



Artificial & Computational Intelligence

AIMLCZG557

Contributors & Designers of document content : Cluster Course

Faculty Team

M3 : Game Playing



BITS Pilani

Pilani Campus

Presented by

Faculty Name

BITS Email ID

Artificial and Computational Intelligence

Disclaimer and Acknowledgement



- Few content for these slides may have been obtained from prescribed books and various other source on the Internet
- I hereby acknowledge all the contributors for their material and inputs and gratefully acknowledge people others who made their course materials freely available online.
- .I have provided source information wherever necessary
- This is not a full fledged reading materials. Students are requested to refer to the textbook w.r.t detailed content of the presentation deck that is expected to be shared over e-learning portal - taxilla.
- I have added and modified the content to suit the requirements of the class dynamics & live session's lecture delivery flow for presentation
- **Slide Source / Preparation / Review:**
- From BITS Pilani WILP: Prof.Raja vadhana, Prof. Indumathi, Prof.Sangeetha
- From BITS Oncampus & External : Mr.Santosh GSK

Course Plan

- M1 Introduction to AI
- M2 Problem Solving Agent using Search
- M3 Game Playing
- M4 Knowledge Representation using Logics
- M5 Probabilistic Representation and Reasoning
- M6 Reasoning over time
- M7 Ethics in AI

Module 3 : Searching to play games

A. Min-max Algorithm

B. Alpha-Beta Pruning

C. Making imperfect real time decisions

Learning Objective

At the end of this class , students Should be able to:

1. Convert a given problem into adversarial search problem
2. Formulate the problem solving agent components
3. Design static evaluation function value for a problem
4. Construct a Game tree
5. Apply Min-Max
6. Apply and list nodes pruned by alpha pruning and nodes pruned by beta pruning

Task Environment

innovate

achieve

lead

Phases of Solution Search by PSA

Assumptions – Environment :

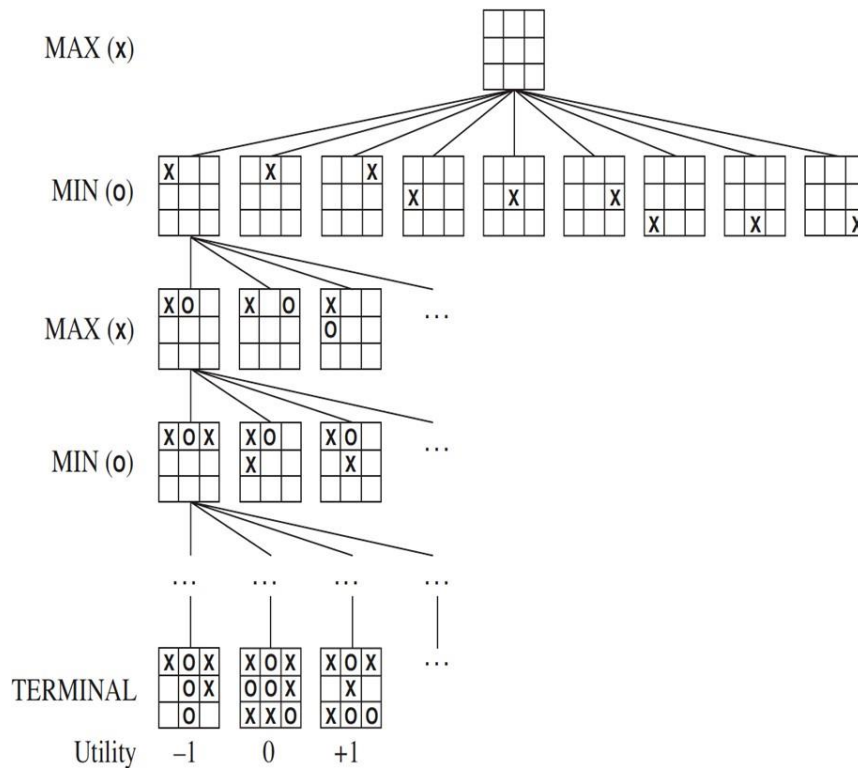
Static (4.5)

Observable

Discrete (4.4)

Deterministic

Number of Agents



Game Problem

Study & design of games enables the computers to model ways in which humans think & act hence simulating human intelligence.

AI for Gaming:

- Interesting & Challenging Problem
- Larger Search Space Vs Smaller Solutions
- Explore to better the Human Computer Interaction



Characteristics of Games:

- Observability
- Stochasticity
- Time granularity
- Number of players



Adversarial Games:

Goals of agents are in conflict where one's optimized step would reduce the utility value of the other.

Games as Search Problem

PSA : Representation of Game:

INITIAL STATE: S0

PLAYER(s)

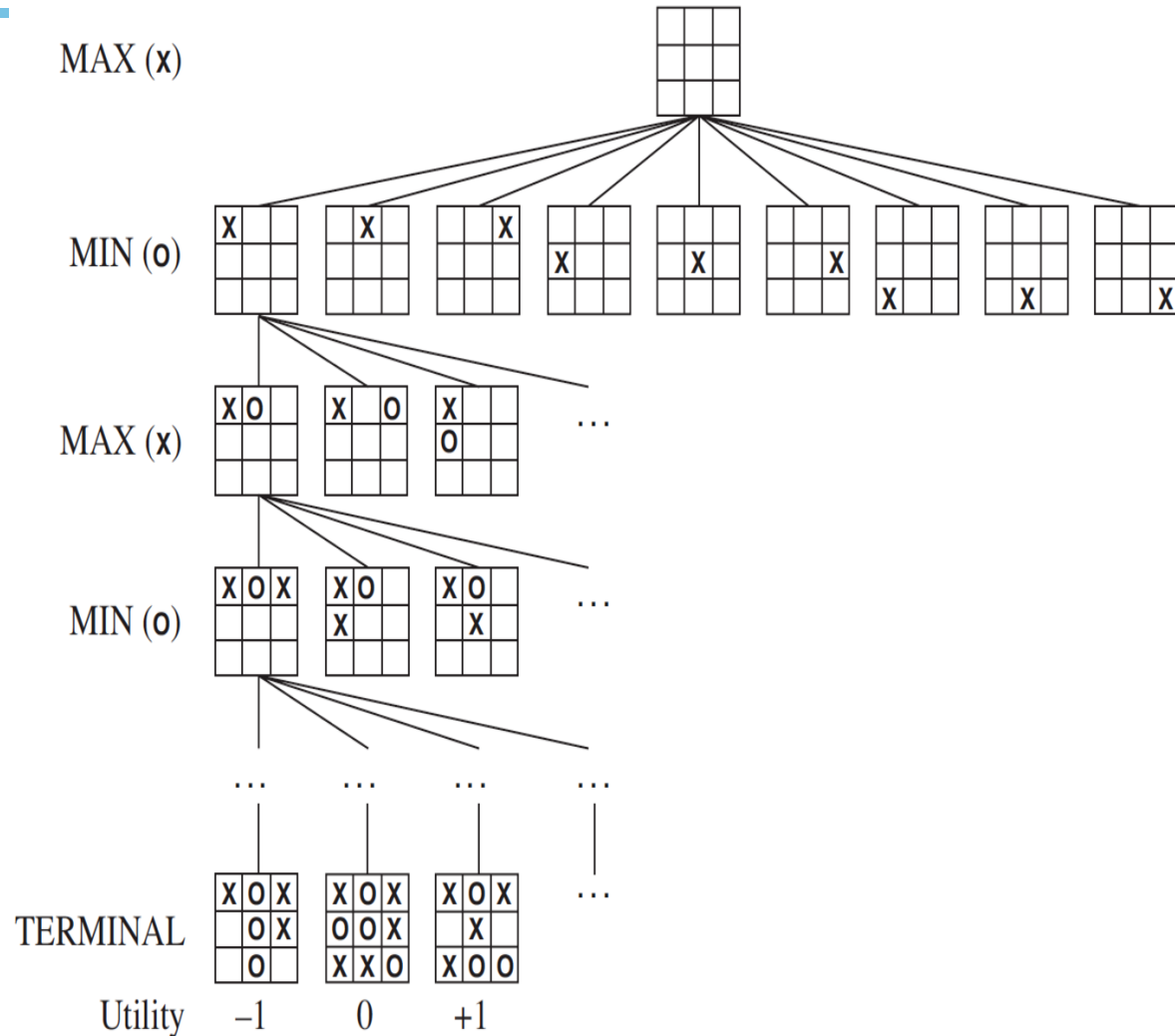
ACTIONS(s)

