 1, 1);
plot(times, data.x_msd20(1, :), 'DisplayName', 'x_1', "LineWidth", 2);
grid on;
hold on;
plot(times, data.r, '--k', 'DisplayName', 'reference', "LineWidth", 1);
yline(0.2, '--r', 'DisplayName', 'x_1 max', "LineWidth", 1);
```

```
yline(-0.2, '--r', 'DisplayName', 'x_1 min', "LineWidth", 1);
xlabel('t [s]');
ylabel('x_1');
xlim tight;
ylim([-0.25, 0.25]);
legend("Location", "best");

subplot(3, 1, 2);
plot(times, data.x_msd20(2, :), 'DisplayName', 'x_2', "LineWidth", 2);
grid on;
xlabel('t [s]');
ylabel('x_2');
xlim tight;
ylim([-0.40, 0.40]);
legend("Location", "best");

subplot(3, 1, 3);
stairs(times, data.u20, 'DisplayName', 'u', "LineWidth", 2);
grid on;
xlabel('t [s]');
ylabel('u');
yline(0.25, '--r', 'DisplayName', 'u max', "LineWidth", 1);
yline(-0.25, '--r', 'DisplayName', 'u min', "LineWidth", 1);
xlim tight;
ylim([-0.30, 0.30]);
legend("Location", "best");
```



After reducing the mass an increasing the stiffness, we can see that the controller is not only robust to these changes, but it also performs better. We can notice that from the initial state, the controller now is able to reach the reference faster at 2.2s vs 2.6s. Additionally, we can observe that the oscillations in the state and input are reduced. However, these differences are mostly due to the fact that the mass is reduced and the stiffness is increased, making the dynamics of the mass spring system faster. The controller is also given more control authority so it can reach the reference by pushing the control input harder. Overall, the controller is still robust while facing a model mismatch by being able to push the control input more.

```
disp('1.i Simulate MPC closed loop with different mass and stiffness');
fprintf('x_1 limits = [%f  %f]\n', min(data.x_msd20(1, :)),
max(data.x_msd20(1, :)));
fprintf('u limits = [%f  %f]\n', min(data.u20), max(data.u20));
```

```
1.i Simulate MPC closed loop with different mass and stiffness
x_1 limits = [-0.199101  0.194064]
u limits = [-0.259843  0.255778]
```

x_1 limits = [-0.199101 0.194064] u limits = [-0.259843 0.255778] There is a very slight constraint violation for the control input, but the state is well within the limits

1.a Standard dual projected gradient Function

```
function [U, lam] = myQP(H, q, A, b, lam0)
  % This function implements the dual projected
```

```
  % gradient algorithm for solving a QP problem.
  % Minimize 1/2 * U' * H * U + q' * U subject to G * U <= Wtilde

  % compared to his notes, G=A, U=x, W=b
  G = A; Wtilde = b;
  invH = inv(H); G_invH = G * invH; % see Note 1
  Hd = G_invH * G';
  % see Note 1
  qd = G_invH * q + Wtilde;
  Nit = 30;
  % maximum number of iterations
  lam = lam0;
  L = norm(Hd);
  k = 1;
  df = Hd * lam + qd;

  while k <= Nit % see Note 2
    lam = max(lam - 1 / L * df, 0);
    df = Hd * lam + qd;
    k = k + 1;
  end

  U = -invH * (G' * lam + q);
  return;
  % Note 1: Can pre-compute these quantities and pass as arguments to
  % myQP. In LQ-MPC setting, only q and Wtilde depend on x_0, makes no
  % sense to constantly re-compute these
  % Note 2: Change to another criterion as desired
end
```

```
myQP:
Solution =
          0.5
            1

Lagrange multipliers =
            3
            0
            0
```

1.b Mass-spring dynamics

```
function xdot = msd(t, x, u, m_msd, k_msd)
  xdot = [
          x(2);
          (-k_msd / m_msd) * x(1) + (u / m_msd);
          ];
end
```

