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Modelling and Control Strategies in Path Tracking Control for Autonomous Ground Vehicles: A Review of State of the Art and Challenges

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Abstract Autonomous vehicle field of study has seen considerable researches within three decades. In the last decade particularly, interests in this field has undergone tremendous improvement. One of the main aspects in autonomous vehicle is the path tracking control, focusing on the vehicle control in lateral and longitudinal direction in order to follow a specified path or trajectory. In this paper, path tracking control is reviewed in terms of the basic vehicle model usually used; the control strategies usually employed in path tracking control, and the performance criteria used to evaluate the controller's performance. Vehicle model is categorised into several types depending on its linearity and the type of behaviour it simulates, while path tracking control is categorised depending on its approach. This paper provides critical review of each of these aspects in terms of its usage and disadvantages/advantages. Each aspect is summarised for better overall understanding. Based on the critical reviews, main challenges in the field of path tracking control is identified and future research direction is proposed. Several promising advancement is proposed with the main prospect is focused on adaptive geometric controller developed on a nonlinear vehicle model and tested with hardware-in-the-loop (HIL). It is hoped that this review can be treated as preliminary insight into the choice of controllers in path tracking control development for an autonomous ground vehicle.

Keywords Path tracking · Autonomous vehicle · Steering control · Trajectory following

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1 Introduction

Autonomous vehicles are smart vehicles that have the capability to have automatic motions and navigate itself depending on its environments and scheduled tasks. Autonomous vehicle systems may differ depending on the environment it is operating on. Flying and aerial vehicles are autonomous vehicles that operate above ground in higher altitude which usually known as unmanned aerial vehicle (UAV). There are also unmanned and autonomous vehicles operating below sea level which also known as unmanned underwater vehicle (UUV). This study focus on the autonomous vehicles that are operating on the ground which also known as autonomous ground vehicle (AGV). AGV can be regarded as a big subset under unmanned ground vehicle category where autonomous vehicle possess the higher level of intelligence to decide on its own and zero human operator needed to operate the vehicle. History of the autonomous vehicle can be traced back to its first successful implementations in the 80's by the pioneer institution in autonomous vehicle, Carnegie Mellon University (CMU) [1, 2]. In recent years, autonomous ground vehicles have been a research trend in automotive field. Several notable automotive companies have been conducting researches and technology advances in producing smart autonomous vehicles [3].

For an automated vehicle system, level of autonomy may vary according to six different levels as outlined by SAE-J3016 [4].

- Level 0: Vehicle control lies solely on human operator. Automated system may issue warnings.
- Level 1: Several assistance may be available automatically such as Adaptive Cruise Control (ACC), Parking Assistance with automated steering, and Lane Keeping Assistance (LKA).
- Level 2: The automated vehicle system executes accelerating, braking, and steering. The automated system can deactivate immediately upon takeover by the driver.
- Level 3: The automated vehicle may perform auto-pilot within known, limited environments (such as freeways).
- Level 4: The automated vehicle system can control the vehicle in all direction throughout all but a few environments such as severe weather. The driver must enable the automated system only when it is safe to do so. When enabled, driver attention is not required.
- Level 5: The automated vehicle system has full autonomy with no human intervention required. The autonomous can drive itself to any location where it is legal to be operated.

In developing a successful autonomous ground vehicles, few challenges need to be addressed. According to Amidi [5], to develop an autonomous vehicle, the overall vehicle system needs to be able to answer the following questions:

- Where am I?
- Where do I want to go?
- How do I get there without hurting myself?

Looking at the above criteria, one can deduce that the overall autonomous system should consist of different modules and tasks to complete as depicted in Fig. 1. There are three basic stages and modules in the system which are:

- ***Sensing and Perception*** – To provide real time data to let the system know the real time location and environment around the vehicle and prepare the raw data in format that is feasible for the system to process.
- ***Planning*** – To use the data provided by Sensing & Perception to dictate the safe and feasible path for the vehicle to follow.
- ***Control*** – Contain control strategies to move vehicle to the desired path. This include actuator control of each sub-system

Each of these modules are important and several publications have already presented overviews on some of these topics, such as review on the overall construction of an autonomous vehicle [6], path planning methods and strategies in autonomous vehicles [7], sensing and guidance technologies used in agricultural autonomous vehicle [8], and intelligence and decision making strategies in autonomous vehicles [9]. In this paper, a critical review on control strategies used for trajectory tracking or path following controller for autonomous vehicles is presented. Referring to Fig. 1, this review will be focusing on trajectory tracking control in the control phase of the autonomous vehicle system and vehicle model which will include research works from earlier stages in 1980s to the latest as of 2016. It can be said that commonly for all of the levels of autonomy outlined by SAE-J3016 [4] above, the autonomous vehicle will be required to follow a specific path and trajectory given by an on-board planner. That is why this area of research can be seen as important aspect and a review paper on this aspect will certainly assist on the clarity of knowledge and technologies already exists today.

Path following or trajectory tracking controller is usually developed to ensure the vehicle to follow a predefined path and trajectory by determining and calculating the desired actuating input for the vehicle to follow. This can be a correctional steering input to adjust the vehicle's position in lateral direction or correctional braking or throttle setting to adjust vehicle's motion in longitudinal direction.

In this paper, critical review of control strategies used in trajectory tracking and path following

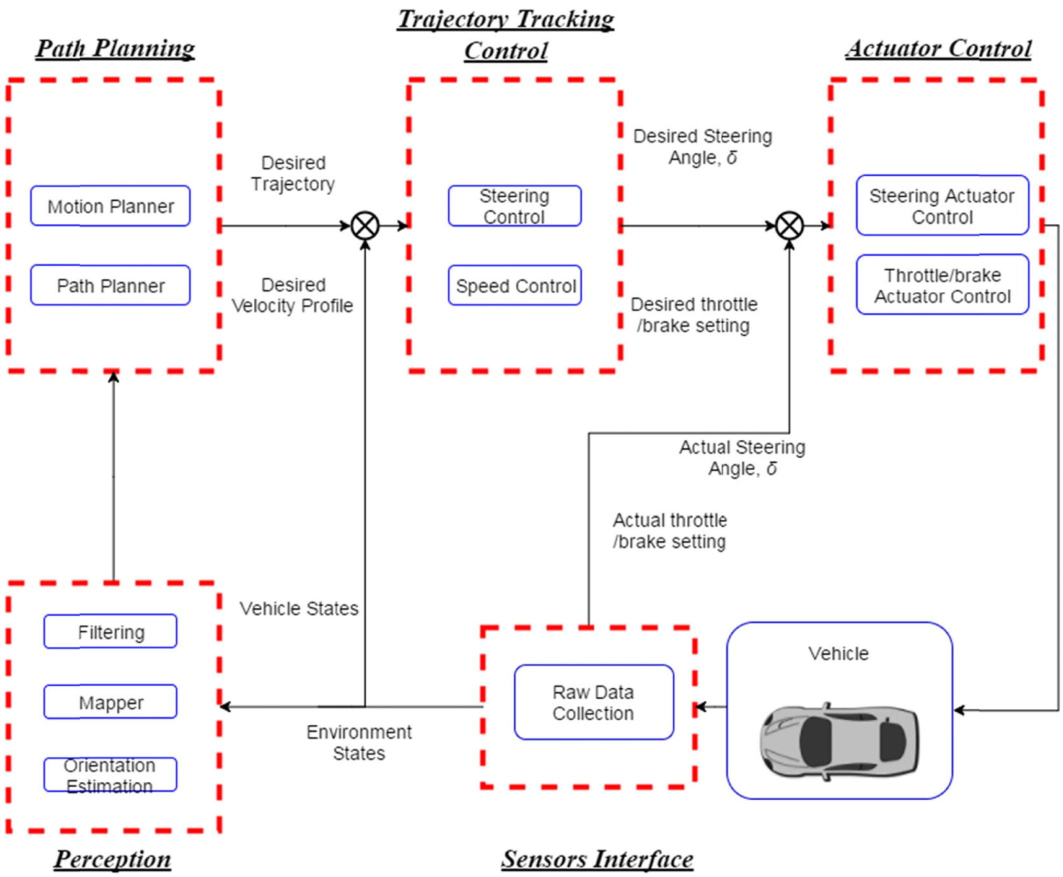

Fig. 1 Overview of autonomous vehicle system

control for autonomous vehicles is presented. First section presents brief introduction on autonomous vehicle, path following and existing reviews by other researches throughout the years. Section 2 will describe the vehicle model usually used in the controller development, either as the basis of the controller formulation or to simulate the vehicle behaviours during controller simulations. The main focus of this paper, review of trajectory control strategies and its classifications will be described next in Section 3. Next section detailed the performance criteria methods to quantify the controller's performance in path tracking control for evaluation and comparison purposes. This is followed by the fifth section where the main challenges and future prospects of path following and trajectory tracking control researches are addressed. Finally, overall conclusions of this paper is presented in final section.

2 Vehicle Model Used in Path Tracking Control

Going through past publications in path tracking control for autonomous vehicle, one can notice that a good portion of the studies employed linear handling model also known as bicycle model in order to develop steering controller. To put things into perspective, vehicle model usually plays a crucial role in two aspects. One aspect is for the vehicle system simulation. In developing any controller, simulation stage is one of the most crucial stage. During this stage, the controller properties are investigated and controller parameters are tuned to yield the best performance for the controller. For this, a vehicle model is usually used to simulate vehicle behaviours under the influence of the proposed controller. Second aspect, control law in trajectory tracking control is sometimes derived from the mathematical representation of the vehicle system.

Usually, a linearized vehicle model is used for this purpose. The equations for vehicle model are manipulated to form state space representation before further developed into a control law for the controller. In this section, both aspects will be addressed and existing approaches for the vehicle modelling will be reviewed from the previous researches.

In general, car-like robot can be classified as a non-holonomic system. Holonomic properties of a system refers to the relationship between the controllable DOF and total number of DOF for the system. A system is holonomic if the controllable DOF if equal to the total number of DOF and non-holonomic if the controllable DOF is less than the total DOF. As stated in Katrakazas et al. [7], normal vehicle is considered non-holonomic due to its total of four DOF which are motion in the two Cartesian coordinates direction, orientation, and heading but only two controllable DOF which are the longitudinal direction (forward and backward) and lateral direction (bounded steering input). In this section, only non-holonomic model of a vehicle will be addressed. It is worth to note that common models for small mobile robot will not be reviewed here.

Vehicle model usually assumed the vehicle body as a rigid body with concentrated sprung mass at the centre of gravity. Path tracking control is similar to any handling control where the behaviour of the vehicle is observed in longitudinal plane. Usually known as handling model, important properties usually observed are the lateral and yaw motions. For handling model, vertical motions are always considered constant by assuming flat road surface where ride properties and vertical forces from suspension is negligible. Most common vehicle model adopt bicycle concept where the vehicle is reduced to a two-tire configuration at the front and rear by assuming same behaviour for left and right tires. With this assumption, the right and left tire is merged and represented by one tire at front and rear, each. Figure 2 shows the common representation of vehicle model usually used in handling studies depicting the full vehicle model.

