Artificial and Computational Intelligence

AIMLCLZG557

Contributors & Designers of document content : Cluster Course Faculty Team

M2 : : Problem Solving Agent using Search

BITS Pilani
Pilani Campus

Presented by
Faculty Name
BITS Email ID

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- I hereby acknowledge all the contributors for their material and inputs and gratefully acknowledge people others who made their course materials freely available online.
- .I have provided source information wherever necessary
- This is not a full fledged reading materials. Students are requested to refer to the textbook w.r.t detailed content of the presentation deck that is expected to be shared over e-learning portal - taxilla.
- I have added and modified the content to suit the requirements of the class dynamics & live session's lecture delivery flow for presentation

- **Slide Source / Preparation / Review:**
- From BITS Pilani WILP**:** Prof.Raja vadhana, Prof. Indumathi, Prof.Sangeetha
- From BITS Oncampus & External : Mr.Santosh GSK

Course Plan

M1 Introduction to AI

M2 Problem Solving Agent using Search

M3 Game Playing

M4 Knowledge Representation using Logics

M5 Probabilistic Representation and Reasoning

M6 Reasoning over time, Reinforcement Learning

M7 Ethics in AI

Learning Objective

At the end of this class , students Should be able to:

1. Compare given heuristics for a problem and analyze which is the best fit
2. Design relaxed problem with appropriate heuristic design
3. Prove the designed relaxed problem heuristic is admissible
4. Differentiate which local search is best suitable for given problem
5. Design fitness function for a problem
6. Construct a search tree
7. Apply appropriate local search and show the working of algorithm at least for first 2 iterations with atleast four next level successor generation(if search tree is large)
8. Design and show Genetic Algorithm steps for a given problem

Module 2 : Problem Solving Agent using Search

A. Uninformed Search

B. Informed Search

C. Heuristic Functions

D. Local Search Algorithms & Optimization Problems

Design of Heuristics

Heuristic Design

- **Effective Branching Factor**
- Good Heuristics
- Notion of Relaxed Problems
- Generating Admissible Heuristics

Effective branching factor (b^*):

If the algorithm generates N number of nodes and the solution is found at depth d, then

$$N + 1 = 1 + (b^*) + (b^*)^2 + (b^*)^3 + \ldots + (b^*)^d$$

Heuristic Design

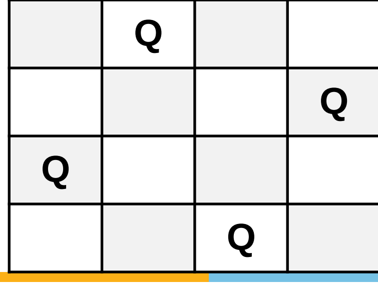
- Effective Branching Factor
- Good Heuristics
- **Notion of Relaxed Problems**
- Generating Admissible Heuristics

Simplify the problem

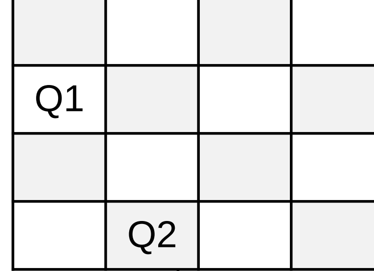
Assume no constraints

Cost of optimal solution to relaxed problem ≤ Cost of optimal solution for real problem

N-Queen

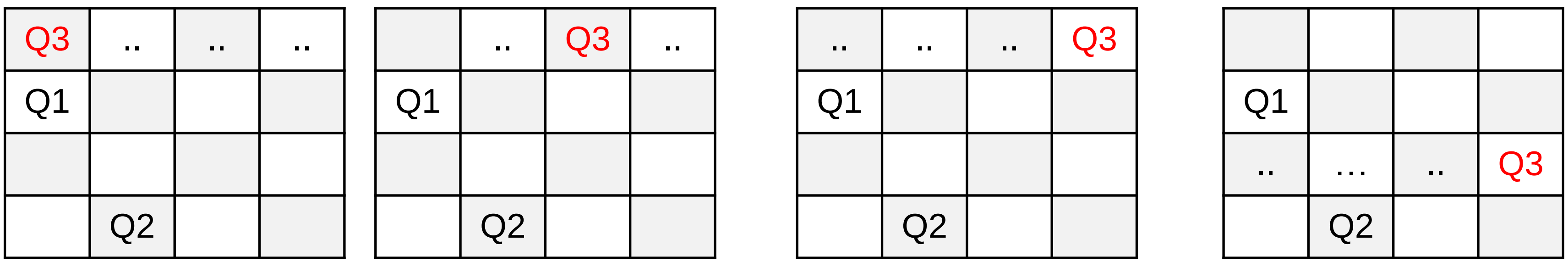


Goal State



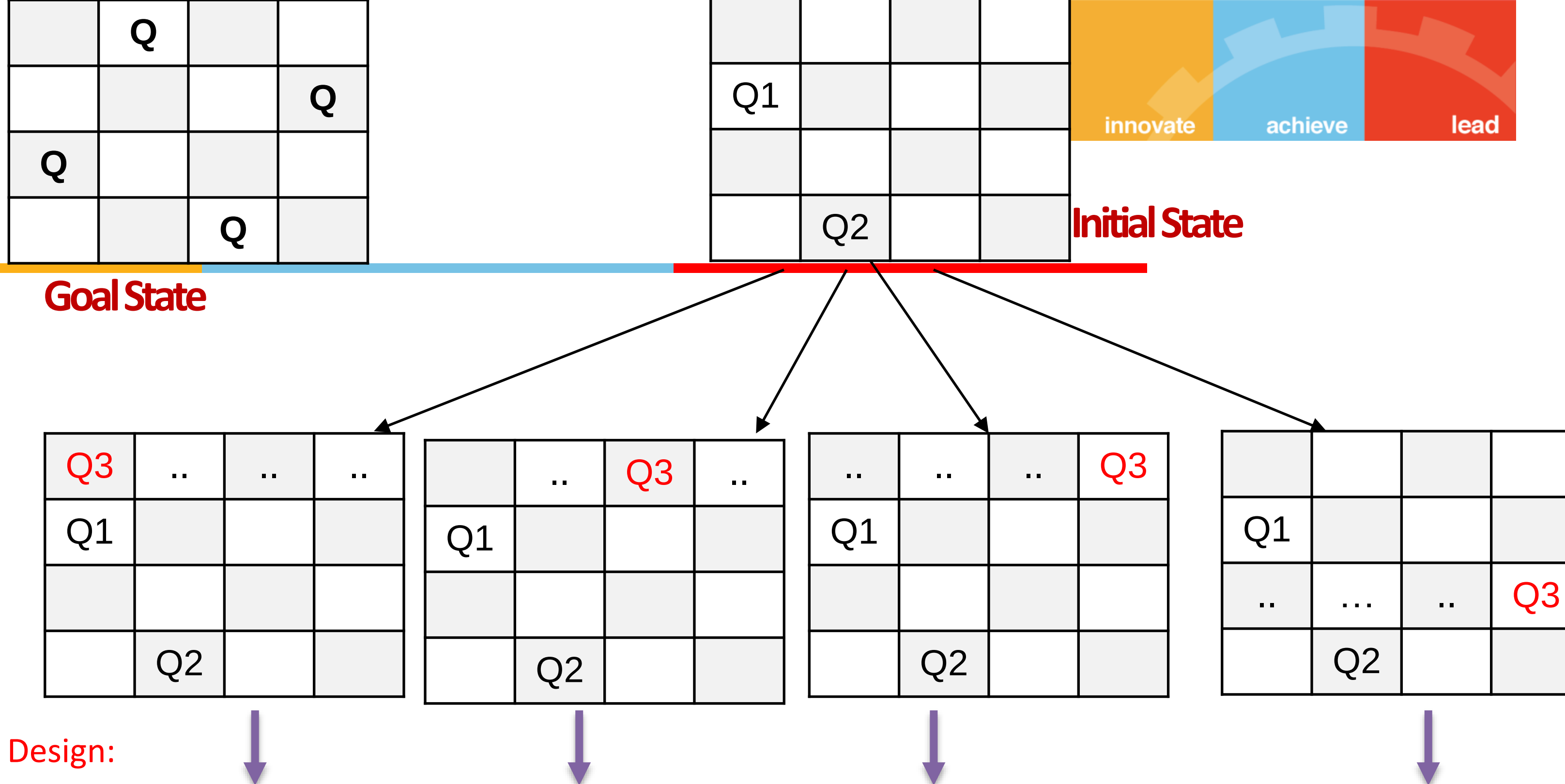
Initial State

- Construct the search tree by considering one row of the board at a time
- State space graph of relaxed problem is a super graph of original state space because of removal of restrictions



Initial State	Possible Actions	Transition Model	Goal Test	Path Cost	No.Of.States
< Xi , Yi >	Place in any non-occupied row in board		isValid Non-Attacking	Transition + Valid Queens	n!