```
1.h Simulate MPC closed loop
x_1 limits = [-0.201262  0.194408]
u limits = [-0.213918  0.209788]
```

1.g function that forms matrices needed for QP

```
function [H, L, G, W, T, IMPC] = formQPMatrices(A, B, Q, R, P, xlim, ulim, N)
  % This function forms the matrices needed for the constrained QP
  % Inputs:
  %   A, B: state-space matrices
  %   Q, R, P: cost function matrices
  %   xlim, ulim: state and input constraints
  %   N: prediction horizon

  nx = size(A, 1);
  nu = size(B, 2);

  S = zeros(N * nx, N * nu);
  % Compute the first column of S
  for i = 1:N
    rowStart = (i - 1) * nx + 1;
    rowEnd = i * nx;
    S(rowStart:rowEnd, 1:nu) = A^(i - 1) * B;
  end

  % Pad the first column and set it to other columns of S
  for i = 2:N
    colStart = (i - 1) * nu + 1;
    colEnd = i * nu;
    zeroRows = (i - 1) * nx;
    zeroCols = nu;
    S(:, colStart:colEnd) = [zeros(zeroRows, zeroCols); S(1:end - zeroRows,
1:nu)];
  end

  M = zeros(N * nx, nx);
  % Compute first row of M
  M(1:nx, :) = A;
  % Compute the rest of M
  for i = 2:N
    rowStart = (i - 1) * nx + 1;
    rowEnd = i * nx;
    % just multiply the previous rows by A to get higher powers
    M(rowStart:rowEnd, :) = A * M(rowStart - nx:rowEnd - nx, :);
  end

  Qbar = zeros(N * nx, N * nx);
  % Compute Qbar and set the last row to P
  for i = 1:N
    % Q is square so we can reuse indices
    rowStart = (i - 1) * nx + 1;
    rowEnd = i * nx;
    temp = Q;

    if i == N
      temp = P;
    end
```

```
    Qbar(rowStart:rowEnd, rowStart:rowEnd) = temp;
  end

  Rbar = zeros(N * nu, N * nu);
  % Compute Rbar
  for i = 1:N
    % R is square so we can reuse indices
    rowStart = (i - 1) * nu + 1;
    rowEnd = i * nu;
    Rbar(rowStart:rowEnd, rowStart:rowEnd) = R;
  end

  H = S' * Qbar * S + Rbar;
  L = S' * Qbar * M;
  G = [S;
       -S;
       eye(N * nu);
       -eye(N * nu)];
  W = [
       repmat(xlim.max', N, 1);
       repmat(-xlim.min', N, 1);
       repmat(ulim.max', N, 1);
       repmat(-ulim.min', N, 1);
       ];
  T = [
       -M;
       M;
       zeros(N * nu, nx);
       zeros(N * nu, nx);
       ];
  IMPC = [eye(nu, nu), zeros(nu, (N - 1) * nu)];

end
```

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```
clc;
clear;
close all;
clear variables
format shortG;
```