Usual configurations consists of geometric, kinematic and dynamic model. Geometrical dimensions of the vehicle are considered in geometric model with no regard on the kinematic (acceleration and its derivatives) and dynamic (forces, inertia, and energy) properties. Similarly, kinematic model consider the motion of the vehicle in terms of its acceleration,

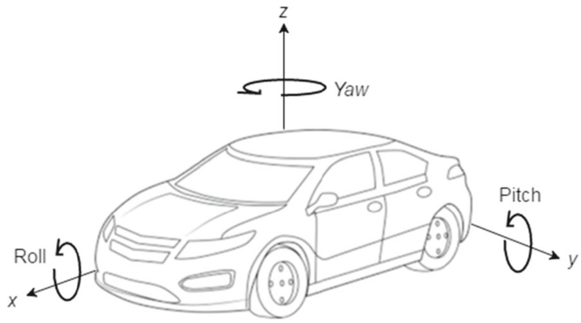

Fig. 2 Vehicle view and common nomenclature used in vehicle Modelling

velocity and position related to the geometric of the whole vehicle. Dynamic model, on the other hand, considers the motion of the vehicle in terms of its internal forces, inertia and energy properties. Each of the model configuration has its own benefits and purpose depending on usage and which properties do we want to study. Therefore, this section will be split into three subsection of these 3 model configurations.

2.1 Geometric Vehicle Model

Geometric vehicle model is particularly important in order to relate the vehicle's dimensions, radius of turn and radius of curvature of road undertaken by the vehicle during turning. As stated earlier, this model only consider the dimension and positions of the vehicle during manoeuvring with no regard to its velocity and acceleration. It is developed based on the Ackerman steering configurations that the line perpendicular to each of the vehicle wheel should intersect at the centre point of the vehicle cornering arc where the radius of turn is R. The importance of this model is in developing one of the most popular path tracking controller, Pure Pursuit [5, 10–16]. This model is rarely used to simulate any of the vehicle states. From Fig. 3, angle of turn, δ is given by the following equation.

$$\delta = \tan^{-1}\left(\frac{L}{R}\right) \quad (1)$$

Another geometric model relates the vehicle position with respect to the trajectory and its subsequent error. This model is particularly important in order to estimate the tracking error between the actual vehicle position and the desired trajectory which is the basis of another geometric-type Stanley controller as well as one of the most common quantification method to

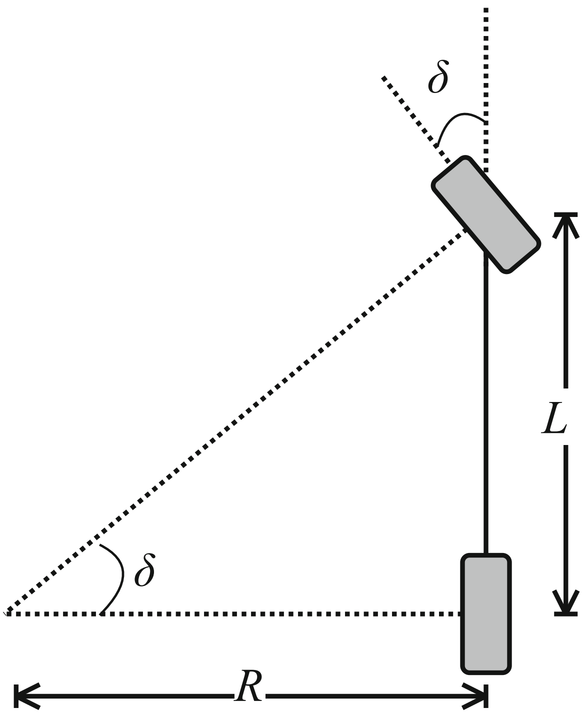

Fig. 3 Geometric model based on Ackerman Steering Configuration

evaluate a path tracking performance. Development of both the model and quantification method will be described in details within its respective sections.

2.2 Kinematic Vehicle Model

In mechanics field, kinematic is defined as the study of motion of a body without regards of the body's internal forces, inertia and energy. Therefore, kinematic vehicle model can be defined as vehicle model that is describing the motion of the vehicle in terms of its position, velocity and acceleration without any regard of its internal dynamics. The motion will be described solely based on its geometry. Kinematic vehicle modelling can be found in most of the studies on path tracking controller due to its simplicity and important relationship to describe the vehicle motion. This model is usually used to describe the vehicle velocity and acceleration in lateral direction as well as its yaw motion with respect to the vehicle fixed axes (also known as local coordinates) and global axes (also known as global coordinates). While it is always used together with geometric model to develop controller algorithm as well as to formulate the tracking error for quantification method, this model is also often used to simulate the vehicle's position, velocity, and acceleration during manoeuvring.

There are several approaches used previously in order to develop and represent the vehicle kinematics. A simple kinematic model is commonly used [17–19] for this purpose as depicted in Fig. 4. The vehicle is described with 2 sets of Cartesian coordinates, local (x-y) and global (X-Y). Local coordinates are set to be fixed on the vehicle body. Global coordinates, on the other hand is set to be fixed on the earth coordinates. Local coordinates are always regarded as moving coordinates with respect to the global coordinates since the vehicle is constantly moving. Based on Fig. 4, kinematic equation of the vehicle describing the velocity in global coordinates, v_X and v_Y as well as local coordinates, v_x and v_y can be deduced as in Eq. 2. Here, v is the total velocity for the vehicle, θ is the vehicle heading with respect to the local coordinates and ψ is the vehicle's orientation with respect to the global coordinates as depicted in Fig. 4.

$$\begin{bmatrix} v_X \\ v_Y \end{bmatrix} = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix} \tag{2}$$

where

$$v_x = v\cos\theta$$
$$v_y = v\sin\theta$$

A simpler kinematic model can be found in which the vehicle is modelled a bicycle. With this, responses

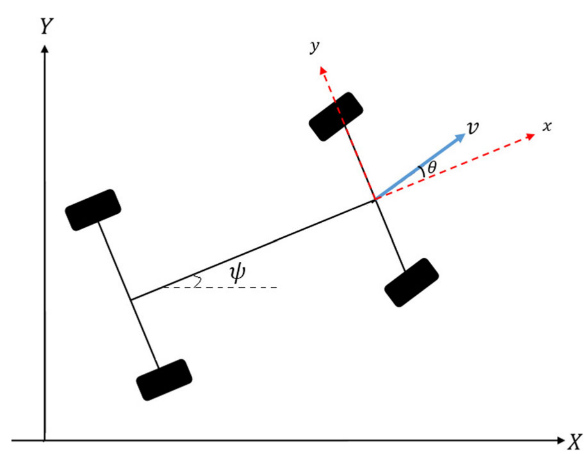

Fig. 4 Full vehicle in kinematic model

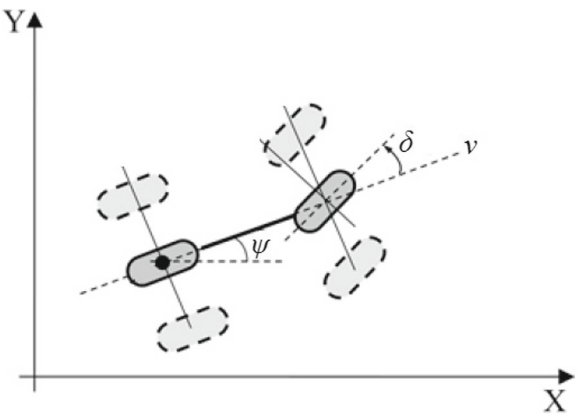

Fig. 5 Reduction of full vehicle model into bicycle model and its associated kinematic properties

from each wheel is simplified to be only one wheel per axle [10, 20–23]. By assuming only one wheel per axle, this model neglect the effect of slip from each wheel to the direction of the vehicle. Therefore, for this model, direction of vehicle's velocity is the same as vehicle heading direction. Referring to notation in Fig. 5, θ is the vehicle heading with respect to the local coordinated and ψ is the vehicle's orientation with respect to the global coordinates and r is the vehicle's yaw rate or $\dot{\psi}$ as depicted kinematic model for the bicycle model is shown in Eq. 3.

$$\begin{bmatrix} v_X \\ v_Y \\ r \end{bmatrix} = \begin{bmatrix} \cos\psi & 0 \\ \sin\psi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \dot{\psi} \end{bmatrix} \tag{3}$$

Another studies [24, 25] extended the kinematic bicycle model by including side slip angle of both front and rear wheel to account for slippery terrain the study needs to address. The equation is as given in Eq. 4 [25] where $V_{LR}$is rear wheel longitudinal slip velocities, $V_{SR}$is rear wheel side slip velocities, β_R is the sideslip angle for rear wheel and β_F is the sideslip angle for front wheel.

$$\begin{aligned} v_X &= v\cos\psi - V_{LR}\cos\psi - V_{SR}\sin\psi \\ v_Y &= v\sin\psi - V_{LR}\sin\psi + V_{SR}\cos\psi \\ \dot{\psi} &= \frac{v - V_{LR}}{L}\left[\tan\beta_R + \tan(\delta - \beta_F)\right] \end{aligned} \tag{4}$$

2.3 Dynamic Vehicle Model

Opposite to kinematic model, dynamic vehicle model is the vehicle model that is describing the motion of the vehicle in terms of its position, velocity and acceleration by considering the internal forces, energy or momentum within the system. In this model, forces from tire and mass of the vehicle body are considered and may include the overall geometric and kinematic relationships as described in previous sections. Common method in deriving the mathematical model of the vehicle handling dynamics is using Newtonian equation of motions. In longitudinal plane, handling model usually contains the translational motion in lateral, x, longitudinal, y, and rotational motion about $z-$axis also known as yaw, ψ, and yaw rate, r Figs. 6 and 7 show the schematic representation of the handling model of full vehicle and half vehicle (bicycle) respectively.

Equation of motion for dynamic vehicle handling model can be shown in Eq. 5 for full vehicle [17] and Eq. 6 for half vehicle model (Bicycle) [26]. This is considering the vehicle with sprung mass m_b, moment of inertia of the sprung mass about z-axis, I_{CG}, acceleration a_x of longitudinal motion in x-direction, a_y of lateral motion in y-direction, and yaw motion with yaw angle, ψ, about z-axis. Position of each vehicle's wheel is denoted with subscripts i, j, where i =*front/rear*, j =*left/right*.

$$\begin{aligned} \sum F_x &= m_b a_x \\ \sum F_y &= m_b a_y \\ \sum M_z &= I_z \ddot{\psi} \end{aligned}$$

<u>For Full Vehicle Model</u>

$$\begin{gathered} F_{xrr} + F_{xrl} + F_{xfl}\cos\delta + F_{yfl}\sin\delta + F_{xfr}\cos\delta + F_{yfr}\sin\delta = m_b a_x \\ F_{yrr} + F_{yrl} - F_{xfl}\sin\delta - F_{xfr}\sin\delta + F_{yfl}\cos\delta + F_{xfr}\cos\delta + F_{yfr}\cos\delta = m_b a_y \\ \left(\begin{aligned} &\sum M_{zij} + \left[-F_{yrr} - F_{yrl}\right] l_r + \left[F_{xfl}\sin\delta + F_{yfl}\cos\delta + F_{xfr}\sin\delta + F_{yfr}\cos\delta\right] l_f \\ &+ \left[F_{xfr}\cos\delta - F_{xfl}\sin\delta + F_{yfl}\sin\delta - F_{yfr}\sin\delta - F_{xrl} + F_{xrr}\right]\tfrac{t}{2} \end{aligned} \right) = I_{z,CG}\ddot{\psi} \end{gathered} \tag{5}$$

<u>For Bicycle Vehicle Model</u>

$$\begin{aligned} &F_{xr} + F_{xf}\cos\delta + F_{yf}\sin\delta = m_b a_x \\ &F_{yr} - F_{xf}\sin\delta + F_{yf}\cos\delta = m_b a_y \\ &F_{yr} l_r + \left[F_{xf}\sin\delta + F_{yf}\cos\delta\right] l_f = I_{z,CG}\ddot{\psi} \end{aligned} \tag{6}$$

Looking at above equations, one can notice that the main contributing external factor in this model is the tire forces which is the main source of external disturbance as well as the traction governing the