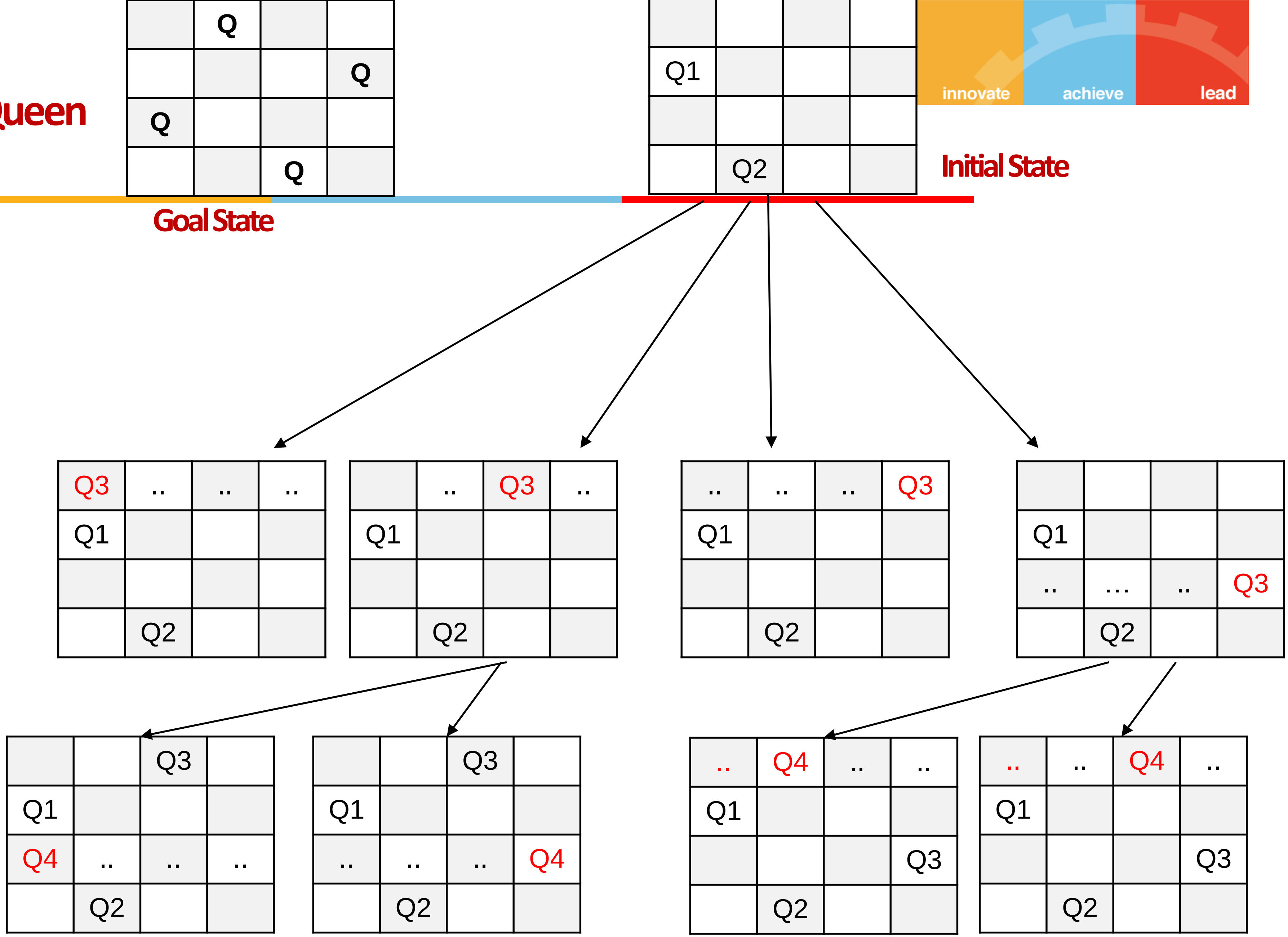
N-Queen



Possible Heuristic Design:

H1(n) : Number of Non conflicting Pairs of Queen (MAX)				
H2(n) : Number of Conflicting Pairs of Queen (MIN)				
H1(n) : Number of safest Queen (MAX)				

