BITS Pilani
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Introduction to Statistical Methods

Team ISM

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BITS Pilani

Overview of the course & Basic of Statistics (CS -1)

23rd and 24th November 2024

IMP Note to Self

Overview of the course

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- M 1 : Basic Probability & Statistics
- M 2 : Conditional Probability & Bayes' theorem
- M 3 : Probability Distributions
- M 4 : Hypothesis Testing
- M 5 : Prediction & Forecasting
- M 6 : Prediction & Forecasting Gaussian Mixture model & Expectation Maximization

TEXT BOOKS

T1 : Statistics for Data Scientists, An introduction to Probability, Statistics and Data Analysis, Maurits Kaptein et al,Springer 2022

T2 : Probability and Statistics for Engineering and Sciences, 8th Edition, Jay L Devore, Cengage Learning

T3 : Introduction to Time Series and Forecasting, Second Edition, Peter J Brockwell, Richard A Davis, Springer.

Evaluation Components

No	Name	Type	Weight
EC-1(a)	Quizzes – 1 & 2	Online	10%
EC-1(b)	Assignments – 1 & 2	Online	20%
EC-2	Mid-Semester Test	Closed Book	30%
EC-3	Comprehensive Exam	Open Book	40%

Module 1: (Basic Probability & Statistics)

Contact Session	List of Topic Title	Reference
CS - 1	Measures of Central Tendency & Measures of Variability, Data – Symmetric & Asymmetric, outlier detection, 5 point summary, Introduction to probability(Session-2)	T1 & T2

"Statistical thinking will be one day as necessary for efficient citizenship as the ability to read and write"

H G Wells

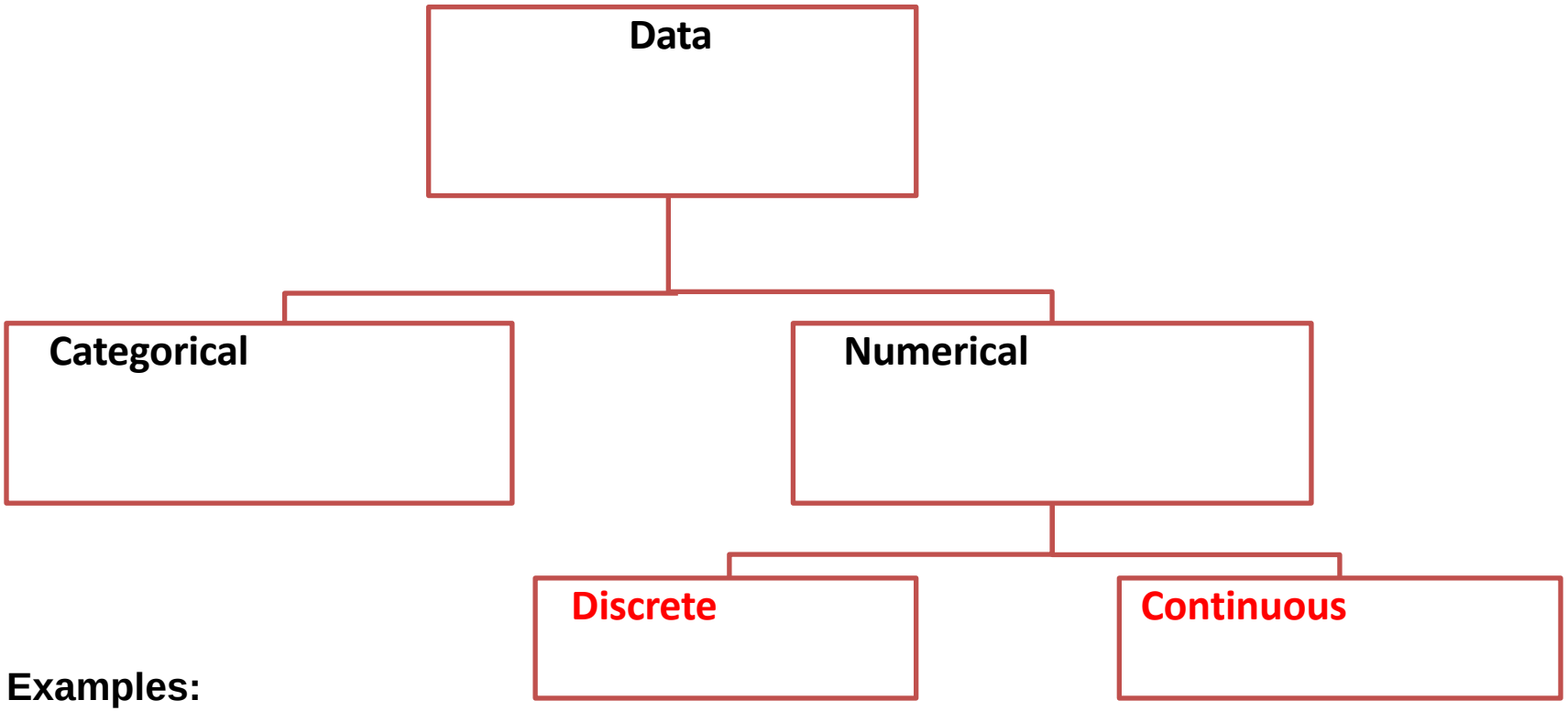
Statistics

Statistics may be defined as science that is employed to

- Collect the data
- Present and organize the data in a systematic manner
- Analyse the data
- Infer about the data
- Take decision from the data.

In other words, Statistics can also be defined as numerical data with a view to analyse it.

Types of Variables



Examples:
Marital Status
Political Party
Eye Color
(Defined categories)

Examples:

- **Number of Children**
- **Defects per hour**

(Counted items)

Examples:

- **Weight**
- **Voltage**

(Measured characteristics)

Levels of Data Measurement

- Nominal — Lowest level of measurement
- Ordinal
- Interval
- Ratio — Highest level of measurement

Nominal

- A **nominal scale** classifies data into distinct categories in which no ranking is implied
- Example : Gender, Marital Status, voting to different party

Ordinal scale

- An **ordinal scale** classifies data into distinct categories in which ranking is implied

- Example:

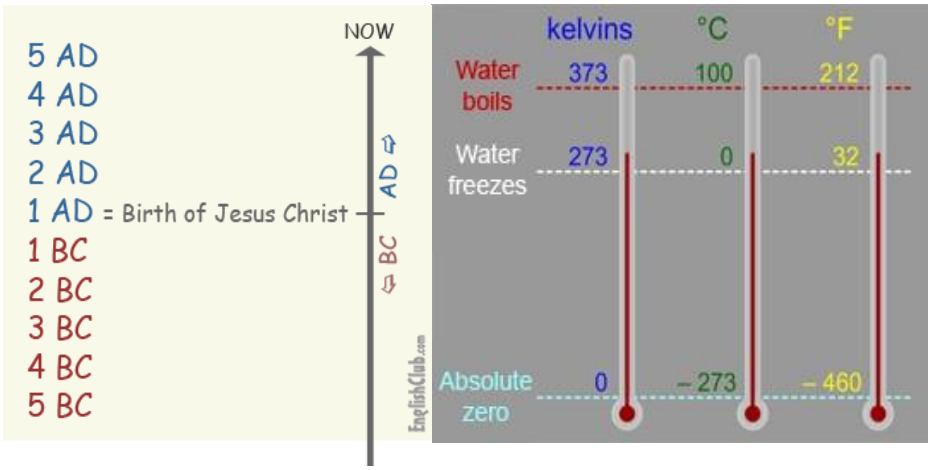
1. Product satisfaction: Satisfied, Neutral, Unsatisfied
2. Faculty rank: Professor, Associate Professor, Assistant Professor
3. Student Grades: A, B, C, D, F
4. Medals won: Gold, Silver, Bronze

Interval scale

- An **interval scale** is an ordered scale in which the difference between measurements is a meaningful quantity but the measurements do not have a true zero point.