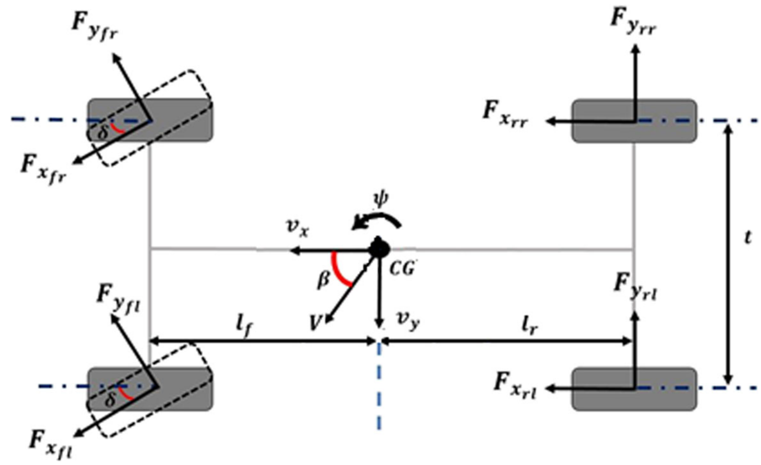

Fig. 6 Full vehicle representation in dynamic model

vehicle motion. Tire forces come from the interaction between the tire and road surfaces mainly caused by the deformation of the tire during different manoeuvring in both the longitudinal and lateral directions. While the forces are non-linear function, linearization of the dynamic model is commonly employed by assuming linear forces acting on each of the tires. This is suitable if the wheel angle, δ,is small which correspond to small lateral slip angle. Such linearization has been carried out on a full vehicle dynamic model [17, 27, 28] and bicycle model [14, 20, 22–24, 26, 29, 30] previously. Linearization of lateral and longitudinal tire forces, $F_x$and F_y respectively is done by assuming proportionality between tire forces and tire slips. This model is valid for small value of lateral and longitudinal slips. Previous studies outlined the region as below $0.5g$acceleration for low longitudinal slip [27] and lateral slip angle lower than 5°[31]. Linearization of lateral and longitudinal forces, F_x and F_y are shown respectively in Eq. 7. Here, $C_x$andC_y are defined as the longitudinal and lateral (cornering) stiffness of the tire, $s$is the longitudinal slip and α is the lateral slip angle.

$$\begin{aligned} F_x &\approx C_x s \\ F_y &\approx C_y \alpha \end{aligned} \tag{7}$$

Non-linear vehicle model, on the other hand, utilised tire model to simulate the lateral and longitudinal forces generated on each wheel during manoeuvring. Consideration of tire non-linear dynamics will better simulate the vehicle response especially in high speed and large steering angle. This approach can be seen in several studies previously used in both full vehicle model [19] and the simplified bicycle model [32]. Most of these studies are using the well-known Pacejka tire model also known as the Magic Formula. In Eq. 8, function P may represent either F_x, F_y, or $M_z$where B, C, D, ϕ, S_h, and S_v represent the stiffness factor, shape factor, peak value, curvature factor, horizontal shift, and vertical shift properties of the function, respectively. These are parameters-dependent functions in terms of various constants where for each set of functions, there will be a different set of B, C, D, ϕ, S_h, and S_v. Another study

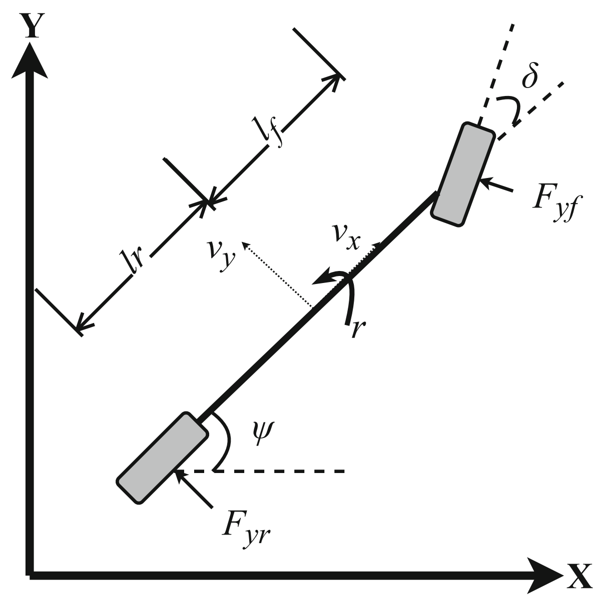

Fig. 7 Half vehicle (Bicycle) representation in dynamic model

can be found using Calspan tire formula [19]. This model estimate the normalised lateral and longitudinal forces, F_x and F_y respectively using a polynomial expression of a saturation function $f(\sigma)$. In order to use this model, one must have the knowledge of tyre vertical force, F_z, coefficient of friction for the road surface, μ, lateral stiffness coefficient, K_s, modified longitudinal stiffness coefficient, K_c', constant for tyre camber angle, Y_γ, and the tire camber angle, γ and the normalised lateral and longitudinal forces can be evaluated using Eq. 9.

$$P\left(F_z, \alpha, \sigma\right) = D \sin\left(C \arctan\left(B\phi\right)\right) + S_v \tag{8}$$

$$\begin{aligned} \frac{F_y}{\mu F_z} &= \frac{f(\sigma) K_s \tan\alpha}{\sqrt{K_s^2 \tan^2\alpha + K_c' s^2}} + Y_\gamma \gamma \\ \frac{F_x}{\mu F_z} &= \frac{f(\sigma) K_c' s}{\sqrt{K_s^2 \tan^2\alpha + K_c' s^2}} \end{aligned} \tag{9}$$

Both of these tire models are depending on several inputs; most importantly, the tire vertical force, F_z. Usually, vertical force acting on each tire was considered constant [32]. Nevertheless, studies by Ping et al. [19], Amer et al. [33] and Wang et al. [17] extended the nonlinearity of the model by including a load distribution model to estimate the vertical tire forces especially during braking and acceleration where the vehicle will experience significant pitch and hence, load transfer between rear and front part of the vehicle. Also, most of the full vehicle models incorporate the effect of yaw rate to the total acceleration for the vehicle [17, 18, 30]. Referring to Fig. 6 and Eq. 5, total lateral and longitudinal acceleration of the vehicle, a_y and a_x, respectively, can be written as follows.

$$\begin{aligned} a_x &= \dot{v}_x - r v_y \\ a_y &= \dot{v}_y + r v_x \end{aligned} \tag{10}$$

Another dynamic approach usually employed in vehicle modelling is Euler-Lagrange method. Different from Newtonian method that focus on analysis of forces and moments acting within the vehicle system, Euler-Lagrange method focuses on the changes in potential and kinetic energies due to outside disturbances into the system. For ground vehicles, Lagrangian approach usually used for models that include ride models, i.e. vehicle model for motions in vertical plane [34–36]. In path tracking studies, this method was seldom used for ground vehicles due to the complex derivations that may entail, according to the various subsystems in lateral dynamics. However, it can be found used in various studies [37–40] to derive mathematical model of smaller scales autonomous mobile robots. Also, Lagrangian method is used in the multi-body dynamics software MSC ADAMS in deriving the mathematical models of the vehicle system. Users can also obtain the Lagrange equations used in modelling the system [41].

In general, the Euler-Lagrange formula can be generalised in Eq. 11.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i^x \tag{11}$$

Where q_i are generalized coordinates and Q_i^x are corresponding external forces. The Lagrangian, L is given in Eq.12, where T and U are the kinetic and potential energy of the system, respectively. In vehicle handling model application for motions in lateral direction, a set of local coordinates based on vehicle-fixed frame usually used, where $q' = \begin{bmatrix} v_x & v_y & \dot{\psi} & \varphi \end{bmatrix}$. Here, v_x and v_y are vehicle velocity in lateral and longitudinal direction respectively, $\dot{\psi}$ is the vehicle yaw rate about vertical axis and φ is the vehicle roll angle about the longitudinal axis [34]. Detail derivations of the Euler-Lagrange approach in vehicle modelling can be referred to [34, 36, 42].

$$L = T - U \tag{12}$$

Both Newtonian and Lagrangian approaches will give the same end results where the vehicle motion is equated into a set of equations of motions. The main difference lies on the process of deriving the equations of motions and the properties considered during the process. Main advantage of the Newtonian method is the less complicated mathematical process involved to derive the equations of motions. Complication may be faced during the identification of forces and in resolving the forces into desired direction of motion. This is still mild compared to the amount of calculus operation in deriving the kinetic and potential energy equations and determining the derivatives of L with respect to each generalized coordinates. However, Newtonian method requires one to recognise each force within the system which include frictions and reaction forces from various interactions and constraints among different subsystem. Here is where the main advantage to use the Euler-Lagrange approach over the Newton approach [43]. Lagrangian approach will analyse motions in terms of the energies within the vehicle system which are scalar quantities that can be equated in terms of the generalised coordinates [42]. This provides a powerful analysis tool to analyse motions of a holonomic system. For non-holonomic systems however, several modifications should be carried out to determine the independent generalized coordinates. These procedures can be found in standard text-books on non-holonomic systems mechanics [44, 45].

2.4 Summary on Vehicle Models

In this section, several vehicle model usually used for path tracking control were presented. Vehicle modelling is particularly an important first step used to (a) simulate the vehicle responses and (b) derive a controller algorithm. Three categories of vehicle model were presented in this section namely geometric model, kinematic model and dynamic model. In all categories, vehicle modelling was done in two ways, full vehicle as well as the half vehicle model also known as bicycle model. All of the basic models were presented where its lengthy derivations can be found in the cited references. However, there is a possibility that path tracking study did not involve vehicle model development. Recent technologies in software development has given rise to many notable software that is capable to predict and simulate the overall vehicle response such as CarSim and MSC ADAMS. These software can be used to replace the vehicle plant module and simulate the vehicle behaviour under various manoeuvrings in testing and tuning stages of controller development [10, 14, 17, 23]. The software is equipped with standard testing procedures as well as standard road courses which will be very feasible in path tracking control. However, to use any tool, one must really have the detailed grasp on how the internal calculations and derivations were developed to have basic knowledge on what is happening in their studied vehicle.

Each approach in vehicle modelling serve different purpose. In selecting which modelling approach one should use, one should go back to basic requirement of the vehicle model. Is the model required to simulate vehicle response? Will the studies analyse response that is outside of linear operating points region envelops? What performance one need to model? A careful selection on vehicle model to be used is really crucial to ensure good controller is produced. To summarise, Table 1 notes the reviewed vehicle model and criteria for each of the approaches.

3 Trajectory Control Strategies

Trajectory tracking control in autonomous vehicles is mainly aimed to provide sufficient steering input as well as throttle and braking input to control the direction and speed of the vehicle to guide the controlled vehicle along a pre-defined path. For the purpose of limiting this review, path planning method and strategies are not included. Detail reviews on these fields can be found in previous studies [7, 9]. This review particularly will cover the control strategy used in order to control the steering angle, δ and vehicle speed v along the pre-defined path. Therefore, this section is presenting the trajectory tracking control covering the available method and strategies to control both the steering and speed inputs.

De Luca et al. [46] had conveniently classified the overall trajectory tracking control into three motion tasks. This classification is intended to view the overall problem into three separate tasks and therefore, a suitable controller can be developed by solving these three tasks together, namely *Point-to-point motion*, *path following*, and *trajectory tracking*. Point-to-point motion treat the whole motion as a basic task of reaching a defined goal point from an initial point of starting, regardless the path and trajectory. Path following is related to the path connecting the starting point and goal point in such that the task is to reach and follow the given geometric path. Trajectory following, on the other hand, concerns on the task for the vehicle to reach and follow the given geometric path within an associated timing law. In other words, the first task concern on the convergence and stabilisation problem to a final position value. Second task require the vehicle to be guided following the path, concerning the steering control of the vehicle. Third task concerns on the time taken for the vehicle to complete the given path, concerning the speed control of the vehicle.