N-Queen

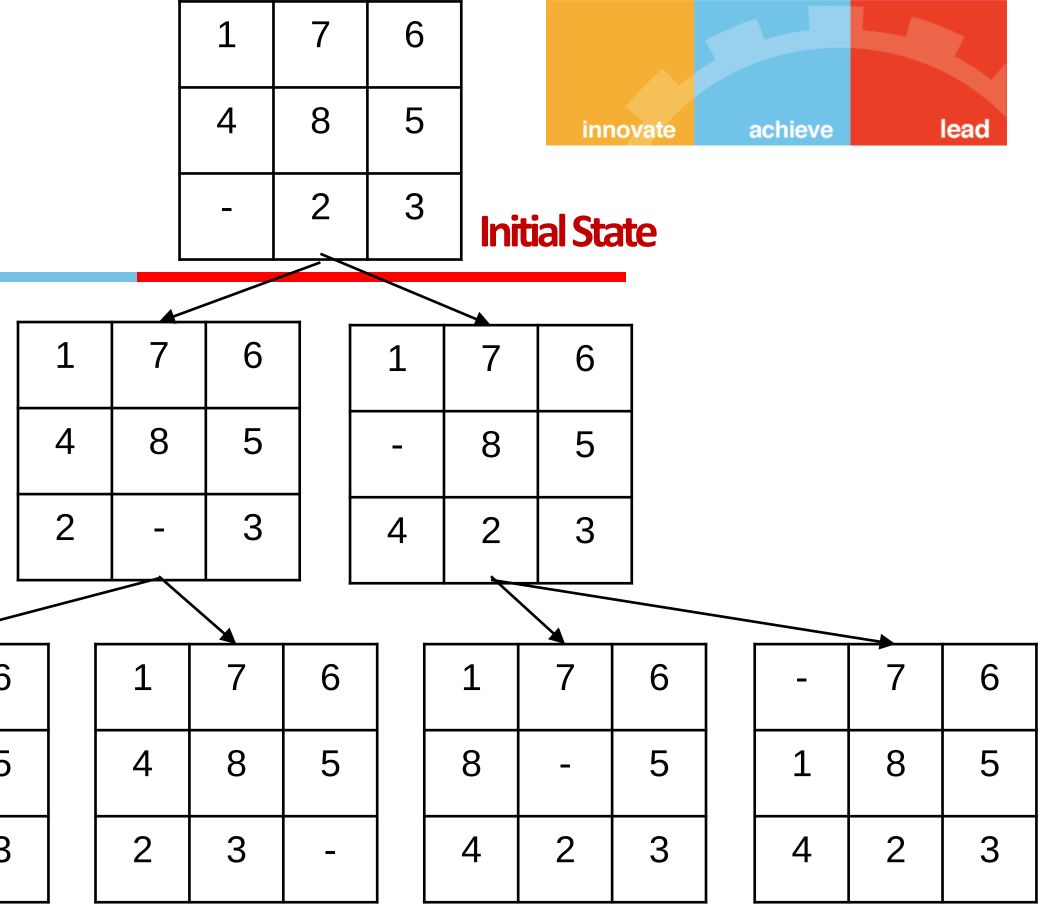


N-Tile

-	1	2
3	4	5
6	7	8

Goal State

Initial State



- Effective Branching Factor

: ~3

: Avg.cost = 18

: No.of.States = ~ 3^{18}

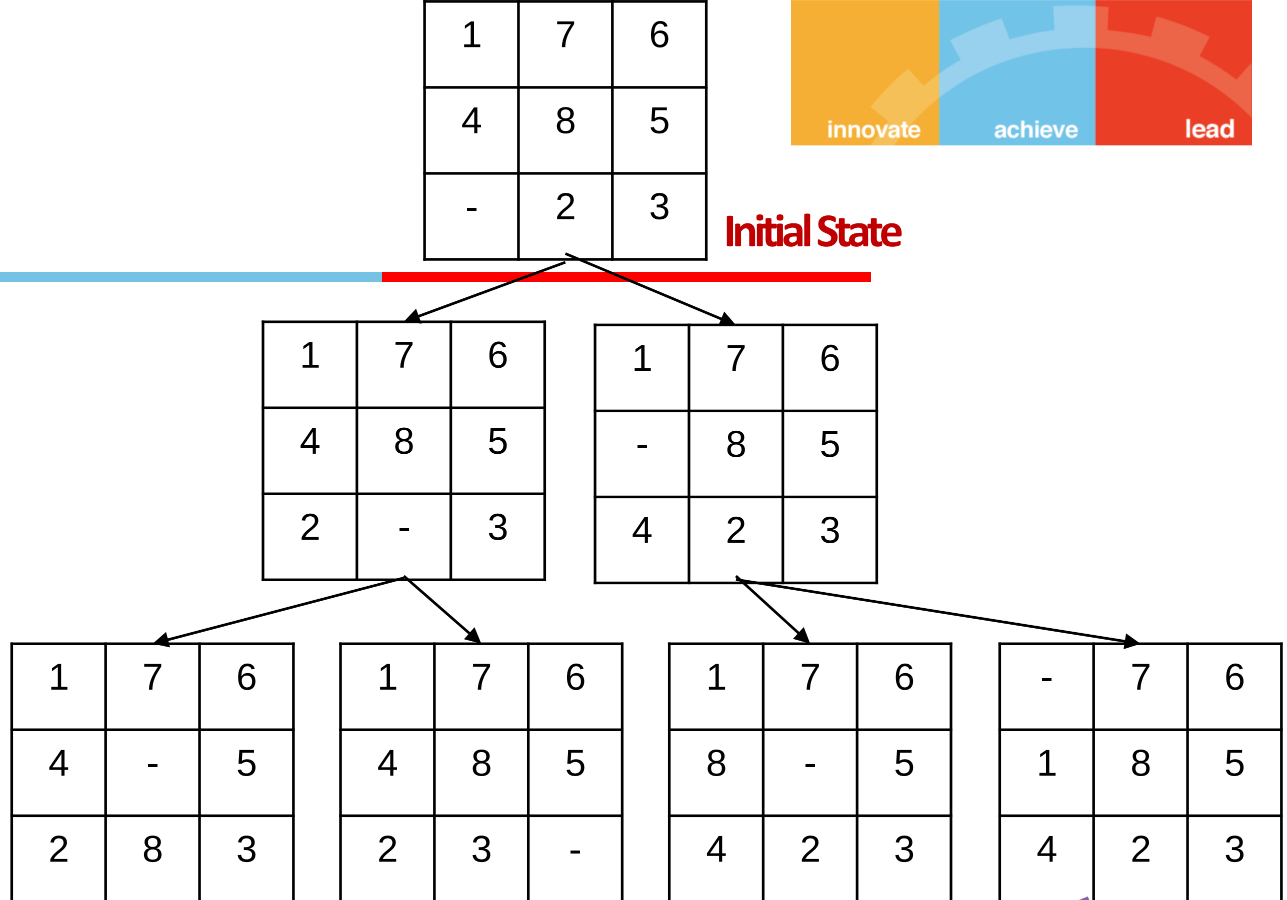
: Graph states : 9!/2 = 181, 440 states

Initial State	Possible Actions	Transition Model	Goal Test	Path Cost	No.Of.States
<LOC, ID>	Move Empty to near by Tile		ID=LOC+1	Transition + Positional + Distance+ Other approaches	9!

N-Tile

-	1	2
3	4	5
6	7	8

Goal State



Initial State

Possible Heuristic Design:

H1(n) : Manhattan distance of Empty tile	
H1(n) : Manhattan distance of all labelled tile	
H1(n) : Number of Misplaced tile (MAX)	

Learn from experience

Trail / Puzzle	X1(n) : No.of.Misplaced Tiles	X2(n): Pair of adjacent tiles that are not in goal	X3(n): Position of the empty tile	………..h`(n)
Example 1	7	10	7	…………
Example 2	5	6	6	…………
………	..	..	..	…………

-	1	2
3	4	5
6	7	8

1	7	6
4	8	5
-	2	3

Create a suitable model:

h(n) = c1*X1(n) + c2*X2(n) + ……….

Local Search & Optimization

Optimization Problem

Goal : Navigate through a state space for a given problem such that an optimal solution can be found

Objective : Minimize or Maximize the objective evaluation function value

Scope : Local

Objective Function : Fitness Value evaluates the goodness of current solution

Local Search : Search in the state-space in the neighbourhood of current position until an optimal solution is found

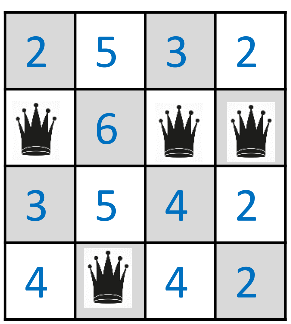
Single Instance Based	Multiple Instance Based
Hill Climbing	Genetic Algorithm
Simulated Annealing	Particle Swarm Optimization
Local Beam Search	Ant Colony Optimization
Tabu Search	

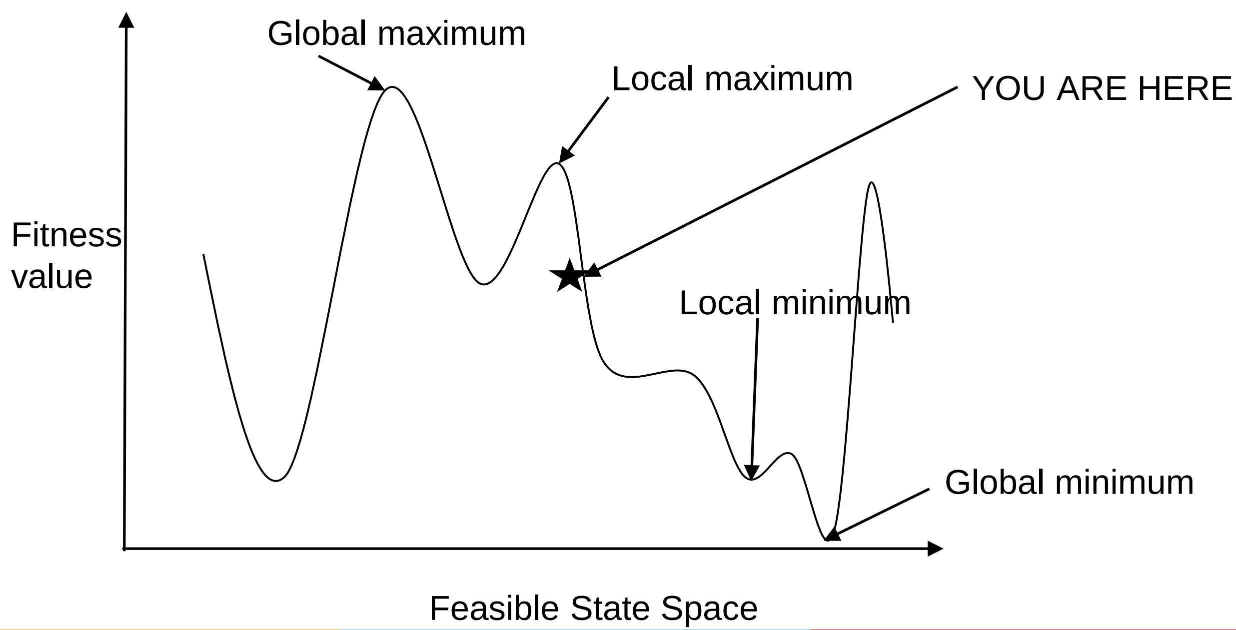
Terminology

Local Search : Search in the state-space in the neighbourhood of current position until an optimal solution is found