RESULT(s, a)

TERMINAL-TEST(s)

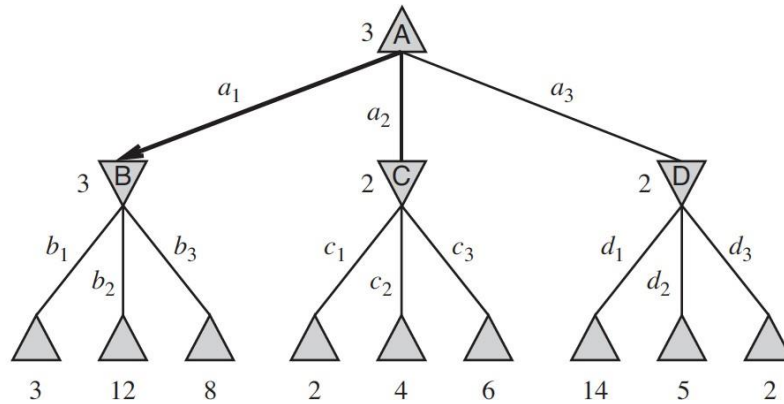
UTILITY(s, p)

Eg., Tic Tac Toe



MAX

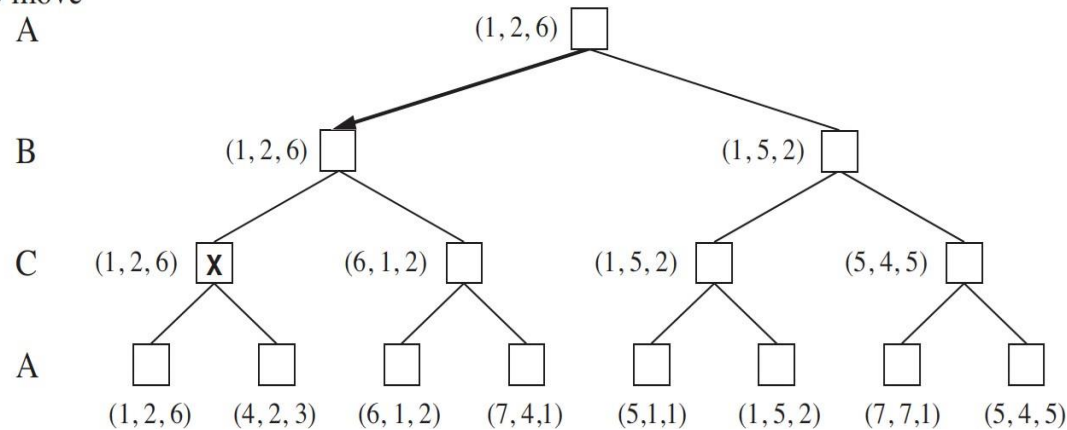
MIN



Two Player Game : 1-Ply Game

to move

Multiplayer Game



Min-Max Algorithm

function MINIMAX-DECISION(*state*) **returns** *an action*
 return $\arg \max_{a \in \text{ACTIONS}(s)} \text{MIN-VALUE}(\text{RESULT}(\text{state}, a))$

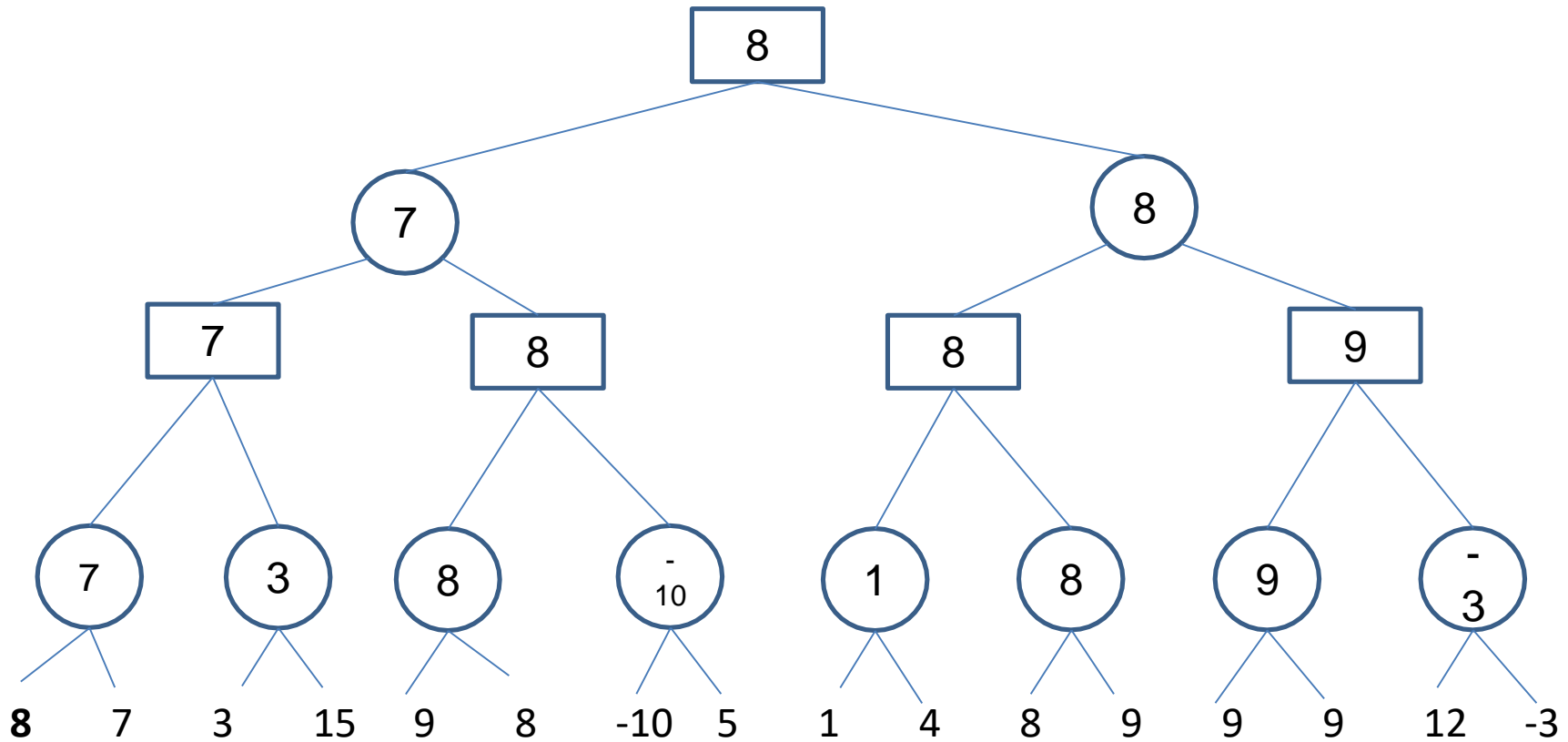
function MAX-VALUE(*state*) **returns** *a utility value*
 if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
 $v \leftarrow -\infty$
 for each *a* **in** ACTIONS(*state*) **do**
 $v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(\text{RESULT}(s, a)))$
 return *v*

function MIN-VALUE(*state*) **returns** *a utility value*
 if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
 $v \leftarrow \infty$
 for each *a* **in** ACTIONS(*state*) **do**
 $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(\text{RESULT}(s, a)))$
 return *v*