3.1 Geometric and Kinematic Controller

This type of controller is developed based on the geometric vehicle model as reviewed in section 2.1. It is by far the most popular type of controller used in autonomous steering control due to its simplicity and stability. Three basic controllers and its derivative will be considered in this section followed by any notable geometric controllers.

3.1.1 Follow the Carrot

This is the most basic geometric controller, based on the analogy of riding on a donkey and guiding the donkey using a carrot directed to the intended direction. This approach can be seen reviewed and applied in several publications previously [47, 48]. In this approach, the nearest point on path at one

Table 1 Summary of different type of vehicle model used in path tracking control

Model Type	Vehicle Model	Strength	Weakness	Comment(s)
Geometric	Geometric model based on Ackermann steering configuration	Simple, less parameter needed	No indication on internal forces	Simple configuration and parameters
		Sufficient to describe relationship between vehicle position and path.		Suitable for purposes that does not require velocity and acceleration responses. Only vehicle position indication
		Usually used to develop geometric controller		
Kinematic	Full vehicle kinematic model	Take into account the possibility of different direction between vehicle heading relative to local coordinates, θ and global coordinates, ψ	No indication on internal forces	Consider left and right wheel
			Slightly more complicated than geometric model	Suitable for purposes that does not require dynamics responses.
	Half (bicycle) vehicle kinematic model	Simple, less complex configurations	No indication on internal forces	Consider only one wheel per axle.
			Assume vehicle heading with respect to local coordinates is the same as heading with respect to global coordinates	Suitable for purposes that does not require dynamics responses.
	Kinematic model with slip angle	Consider slip in the model for manoeuvrings	No indication on internal forces	Suitable for studies on vehicles operating on slippery surfaces and high speed range
			Added complexity	
Dynamic (linear)	Full vehicle kinematic model	Consider forces in all wheels, especially in cornering manoeuvres	Does not consider non-linear function of tire properties	Suitable for studies where dynamic observation is important but small slip $\approx$0.5g acceleration and $\approx$ 5 slip angle
	Half vehicle kinematic model	Less complicated model	Does not consider non-linear function of tire properties	Same as above
			Neglect the effect of different responses for left and right tires	The difference of behaviour in right and left tire may become significant during cornering

Table 1 (continued)

Model Type	Vehicle Model	Strength	Weakness	Comment(s)
Dynamic (nonlinear)	Full vehicle kinematic model	Consider forces in all wheels, especially in cornering manoeuvres	More forces included. More complex	One of the complete handling model
		Consider the nonlinearity of tire responses with respect to slip angle		Suitable for studies on vehicle subject to different manoeuvrings
	Half vehicle kinematic model	Less complicated model	Does not consider the effect of inner and outer wheel during cornering	Suitable for studies on vehicle subject to different manoeuvrings
		Consider the nonlinearity of tire responses with respect to slip angle		

look-ahead distance, L is chosen to be the instantaneous goal point, known as the "carrot point". Orientation error with respect to the global coordinates, ψ_e, is determined by the difference between the vehicle orientation, ψ and the orientation of carrot point, ψ_c as shown in Fig. 8. The overall steering command is shown in Eq. 13. The steering angle required to correct the vehicle heading and orientation can be determined using proportional controller, as shown in Eq. 14. A PID controller triggered by the error ψ_e, can be used to replace the proportional controller which will improve the effectiveness of the vehicle [48].

$$\psi_e = \psi_c - \psi \tag{13}$$

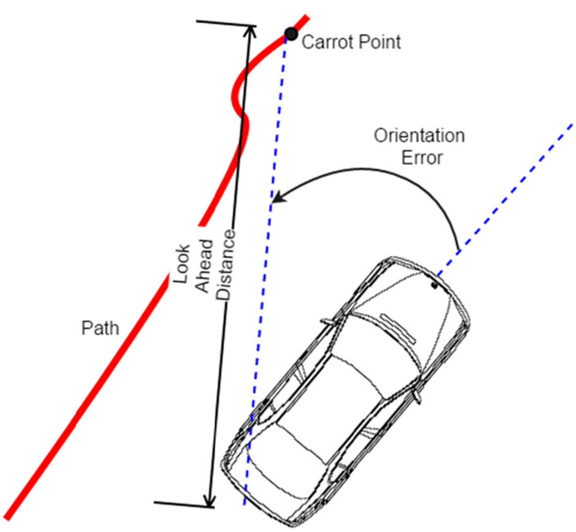

Fig. 8 Representation of Follow-the-Carrot method

$$\delta = K_{carrot} \times \psi_e \tag{14}$$

3.1.2 Pure Pursuit

Pure pursuit is the most popular geometric controller among the existing methods. The name Pure Pursuit comes from the way the method was formulated. The vehicle is understood to be chasing a moving point throughout the duration. Therefore, it is constantly "in pursuit" towards the moving point (carrot point). In understanding pure pursuit method, it is the extension of the earlier follow-the-carrot method where a carrot point is chosen based on the nearest point on path within one look ahead distance from the vehicle. Similarly, heading error is calculated based on Eq. 13. Pure pursuit method extends this approach at this stage by fitting a smooth circular curve from vehicle to the carrot point for the vehicle to follow. This circular curve is defined such as its line passes through two points; the vehicle position and the carrot points. This curvature will ensure smooth steering for the vehicle and reduce the oscillatory nature of the basic follow-the-carrot method as reported by Wit [15]. The overall method is as depicted in Fig. 9.

Given that the circular arc goes through the carrot point and control point on the vehicle with centre at O; a local coordinate axes with origin on the control point; and intended steering angle δ as shown in Fig. 9. One should notice that the triangle produced is an isosceles triangle with common length R. Using the rule of sine involving look-ahead length, l_d and the angle α between the vehicle's heading vector and

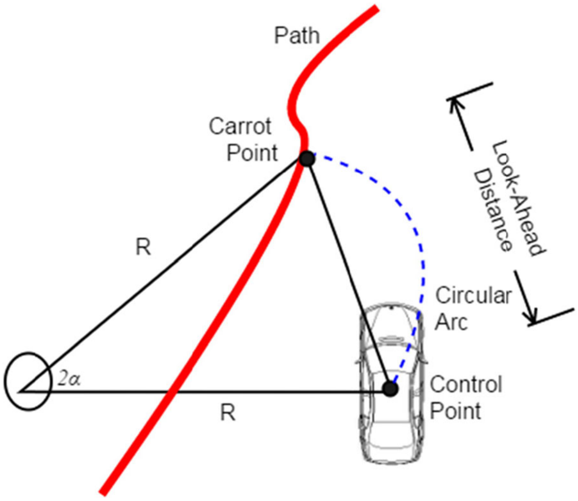

Fig. 9 Pure Pursuit Strategy

the look-ahead vector, a simple mathematical equation can be derived in Eq. 17 [14].

$$\begin{aligned}\frac{l_d}{\sin 2\alpha} &= \frac{R}{\sin(\frac{\pi}{2}-\alpha)}\\ \frac{l_d}{\sin\alpha\cos\alpha} &= \frac{R}{\sin\frac{\pi}{2}\cos\alpha-\cos\frac{\pi}{2}\sin\alpha}\end{aligned} \tag{15}$$

$$\text{Simplifying,}\ \ R = \frac{l_d}{2\sin\alpha} \tag{16}$$

Thus, steering angle can be derived by substituting Eq. 16 into Eq. 1 previously such that:

$$\begin{aligned}\delta &= \tan^{-1}\left(\frac{L}{R}\right) = \tan^{-1}\left(\frac{2L\sin\alpha}{l_d}\right)\\ &= \tan^{-1}\left(\frac{2L\left(\frac{e_y}{l_d}\right)}{l_d}\right) = \tan^{-1}\left(\frac{2Le_y}{l_d^2}\right)\end{aligned} \tag{17}$$

It can be seen that the steering angle generated in Eq. 17, is a function of look-ahead distance l_d and lateral error, e_y. Snider [14] has detailed the method to tune this controller by equating the look-ahead distance as a function of the vehicle velocity such that $l_d^2 = kv_x$. This way, the controller will be tuned with only one parameter. Basic algorithm of pure pursuit is as depicted in Fig. 10.

Implementation of Pure Pursuit strategy can be traced back to a study in late 1960's used for missile tracking where the missile is directed towards an instantaneous goal point (carrot point) [49]. A study by Wallace et al. [1] was the first study that implemented this strategy on a mobile robot together with a vision method that is keeping the vehicle at the centre of road image captured. Amidi [5] then implemented

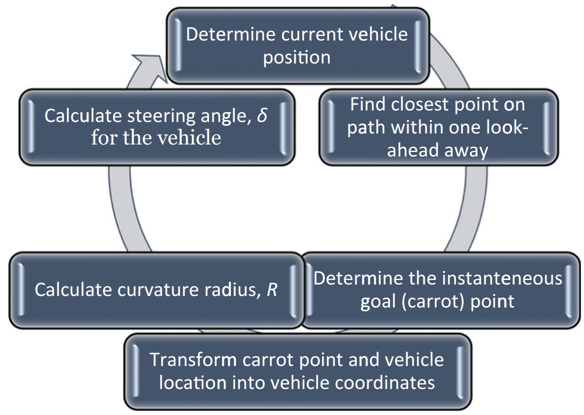

Fig. 10 Pure Pursuit Algorithm

Pure Pursuit and compare it against quintic polynomial fitting method and "Control Theory" method to prove that Pure Pursuit is the most robust and reliable method at that time. A technical report from the same institution by Coulter [13] described in details the implementation of this method on the autonomous vehicle in CMU. Also, Coulter [13] pointed out the main properties of Pure Pursuit that the efficiency and reliability of the algorithm are based on the selection of the look-ahead distance. Too large and the vehicle will "cut corners" and too small will cause the vehicle trajectory to be too oscillatory. Ollero et al. [50] presented an implementation of Pure Pursuit on autonomous vehicle equipped with a perception module that employs different position estimation methods. These had kick-started series of successful implementations and performance evaluations of the pure pursuit algorithm e.g. [14, 21, 22, 51–58]. Also, this method has been reported to be employed in at least two vehicles in the DARPA Grand Challenge [59] and three vehicles in the DARPA Urban challenge [60].

Due to its popularity, Pure Pursuit has been a standard benchmark to validate a new controller proposed by active researches. Hellstrom, Ringdahl [61] compared their novel controller for forest vehicles, "follow the past" against the existing Follow the carrot and Pure Pursuit method. This study incorporated the recorded steering behaviour data and used command fusion to generate steering input based on three different tasks considered by the controller. The tasks were based on recorded orientation, recorded steering angle, and It was presented in details within a thesis by Ringdahl [52]. An interesting approach by Wit et al. [16] and his thesis [15] introduced "Vector Pursuit" which relates a screw theory to the pursuit algorithm. The screw theory considers the instantaneous motion of the vehicle from current location to desired position relative to a given global coordinates. This will incorporate orientation information which was absent in the original Pure Pursuit strategy. The improvement was compared against the existing Pure Pursuit. Also, recent study by Bayar et al. [53] compared their proposed path tracking controller for orchard vehicles against Pure Pursuit. This study incorporated yaw rate and wheel slip control to cater the slippery terrain in orchards.