- Example

1. Temperature in Fahrenheit and Celsius
2. Months of the Year: there's no month called zero and we can't say January is twice as much as June.



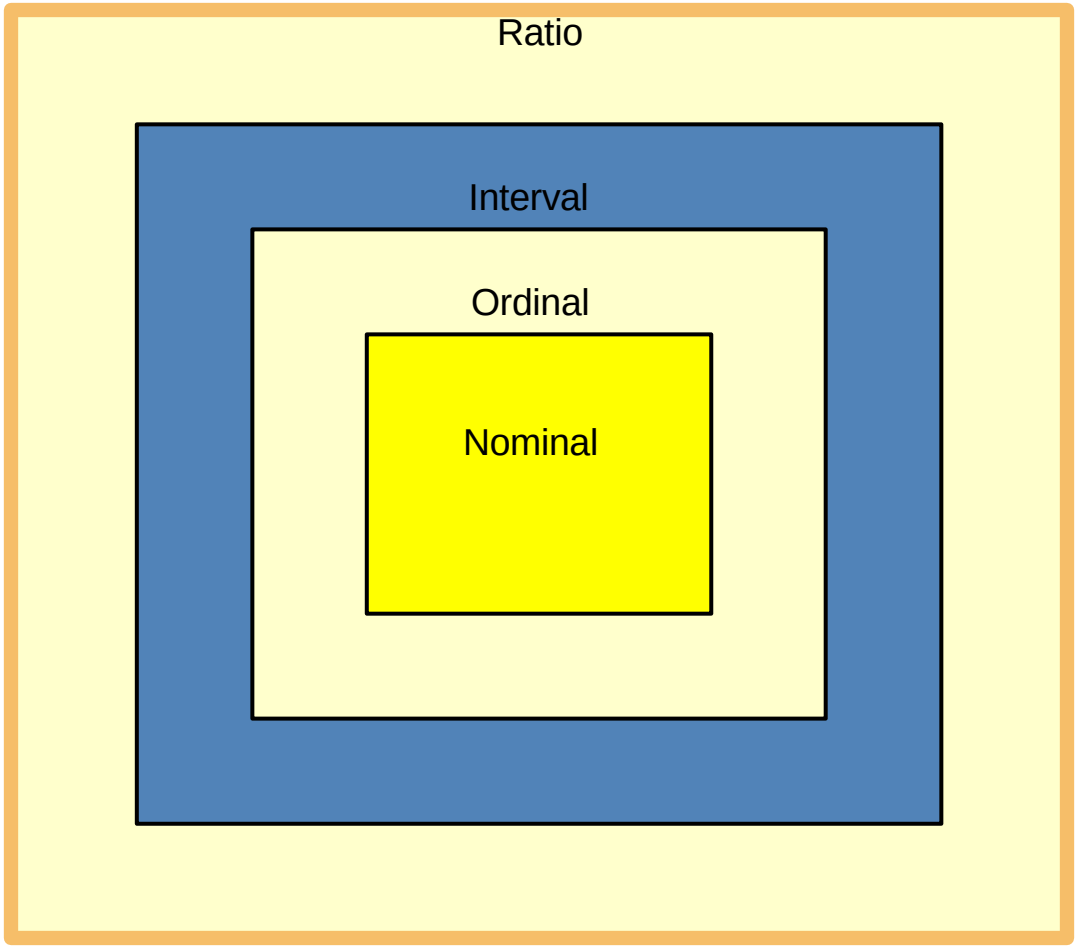
Ratio scale

- A **ratio scale** is an ordered scale in which the difference between the measurements is a meaningful quantity and the measurements have a true zero point.

- Example

1. Weight
2. Age
3. Salary

Usage Potential of Various Levels of Data



Impact of choice of measurement scale

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Data Level	Meaningful Operations	Statistical Methods
Nominal	Classifying and Counting	Nonparametric
Ordinal	All of the above plus Ranking	Nonparametric
Interval	All of the above plus Addition, Subtraction	Parametric
Ratio	All of the above plus multiplication and division	Parametric

Types of Variable

Qualitative (Categorical): express a qualitative attribute such as hair color, eye color, religion.

Quantitative(Numerical): measured in terms of numbers such as height, weight, number of people.

Nominal:
No ordering is possible such as hair color, eye color, religion.

Ordinal:
Ordering is possible such as health, which can take values such as poor, reasonable, good, or excellent.

Discrete:
countable and have a **finite number of possibilities such as** number of people

Continuous: not countable and have an **infinite number of possibilities** such as height

INTERVAL: ratio of values of variable do not have any meaning and it does not have an inherently defined zero value such as temperature

RATIO: ratio of values of variable have meaning and it have an inherently defined zero value such as length

Example

Question	Type of Variable
Types of vehicle owned Two wheeler, four wheeler	Categorical : Nominal
Product satisfaction Unsatisfied, neutral, fairly satisfied, satisfied	Categorical : Ordinal
To how many magazines do you currently subscribed Zero, One, Two, Three, Four	Discrete
How tall are you (in inches)	Continuous : Ratio
Weight (in Kilograms)	Continuous : Ratio
Temperature (in degrees Celsius or degrees Fahrenheit)	Continuous : Interval

Measures of Central Tendency

- Measure of central tendency provides a very convenient way of describing a set of scores with a single number that describes the **PERFORMANCE** of the group.

- Also defined as a single value that is used to describe the "**center**" of the data.

- Three commonly used measures of central tendency:
  1. Mean
  2. Median
  3. Mode

Mean

- Also referred as the "**arithmetic average**"
- The most commonly used measure of the center of data
- Numbers that describe what is average or typical of the distribution
- Computation of Sample Mean:

$$\bar{Y} = \frac{\sum Y}{N}$$ = Σ Y/ N = (Y1 + Y2 + Y3 + … Yn) / N where

"Y bar" equals the sum of all the scores, Y, divided by the number of scores, N.

- Computation of the Mean for grouped Data

$$\bar{Y} = \frac{\sum f Y}{N}$$ Where f Y = a score multiplied by its frequency

Mean: Ungrouped Data

Suppose you define the time to get ready as the time (rounded to the nearest minute) from when you get out of bed to when you leave your home. You collect the times shown below for 10 consecutive work days:

Day	1	2	3	4	5	6	7	8	9	10
Time (min)	39	29	43	52	39	44	40	31	44	35

The mean time is 39.6 minutes, computed as follows:

$$\overline{X} = \frac{\text{Sum of the values}}{\text{Number of values}}$$

$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

$$\overline{X} = \frac{39 + 29 + 43 + 52 + 39 + 44 + 40 + 31 + 44 + 35}{10}$$

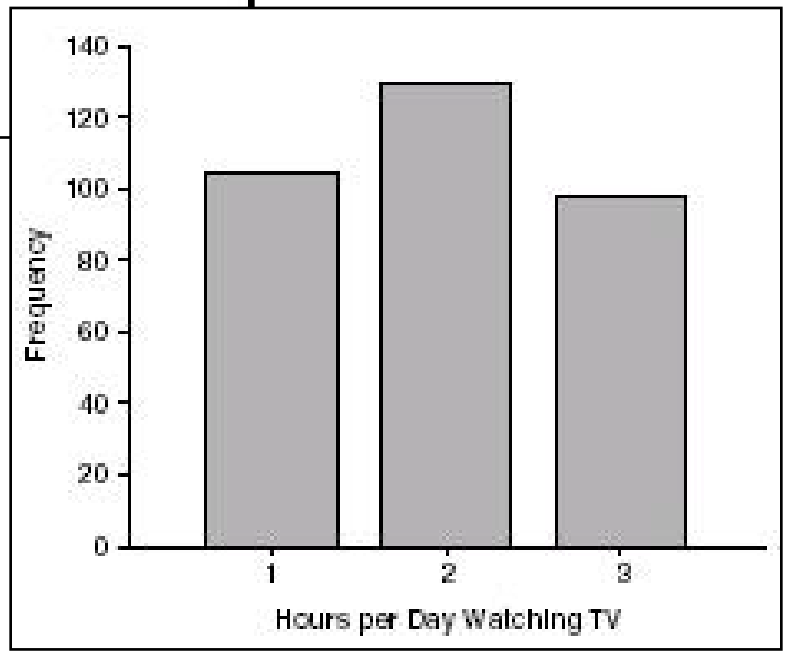
$$\overline{X} = \frac{396}{10} = 39.6$$