Algorithms:

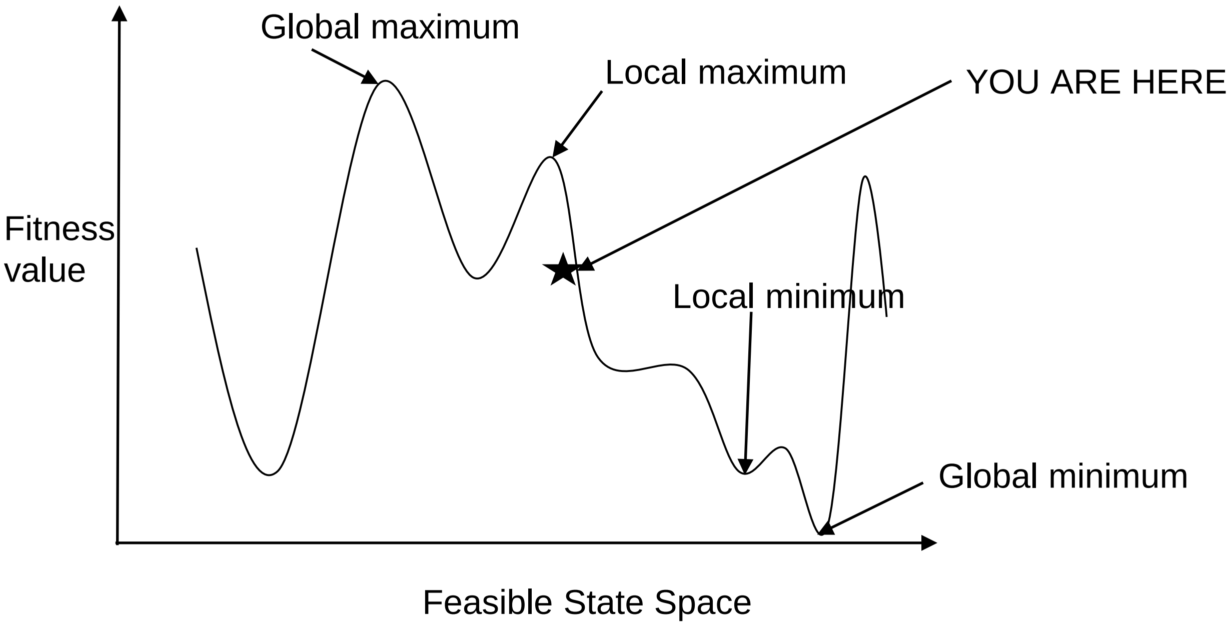
- Choice of Neighbor
- Looping Condition
- Termination Condition





Hill Climbing

Hill Climbing



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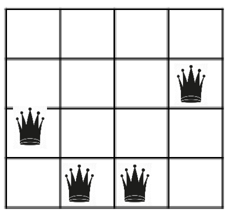
Hill Climbing

Random Restart

1. Select a random state
2. Evaluate the fitness scores for all the successors of the state
3. Select the next state based on the highest fitness
4. Repeat from Step 2



3	4	4	2	3

function HILL-CLIMBING(*problem*) **returns** a state that is a local maximum

  current ← MAKE-NODE(*problem*.INITIAL-STATE)
  loop do
    neighbor ← a highest-valued successor of *current*
    if neighbor.VALUE ≤ current.VALUE **then return** *current*.STATE
    current ← *neighbor*

Hill Climbing

1. Select a random state
2. Evaluate the fitness scores for all the successors of the state

h(n) = No.of non-conflicting pairs of queens in the board.

Q1-Q2

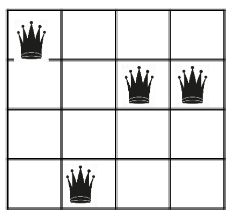
Q1-Q3

Q1-Q4

Q2-Q3

Q2-Q4

Q3-Q4

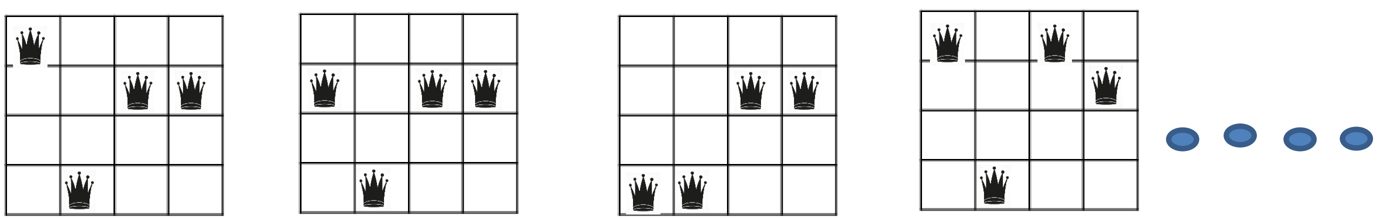


1	4	2	2	4

Note : Steps 3 & 4 in the above algorithm will be a part of variation of Hill climbing

Hill Climbing

1. Select a random state
2. Evaluate the fitness scores for all the successors of the state

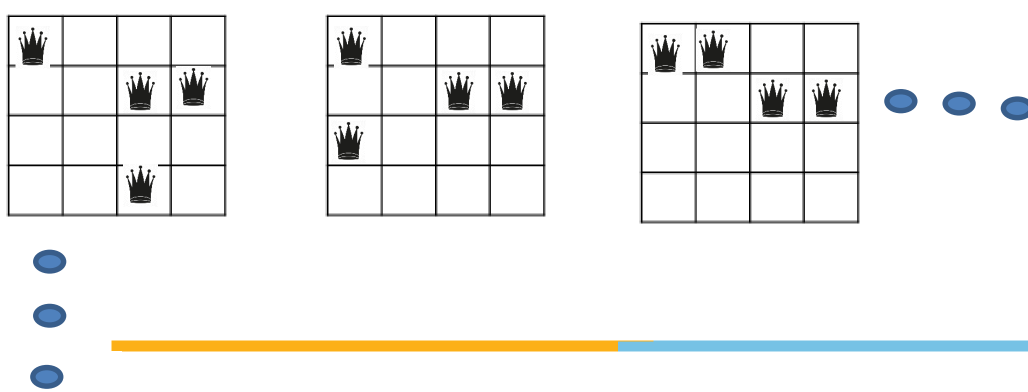


1	4	2	2

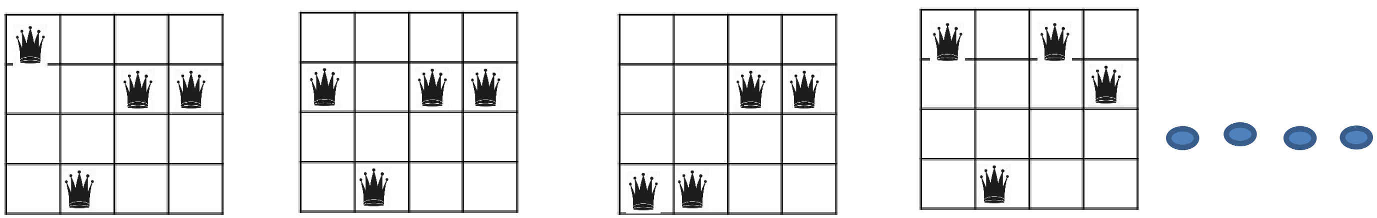
2	4	2	2

4	4	2	2

1	4	1	2



Hill Climbing



1	4	2	2	4

2	4	2	2	2

4	4	2	2	2

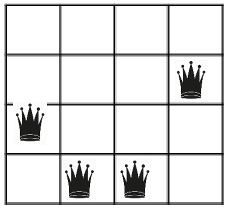
1	4	1	2	3

Local Maxima → Random Restart

Hill Climbing

Random Restart

1. Select a random state
2. Evaluate the fitness scores for all the successors of the state
3. Calculate the probability of selecting a successor based on fitness score
4. Select the next state based on the highest probability
5. Repeat from Step 2