As mentioned earlier, the main shortcoming of Pure Pursuit is in the selection of look-ahead distance. Optimum compromise between stability and tracking performance is very difficult to achieve which make this strategy seems to be course-dependent [14]. Besides, this controller was observed to neglect the dynamics of the vehicle [62], the curvature of the path [14] as well as the overall global coordinate axes. All dimensions are determined based on the local coordinates of the vehicle. Another shortcoming of this controller is that it may not be able to negotiate discontinuous paths. This is the common problem in geometric controller [14]. Also, rapid changes in path that may cause sudden change in vehicle heading is known to make the controller to be unstable. This was due to the fact that the steering command neglects the dynamic properties of the vehicle and steering systems, and rapid changes in heading error will trigger the controller to generate large steering demand which may cause the rear vehicle to skid [13]. Neglecting steering dynamics causes the controller to overestimate the ability of the system to deliver correctional steering inputs due to the delay in the system.

Improvement of this method had been proposed previously. Fuzzy supervisory system was proposed to automatically tune look-ahead distance depending on curvature and speed [12, 63] where Shan et al. [12] proposed further improvement to fit a clothoid function as the smoother path compared to circular arc proposed in the original strategy. Campbell [55] proposed an analytical method to tune the look-ahead distance based on linearized model for path tracking control whereas a study by Raffo et al. [22] developed an adaptive look-ahead selection with a nonlinear model predictive control (NMPC) which shown good compromise between performance and computational cost. Automatic tuning for look-ahead distance also proposed earlier by decoupling yaw motion and vehicle lateral dynamics which give a dynamic look-ahead distance that is independent of vehicle speed [64, 65]. More recently, Park et al. [66] proposed an advanced Pure Pursuit which incorporated a PID control to provide additional steering control based on the lateral error.

3.1.3 Stanley Method

“Stanley” is an autonomous vehicle developed by Stanford University that won the second DARPA Grand Challenge in 2005 [59]. With the total of 195 teams registered, only 23 raced and of these, only five teams managed to finish the race course in the Mojave Desert, Southwest of USA. Stanford team and their robot vehicle, Stanley, led by Sebastian Thrun emerged as winner by finishing first in 6 hours 45 minutes and average speed of 19.1mph (8.5 m/s, 30.7 kmph). The overall architecture of the vehicle was described by Thrun et al. [67] and the steering control for path tracking controller was detailed by Hoffmann et al. [26]. The path tracking system employed a nonlinear geometric controller considering heading error and lateral error. A basic controller was presented by Thrun et al. [67] considering these two properties as shown in Eq. 18. Here, ϕ is the heading error between the trajectory direction and vehicle direction of motion, determined by evaluating the difference between instantaneous orientation of the vehicle and trajectory $\phi = \psi - \psi_{traj}$. e is the lateral error, measured from the centre of steering wheel axle to the nearest point on trajectory and v is the instantaneous velocity of the vehicle as illustrated in Fig 11.

$$\delta(t) = \phi + \tan^{-1}\left(\frac{ke(t)}{v(t)}\right) \tag{18}$$

Global asymptotic stability of this control law has been evaluated in the original publication [26] at $e = 0$, $v > 0$ and $0 < \delta_{\max} < \frac{\pi}{2}$. This means that the control law in Eq. 18 is saturated about δ_{max} as shown in Eq. 19. In the same paper, they improved the control law by augmenting Eq. 18 to consider the dynamics of vehicle and trajectory. The augmentation of control law was done by considering the steady state yaw, ϕ_{ss}, relative to a constant curvature path, yaw rate

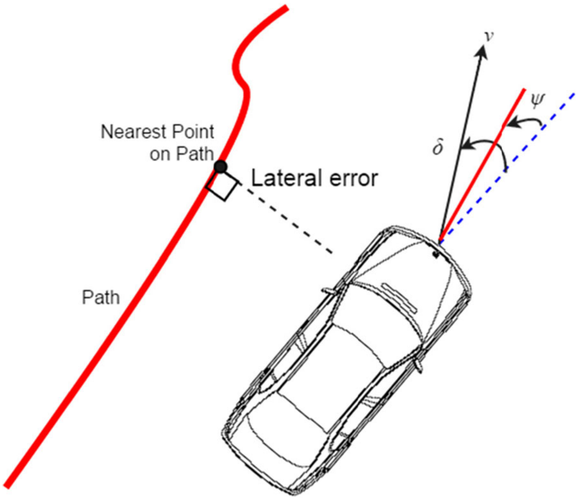

Fig. 11 Configurations and parameters for Stanley controller

error between vehicle's yaw rate, $\dot{\psi}$, and trajectory yaw rate, $\dot{\psi}_{traj}$, and steering correction term to compensate the steering servo delay and overshoot were added as shown in Eq. 20. Here, δ_{meas} is the steering angle measured in discrete time step; k and k_ψ are tuneable parameters; and k_{soft} was added to stabilize controller during low velocities. This augmented control law is still saturated within the same δ_{max} as before. Performance of this controller was tested exhaustively months before the race and it was also proven to be able to guide vehicle in reverse as well. Although Stanley winning was mainly attributed to its stellar performance in data perception and path planning [68], its nonlinear path tracking controller was proven to be a great asset.

$$\delta(t) = \begin{cases} \delta_{\max} & \text{if} \left|\phi + \tan^{-1}\left(\frac{ke(t)}{v(t)}\right)\right| \geq \delta_{\max} \\ \phi + \tan^{-1}\left(\frac{ke(t)}{v(t)}\right) & \text{if} \left|\phi + \tan^{-1}\left(\frac{ke(t)}{v(t)}\right)\right| < \delta_{\max} \\ -\delta_{\max} & \text{if} \left|\phi + \tan^{-1}\left(\frac{ke(t)}{v(t)}\right)\right| \leq \delta_{\max} \end{cases} \tag{19}$$

$$\delta(t) = (\phi - \phi_{ss}) + \arctan\left(\frac{ke(t)}{k_{soft} + v(t)}\right) + k_\psi\left(\dot{\psi} - \dot{\psi}_{traj}\right) + k_{d,steer}\left(\delta_{meas}(i) - \delta_{meas}(i+1)\right) \tag{20}$$

Stanley method can be found studied and implemented in a number of previous studies since its successful implementations in the 2005 DARPA Challenge. Snider [14] has developed, tuned, and implemented different tracking controller from geometric, kinematic, and dynamic type of controllers as well as optimal controllers and tested them all on three road courses within CARSIM. The study found that Stanley control law from Eq. 18 performed surprisingly well compared to other controllers despite its simplicity. In their improvement of Pure Pursuit strategy, Shan et al. [12] compared their proposed strategy with Stanley Method and the improved Pure Pursuit managed to outperform Stanley in all tests. Recently, Töro et al. [10] has implemented the nonlinear Stanley controller on a Hardware-in-the-loop testing and compare it against a linear method of double-loop control with feed forward load disturbance compensation. It was found that the Stanley controller was more superior with better transient response and lower steady state errors in curves. One can notice by going through these publications that all of the studies involving Stanley method was employing the control law as in Eq. 19 while the original implementation in Hoffmann et al. [26] considered more factors in their control law as stated in Eq. 20. The absence of look-ahead distance was compensated by the yaw rate term, dynamics of the vehicle motion was compensated by the inclusion of steady state yaw, ϕ_{ss}, as controller set point; delay and dynamics of the steering mechanism was compensated by the δ_{meas} term; and instability during low speed (which will make the second term be too large) was compensated by k_{soft}. Neglecting these terms may affect the ability of the control law to perform like Stanley implementation in 2005 DARPA.

3.1.4 Other Notable Geometric and Kinematic Controllers

Around the same time CMU came up with studies on Pure Pursuit, a kinematic controller was designed by Kanayama et al. [69] using Lyapunov control theory. Main focus of the study was to determine a suitable referenced linear and rotational velocities for a non-holonomic mobile robot based on tracking error in terms of lateral error, y_e, longitudinal error, x_e, and orientation (heading) error, θ_e, as shown in Eq. 21. Here, tracking errors are defined as the difference between the reference trajectory and orientation (x_{ref}, y_{ref}, θ_{ref},) and actual responses (x, y, θ,).

$$\begin{pmatrix} x_e \\ y_e \\ \theta_e \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{ref} - x \\ y_{ref} - y \\ \theta_{ref} - \theta \end{pmatrix} \tag{21}$$

A new tracking control was proposed by Sun et al. [70] called the "Ribbon Tracking Method". In this strategy, a new road-vehicle model was developed called the "Ribbon Model" in order to determine the optimal curvature for the vehicle to take on. Then, a two-stage path tracking controller was developed consists of a feed-forward steering command and a feedback disturbance compensator. This controller was developed using steering curvature as the output variables which avoids nonlinear consideration as opposed to having Cartesian coordinates, (x, y) as output variables. It was claimed to solve the problems of having to choose an optimal look-ahead distance and managed to consider the dynamics of the vehicle with the compensator. MIT team in the DARPA Urban Challenge, the third DARPA autonomous race was employing a path tracking controller based on Pure Pursuit with a significant difference in their algorithm [58]. Instead of using rear axle as control point, they used an anchor point situated in variable places which change the overall algorithm to be significantly different than stated in Eq. 17. The vehicle managed to finish the race course and placed 4th overall.

Back stepping methods in path tracking controllers considering the geometric and kinematic of the vehicle can be found in previous publications [29, 71, 72]. In a notable technical report by Snider [14], a kinematic controller taken from De Luca et al. [46] was derived, tuned and implemented. The controller was using state feedbacks considering the vehicle kinematics such as velocity in longitudinal and lateral direction, yaw rate, and path parameters using methods such as smooth time-varying feedback control as well as input scaling. This gives us a three-term feedback control law, each with a tuneable gains. The gains were tuned based on stability relationship which results in only one parameter to choose as presented in De Luca et al. [46]. Another study by Zakaria et al. [73] employed a feedback control known as "Future Prediction Control" to determine the vehicle's future position with respect to the path and adjust the steering input to reduce the future error. The control law then consists of heading error, ϕ, future lateral error, e_f, and vehicle velocity, v, in a somewhat similar strategy as Stanley control law in Eq. 18. Control law used in this study is stated in Eq. 22. The study is then enhanced in a later publication [20] where a spike detection is employed to avoid erratic behaviour by the controller due to GPS signal lost.

$$\delta(t) = \sin\phi + \frac{ke_f}{v} \tag{22}$$

Scaglia et al. [74] and their subsequent publication [75] presented a linear interpolation method for path tracking control. In this method, the nonlinearity behaviour of the system was compensated by a linear interpolation of controlled variables which made it suitable to be used in any nonlinear multivariable system of non-holonomic mobile robots which may include autonomous vehicle. Stability and suitability of the control scheme was proven in Scaglia et al. [74]. Overall, geometric controllers offer a simple steering command with less complicated algorithms and required state variables. However, this type of controller neglects the dynamic effects of the vehicle system that is crucial especially in extreme manoeuvrings with rapid changes in path headings and subsequently, rapid changes in steering input demands.

3.2 Dynamic Controllers

Dynamic controllers in path tracking control include the dynamic properties of the vehicles in their control law. Derivation of the control law may begin with the dynamic model explained in Section 2.3 or simply including dynamic properties of the vehicle in the control law. Rossetter [27] in his work proposed a new lane keeping strategy using virtual force concept. Kalman Filters were used to process position and yaw rate data for the controller to determine sufficient virtual force to keep the vehicle on intended path which will affect both the rotational and lateral dynamics of the vehicle. Stability of this controller was validated using Lyapunov theory on the vehicle motion in lateral direction subjected to time-varying disturbances. On the downside, this method is still using look-ahead distance to determine the virtual force and hence subjected to the problem of choosing an optimal value of look-ahead distance as faced in Pure Pursuit technique.