Mean: Grouped Scores

Hours Spent Watching TV	*Frequency (f)*	*fY*	*Percentage*	*C%*
1	104	104	31.3	31.3
2	130	260	39.2	70.5
3	98	294	29.5	100.0
Total	332	658	100.0	

$$\bar{Y} = \frac{\sum fY}{N} = \frac{658}{332} = 1.98$$

Data of Children watching TV in Bengaluru



Mean

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Properties

- It measures stability. Mean is the most stable among other measures of central tendency because every score contributes to the value of the mean.
- It may easily affected by the extreme scores.
- The sum of each score's distance from the mean is zero.
- It can be applied to interval level of measurement
- It may not be an actual score in the distribution
- It is very easy to compute.

Mean

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When to Use the Mean

- Other measures are to be computed such as standard deviation, coefficient of variation and skewness

- Sampling stability is desired.

The Mode

- The category or score with the largest frequency (or percentage) in the distribution.
- The mode can be calculated for variables with levels of measurement that are: nominal, ordinal, or interval-ratio.

Example: A systems manager in charge of a company s network keeps track of the number of server failures that occur in a day. Compute the mode for the following data, which represents the number of server failures in a day for the past two weeks:.

1	3	0	3	26	2	7	4	0
2	3	3	6	3				

Because 3 appears five times, more times than any other value, the mode is 3. Thus, the systems manager can say that the most common occurrence is having three server failures in a day.

Mode

Properties

- It can be used when the data are qualitative as well as quantitative.
- It may not be unique.
- It is not affected by extreme values.

When to Use the Mode

  - When the data set is measured on a nominal scale

The Median

- The score that divides the distribution into two equal parts, so that half the cases are above it and half below it.
- You compute the median value by following one of two rules:

  Rule 1 If there are an *odd* number of values in the data set, the median is the middle-ranked value.
  Rule 2 If there are an *even* number of values in the data set, then the median is the *average* of the two middle ranked values.
- The median is the middle score, or average of middle scores in a distribution.
  - Fifty percent (50%) lies below the median value and 50% lies above the median value.
  - It is also known as the middle score or the 50th percentile.

Median

Example-1: The three-year annualized returns for the seven small-cap growth funds with low risk are ranked from the smallest to the largest:

19.0 20.8 22.3 22.4 24.9 26.0 29.9

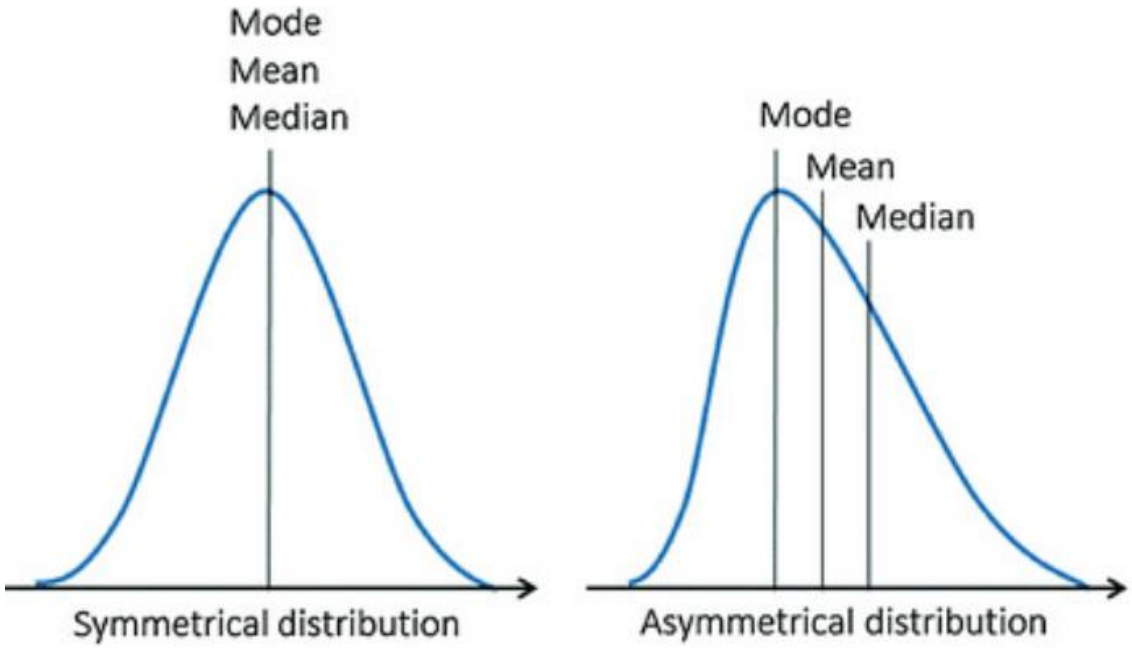
The Median is the 4th position value i.e. 22.4.

Example-2: Consider the time taken for reaching office from home in minutes arranged from lowest to highest

29 31 35 39 39 40 43 44 44 52

The number of observations are 10. Hence the median is the average of values at the 5th and 6th Position, i.e. average of 39 and 40 = **39.5**

Data : Symmetrical and Asymmetrical



- Central tendency Median is used for both Symmetrical and Asymmetrical data
- While Mean or Mode or Median can be used as central tendency for Symmetrical data.