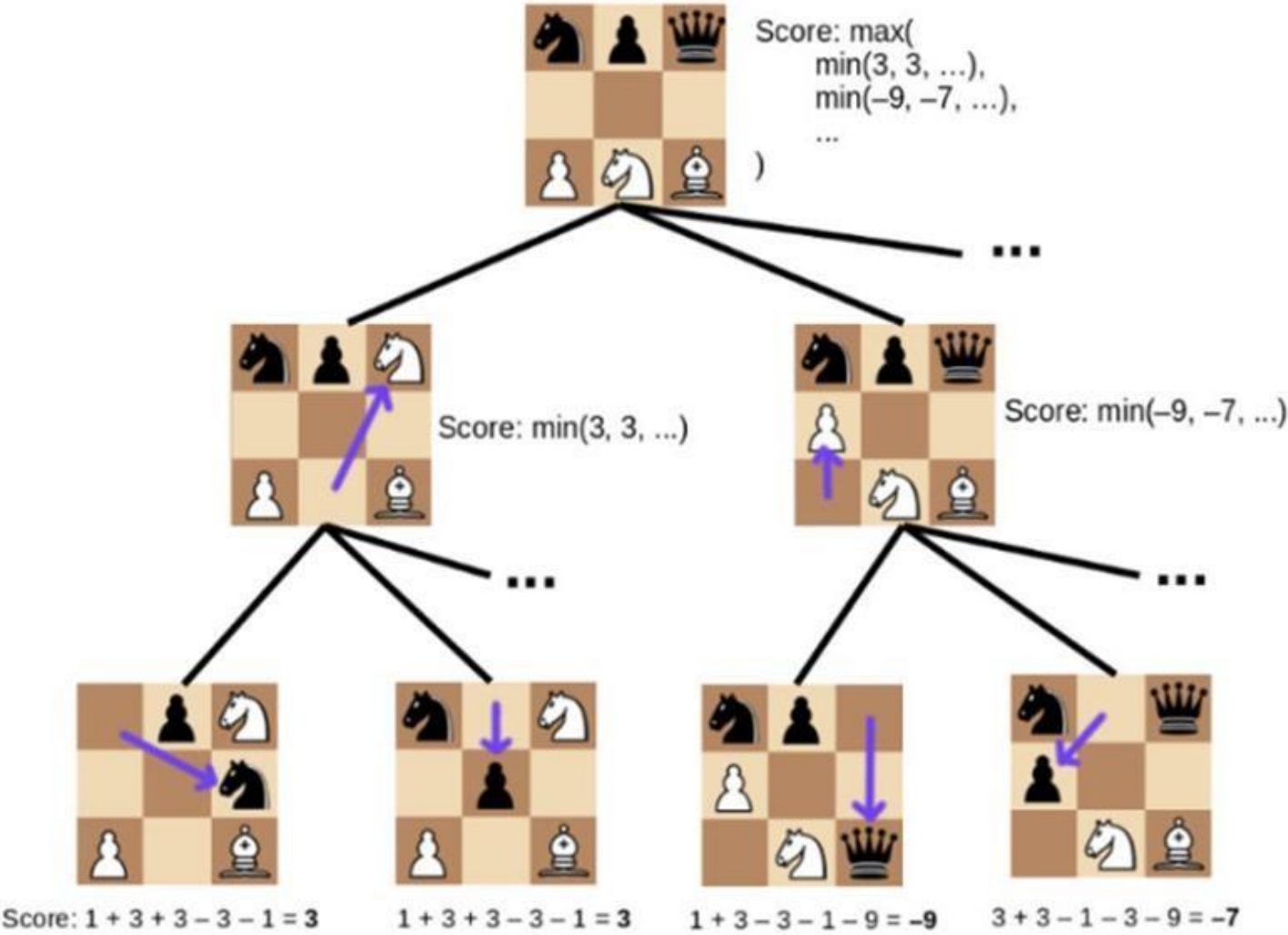
Min-Max Algorithm – Example -1

Squares represent MAX nodes

Circles represent MIN nodes



Design of Static Evaluation Values



Design of Static Evaluation Values

N-Queens

♚			
		♚	♚
	♚		

1	4	2	2	4
---	---	---	---	---

Tic-Tac-Toe

0	0	x
x		0
		x

Max's Share	2
Min's Share	1
Board Value	1

N-Tile

2	8	3
1	6	4
7		5

1	2	3
8		4
7	6	5

No.of.Tiles Out of Place	5
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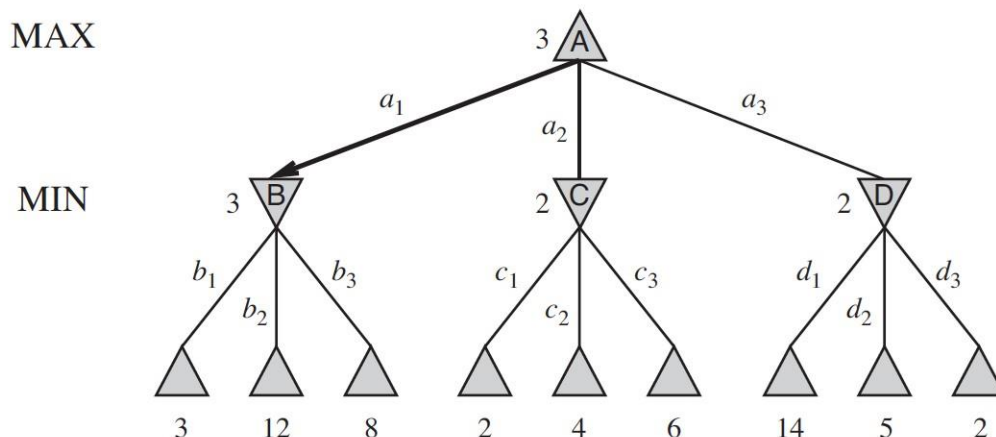
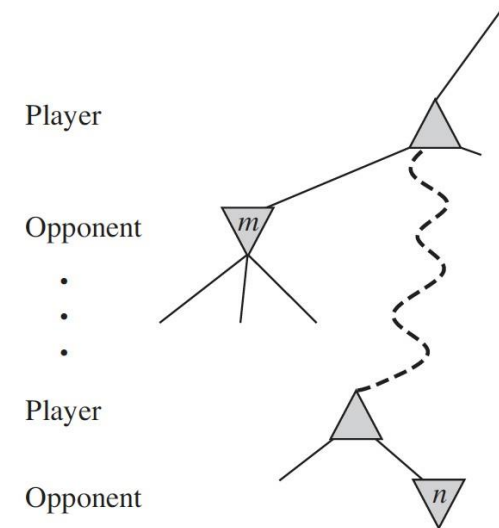
$$\text{Eval}(S) = w_1 f_1(s) + w_2 f_2(s) + \dots + w_n f_n(s)$$

$$= 0.6 (\text{MaxChance} - \text{MinChance}) + 0.4 (\text{MaxPairs} - \text{MinPairs})$$

Alpha – beta Pruning

General Principle:

At a node n if a player has better option at the parent of n or further up, then n node will never be reached .Hence the entire subtree pruned



$$\begin{aligned}
 \text{MINIMAX}(\text{root}) &= \max(\min(3, 12, 8), \min(2, x, y), \min(14, 5, 2)) \\
 &= \max(3, \min(2, x, y), 2) \\
 &= \max(3, z, 2) \quad \text{where } z = \min(2, x, y) \leq 2 \\
 &= 3.
 \end{aligned}$$