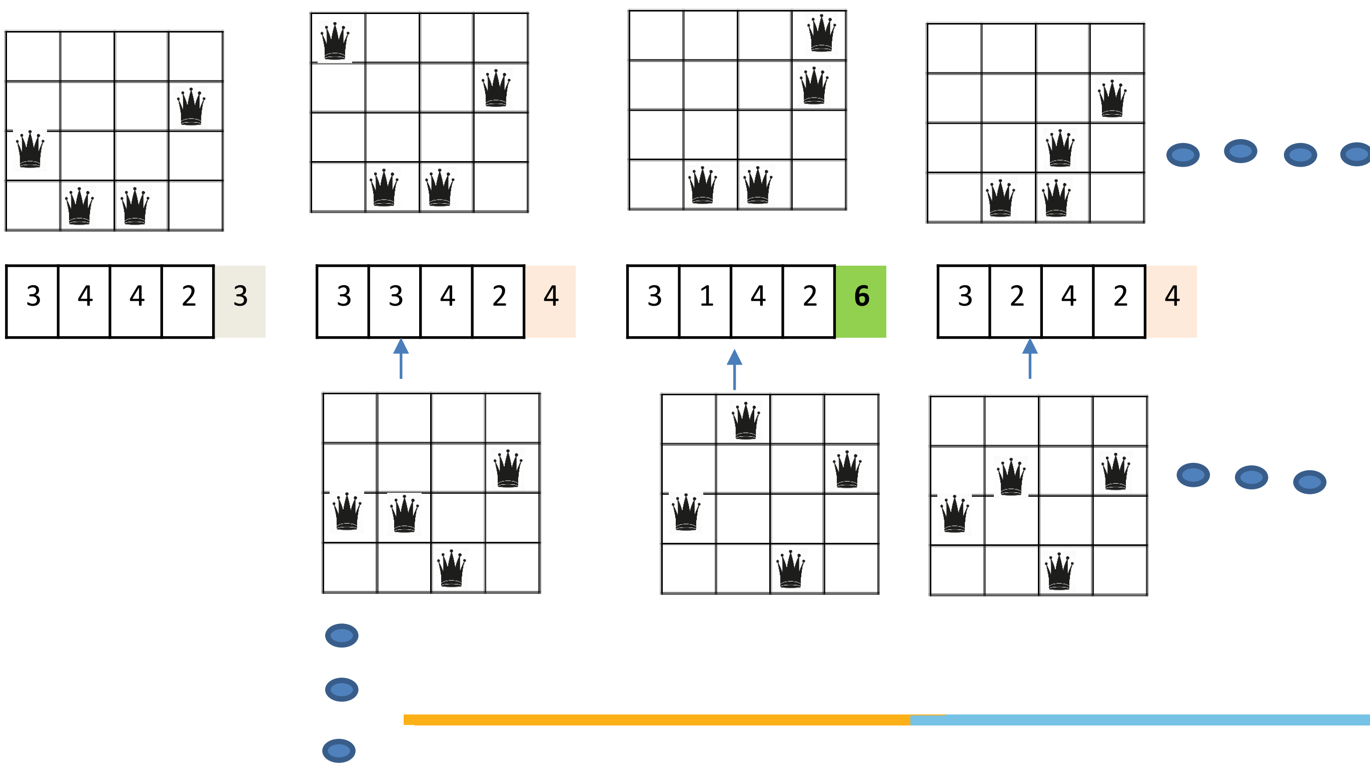


3	4	4	2	3

```
function HILL-CLIMBING(problem) returns a state that is a local maximum

  current ← MAKE-NODE(problem.INITIAL-STATE)
  loop do
     neighbor ← a highest-valued successor of current
     if neighbor.VALUE ≤ current.VALUE then return current.STATE
     current ← neighbor
```

Hill Climbing

1. Select a random state
2. Evaluate the fitness scores for all the successors of the state
3. Calculate the probability of selecting a successor based on fitness score
4. Select the next state based on the highest probability
5. Repeat from Step 2

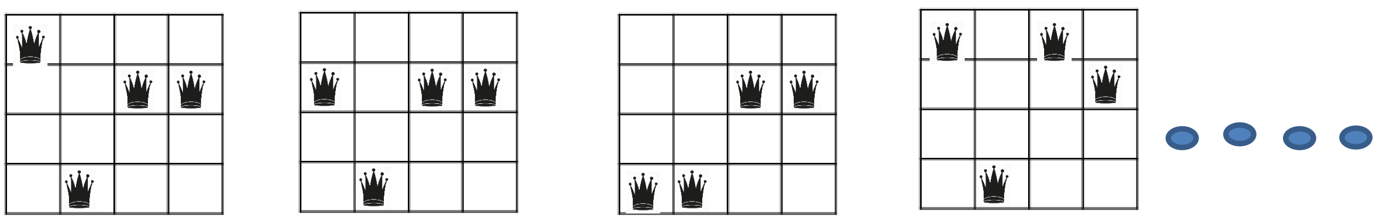


Stochastic Hill Climbing

$next \leftarrow$ a randomly selected successor of $current$
$\Delta E \leftarrow next.\textsc{Value} - current.\textsc{Value}$
if $\Delta E > 0$ **then** $current \leftarrow next$
else $current \leftarrow next$ only with probability $e^{\Delta E/T}$

Stochastic Hill Climbing

1. Select a random state
2. Evaluate the fitness scores for all the successors of the state
3. Calculate the probability of selecting a successor based on fitness score
4. Select the next state based on the highest probability
5. Repeat from Step 2



1	4	2	2	4	2	4	2	2	2	4	4	3	3	2	1	4	1	2	3
				0.16					0.08					0.08					0.13
				0.08					0.42					0.42					0.25

12 N = {4,2,2,3,3,2,1,3,2,1,3,2}

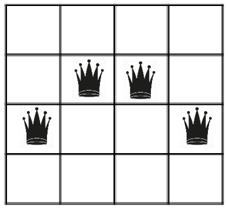
Simulated Annealing

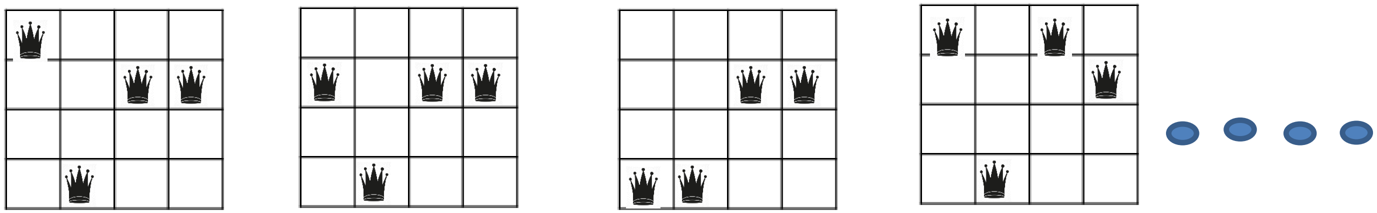
function SIMULATED-ANNEALING(*problem*, *schedule*) **returns** a solution state
  inputs: *problem*, a problem
      schedule, a mapping from time to "temperature"

  current ← MAKE-NODE(*problem*.INITIAL-STATE)
  for $t = 1$ **to** ∞ **do**
    $T \leftarrow$ *schedule*(t)
    if $T = 0$ **then return** *current*
    next ← a randomly selected successor of *current*
    $\Delta E \leftarrow$ *next*.VALUE – *current*.VALUE
    if $\Delta E > 0$ **then** *current* ← *next*
    else *current* ← *next* only with probability $e^{\Delta E/T}$

Simulated Annealing

1. Select a random state
2. Evaluate the fitness scores for all the successors of the state
3. Calculate the probability of selecting a successor based on fitness score
4. Select the next state based on the highest probability
5. Repeat from Step 2





1	4	2	2	4	0.55
2	4	2	2	2	0.5
4	4	3	3	2	0.5
1	4	1	2	3	0.525

12 N = {4,2,2,3,3,2,1,3,2,1,3,2}

Init = 2

Simulated Annealing

Current Value = 4 (Local Maxima)

Global Maxima = 6

Next Value	ΔE	ΔE/t	$e^{\Delta E/t}$	$\frac{1}{1+e^{\Delta E/t}}$	ΔE/t	$e^{\Delta E/t}$	$\frac{1}{1+e^{\Delta E/t}}$
2	2	0.1	1.12	0.47	0.4	1.49	0.40
3	1	0.05	1.05	0.49	0.2	1.22	0.45
5	-1	-0.05	0.95	0.51	-0.2	0.82	0.55

Simulated Annealing

Maximization problem design to achieve global minima

Set Temp to very high temp t

Set n as number of iteration to be performed at a particular t

L1: Randomly select a random neighbour

Calculate Energy barrier E = f(N)-f(C)

If **E > 0** then its a good move

    Move ahead for next tree search level

Else

    Create a random number r:[0-1]

    If **$r < e^{-E/t}$**

        Choose this bad state & move downhill

    Else

        Go to L1.