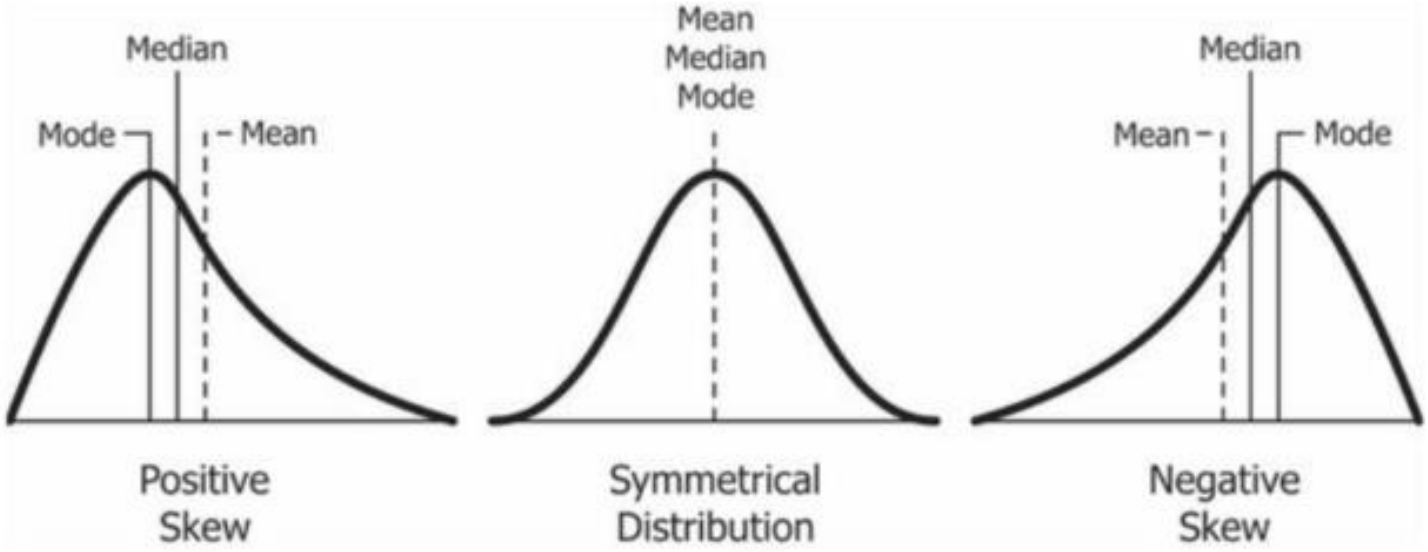
Shape of the distribution of data

- Symmetrical : Mean is equal to median
- Skewed
- Negatively (Left): mean < median

  Distributions with a left skew have long left tails;
- Positively (Right) : mean > median

  Distributions with a right skew have long right tails.
- Bimodal : has two distinct modes
- Multi-modal : has more than 2 distinct modes

Types of Distribution



Using Mode: $\frac{\bar{x} - \text{Mode}}{s}$

Using Median: $\frac{3(\bar{x} - \text{Median})}{s}$

Note: In science, an empirical relationship is a relationship that is supported by experiment and observation but not necessarily supported by theory. For moderately skewed data (Skewness between -0.5 to 0.5) empirical relationship is

3Medain= Mode+2Mean

Why mean is not enough?

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Sl. No.	X_1
1	2
2	8
3	5
4	3
5	7
6	8
7	5
8	2
9	5
Total	45

Statistical measures	Group 1
Mean	5
Median	5
Mode	5

Sl. No.	X_2
1	1
2	15
3	5
4	5
5	6
6	3
7	5
8	2
9	3
Total	45

Statistical measures	Group 2
Mean	5
Median	5
Mode	5

Sl. No.	X_1	X_2
1	2	1
2	8	15
3	5	5
4	3	5
5	7	6
6	8	3
7	5	5
8	2	2
9	5	3
Total	45	45

Statistical measures	Group 1 & 2
Mean	5
Median	5
Mode	5

Do we need any other measure?

Answer: Yes

Measures of variability

Three Measures of Variability:

- The Range
- The Variance
- The Standard Deviations

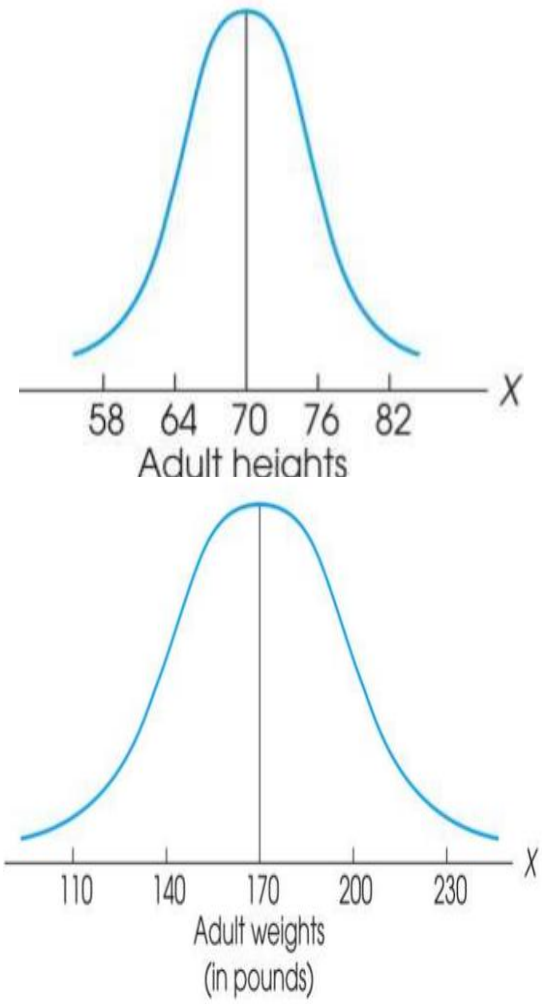
Measure of Variability

Variability can be defined several ways:

- A quantitative distance measure based on the differences between scores
- Describes distance of the spread of scores or distance of a score from the mean

Purposes of Measure of Variability:

- Describe the distribution
- Measure how well an individual score represents the distribution



The Ranges

- The distance covered by the scores in a distribution – From smallest value to highest value
- For continuous data, real limits are used

$$\text{Range} = \text{URL for } X_{max} - \text{ LRL for } X_{min}$$

- Based on two scores, not all the data – An imprecise, unreliable measure of variability

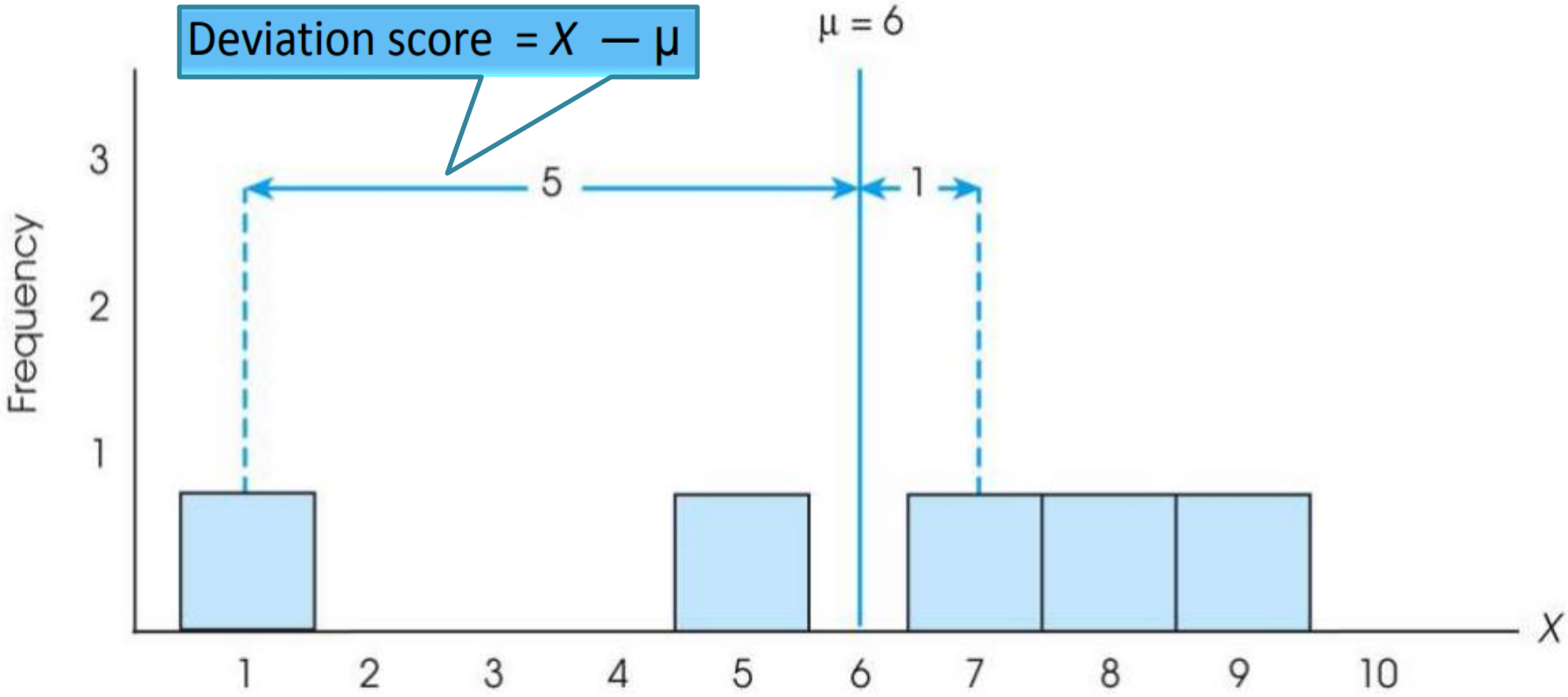
Example: For a set of scores: 7, 2, 7, 6, 5, 6, 2