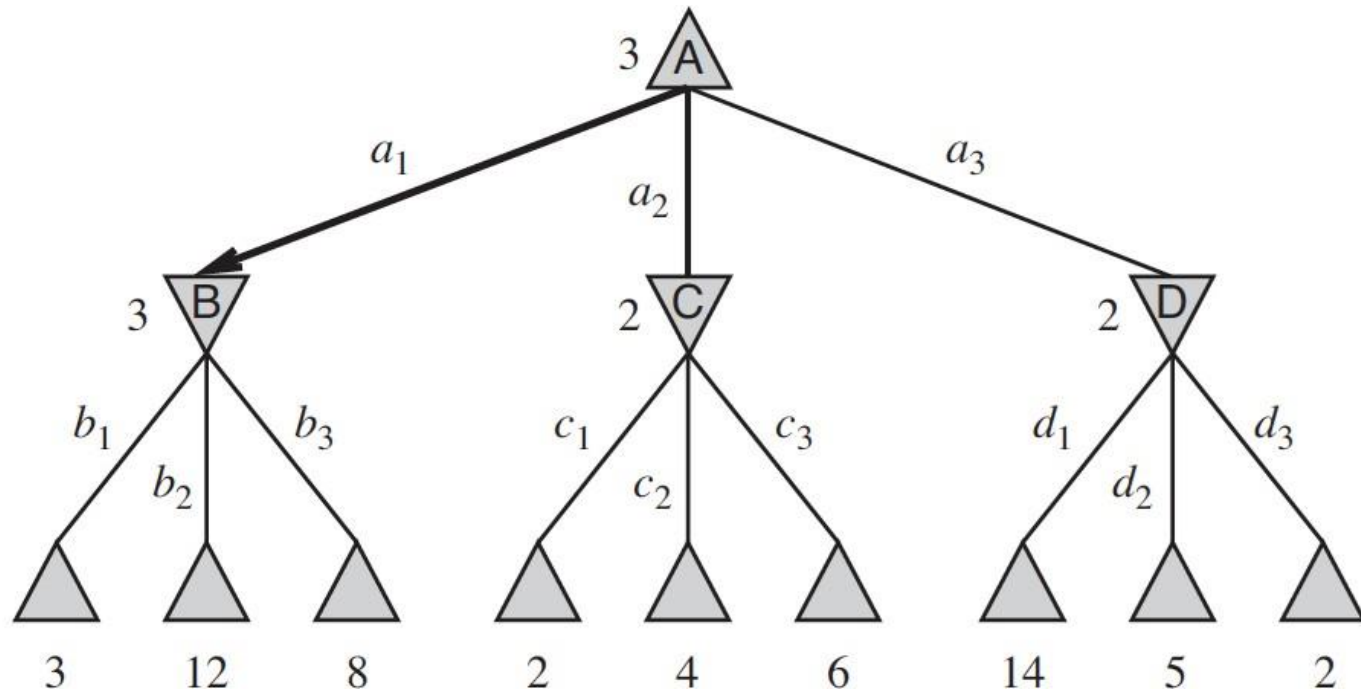
Alpha Beta Pruning



Book Example

MAX

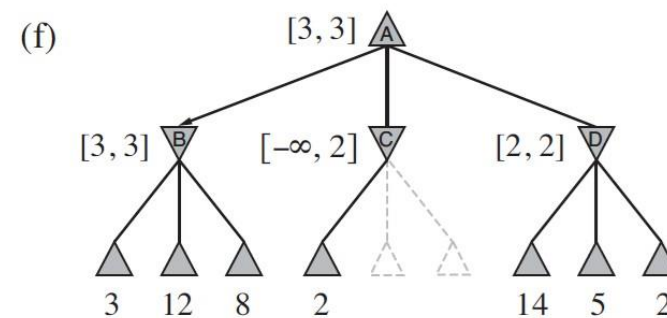
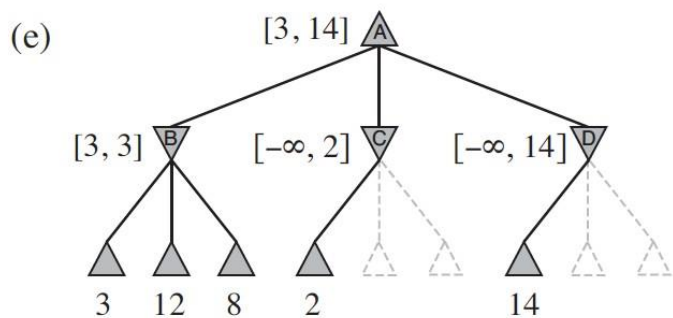
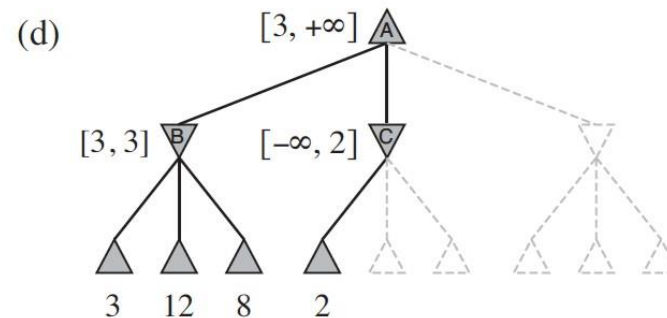
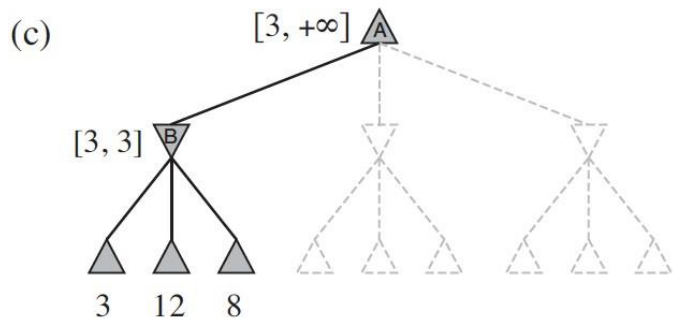
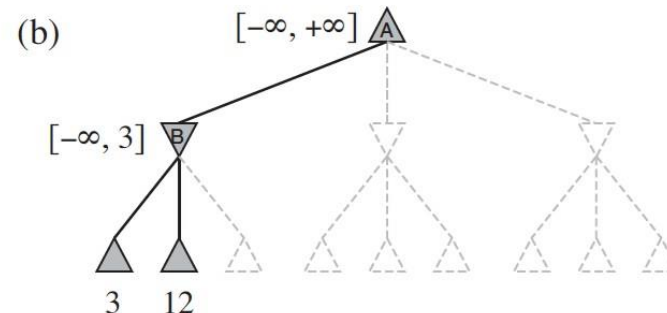
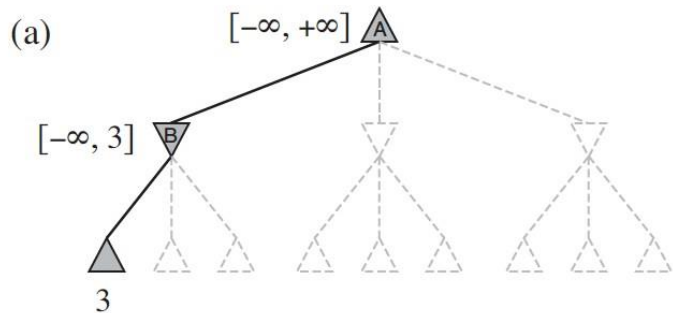
MIN



Alpha Beta Pruning



Book Example



Alpha beta Modifications

function ALPHA-BETA-SEARCH(*state*) **returns** an action
 $v \leftarrow \text{MAX-VALUE}(\text{state}, -\infty, +\infty)$
 return the *action* in ACTIONS(*state*) with value *v*

function MAX-VALUE(*state*, α , β) **returns** a utility value
 if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
 $v \leftarrow -\infty$
 for each *a* **in** ACTIONS(*state*) **do**
 $v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(\text{RESULT}(s, a), \alpha, \beta))$
 if $v \geq \beta$ **then return** *v*
 $\alpha \leftarrow \text{MAX}(\alpha, v)$
 return *v*

function MIN-VALUE(*state*, α , β) **returns** a utility value
 if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
 $v \leftarrow +\infty$
 for each *a* **in** ACTIONS(*state*) **do**
 $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(\text{RESULT}(s, a), \alpha, \beta))$
 if $v \leq \alpha$ **then return** *v*
 $\beta \leftarrow \text{MIN}(\beta, v)$
 return *v*

Is it possible to compute the minimax decision for a node without looking at every successor node?