Fierro, Lewis [39] has modified the standard kinematic controller to include a dynamic back stepping extension to the control structure. The proposed structure contains a torque controller in order to guide the mobile robot along a path which was claimed to solve all three motion tasks as outlined in De Luca et al. [46]. Asymptotic stability was ensured and validated using Lyapunov theory. The overall structure was designed to accommodate wide range of future controller development from the conventional to an adaptive type controllers. This type of controllers considered the crucial effects of system dynamics which can increase the controller's ability compared to geometric/kinematic controllers. However, a dynamic controller requires dynamic feedbacks such as force and torque which require expensive dedicated sensors. Alternative ways to determine these states may require additional data processing stages which can take its toll on the computational power.

3.3 Optimal Controller

LQR is one of the most popular optimal control theory where the controller gain was determined using linear quadratic optimisation approach. Standard method to derive the control law can be found in text books [76, 77]. Snider [14] outlined the implementation of LQR controller in path tracking control with dynamics feedbacks. However, due to the absence of path feedback, the controller performed badly. The method was modified to include feed forward term of steering angle to zero steady state error. However, the result was still worse since the look-ahead distance is still absent and hence, the controller can only be reactive. Solution to this problem can be found by implementing optimal preview as proposed by Sharp in at least three publications [78–80].

In Sharp et al. [80], mathematical modelling and the optimal controller was presented. The controller was based on linear discrete time optimal preview control theory where it samples several values of preview path errors, attitude error and lateral position error and converts the recorded data into the desired steering angle. The method was demonstrated later in autonomous vehicle [79] and motorcycle nearing its cornering limit [78]. In Sharp [79], the study developed a linear optimal preview approach and was shown to display excellent path tracking performance and robust against changes in the vehicle dynamics. In Sharp [78], the study focused on gain scheduling of the controller parameters according to speed and lateral acceleration. However, this method did not performed as perfectly as one would think even though it considers the dynamics of the vehicle as well as optimal preview consideration which, in theory, will compensate the absence of look-ahead distance. This finding was supported by Snider [14] and Zakaria [31]. Optimal controllers may provide simpler structure of $U = -kx$ compared to other controllers, where U is the control signal, k is the controller gains and x is the vehicle's states. Determination of k is done offline in the controller development stage, which give us the simple structure. However, this may not be the case for controllers that employ online tunings. Also, most of the optimal controllers were developed from the linear system which may limit the ability of the controller within linearity operating envelops.

3.4 Adaptive and Intelligent Controller

Adaptive controllers are usually developed for studies or applications that require high robustness to certain disturbance and changes. An adaptive path tracking controller was designed and fully tested by Martins et al. [81]. This controller takes the linear and angular velocities as input reference signals with an adaptive parameter update law is used to avoid parameter drift. Although the implementation was done on a unicycle mobile robot, it can be modified for implementation on a vehicle platform with single control point. Fukao et al. [40] extended the back stepping kinematic/dynamic concept presented before in Fierro, Lewis [39] by including an adaptive law that caters for mobile robot with unknown properties. In this study, a new kinematic controller was proposed, coupled with a torque controller as in Fierro, Lewis [39]. Another notable adaptive controllers can be found where each may cater different aspects of vehicle working conditions [82–85]. An interesting study by Lucet et al. [24] proposed an adaptive controller for slippery road using yaw stabilization approach to drive the front and rear wheel. The effects of wheel-ground skidding in slippery terrain was compensated using an adaptive law that tunes the cornering stiffness by observing the dynamic wheel sideslip angle. Another adaptive controller for vehicle with unknown skidding was proposed in [86].

Just recently, an adaptive controller was proposed by Huang et al. [87] for wheeled mobile robot which consists of an adaptive virtual velocity controller and torque control law. This controller was claimed to be able to adapt to any external disturbances by including a feed forward compensator by estimating the lumped disturbances. A virtual linear and angular velocity was determined using the virtual kinematic controller. This velocity will act as a set point for dynamic controller which will generate torque required to track the virtual velocities. Khatib et al. [88] compared two adaptive controllers, namely an adaptive proportional controller (APC) and Universal Adaptive Stabilization (UAS) in order to control the linear and rotational velocities of the mobile robot to automatically adapt against noise in sensors measurement. The robustness of the controllers was evaluated using numerical approach in terms of the Integral of time multiplied by the absolute value of the error (ITAE) criteria, the Integral of square of the error (ISE), and the Integral of absolute magnitude of the error (IAE). It was found that the UAS approach performed better in noisy measurement with faster convergence compared to the APC controller and conventional state feedback linearization approach.

Dørum et al. [89] developed and evaluated several path tracking controllers in order to control the vehicle velocity and two of them were adaptive controllers. A dynamic controller was derived from nonlinear vehicle model where an adaptive law was applied to estimate the nonlinear vehicle parameters, as illustrated in Fig 12. Also, a linear Model Reference Adaptive Controller (MRAC) was developed and evaluated experimentally. It was found that the MRAC delivered the best performance of all controllers on a system with fairly slow dynamics. However, it can be seen that this controller was developed using complex algorithm which may take a lot of computational effort.

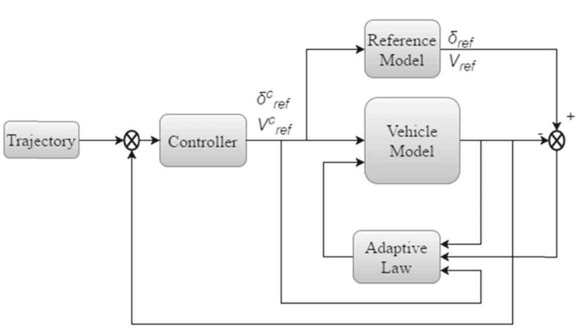

Fig. 12 Adaptive Control structure as proposed in Dørum et al. [89]

Neural networks and fuzzy sets can be seen used in a lot of adaptive and intelligent control due to its ability in guiding the controller to make fast decisions. Implementations of decision making algorithm using these two approaches can be categorised into two, namely supervisory or controller type implementation. General structure of these two approaches can be seen in Fig. 13. In supervisory type of fuzzy or neural networks, the decision making algorithms were used to tune selected parameters in the dominant controller which, in turn, may be categorised as an adaptive controller. In controller type implementations, the algorithm were used as the main controller to decide the control input for vehicle system plant. Controller type implementation of Fuzzy controller can be seen in Bin et al. [90] where the fuzzy controller was used in the speed control module to determine the desired velocity based on the instantaneous vehicle distance to goal point. Most of the Fuzzy/Neural Network approach in literatures proposed the supervisory type of intelligent control. In Ping et al. [19, 91], the author proposed an adaptive yaw rate feedback controller with adaptive fuzzy supervisory control. This approach consists of two controller loops to control the lateral error and yaw rate, respectively to control the steering input. PID and proportional controllers were used in the lateral error control loop and yaw rate control loop respectively. The proportional gain in the yaw rate controller were tuned using a fuzzy supervisory system with tuneable defuzzification parameters which tuned using back-propagation method. The study was equipped with a steering system control development using PID controller to ensure the steering controller will generate the desired steering input. Same approach was used with Neural Network supervisory system by Bittanti et al. [92] and Yang et al. [93] that employed the intelligent algorithm to tune online parameters to deal with changes in operating conditions.

Another notable intelligent control approach was proposed by Ahmed, Petrov [94] which was an extension of an earlier controller by that used a conventional control algorithm for linear/rotational velocity rules, local error coordinate system, and reference and current postures for vehicle control. They proposed a type-2 Takagi-Sugeno fuzzy-neural to tune

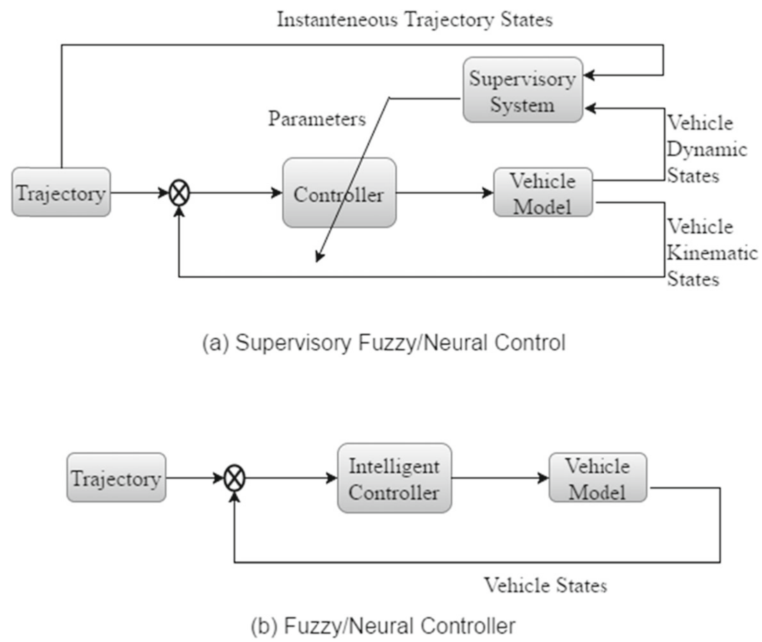

Fig. 13 Control structure of **(a)** Supervisory intelligent control and **(b)** Intelligent controller

the PID controller used to improve the performance of the trajectory tracking in presence of disturbances and unmodelled friction. In this approach, the learning rate for neural network was based on instantaneous minimization algorithm as outlined in Gomi, Kawato [95]. Benzaoui et al. [96] proposed a Fuzzy Inference System to estimate unknown model parameters for path tracking controller specific to obstacle avoidance applications. Stability of the controller was further validated using Lyapunov theory. Other than these, there are many more previous publications detailing on fuzzy supervisory control applied in path tracking controller [12, 15, 50, 63, 97–99]. Granted that most of these controller were developed for mobile robot, with slight modifications, the same strategy will be suitable to be used in a car-like vehicles.

Although adaptive/intelligent controllers may promise a robust solution, the process of designing and developing each controller can be exhaustive. In order for the overall controller, e.g. Neural Network or Fuzzy systems to be “experienced”, it should be equipped with “expertise” or “experiences” to make suitable decisions. The algorithms within the intelligent controllers need to be trained or programmed with these expertise. This algorithm may be programmed to continuously learning and gain more “expertise” which may need adequate computational power on-board.

3.5 Model Based Controllers

Model based controllers are sometimes regarded as adaptive controller. While the claim is valid, especially for controllers that employed model based predictions to estimate the ideal input to adapt to any changing vehicle conditions, model based controller can be regarded as one whole class. The application and advancement of model based controllers have been an active research interests especially due to the increase availability of portable super computers that can handle complex algorithms and computational power requirements. Model predictive controller (MPC) usually used a linear or nonlinear plant model to predict the required control input for the plant. This method usually involves optimisation procedure in order to get the optimal value for the plant input. Development of MPC for path tracking control can be found in previous publications. Perhaps the most popular model predictive control (MPC) approach was Ollero, Amidi [100] which proposed a generalized predictive controller that was intended to minimize a cost index containing errors between predicted steering angle and the desired steering angle. Heuristic method was used to solve the optimisation problem. This method was implemented by the same author later [50, 63]. In Falcone et al. [32], model predictive control was implemented to predict an optimal steering input for the vehicle to follow desired path in obstacle avoidance task using dSPACE rapid prototyping module. Both nonlinear MPC and linear-time-varying (LTV) MPC for approximation of the vehicle states were implemented on a vehicle platform and states optimisation was carried out with heuristic method. The nonlinear MPC used a nonlinear optimisation algorithm for minimization of a quadratic performance index while the LTV MPC used time varying convex quadratic optimization problem for the same task. One can see that the implementation of MPC controller will require high computational resources especially in solving the optimisation problem in real time. Even though one can reduce the horizon or calculation time steps to reduce the computational demands, but this may compromise the accuracy and the reliability of MPC to predict the optimal steering input for vehicle. Nevertheless, MPC can be seen implemented in many applications. Bayar et al. [25, 53] implemented MPC in autonomous vehicles for orchard environment, while Tomatsu et al. [101] implemented MPC for path tracking on an excavator in digging operation. More applications can be seen in previous studies [22, 102, 103].