Range = Highest Score minus Lowest score = 7 - 2 = 5

The Standard Deviation

- Most common and most important measure of variability is the standard deviation
    - A measure of the standard, or average, distance from the mean
    - Describes whether the scores are clustered closely around the mean or are widely scattered
- Calculation differs for population and samples
- Variance is a necessary *companion concept* to standard deviation but *not the same* concept

The Standard Deviation



Exercise : Find out the deviations of all the data points with the mean….and then find the ‘mean deviation’.

The Standard Deviation

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- Mean deviations will always be ‘zero’ !
  (because Mean is a balance point)

Then, how do you find ‘Standard Deviation’ ?

Need a new strategy

The Standard Deviation

New Strategy :

a) First square each deviation score
b) Then sum the Squared Deviations (SS)
c) Average the squared deviations

- Mean Squared Deviation is known as **"Variance"**
- Variability is now measured in squared units

$$Standard\ Deviation = \sqrt{Variance}$$

The Variance

Variance equals mean (average) squared deviation (distance) of the scores from the mean

$$\text{Variance} = \frac{\text{sum of squared deviations}}{\text{number of scores}}$$

where $SS = \sum (X - \mu)^2$

The Population Variance

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- Population variance equals mean (average) squared deviation (distance) of the scores from the population mean
- Variance is the average of squared deviations, so we identify population variance with a lowercase Greek letter sigma squared: σ^2
- Standard deviation is the square root of the variance, so we identify it with a lowercase Greek letter sigma: σ

Sl. No.	X_1	X_2
1	2	1
2	8	15
3	5	5
4	3	5
5	7	6
6	8	3
7	5	5
8	2	2
9	5	3
Total	45	45

Statistical measures	Group 1 & 2
Mean	5
Median	5
Mode	5

Sl. No.	X_1
1	2
2	8
3	5
4	3
5	7
6	8
7	5
8	2
9	5
Total	45

$$\overline{X} = \frac{\sum_{1}^{9} x_i}{N} = \frac{45}{9} = 5$$

$$S = \sqrt{\frac{\sum (X - \overline{X})^2}{n - 1}}$$

$$\mathbf{S = \sqrt{\frac{44}{8}} = 2.345}$$

Sl. No.	X_2
1	1
2	15
3	5
4	5
5	6
6	3
7	5
8	2
9	3
Total	45

$$\overline{X} = \frac{\sum_{1}^{9} x_i}{N} = \frac{45}{9} = 5$$

$$S = \sqrt{\frac{\sum (X - \overline{X})^2}{n-1}}$$

$$\mathbf{S = \sqrt{\frac{134}{8}} = 4.093}$$

Learning Check

a) If all the scores in a data set are the same, the Standard Deviation is equal to 1.00

True / False ?

Select the correct option

b) The standard deviation measures …

(1) Sum of squared deviation scores

(2) Standard distance of a score from the mean

(3) Average deviation of a score from the mean

(4) Sq. root of the average squared distance of a score from the mean

Solution

a) If all the scores in a data set are the same, they are equal to the mean and hence the deviation from mean = 0 therefore, Standard Deviation is equal to **zero**

False

b) The standard deviation measures …

(1) Sum of squared deviation scores

(2) Standard distance of a score from the mean

(3) Average deviation of a score from the mean

(4) Sq. root of the average squared distance of a score from the mean

Standard Deviation and Variance for a Sample

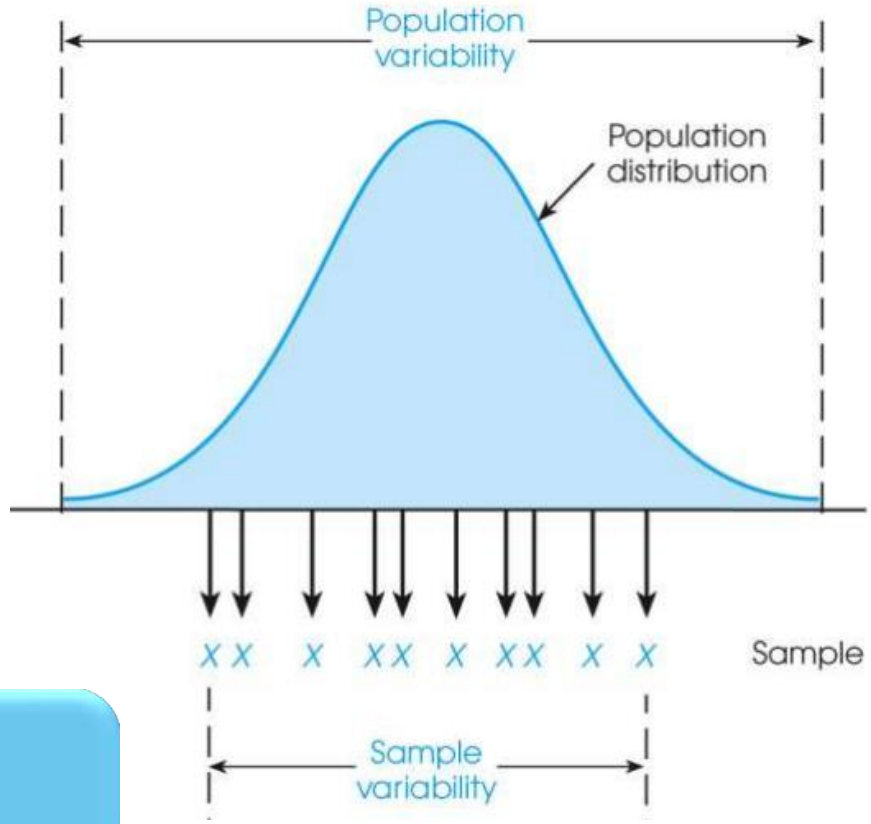
- Goal of inferential statistics:
  - Draw general conclusions about population based on limited information from a sample

- Samples differ from the population
  - Samples have less variability

  - Computing the Variance and Standard Deviation in the same way as for a population would give a biased estimate of the population values

Sample Standard Deviation and Variance

- Sum of Squares (SS) is computed as before
- Formula for Variance has n-1 rather than N in the denominator
- Notation uses s instead of σ



$$variance\ of\ sample = s^2 = \frac{SS}{n-1}$$

$$standard\ deviation\ of\ sample = s = \sqrt{\frac{SS}{n-1}}$$

Degrees of Freedom

- Population variance
  - Mean is known
  - Deviations are computed from a known mean

- Sample variance as estimate of population
  - Population mean is unknown
  - Using sample mean restricts variability

- Degrees of freedom
  - Number of scores in sample that are independent and free to vary
  - Degrees of freedom (df) = n – 1

Learning Check

Select the correct option

a) A sample of four scores has SS = 24. What is the variance?
   (1) The variance is 6
   (2) The variance is 7
   (3) The variance is 8
   (4) The variance is 12

b) A sample systematically has less variability than a population

True / False ?

c) The standard deviation is the distance from the Mean to the farthest point on the distribution curve

True / False ?