2 Soft Constraints

2.b Simulate MPC with soft constraints

```
% Define the base system
Ts = 0.1;
m = 1;
k = 1;
Ac = [0 1;
      -k / m 0];
Bc = [0;
      1 / m];

% Discretize the system
[Ad, Bd, Cd, Dd] = c2dm(Ac, Bc, [], [], Ts, 'zoh');

% construct the extended system for the LQ-MPC problem
A = [Ad, zeros(2, 3);
     1, 0, 1, 0, 0;
     0, 0, 0, 1, 0;
     1, 0, 0, 0, 1];
B = [Bd; 0; 1; 0];

Q = diag([0, 0, 1, 0, 0]);
R = 1;
[K, Pdxu, E] = dlqr(A(1:3, 1:3), B(1:3), Q(1:3, 1:3), R);
P = blkdiag(Pdxu, zeros(2, 2));

cmds = [-0.19 0.19 -0.25];
cmd = cmds(1);
prev_cmd = cmd;
xk_msd_sm = [0; 0];
xk_msd_lg = [0; 0];
xk_sm = [0; 0; -cmd; 0; 0];
xk_lg = [0; 0; -cmd; 0; 0];
```

```
N = 15;
lN = 10;
% large number
XLIM.max = [lN, lN, lN, 0.2, 0.2]; % [dx1, dx2, e, u, x_1]
XLIM.min = -XLIM.max;
% Note our "control" is "control increment", actual control is the fourth
state
ulim.max = [lN];
ulim.min = -ulim.max;
[H_sm, L_sm, G_sm, W_sm, T_sm, IMPC_sm] = formQPMatrices(A, B, Q, R, P, XLIM,
ulim, N, 1e-3);
[H_lg, L_lg, G_lg, W_lg, T_lg, IMPC_lg] = formQPMatrices(A, B, Q, R, P, XLIM,
ulim, N, 1e3);

lam_sm = ones(size(G_sm, 1), 1);
lam_lg = ones(size(G_lg, 1), 1);

Tcmd = 12;
Tsim = Tcmd * length(cmds);
times = 0:Ts:Tsim;
fidelity = Ts / 10;

data.x_msd_sm = zeros(2, Tsim / Ts);
data.x_msd_lg = zeros(2, Tsim / Ts);
data.u_sm = zeros(1, Tsim / Ts);
data.u_lg = zeros(1, Tsim / Ts);
data.r = zeros(1, Tsim / Ts);
data_idx = 1;

for t = times
  % get the current command
  cmd = cmds(min(1 + floor(t / Tcmd), length(cmds)));

  if cmd ~= prev_cmd
    % update the error in the state
    xk_sm(3) = xk_sm(3) - cmd + prev_cmd;
    xk_lg(3) = xk_lg(3) - cmd + prev_cmd;
  end

  % solve the QP
  [U_sm, lam_sm] = myQP(H_sm, L_sm * xk_sm, G_sm, W_sm + T_sm * xk_sm,
lam_sm);
  [U_lg, lam_lg] = myQP(H_lg, L_lg * xk_lg, G_lg, W_lg + T_lg * xk_lg,
lam_lg);

  % get the first control increment
  delta_uk_sm = IMPC_sm * U_sm;
  delta_uk_lg = IMPC_lg * U_lg;
  uk_sm = xk_sm(4) + delta_uk_sm; % nu is just 1 so this works
  uk_lg = xk_lg(4) + delta_uk_lg; % nu is just 1 so this works

  % simulate the system
  [~, xk1_msd_ode_sm] = ode45(@(t, x) msd(t, x, uk_sm, 1, 1), [t +
fidelity:fidelity:t + Ts], xk_msd_sm);
```

```
  [~, xk1_msd_ode_lg] = ode45(@(t, x) msd(t, x, uk_lg, 1, 1), [t +
fidelity:fidelity:t + Ts], xk_msd_lg);
  xk1_msd_sm = xk1_msd_ode_sm(end, :)';
  xk1_msd_lg = xk1_msd_ode_lg(end, :)';

  % update the state
  delta_xk_msd_sm = xk1_msd_sm - xk_msd_sm;