In order to reduce the computational cost, Beal [104] implemented model predictive control using a custom C-Code that can solve the optimisation and MPC implementation in under 2ms. Also, metaheuristic algorithms have been chosen in several studies to solve the MPC optimisation problem. This can be seen in two recent studies [105, 106] which are using metaheuristic optimisation method to carry out the optimisation process in real time. Merabti et al. [105] presented three kinds of metaheuristic optimisation algorithms to solve the optimisation of the nonlinear MPC proposed for mobile robot path tracking control. They were Ant Colony Optimisation (ACO), Gravitational Search Algorithm (GSA), and Particle Swarm Optimisation (PSO). These three algorithms were implemented to optimise the nonlinear MPC control input and the performances were compared against each other. It was proven that PSO was the best algorithm with least computational time and faster convergence to optimum solution. However, selection method for the algorithms were unclear. The difference in number of iterations (25, 50, and 1000 for PSO, ACO, and GSA respectively) can be a good indicator to the computational times of the algorithm. Nevertheless, an efficient online metaheuristic algorithm might be a solution to the computational demand for MPC due to its simple algorithms.

3.6 Classical Controllers

In this study, the classification of classical controllers are applied to PID controllers and sliding mode controller (SMC). PID controller is a common controller usually used in industrial applications due to its simple design and theory. It consists of three terms and usually triggered by the error between actual response and a desired response. The three terms are Proportional, P, Integral, I and Derivative, D, which corresponds to the action that each term applied to the triggering error, respectively. In trajectory tracking, it usually used to control the steering input as well as velocity input depending on a desired steering and velocity received from trajectory planner. In Hoffmann et al. [26], main study was on the lateral control which determine the steering input for vehicle using Stanley geometric controller. However, the study also described the longitudinal control which determine the throttle level and brake cylinder pressure input using a PI controller. The controller was triggered by the error between desired velocity and actual velocity of the vehicle. The controller was claimed to be vulnerable to chattering and dead bands, which was solved by employing switching method as guided in Gerdes, Hedrick [107] and [108]. Similar approach in controlling vehicle speed was proposed in several studies recently [29, 84, 109].

Park et al. [66] applied PID controller to the steering actuator control to generate the correct steering angle as provided by the steering controller. A deadband compensator was employed to overcome the dead band and improve tracking performance and controller stability. Ping et al. [19] and Amer et al. [33] employed PID controllers in both of their controller loops for steering control. While PID controllers are particularly easier to implement due to its simplicity, tuning of its parameters can be very difficult. For a fast varying system such as vehicles, one set of parameters may be suitable within a certain range of operating condition and any operating condition outside of this range will need further tuning of the controller. This lack of robustness was usually handled by an adaptive law which will auto-tune the controller parameters based on the changing conditions, usually called adaptive PID controller. This approach can be seen in several studies, e.g. [19, 88].

Sliding mode controller (SMC) is another classical controller reviewed in this study. In SMC, the state feedbacks as well as the control signals are treated as discontinuous functions which make it unaffected to parametric uncertainties and external disturbances [110]. This is the main reason why this type of controller is suitable for nonlinear systems. The name "Sliding Mode" refers to the motion of the system as it slides along the boundaries (called sliding surfaces) of the control structure. The control law utilizes fast switching strategy in order to drive and maintain the system's state trajectory towards the chosen sliding surfaces [111] as depicted in Fig. 14. With this feature, SMC offers a nonlinear controller with fast response and excellent robustness under system uncertainties and external disturbances. However, due to fast switching solution offered by this controller, chattering in control signal is inevitable. With the delay and imperfections in physical actuators, it may lead to serious actuator and plant damages, energy losses and unwanted disturbance due to these chattering [112].

Sliding mode controllers (SMC) have been successfully employed in many researches on path tracking control for mobile robots as well as vehicle platforms [11, 17, 23, 113–119]. Most of these studies chose tracking error as formulated in Eq. 21 to be the error metric in the controller design. Chattering issues were addressed in the majority part of these studies such as: Wang et al. [17] proposed saturation function in the control law and Guo et al. [23] proposed an SMC with neural network based switching algorithm to reduce the controller's chattering. Also, Higher Order SMC (HOSMC) was proposed by Aithal, Janardhanan [114]

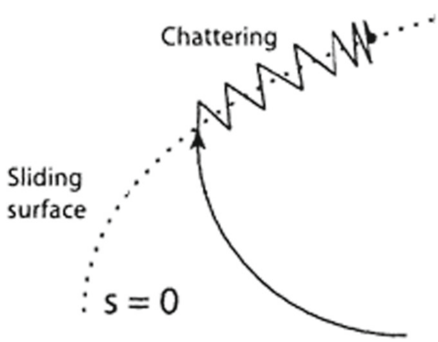

Fig. 14 Sliding surface and effect of chattering in Sliding Mode Controllers

and Solea, Cernega [113] so that the controller will act on the higher order time derivatives of the error metric instead of the first order derivative as in conventional SMC.

3.7 Summary on Path Tracking Controllers

The main scope of this section is to review the control strategies for path tracking control. Several types of controllers that are usually used for path tracking control were presented. Although it can be argued that the main contributor for an efficient path tracking control is the ability of the vehicle to determine the best path in path planning module; longitudinal and lateral control of the vehicle during this motion is particularly important in order to ensure the efficiency of steering, throttle and braking action within the vehicle. This will avoid any error in vehicle direction with respect to the designed path which is the main aim of a path tracking controller. Each of the approaches possesses its own advantages and disadvantages. The selection of control strategies are based on the requirement of the system and properties of the state variables that need to be controlled. Summary of controller types as reviewed in this section is as tabulated in Table 2.

4 Performance Criteria for Path Tracking Controller

In any control strategy development, performance evaluation usually is the final and most important stage. In development of path tracking controller, it is crucial to evaluate the controller's ability to guide the vehicle along a pre-defined path. Performance evaluation can be found in all of the studies on path tracking control. However, performance quantification method differs from work to work. In this section, the performance quantification methods which cover any method used to measure the controller's performance will be reviewed.

Different paths have been used for controller's evaluation, ranging from as simple as straight road, standard lane change, to a sharp curve. As reviewed before, path tracking controller will provide the correctional steering input or throttle/braking input in order to control the lateral and longitudinal direction of the vehicle relative to a desired path. The path may be generated by a path planner or pre-defined

Table 2 Summary of different type of controller in path tracking control

Controller Types	Strength	Weakness	Comments
Geometric & Kinematic	Relatively the simplest type of controller due to least complex state variables	Does not regard the dynamics of the vehicle in controller	Most suitable in applications where dynamics of the vehicle can be ignored
		Controller parameters can be over-tuned and path dependent	
Dynamic	Dynamic effect of the vehicle is included in control law	Attainment of vehicle's dynamic states (e.g. Forces, torques) may not be straightforward in experiment.	Most suitable in applications where dynamics of the vehicle is crucial to control and the computational power on-board is sufficient to obtain and process dynamic states
Optimal	Certain optimal controller (e.g. LQR) carry out offline gain optimisation →simple online controller	Some optimal control may require online optimisation (e.g. Sharp et al. [80]) and can be computationally demanding	Suitable for robust applications
		Development of the controller is based on linear assumptions. May limit controller's ability	
Adaptive	Adapt to various operating conditions making the controller robust to changes	Robustness may be catered only for a specific condition	
	If intelligence algorithms (e.g. NN or fuzzy) is included, will simplify controller and need less computational effort	Some adaptive algorithms will be complex and hence, increasing the computational cost of the overall approach	
Model Based	Consider the overall vehicle model in determining control signals	Involve online optimisation problem within the controller. Heuristics method will require a computational resources	
	Adapt to changes in vehicle parameters making the controller more robust		
Classical	Established method in control field especially good in nonlinear system control.	May need complex derivations and selections (e.g. SMC and H infinity)	
	Solution to common problems can be found in standard literature		

at the beginning of the study. Jung-Min, Jong-Hwan [119] used straight road while Snider [14] and Hima et al. [29] used standard double lane change road course to test their controller. Although some may say that straight roads lack the challenge to the controller due to the absence of multiple curvature, this kind of path may serve the purpose of testing the controller's stability in keeping the vehicle on track.

Curved roads were used as testing environment in several studies with different approaches in introducing curvatures in the path to be tracked by the path tracking controller. Continuous curves can be defined in a circular shaped paths [14, 22, 28, 70, 74, 78, 86, 115] with Snider [14] introduced "Fig. 8" (or infinity) path by combining two separate circles. Ollero et al. [63] introduced very difficult path with multiple sharp curves and proved that its controller with Fuzzy supervisory system was capable to manoeuvre the path. Also, Snider [14], Thrun et al. [67] and Hoffmann et al. [26] used road course with multiple sharp curves as one of the paths to evaluate controllers in their technical reports while Sun et al. [70] used sinusoidal roads and large turn roads with different width.

In evaluating the proposed controller, most of the studies used observation method by graphically comparing the actual vehicle trajectory against the desired trajectory as given by path planner or pre-defined earlier [5, 19, 39, 113, 115]. However, this observation method may not provide the full insights into the controller's performance, especially if the defined path covers large range (around 500m) which will make the relatively smaller tracking errors to be invisible. Therefore, several quantification methods were used to evaluate the controller's performance. One of the most common quantification methods is *Tracking Error*. This is usually described as the difference between the desired path and the actual vehicle position. Generally, tracking error is determined as the distance from the control point to the point on path. However, the real definition of tracking error may differ depending on the definition of point on path as well as the location of control point. The location of control point can be located on the centre of front tire axle, centre of rear tire axle, and centre of gravity (COG) of the vehicle. Point on path may be defined by the look-ahead distance or nearest point from control point. One of the most common tracking error formula is stated in Eq. 23 with lateral (y_e), longitudinal (x_e), and orientation errors (θ_e) relative to the local coordinates were formulated in terms of the distance between rear axle control point and a point of path within one look-ahead away as shown in Fig. 15a. The same formula can be used with different location of control point. Stanley Method by Stanford University [26] used only lateral error in their formulation as shown in Fig. 11. Here, the control point situated on the front tire axle and the error was defined as the shortest perpendicular distance between the control point and the point on path.

$$\begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\psi_c & \sin\psi_c & 0 \\ -\sin\psi_c & \cos\psi_c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_d - X_c \\ Y_d - Y_c \\ \psi_d - \psi_c \end{bmatrix} \tag{23}$$

Basic tracking error in Eq. 23 has been used widely to measure the controller performance throughout the path tracking manoeuvre [23, 53, 66, 72, 86, 87, 94, 99, 114]. Based on these errors, few derivative