If Goal is reached or {acceptable goal(set criteria to check)node is reached & t is small END}

Else

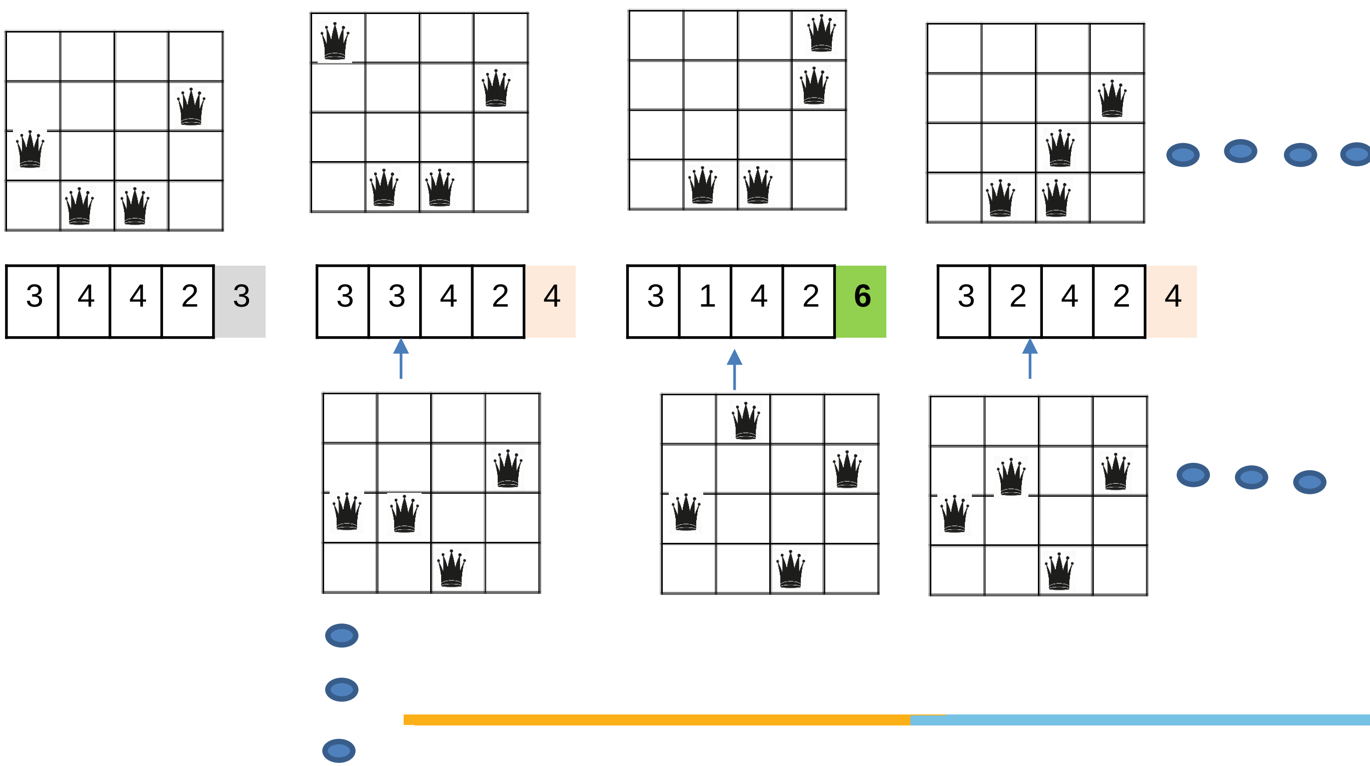
    If no.of.neighbors explored has reached a threshold >=n

        then Lower t and go to L1.

Local Beam Search

Beam Search

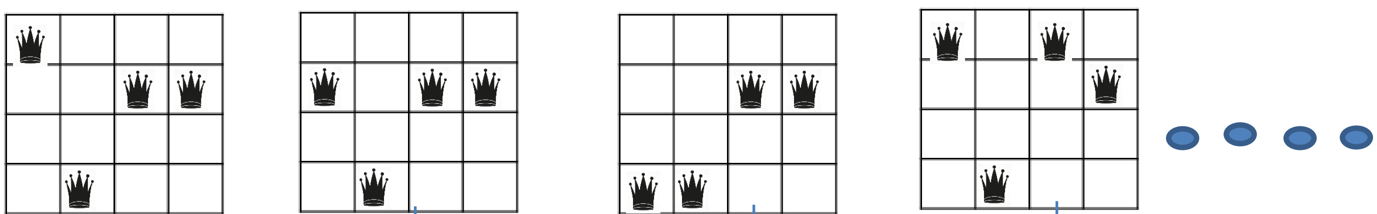
1. Initialize k random state
2. Evaluate the fitness scores for all the successors of the k states
3. Calculate the probability of selecting a successor based on fitness score
4. Select the next state based on the highest probability
5. If the goal is not found, Select the next ‘k’ states randomly based on the probability
6. Repeat from Step 2



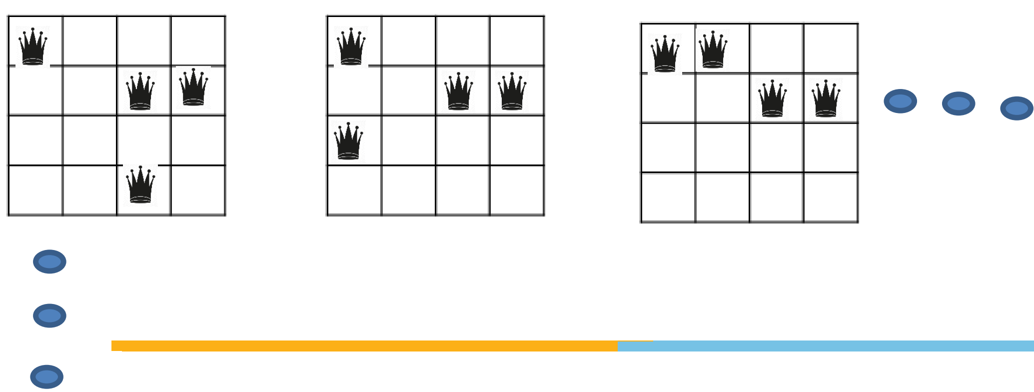
Stochastic Beam Search

Sample from 1st State

1. Initialize k random state
2. Evaluate the fitness scores for all the successors of the k states
3. Calculate the probability of selecting a successor based on fitness score
4. Select the next state based on the highest probability
5. If the goal is not found, Select the next ‘k’ states randomly based on the probability
6. Repeat from Step 2