Solution

Select the correct option

a) A sample of four scores has SS = 24. What is the variance?

(1) The variance is 6
(2) The variance is 7
(3) The variance is 8
(4) The variance is 12

b) Extreme scores affect variability, but are less likely to be included in a sample

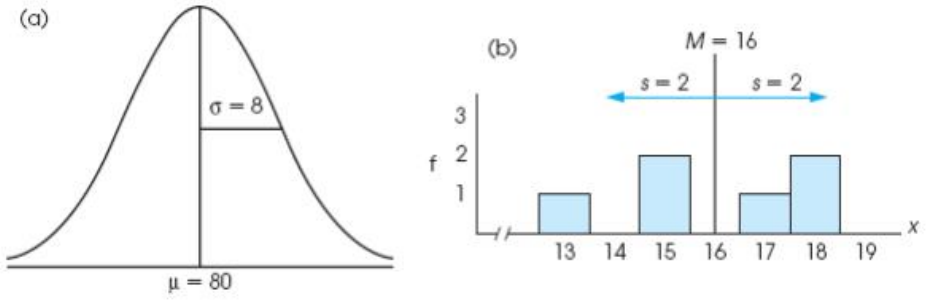
c) The standard deviation extends from the mean approximately halfway to the most extreme score

True

False

Descriptive Statistics

- A standard deviation describes scores in terms of distance from the mean
- Describe an entire distribution with just two numbers (M and s)
- Reference to both allows reconstruction of the measurement scale from just these two numbers
- Means and standard deviations together provide extremely useful descriptive statistics for characterizing distributions



Five point summary of Data

The five number summary of data includes 5 items:

- **Minimum**.
- **Q1** (the first quartile, or the 25% mark).
- **Median**.
- **Q3** (the third quartile, or the 75% mark).
- **Maximum**.

Interquartile range (IQR)

- It is measure of Variation
- Also Known as Mid-spread : Spread in the Middle 50%
- Difference Between Third & First Quartiles:
- Not Affected by Extreme Values

Interquartile Range = IQR = $Q_3 - Q_1$

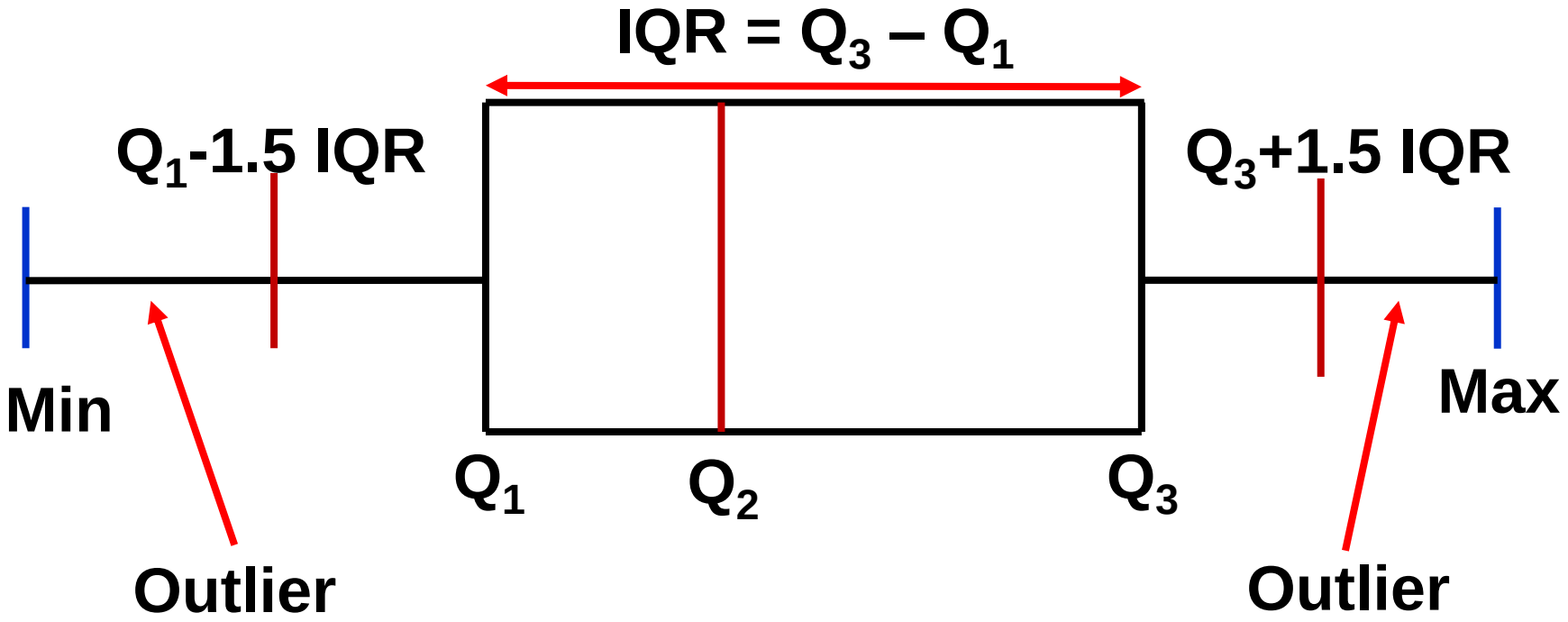
Data in Ordered Array: 11 12 13 16 16 17 17 18 21

Position of $Q_1 = \frac{1 \bullet (9 + 1)}{4} = 2.50$, $Q_1 = 12.5$

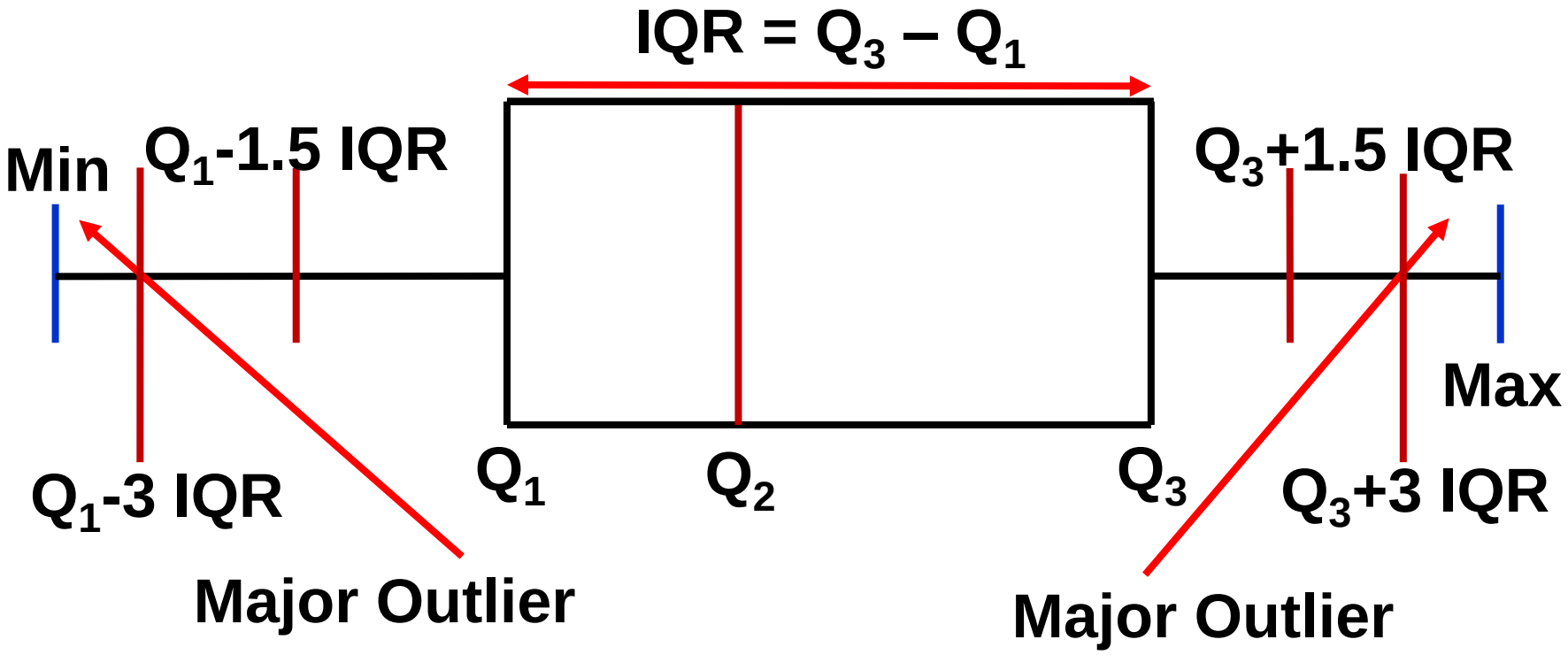
Position of $Q_3 = \frac{3 \bullet (9 + 1)}{4} = 7.50$, $Q_3 = 17.5$