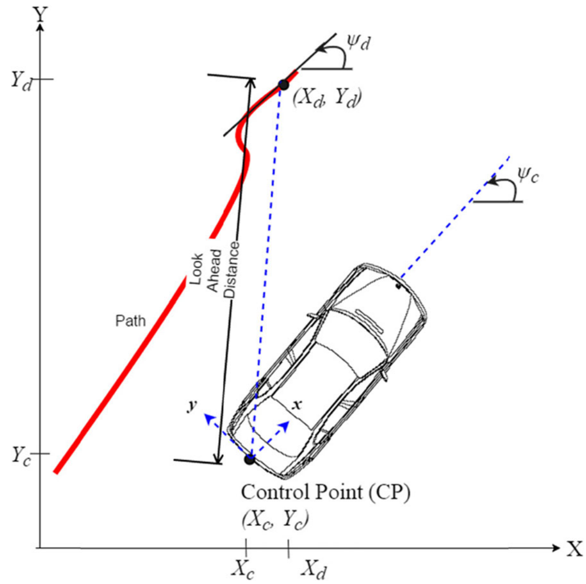

Fig. 15 Parameters to determine tracking error

quantification methods have been formulated. An absolute error, $|e|$ was used by Scaglia et al. [74] as stated in Eq. 24. Elbanhawi et al. [21] used mean tracking error in both lateral and longitudinal directions, as stated in Eq. 25. Here, N is the total number of points considered along the path. Meanwhile, Tzafestas et al. [120] compared the GPS and odometry reading to get the instantaneous error for the path tracking motion. Problem of different sampling time between the responses was solved by plotting the error against normalized length of the path. Shan et al. [12] added another term in which the total cross-track error, E_{cte} is the sum of tracking error between the vehicle path and desired path, $Error_{tracking}$ and error between vehicle path and the fitted clothoid line, $Error_{calculate}$ as shown in Eq. 26. Representation of these parameters are illustrated in Fig. 16. Another interesting approaches were used by Khatib et al. [88] that introduced further evaluation on tracking error, e with three performance indices, S,namely Integral of square of the error (S_{ISE}), Integral of absolute magnitude of the error (S_{IAE}), and Integral of time multiplied by the absolute value of the error (S_{ITAE}) as shown in Eqs. 27, 28, 29.

$$|e| = \sqrt{e_x^2 + e_y^2} \tag{24}$$

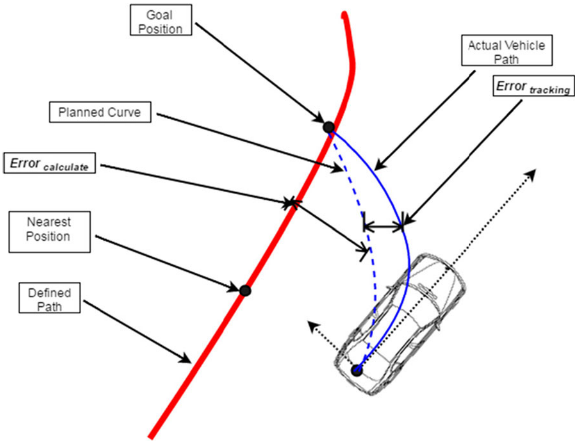

Fig. 16 Cross track error as defined in Shan et al. [12]

$$E_{(x,y)} = \frac{1}{N}\sum_{i=0}^{N}\sqrt{e_x^2 + e_y^2} \tag{25}$$

$$Error_{cte} = Error_{tracking} + Error_{calculate} \tag{26}$$

$$S_{ISE} = \int_0^\infty (e(t))^2 \, dt \tag{27}$$

$$S_{IAE} = \int_0^\infty |e(t)| \, dt \tag{28}$$

$$S_{ITAE} = \int_0^\infty t\,|e(t)| \, dt \tag{29}$$

Upon proposing control strategy, researchers usually evaluate the stability of the controller using Lyapunov theory. This will ensure the stability of the controller and operating envelope of stable region for the controller. Controller stability evaluation and determination of stable region can be seen in many notable studies previously [27, 51, 53, 69, 84, 85, 88, 104]. Another performance criteria can be seen in Elbanhawi et al. [21] that used three criterions to evaluate their proposed controller. First one was stated in Eq. 25 while another two were controller effort and steady state response. Controller effort measured the effort of the controller in correcting the vehicle direction based on correctional steering angle δ_i generated by the controller over Nnumber of points along the path as shown in Eq. 30. Steady state responses were observed in the same study to observe the stability of vehicle under external disturbances.

$$\text{Controller Effort, } E_\delta = \frac{1}{N}\sum_{i=0}^{N}\sqrt{\delta_i^2} \tag{30}$$

5 Research Challenges in Path Tracking Control

Referring to previous studies conducted in proposing solutions to path tracking control for an autonomous ground vehicles, one can see that critical areas are as reviewed in this paper namely; the choice of vehicle model in developing the control strategies; design of controller strategies in guiding the vehicle to follow a pre-defined path; and performance criteria used to evaluate the developed controller. Few challenges were gathered in each of these areas together with the prospect of research which will be outlined herewith according to the different stages in path tracking control, namely vehicle modelling, controller development, and performance evaluation. Finally, overall challenges in path tracking control is presented in final subsection.

5.1 Vehicle Model

It can be seen that most of the previous studies were using linear geometric or kinematic models to represent the vehicle model behaviour. In these models, the dynamic effect from surrounding such as friction forces and tire slips were assumed negligible. Some studies may use dynamic model but excluding the nonlinear tire forces this may still not be enough since the tire effects can be attributed to the main source of forces acting on the vehicle in both lateral and longitudinal direction, especially in higher speed region. Effects of internal/external disturbances such as engine torques, brake torques, drivetrain interactions and drag forces in both lateral and longitudinal directions were rarely included in simulating the vehicle behaviour in handling studies. This type of model may possess a high level of complexity but the importance of acknowledging and considering these effects in the controller's development stage cannot be underestimated. Therefore, while it has been proven that a simplified model was sufficient to simulate vehicle behaviours, developing a nonlinear vehicle model considering realistic tyre model and external/internal disturbances are surely a challenge especially in controller development stage.

5.2 Controller Development

5.2.1 Route/Path Definition

Most of control strategies presented here only support continuous paths especially the geometric type controllers. This may not be a problem by assuming that the motion planner can give continuous path for the vehicle to follow as stated by Snider [14] and the state of the art in path planning can be relied upon as reviewed in detail in Katrakazas et al. [7]. However, developing a controller that will not depend solely on the ability of the motion planner to provide smooth path is surely a legit challenge in this field.

5.2.2 Robustness of controller to navigate various paths.

This is one of the important challenges in controller development aspect. The controller should be able to navigate sharp turns and wide range of roads fed by the motion planner. Efficient and simple controllers have been proposed previously especially among the geometrical type of controllers. However, its simplicity might be causing the controller to ignore most effects and these effects were compensated in the selection of controller parameters. Thus, the simple controllers are highly dependable on variation of controller parameters and making it parameter-dependent. On the other hand, model based adaptive controllers offered good adaptability to different road manoeuvring conditions. However, these intelligent controllers were always associated with high computational cost. Therefore, an adaptive geometric controller may pose a good prospect of research which may offer an adaptive solution that is both simple and require small computational power.

5.2.3 Adaptive Controllers

There were a lot of adaptive controllers proposed throughout the years and each of them were proven to guide the vehicle to follow the defined path. The inclusion of intelligent algorithm in parameter tuning will make the controller more robust and stable under different conditions. However, the robustness will be catered only for a specific condition. For example, a controller that is designed to be robust to the variation in vehicle's behaviour may not be able to cater the unexpected disturbances, or unmodelled frictions. Also, some adaptive algorithm will have an associated complexity to it and hence, increasing the computational cost of the overall approach. Development of an adaptive controller that can cater speed variations, angle of turn variations, unmodelled frictions, and slippery road should be an interesting prospect of research. Also, development of an efficient model based controller is still an active research area since model based controller may be constrained depending on the availability of a reliable computational power. However, in developing an efficient and suitable adaptive controller, one may have to take extra consideration for the controller's stability to ensure that the controller not only can adapt to various surroundings, but it may also will not behave erratically with fluctuating performance.

5.3 Implementation of Path Tracking Control

5.3.1 Sensing and Perception

In order to realize the actual position of vehicle, real-time data is always important to gather. While a lot of sensor based implementations have been proposed [28, 51, 121], sensor configurations and perception methods for the raw data are the ever expanding research areas.

5.3.2 Testing

Among the previous works, testing and validation of the proposed controller usually carried out through simulations, hardware-in-the-loop (HIL) simulations and real application on actual vehicle. Controller testing always carried out through simulation with a validated vehicle model. System actuators were usually modelled mathematically to depict the behaviours. This approach was chosen due to its high repeatability and significantly reducing cost and time consumption that is always associated in real-time testing. Also, it will avoid any safety risk in case the controller failed to guide the vehicle. However, in all controller design studies, real application testing should be aimed. The main challenge is to implement the controller on an actual vehicle. It will enable the controller to be tested to handle algorithms and tasks even during variable weather conditions such as rain, snow, and cloudy. Also, the controller would be tested to handle different kind of roads, such as highways, intersections, and bumpy roads. While simulations are beneficial in terms of repeatability of the experiment, and real-time testing will expose the vehicle (and controller) to real situations, HIL simulations offer compromised solution between these approaches with great repeatability and added considerations of the physical limit for the system actuators. Unexpected dynamics, delays and unmodelled frictions exist in physical actuators are compromised by including a physical actuator within the testing setups. Vehicle behaviours may still be provided by the simulated model. Therefore, hardware-in-the-loop simulations offer the best solution in order to provide good repeatability, ensured safety, and real implementations on physical system.

5.3.3 Performance Criteria

A lot of methods have been utilized to evaluate the controller's ability in guiding the vehicle to follow the desired path. Most studies evaluate their controllers based on tracking errors and several other methods to further quantify theses errors. Since computational requirement play an important criteria in controller development, a method to quantify the computational cost required is yet to be used. Several aspects need to be considered such as sampling rate required and time required to calculate controller signals.

5.4 Overall challenges in Path Tracking Control

Overall, path tracking control focus mainly on guiding the vehicle on a pre-defined path usually given by the on-board path planner or motion planner. Main challenge in this field of study is the ability of the controller to navigate different type of road curvature, with different speed and different road conditions. Integrated control should be developed in order to combine the steering, throttle/braking, and suspension controls for coordinated control in lateral, longitudinal and vertical direction while navigating through various road conditions. In terms of lateral and longitudinal controls, an adaptive solution may offer good progress with a basic geometric controller. This will combine the simplicity of the geometric controller with the robustness aspect that can be offered by an adaptive controller approach.

6 Conclusions

It has been three decades since the first successful implementations of autonomous vehicle in 1980's, which have seen rapid improvement in autonomous vehicle technology. Initiatives from certain agencies (e.g. DARPA) have further fuelled the advancement of this field in the last decade. With driverless vehicles entering market that is currently growing at least 16 % annually [122], autonomous vehicles field of research is expected to grow exponentially.

This paper has reviewed the current state of the technology in path tracking control for an autonomous ground vehicles which covers the vehicle modelling, development of control strategy usually used for direction control and performance criteria used to evaluate any proposed controller. Upon reviewing, it was identified that the vehicle model used to represent the vehicle behaviours mostly neglected the dynamic effect and nonlinearity of the tyre responses. In terms of controller development, most controllers depend on kinematic states of the vehicle while few notable controllers can be seen to include dynamic states in the control law. It can be seen that among the controllers reviewed in this study, geometric type controllers are the most popular and commonly used approach due to its simple configurations and implementation. However, this requires specific tuning with detailed vehicle model to simulate vehicle response while tuning which makes the controller less robust. An efficient nonlinear controller such as MPC and adaptive controller can be a good alternative but come with an associated complexity in algorithms which may raise the computational needs. Since the ability to navigate different road conditions with minimum computational cost is the main challenge here, the development of an adaptive and intelligent metaheuristic geometric controller has been proposed. In terms of performance criteria, tracking errors were the most common characteristic to evaluate the controller's performance. Determination of tracking error may differ from study to study depending on the location of control point and control method used. Finally, it is hoped that this review may provide sufficient introduction and information on the field of trajectory tracking control for autonomous ground vehicles.

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