1	4	2	2	4
2	4	2	2	2
4	4	3	3	2
1	4	1	2	3



Genetic Algorithm

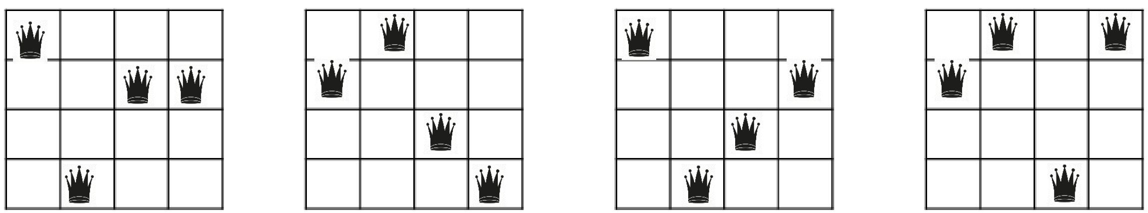
Genetic Algorithm

```
function GENETIC-ALGORITHM(population, FITNESS-FN) returns an individual
  inputs: population, a set of individuals
          FITNESS-FN, a function that measures the fitness of an individual

  repeat
     new_population ← empty set
     for i = 1 to SIZE(population) do
        x ← RANDOM-SELECTION(population, FITNESS-FN)
        y ← RANDOM-SELECTION(population, FITNESS-FN)
        child ← REPRODUCE(x, y)
        if (small random probability) then child ← MUTATE(child)
        add child to new_population
     population ← new_population
  until some individual is fit enough, or enough time has elapsed
  return the best individual in population, according to FITNESS-FN
```

```
function REPRODUCE(x, y) returns an individual
  inputs: x, y, parent individuals

  n ← LENGTH(x); c ← random number from 1 to n
  return APPEND(SUBSTRING(x, 1, c), SUBSTRING(y, c + 1, n))
```

Genetic Algorithm

1. Select ‘k’ random states – **Initialization : k=4**
2. Evaluate the fitness value all states
3. If anyone of the state’s has achieved the threshold fitness value or threshold new states or no change is seen than previous iteration then the algorithm stops
4. Else, use roulette wheel mechanism to select pair/s
5. Pairs selected produces new state (successor) by crossover
6. Successor is allowed to mutate
7. Repeat from Step 2



1	4	2	2

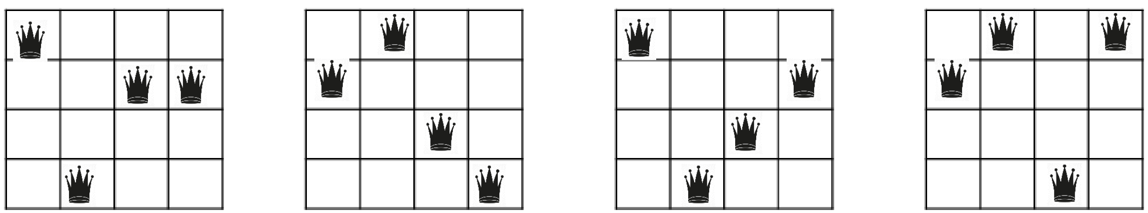
2	1	3	4

1	4	3	2

2	1	4	1

Genetic Algorithm

1. Select 'k' random states – **Initialization : k=4**
2. Evaluate the fitness value all states : Maximizing function : No.of.Non-attacking pairs Queens → Threshold = 6
3. If anyone of the state's has achieved the threshold fitness value or threshold new states or no change is seen than previous iteration then the algorithm stops
4. Else, use roulette wheel mechanism to select pair/s
5. Pairs selected produces new state (successor) by crossover
6. Successor is allowed to mutate
7. Repeat from Step 2



1	4	2	2	4

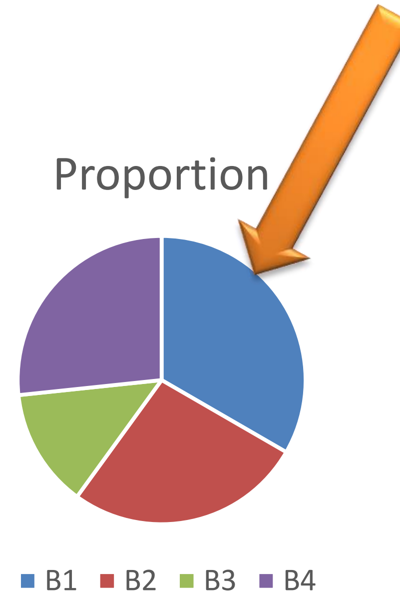
2	1	3	4	4

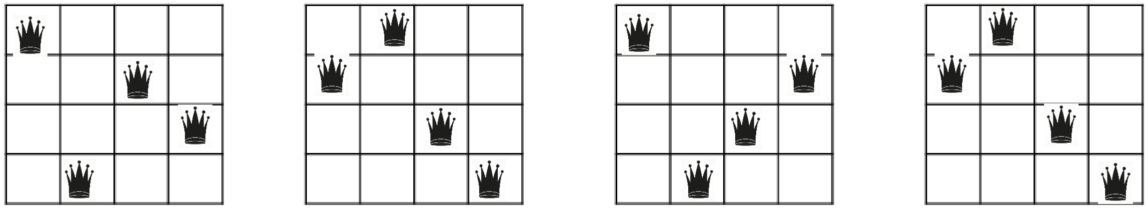
1	4	3	2	2

2	1	4	1	3

Genetic Algorithm – Example 1

Eg., use roulette wheel mechanism to select pair/s





1	4	2	3	5	0.33
2	1	3	4	4	0.27
1	4	3	2	2	0.13
2	1	3	4	4	0.27

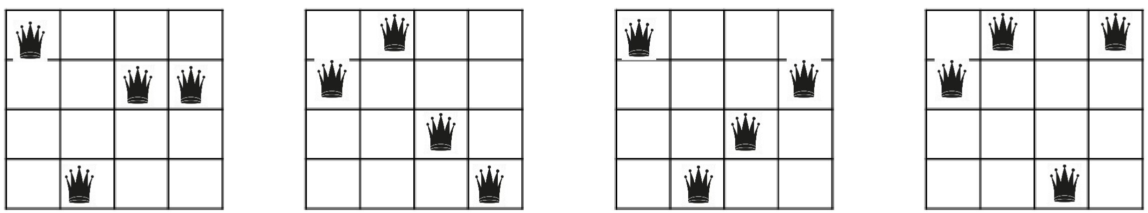
2	1	3	4
1	4	2	3
1	4	2	3
1	4	3	2

Sample winners of game -1 ,2,3,4 : B4, B1, B1, B3

Genetic Algorithm –Example 2

Selection

1. Select ‘k’ random states – **Initialization : k=4**
2. Evaluate the fitness value all states : Maximizing function : No.of.Non-attacking pairs Queens → Threshold = 6
3. ~~If anyone of the state’s has achieved the threshold fitness value or threshold new states or no change is seen than previous iteration then the algorithm stops~~
4. Else, use roulette wheel mechanism to select pair/s
5. Pairs selected produces new state (successor) by crossover
6. Successor is allowed to mutate
7. Repeat from Step 2



1	4	2	2	4	0.31
2	1	3	4	4	0.31
1	4	3	2	2	0.15
2	1	4	1	3	0.23

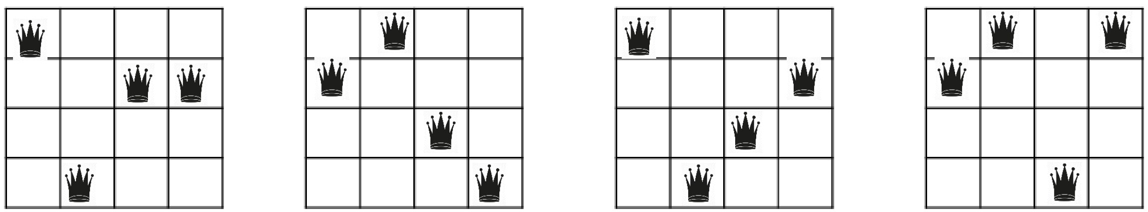
2	1	4	1
1	4	2	2
1	4	2	2
1	4	3	2

Sample winners of game -1 ,2,3,4 : B4, B1, B1, B3

Genetic Algorithm - Example 2

Crossover

1. Select ‘k’ random states – **Initialization : k=4**
2. Evaluate the fitness value all states : Maximizing function : No.of.Non-attacking pairs Queens → Threshold = 6
3. If anyone of the state’s has achieved the threshold fitness value or threshold new states or no change is seen than previous iteration then the algorithm stops
4. Else, use roulette wheel mechanism to select pair/s
5. Pairs selected produces new state (successor) by crossover
6. Successor is allowed to mutate
7. Repeat from Step 2