Interquartile Range = IQR= $Q_3 - Q_1 = 17.5 - 12.5 = 5$

Box and Whisker plot



Box-and-Whisker plot



Potential outliers

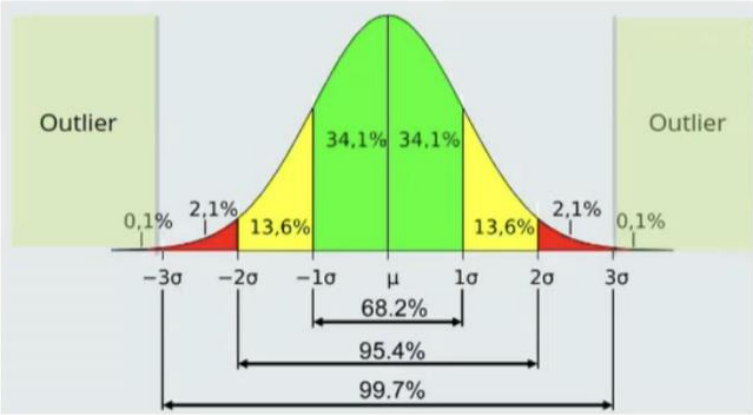
- The lower limit and upper limit of a data set are given by:

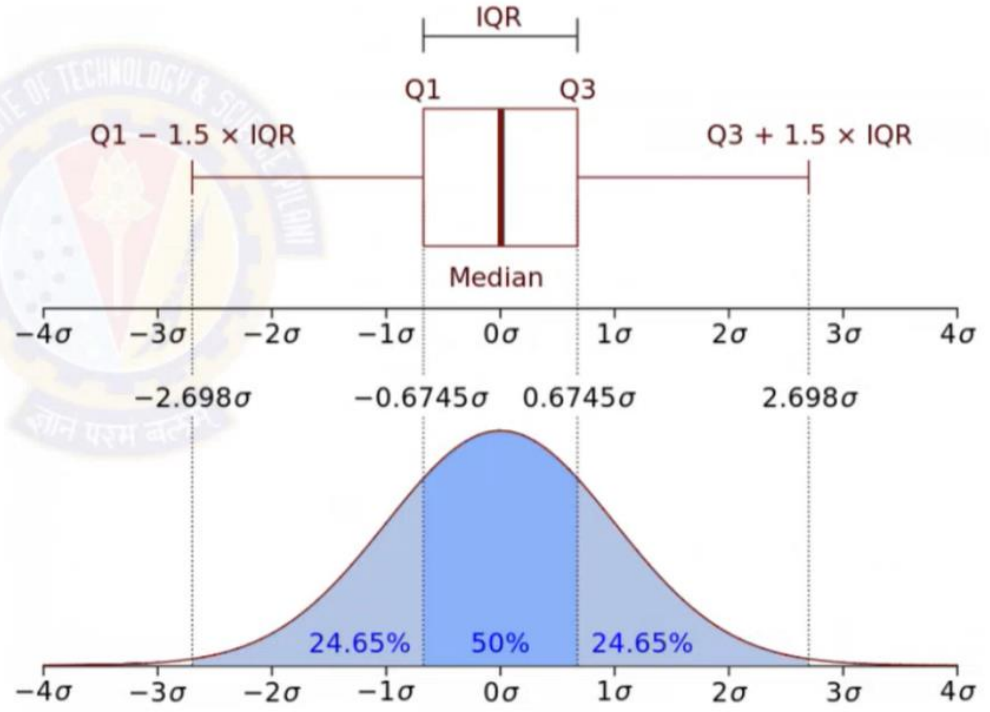
  Lower limit = Q_1 - 1.5 x IQR

  Upper limit = Q_3 + 1.5 x IQR

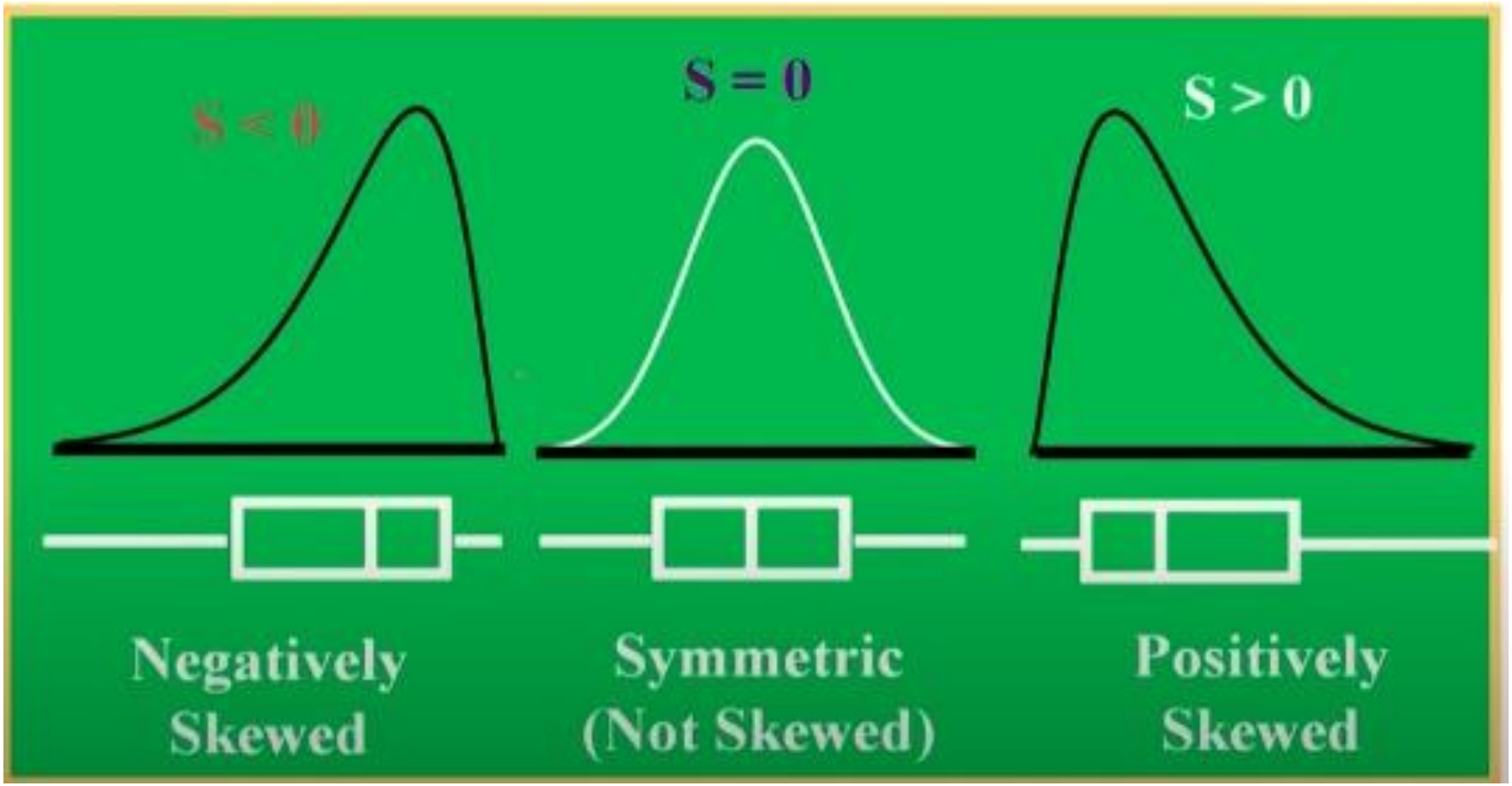
- Data points that lie below the lower limit or above the upper limit are potential outliers.