  delta_xk_msd_lg = xk1_msd_lg - xk_msd_lg;

  xk1_sm = [delta_xk_msd_sm; % delta x1 and x2
            xk_sm(3) + delta_xk_msd_sm(1); % e
            uk_sm; % u
            xk1_msd_sm(1)]; % x1

  xk1_lg = [delta_xk_msd_lg; % delta x1 and x2
            xk_lg(3) + delta_xk_msd_lg(1); % e
            uk_lg; % u
            xk1_msd_lg(1)]; % x1

  % update
  xk_msd_sm = xk1_msd_sm;
  xk_msd_lg = xk1_msd_lg;
  xk_sm = xk1_sm;
  xk_lg = xk1_lg;
  prev_cmd = cmd;

  % save data
  data.x_msd_sm(:, data_idx) = xk_msd_sm;
  data.x_msd_lg(:, data_idx) = xk_msd_lg;
  data.u_sm(data_idx) = uk_sm;
  data.u_lg(data_idx) = uk_lg;
  data.r(data_idx) = cmd;

  data_idx = data_idx + 1;

end

% plot the results
fig = figure();
fig.Position(3:4) = [800, 600];
sgtitle('2.b Simulate MPC with slack variable');
subplot(3, 1, 1);
plot(times, data.x_msd_sm(1, :), 'DisplayName', '$\mu = 1e-3$', "LineWidth",
2);
grid on;
hold on;
plot(times, data.x_msd_lg(1, :), 'Color', '#77AC30', 'DisplayName', '$\mu =
1e3$', "LineWidth", 2);
plot(times, data.r, '--k', 'DisplayName', 'reference', "LineWidth", 1);
yline(0.2, '--r', 'DisplayName', '$x_1$ max', "LineWidth", 1);
yline(-0.2, '--r', 'DisplayName', '$x_1$ min', "LineWidth", 1);
xlabel('t [s]');
ylabel('x_1');
xlim tight;
```

```
ylim([-0.3, 0.3]);
legend("Location", "best", "Interpreter", "latex");

subplot(3, 1, 2);
plot(times, data.x_msd_sm(2, :), 'DisplayName', '$\mu = 1e-3$', "LineWidth",
2);
grid on;
hold on;
plot(times, data.x_msd_lg(2, :), 'Color', '#77AC30', 'DisplayName', '$\mu =
1e3$', "LineWidth", 2);
xlabel('t [s]');
ylabel('x_2');
xlim tight;
ylim([-0.5, 0.5]);
legend("Location", "best", "Interpreter", "latex");

subplot(3, 1, 3);
stairs(times, data.u_sm, 'DisplayName', '$\mu = 1e-3$', "LineWidth", 2);
grid on;
hold on;
stairs(times, data.u_lg, 'Color', '#77AC30', 'DisplayName', '$\mu = 1e3$',
"LineWidth", 2);
xlabel('t [s]');
ylabel('u');
yline(0.2, '--r', 'DisplayName', 'u max', "LineWidth", 1);
yline(-0.2, '--r', 'DisplayName', 'u min', "LineWidth", 1);
xlim tight;
ylim([-0.6, 0.6]);
legend("Location", "best", "Interpreter", "latex");

snapnow;

disp('1.h Simulate MPC closed loop');
fprintf('x_1_sm limits = [%f  %f]\n', min(data.x_msd_sm(1, :)),
max(data.x_msd_sm(1, :)));
fprintf('x_1_lg limits = [%f  %f]\n', min(data.x_msd_lg(1, :)),
max(data.x_msd_lg(1, :)));
fprintf('u_sm limits = [%f  %f]\n', min(data.u_sm), max(data.u_sm));
fprintf('u_lg limits = [%f  %f]\n', min(data.u_lg), max(data.u_lg));
```