Alpha – beta Pruning

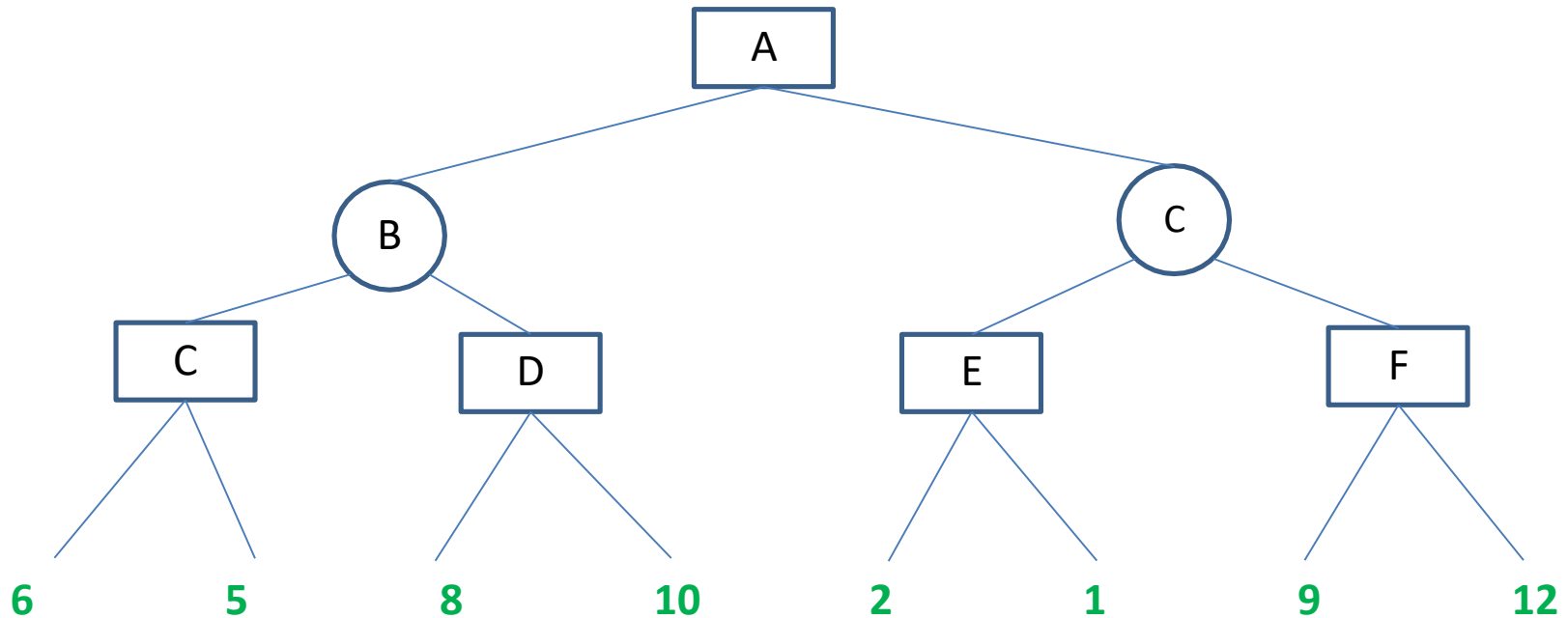
Steps in Alpha – Beta Pruning:

1. At root initialize $\alpha = -\infty$ and $\beta = +\infty$. This is to set the worst case boundary to start the algorithm which aims to increase α and decrease β as much as optimally possible
2. Navigate till the depth / limit specified and get the static evaluated numeric value.
3. For every value VAL being analyzed : Loop till all the leaf/terminal/specified state level nodes are analyzed & accounted for OR until **$\beta \leq \alpha$** .
 1. If the player is MAX :
 1. If $VAL > \alpha$
 2. then reset $\alpha = VAL$
 3. also check **if** $\beta \leq \alpha$ **then** tag the path as unpromising (TO BE AVOIDED) **and** prune the branch from game tree. Rest of their siblings are not considered for analysis
 2. Else if the player is MIN:
 1. If $VAL < \beta$
 2. then reset $\beta = VAL$
 3. also check **if** $\beta \leq \alpha$ **then** tag the path as unpromising (TO BE AVOIDED) **and** prune the branch from game tree. Rest of their siblings are not considered for analysis