Outlier Detection





Box-and-Whisker plot: Skewness and coefficient of skewness



Glossary

Measures of Central Tendency:
Mean, Median, Mode

Measures of Variability: Range, Standard Deviation , Variance

Shape of the distribution

Five point summary

Outliers

Glossary

Measures of Central Tendency:
Mean, Median, Mode

Measures of Variability: Range, Standard Deviation , Variance

Shape of the distribution

Five point summary

Outliers

Practice Problems:

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Q.1 A sample of 77 individuals working at a particular office was selected and the noise level (dBA) experienced by everyone is the following data:

55.3, 55.3, 55.3, 55.9, 55.9, 55.9, 55.9, 56.1, 56.1, 56.1, 56.1, 56.1, 56.1, 56.8, 56.8, 57.0, 57.0, 57.0, 57.8, 57.8, 57.8, 57.9, 57.9, 57.9, 58.8, 58.8, 58.8, 59.8, 59.8, 59.8, 62.2, 62.2, 63.8, 63.8, 63.8, 63.9, 63.9, 63.9, 64.7, 64.7, 64.7, 65.1, 65.1, 65.1, 65.3, 65.3, 65.3, 65.3, 67.4, 67.4, 67.4, 67.4, 68.7, 68.7, 68.7, 68.7, 69.0, 70.4, 70.4, 71.2, 71.2, 71.2, 73.0, 73.0, 73.1, 73.1, 74.6, 74.6, 74.6, 74.6, 79.3, 79.3, 79.3, 79.3, 83.0, 83.0, 83.0.

Find a) Arithmetic Mean, SD, variance, and IQR

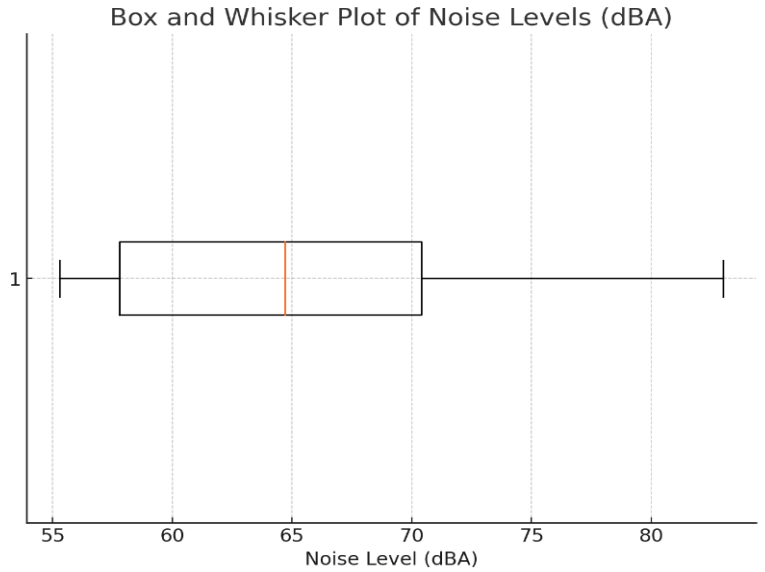
b) Draw box and whisker plot

c) Comment on the outliers, if any.

Solution:

c)

- Arithmetic Mean: 64.89 dBA
- Standard Deviation (SD): 7.80 dBA
- Variance: 60.88 $(dBA)^2$
- Interquartile Range (IQR): 12.60 dBA



- Lower Bound: $Q1 - 1.5 \times IQR = 59.8 - (1.5 \times 12.6) = 40.9$
- Upper Bound: $Q3 + 1.5 \times IQR = 72.4 + (1.5 \times 12.6) = 91.3$

No data points fall below 40.9, but values such as 79.3 and 83.0 are potential upper outliers, as they exceed 91.3.

Practice Problems:

Q.2 The data given below is the total fat, in grams per serving, for a sample of 20 chicken sandwiches from fast-food chains.

7 8 4 5 16 20 20 24 19 30 23 30 25 19 29 29 30 30 40 56

a. Compute the mean, median, first quartile, and third quartile.

b. Compute the variance, standard deviation, range, interquartile range, Are there any outliers? Explain.

c. Are the data skewed? If so, how?

d. Based on the results of (a) through (c), what conclusions can you reach concerning the total fat of chicken sandwiches?

Practice Problems:

Q.3 The following data represent the battery life (in shots) for three pixel digital cameras:

300	180	85	170	380	460
260	35	380	120	110	240

List the Five-point summary.

Q.4 For the data set below:

82	45	64	80	82	74	79	80	80	78	80	80	48	73	80	79	81	70	78	73

a. Obtain and interpret the quartiles.
b. Determine and interpret the interquartile range.
c. Find and interpret the five-number(point) summary.
d. Identify potential outliers, if any.
e. Construct and interpret a boxplot.

Practice Problems:

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Q.5 A bank branch located in a commercial place of a city has developed an improved process for serving customers during the noon-to-1:00 p.m. lunch period. The waiting time, in minutes (defined as the time the customer enters the line to when he or she reaches the teller window), of a sample of 15 customers during this hour is recorded over a period of one week. The results are: 4.21, 5.55, 3.02, 5.13, 4.77, 2.34, 3.54, 3.20, 4.50, 6.10, 0.38, 5.12, 6.46, 6.19, 3.79.

Another branch, located in a residential area, is also concerned with the noon-to-1 p.m. lunch hour. The waiting time, in of a sample of 15 customers during this hour is recorded over a period of one week. The results are listed below: 9.66, 5.90, 8.02, 5.79, 8.73, 3.82, 8.01, 8.35, 10.49, 6.68, 5.64, 4.08, 6.17, 9.91, 5.47.

a. List the five-number summaries of the waiting times at the two bank branches.
b. Construct box-and-whisker plots and describe the shape of the distribution of each for the two bank branches.
c. What similarities and differences are there in the distributions of the waiting time at the two bank branches?

IMP Note to Self

Thank You