With the slack variables added, we can see that the constraints for both values of μ are violated. This makes sense since the slack variables are added to the cost function to allow for some constraint violation. We can also see that the constraints are violated more for the smaller value of μ than for the larger value of μ. This is expected since the larger value of μ penalizes the slack variables more, and thus the cost function will be more sensitive to the constraint violations.

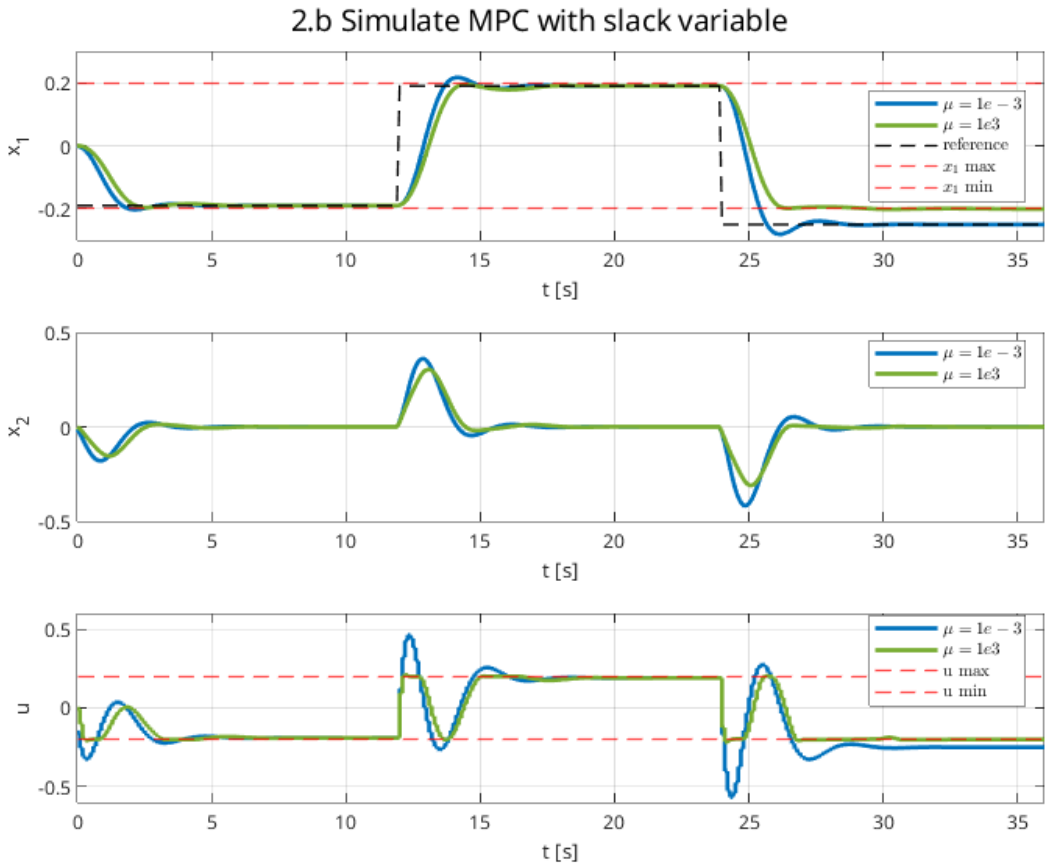
We can also observe that x_1 reaches its reference value faster for the smaller value of μ than for the larger value of μ because the controller is able to be more aggressive with the smaller value of μ by violating the constraints more on the controller input(u). We can also observe that the final referrence value which violates the limits for x_1 is actually reached when using the smaller value of μ. The controller is able to keep violating the constraints on the controller input(u) and x_1 as it balances the cost of following the reference with the cost of violating the constraints on the controller input(u) and x_1.

2.a function that forms matrices needed for QP with soft constraints

```
function [H, L, Gs, W, T, IMPC] = ...
    formQPMatrices(A, B, Q, R, P, xlim, ulim, N, slackPenalty)
  % This function forms the matrices needed for the constrained QP
  % Inputs:
  %   A, B: state-space matrices
  %   Q, R, P: cost function matrices
  %   xlim, ulim: state and input constraints
  %   N: prediction horizon
  %   slackPenalty: penalty for slack variables

  nx = size(A, 1);
  nu = size(B, 2);

  S = zeros(N * nx, N * nu);
  % Compute the first column of S
  for i = 1:N
    rowStart = (i - 1) * nx + 1;
    rowEnd = i * nx;
    S(rowStart:rowEnd, 1:nu) = A^(i - 1) * B;
  end

  % Pad the first column and set it to other columns of S
  for i = 2:N
    colStart = (i - 1) * nu + 1;
    colEnd = i * nu;
    zeroRows = (i - 1) * nx;
    zeroCols = nu;
    S(:, colStart:colEnd) = [zeros(zeroRows, zeroCols); S(1:end - zeroRows,
1:nu)];
  end

  % add padding to account for the slack variables
  Ss = [S, zeros(N * nx, nx)];

  M = zeros(N * nx, nx);
  % Compute first row of M
  M(1:nx, :) = A;
  % Compute the rest of M
  for i = 2:N
    rowStart = (i - 1) * nx + 1;
    rowEnd = i * nx;
    % just multiply the previous rows by A to get higher powers
    M(rowStart:rowEnd, :) = A * M(rowStart - nx:rowEnd - nx, :);
  end

  Qbar = zeros(N * nx, N * nx);
  % Compute Qbar and set the last row to P
  for i = 1:N
    % Q is square so we can reuse indices
```

```
    rowStart = (i - 1) * nx + 1;
    rowEnd = i * nx;
    temp = Q;

    if i == N
      temp = P;
    end

    Qbar(rowStart:rowEnd, rowStart:rowEnd) = temp;
  end

  Rbar = zeros(N * nu, N * nu);
  % Compute Rbar
  for i = 1:N
    % R is square so we can reuse indices
    rowStart = (i - 1) * nu + 1;
    rowEnd = i * nu;
    Rbar(rowStart:rowEnd, rowStart:rowEnd) = R;
  end

  % add penalty for slack variables
  Rbars = [Rbar, zeros(N * nu, nx);
           zeros(nx, N * nu), slackPenalty * eye(nx)];

  H = Ss' * Qbar * Ss + Rbars;
  L = Ss' * Qbar * M;
  Gs = [S, repmat(eye(nx), N, 1); % eye should be nx * ne, but theyre the same
        -S, repmat(eye(nx), N, 1);
        eye(N * nu), zeros(N * nu, nx);
        -eye(N * nu), zeros(N * nu, nx);
        ];

  W = [
       repmat(xlim.max', N, 1);
       repmat(-xlim.min', N, 1);
       repmat(ulim.max', N, 1);
       repmat(-ulim.min', N, 1);
       ];
  T = [
       -M;
       M;
       zeros(N * nu, nx);
       zeros(N * nu, nx);
       ];
  IMPC = [eye(nu, nu), zeros(nu, (N - 1) * nu), zeros(nu, nx)];

end

% Standard dual projected gradient Function
function [U, lam] = myQP(H, q, A, b, lam0)
  % This function implements the dual projected
  % gradient algorithm for solving a QP problem.
  % Minimize 1/2 * U' * H * U + q' * U subject to G * U <= Wtilde
```

```
  % compared to his notes, G=A, U=x, W=b
  G = A; Wtilde = b;
  invH = inv(H); G_invH = G * invH; % see Note 1
  Hd = G_invH * G';
  % see Note 1
  qd = G_invH * q + Wtilde;
  Nit = 30;
  % maximum number of iterations
  lam = lam0;
  L = norm(Hd);
  k = 1;
  df = Hd * lam + qd;

  while k <= Nit
    lam = max(lam - 1 / L * df, 0);
    df = Hd * lam + qd;
    k = k + 1;
  end

  U = -invH * (G' * lam + q);
  return;
end

% Mass-spring dynamics
function xdot = msd(t, x, u, m_msd, k_msd)
  xdot = [
          x(2);
          (-k_msd / m_msd) * x(1) + (u / m_msd);
          ];
end
```




```
1.h Simulate MPC closed loop
x_1_sm limits = [-0.280757  0.216577]
x_1_lg limits = [-0.201903  0.194566]
u_sm limits = [-0.566450  0.463464]
u_lg limits = [-0.214613  0.210270]
```

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