Alpha Beta Pruning - Another Example



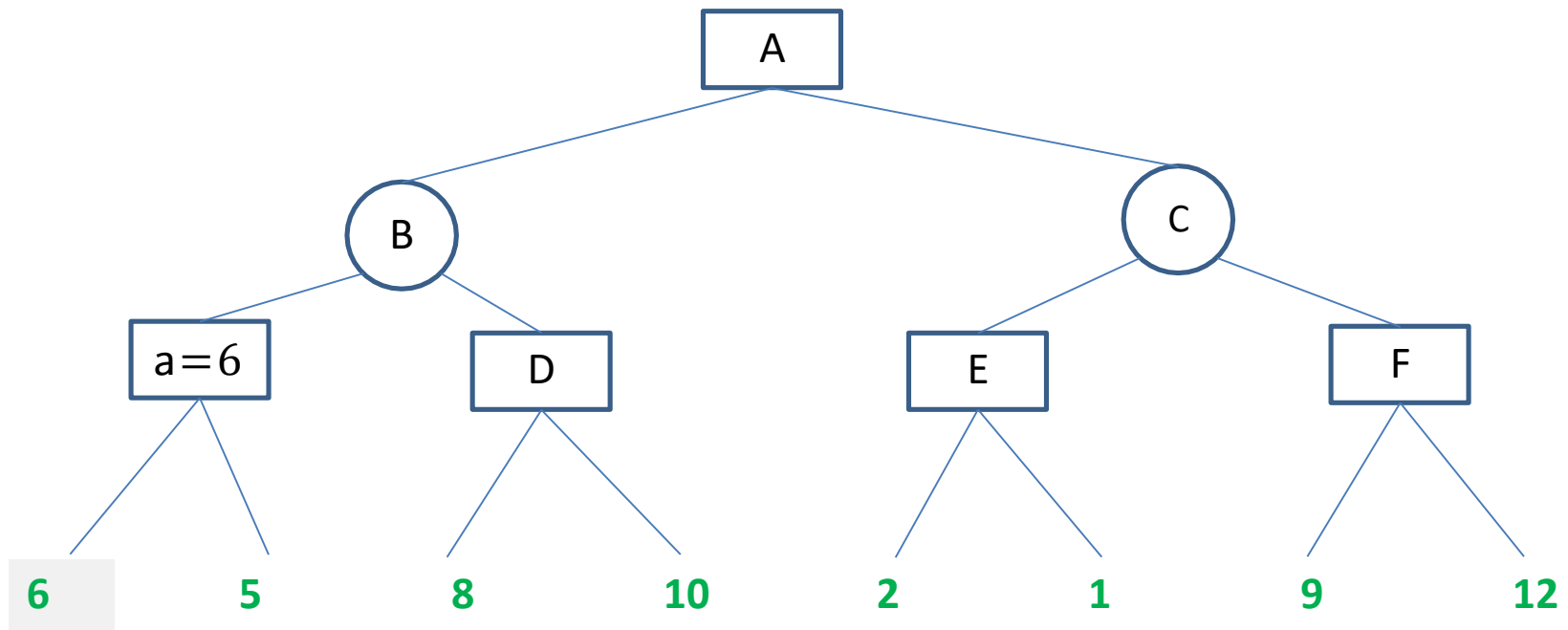
Idea –Pruning



Alpha Beta Pruning



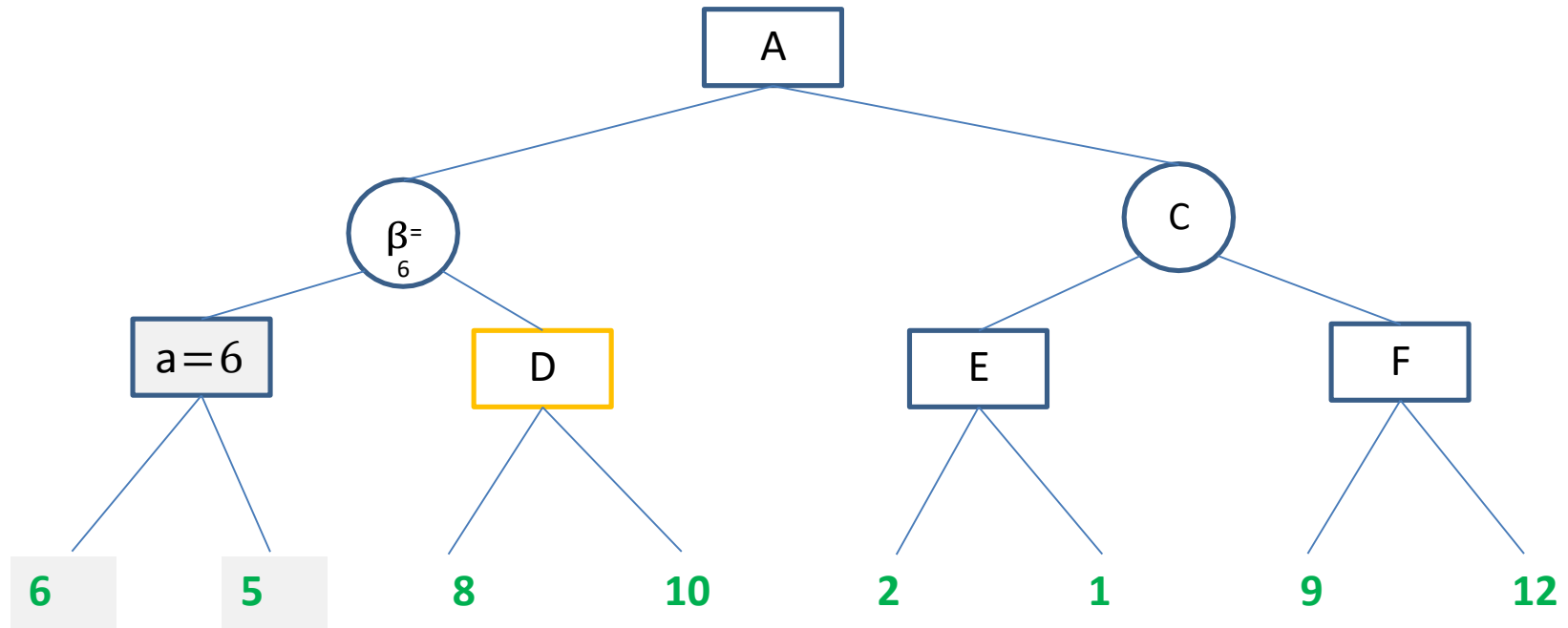
Idea –Pruning



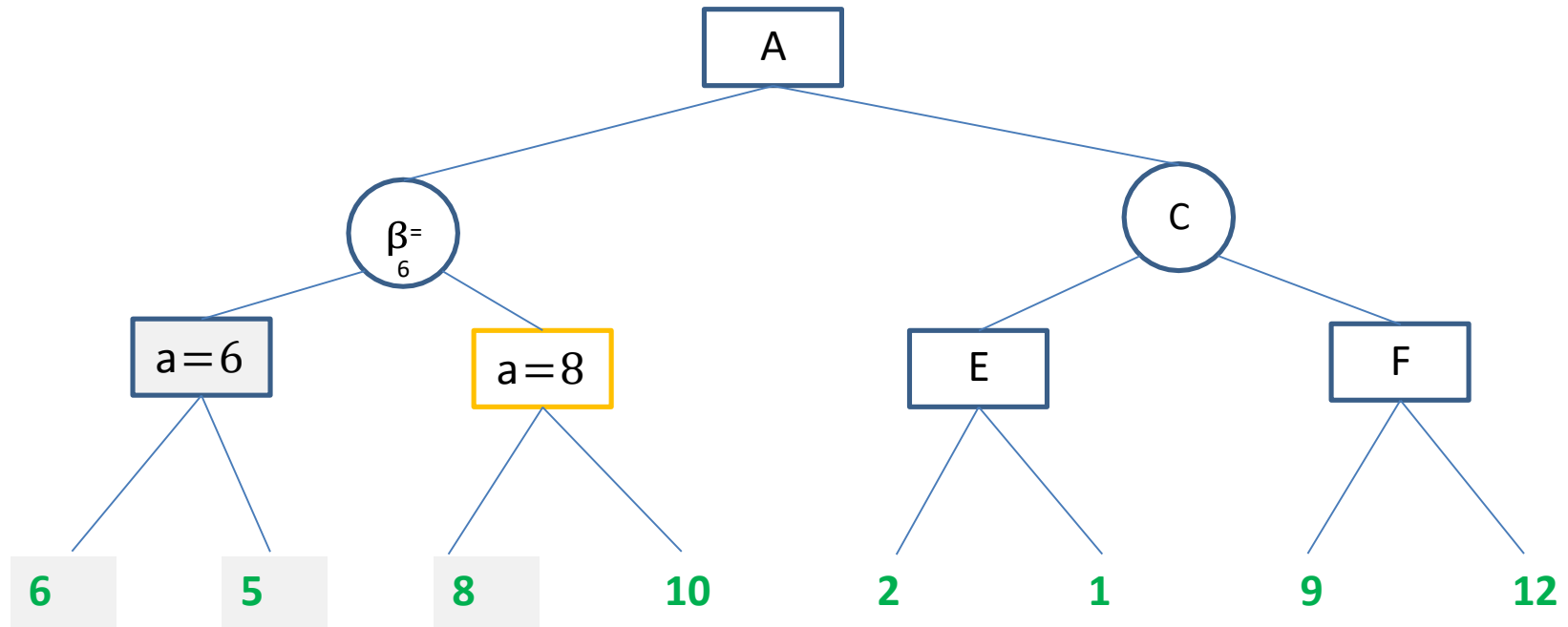
Alpha Beta Pruning