1	4	2	2	4	0.31
2	1	3	4	4	0.31
1	4	3	2	2	0.15
2	1	4	1	3	0.23

2	1 4 1	⟺	1	4 2 2		1	4 2	2	
1	4 3	2							

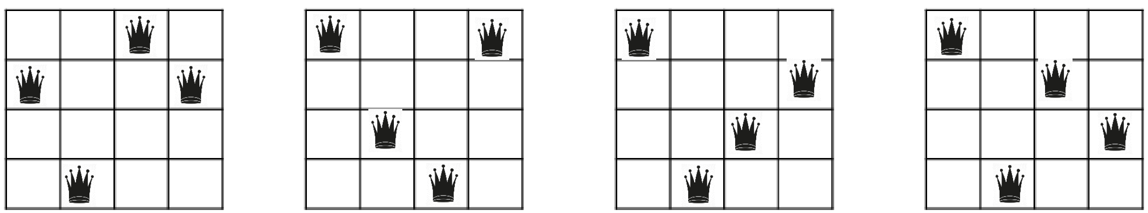
2 | 1 4 1 ⟺ 1 | 4 2 2

1 | 4 2 | 2 ⟺ 1 | 4 3 | 2

2	4	2	2
1	1	4	1
1	4	3	2
1	4	2	2

Genetic Algorithm - Example 2
Mutation

1. Select 'k' random states – **Initialization : k=4**
2. Evaluate the fitness value all states : Maximizing function : No.of.Non-attacking pairs Queens → Threshold = 6
3. ~~If anyone of the state's has achieved the threshold fitness value or threshold new states or no change is seen than previous iteration then the algorithm stops~~
4. Else, use roulette wheel mechanism to select pair/s
5. Pairs selected produces new state (successor) by crossover
6. Successor is allowed to mutate
7. Repeat from Step 2



2	4	2	2

1	1	4	1

1	4	3	2

1	4	2	2

2	4	1	2

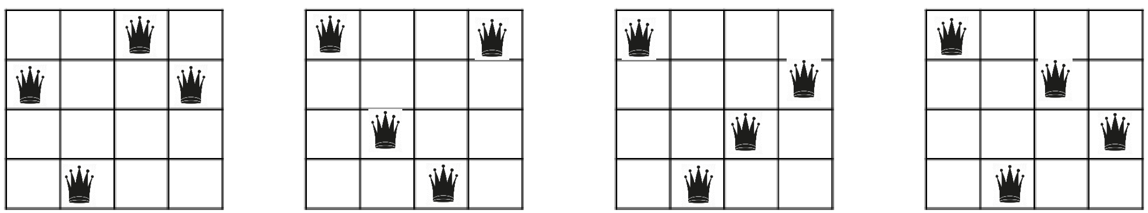
1	3	4	1

1	4	3	2

1	4	2	3

Genetic Algorithm

1. Select ‘k’ random states – **Initialization : k=4**
2. Evaluate the fitness value all states : Maximizing function : No.of.Non-attacking pairs Queens → Threshold = 6
3. If anyone of the state’s has achieved the threshold fitness value or threshold new states or no change is seen than previous iteration then the algorithm stops
4. Else, use roulette wheel mechanism to select pair/s
5. Pairs selected produces new state (successor) by crossover
6. Successor is allowed to mutate
7. Repeat from Step 2



2	4	2	2

1	1	4	1

1	4	3	2

1	4	2	2

2	4	1	2	3	0.23
1	3	4	1	3	0.23
1	4	3	2	2	0.15
1	4	2	3	5	0.39

Genetic Algorithm

1. Select ‘k’ random states – **Initialization : k=4**
2. Evaluate the fitness value all states : Maximizing function : No.of.Non-attacking pairs Queens → Threshold = 6
3. If anyone of the state’s has achieved the threshold fitness value or threshold new states or no change is seen than previous iteration then the algorithm stops
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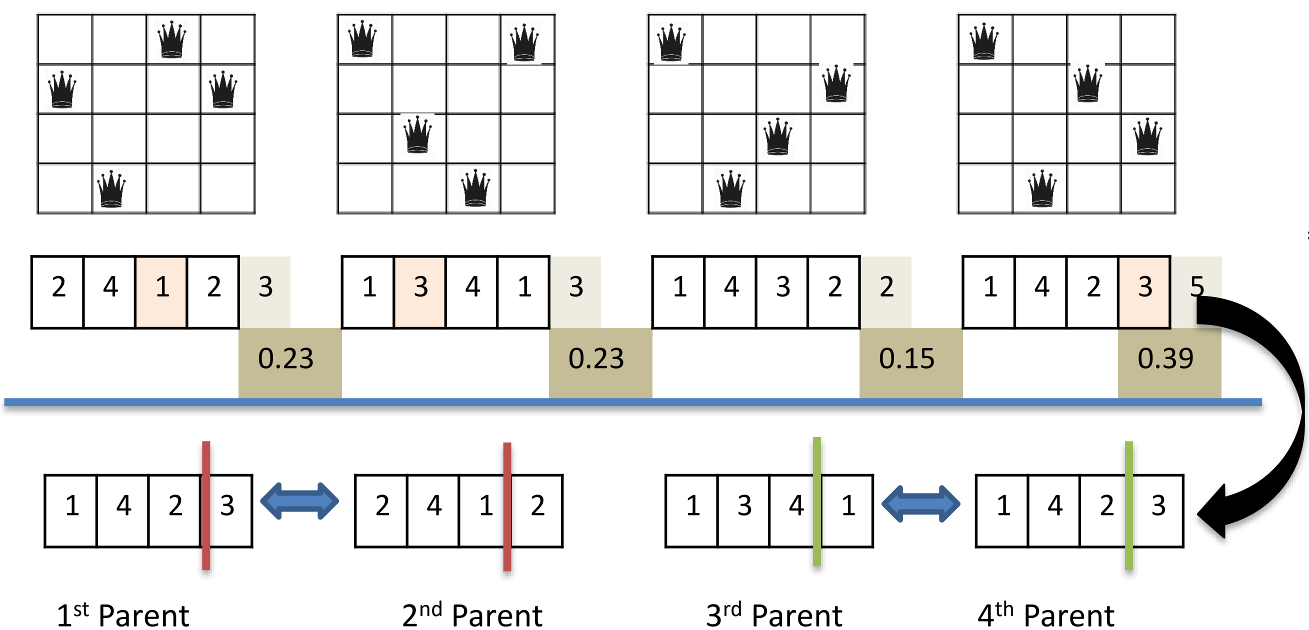

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Other crossover and mutation operators are shared in separate document & uploaded in the elearn portal

Genetic Algorithm

1. Select 'k' random states – **Initialization : k=4**
2. Evaluate the fitness value all states : Maximizing function : No.of.Non-attacking pairs Queens → Threshold = 6
3. If anyone of the state's has achieved the threshold fitness value or threshold new states or no change is seen than previous iteration then the algorithm stops
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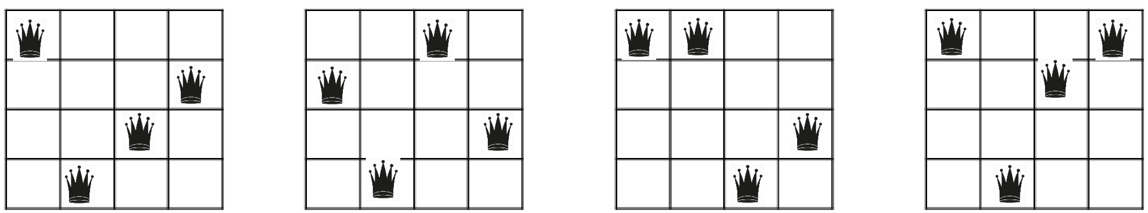

1	4	2	3	2	4	1	2	1	3	4	1	1	4	2	3

1	4	2	2	2	4	1	3	1	3	4	3	1	4	2	1

1	4	3	2	2	2	4	1	3	6	1	1	4	3	3	1	4	2	1	4

Genetic Algorithm

Techniques:

1. Design of the fitness function
2. Diversity in the population to be accounted
3. Randomization

Application:

- Creative tasks
- Exploratory in nature
- Planning problem
- Static Applications

Required Reading: AIMA - Chapter # 4.1, #4.2

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials