The actual state space model of a linear process is given by

$$x_{t+1} = Ax_t + Bu_t \tag{1}$$

$$y_t = Cx_t \tag{2}$$

CVA is employed to identify the state space model. First, the data is stacked as follows

$$p_t = \begin{bmatrix} y_{t-1}^T & \dots & y_{t-l}^T & u_{t-1}^T & \dots & u_{t-l}^T \end{bmatrix}^T \tag{3}$$

$$f_t = \begin{bmatrix} y_t^T & \dots & y_{t+h}^T \end{bmatrix}^T \tag{4}$$

Then, we perform singular value decomposition as follows

$$\Sigma_{pp}^{-1/2} \Sigma_{pf} \Sigma_{ff}^{-1/2} = UDV^T \tag{5}$$

where Σ_{pp} is the covariance of p_t, Σ_{ff} is the covariance of f_t and Σ_{pf} is the covariance between p_t and f_t. The canonical loading is computed by

$$J_k = U_k^T \Sigma_{pp}^{-1/2} \tag{6}$$

where U_k is the first k columns of U. As such, the estimated state by CVA is given by

$$\hat{x}_t = J_k p_t \tag{7}$$

The identified state space model is expressed as

$$\begin{bmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{bmatrix} = \text{cov}(\begin{bmatrix} \hat{x}_{t+1} \\ y_t \end{bmatrix}, \begin{bmatrix} \hat{x}_t \\ u_t \end{bmatrix}) \text{cov}^{-1}(\begin{bmatrix} \hat{x}_t \\ u_t \end{bmatrix}, \begin{bmatrix} \hat{x}_t \\ u_t \end{bmatrix}) \tag{8}$$

Given the identified the state space model in Eq. 8 and the actual state space model in Eqs. 1 and 2, we are interested in finding a rotation matrix R that relates the actual state x_t and the identified state $\hat{x}_t$ as follows

$$R^T \hat{x}_t = x_t \tag{9}$$

where R is a square matrix because it is assumed that the dimension of identified state is same as that of the actual state. The rotation matrix R could be computed through least-squares estimation as follows

$$R = (\hat{\mathbf{X}}^T \hat{\mathbf{X}})^{-1} \hat{\mathbf{X}}^T \mathbf{X} \tag{10}$$

where $\hat{\mathbf{X}} = \begin{bmatrix} \hat{x}_1 & \hat{x}_2 & \dots & \hat{x}_N \end{bmatrix}^T$ and $\mathbf{X} = \begin{bmatrix} x_1 & x_2 & \dots & x_N \end{bmatrix}^T$. Nevertheless, the actual state x_t is not known. Note that Eqs. 1 and 2 are known, the actual state in Eq. 10 is replaced by the stated estimated by the Luenberger observer.

$$\tilde{x}_{t+1} = A\tilde{x}_t + L(y_t - C\tilde{x}_t) + Bu_t \tag{11}$$

where $\tilde{x}_t$ is the state estimated by the Luenberger observer. Therefore, the rotation matrix R relies on A, B, C and L. In order to improve the accuracy in computing R, $\mathbf{X}$ is composed of the converged states estimated by the Luenberger observer. Simulation results from a random linear system:

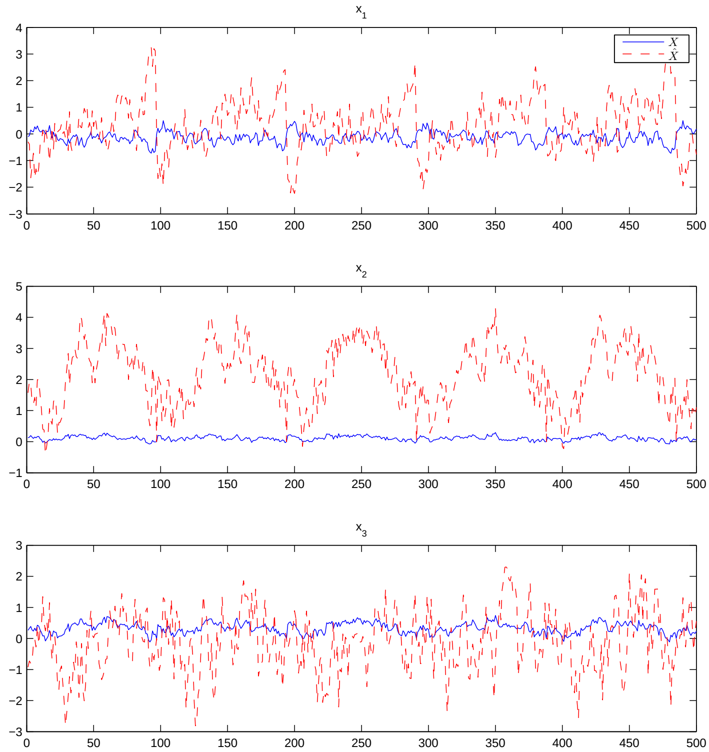

Figure 1: $\mathbf{X}$ vs. $\hat{\mathbf{X}}$

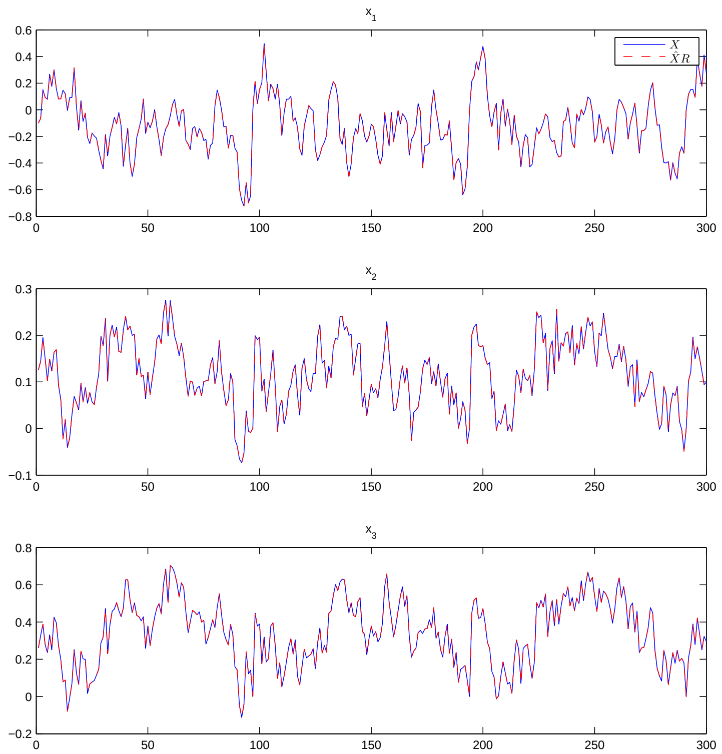

Figure 2: $\mathbf{X}$ vs. $\hat{\mathbf{X}}R$