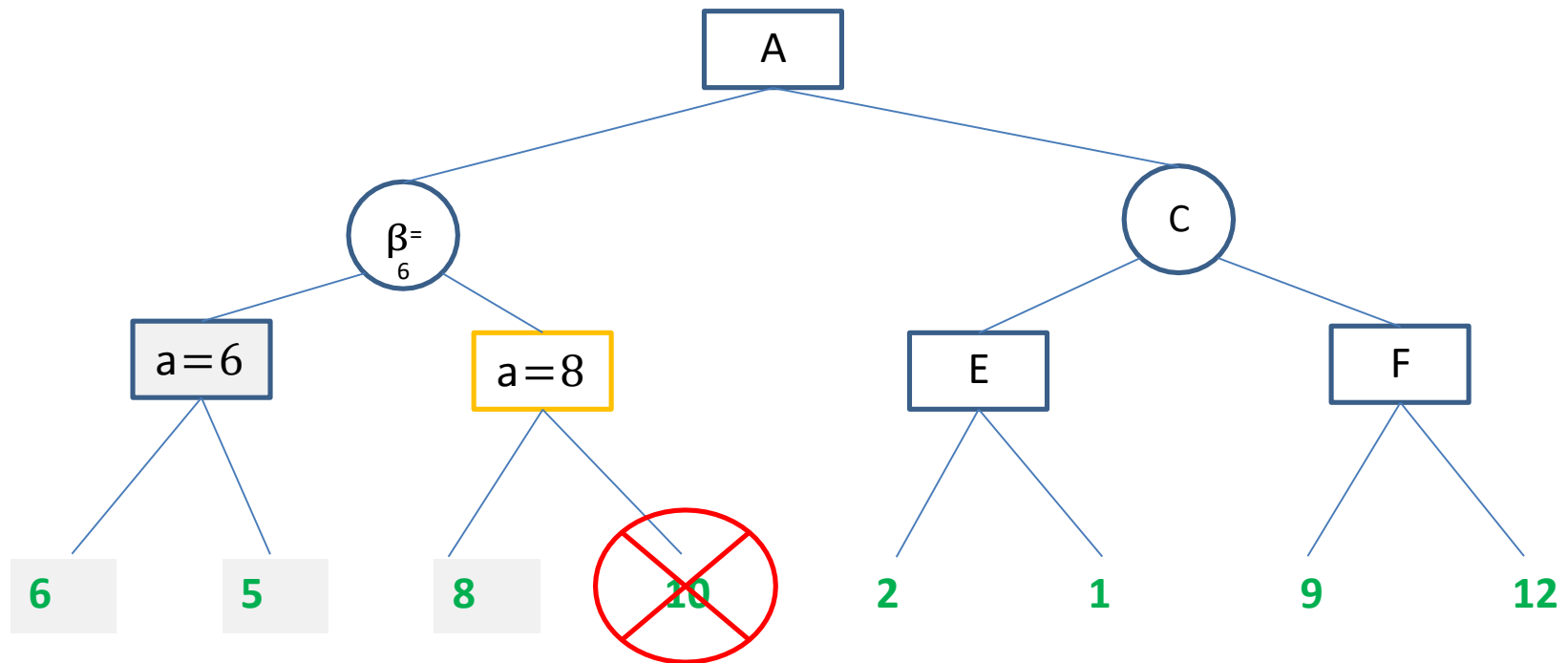
Idea –Pruning



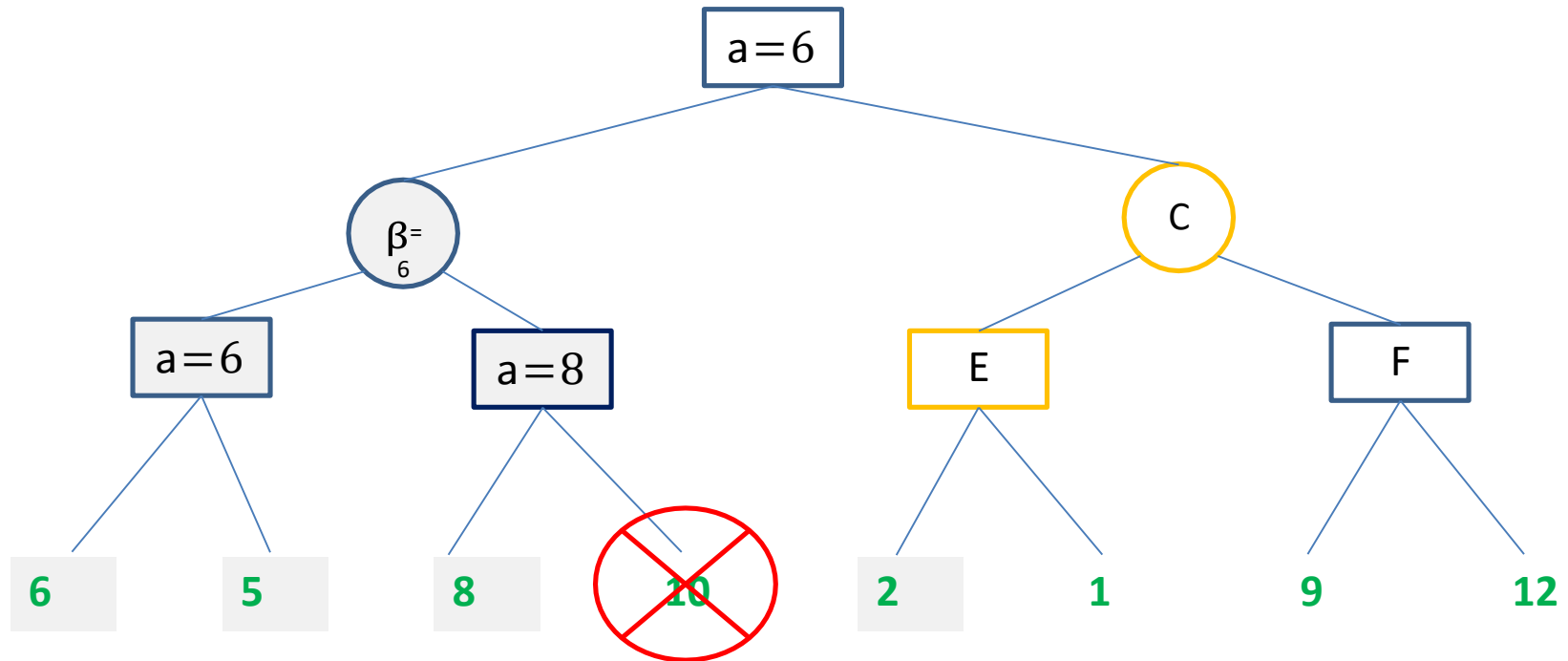
Idea – Alpha Pruning



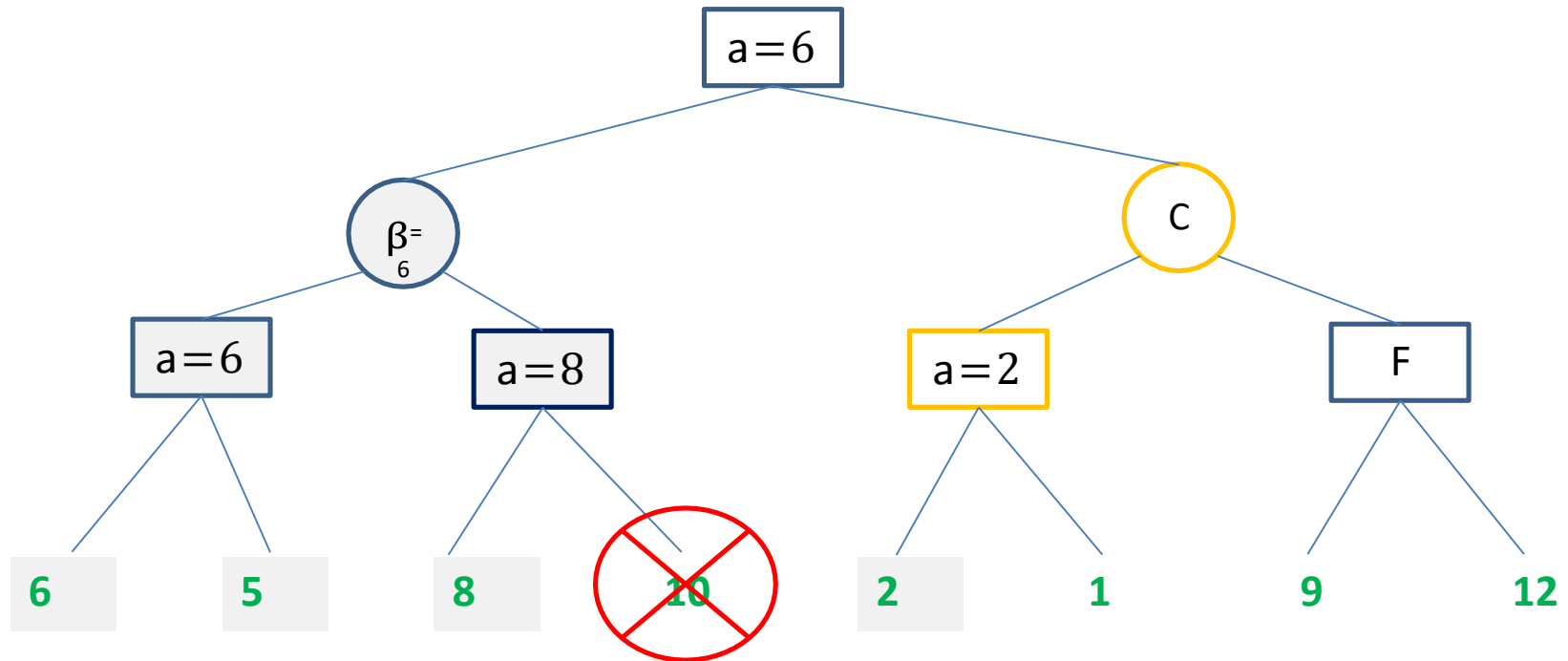
Idea – Beta Pruning



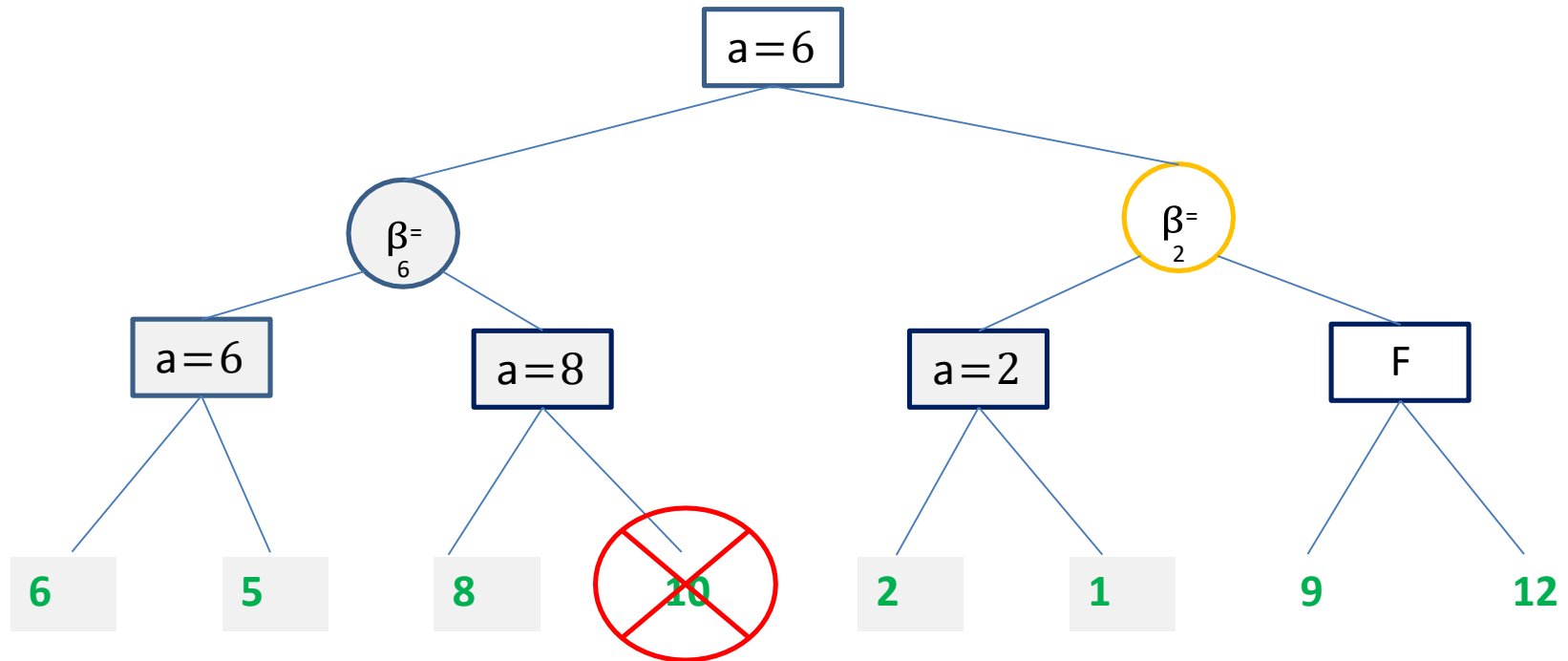
Idea –Pruning



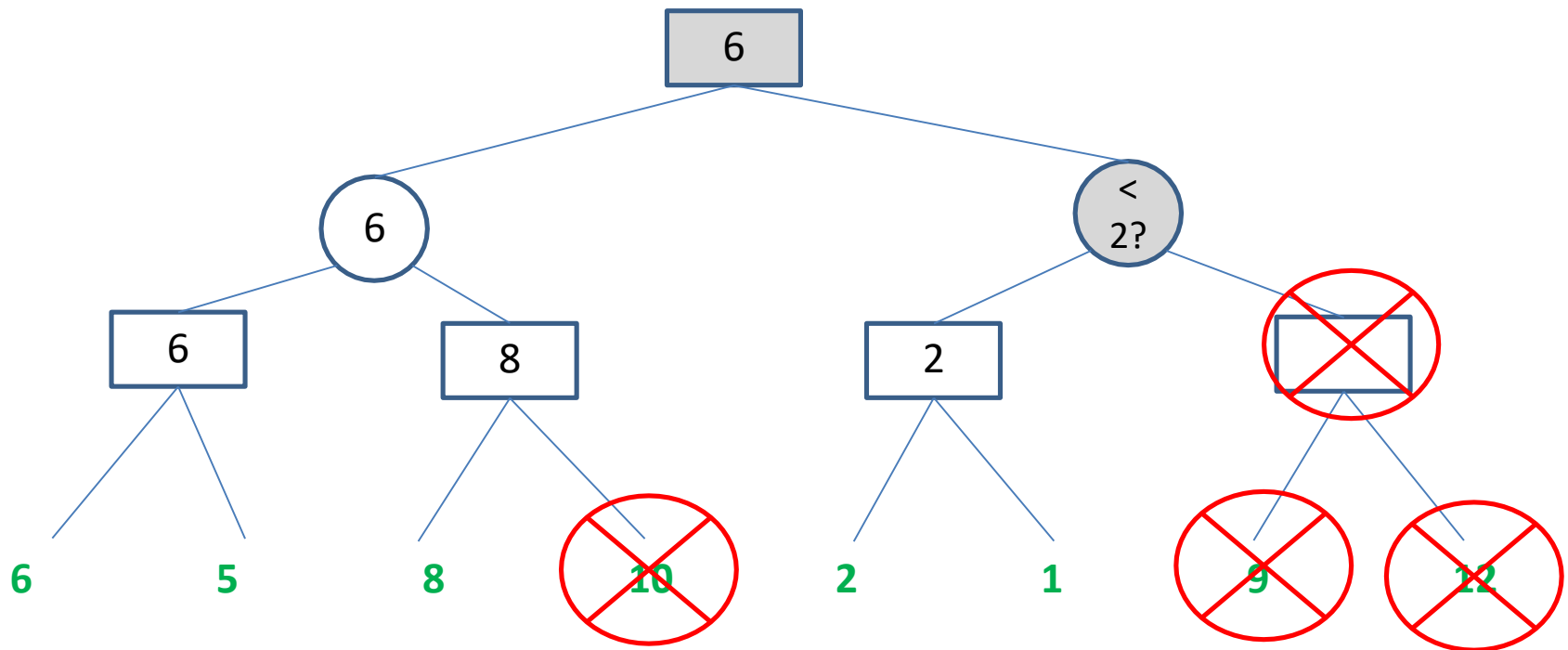
Idea –Pruning



Idea –Pruning



Idea – Alpha Pruning



Alpha – Lower bound of Maximizer's value. Perceived value that Maximizer hopes to against a competitive Minimizer

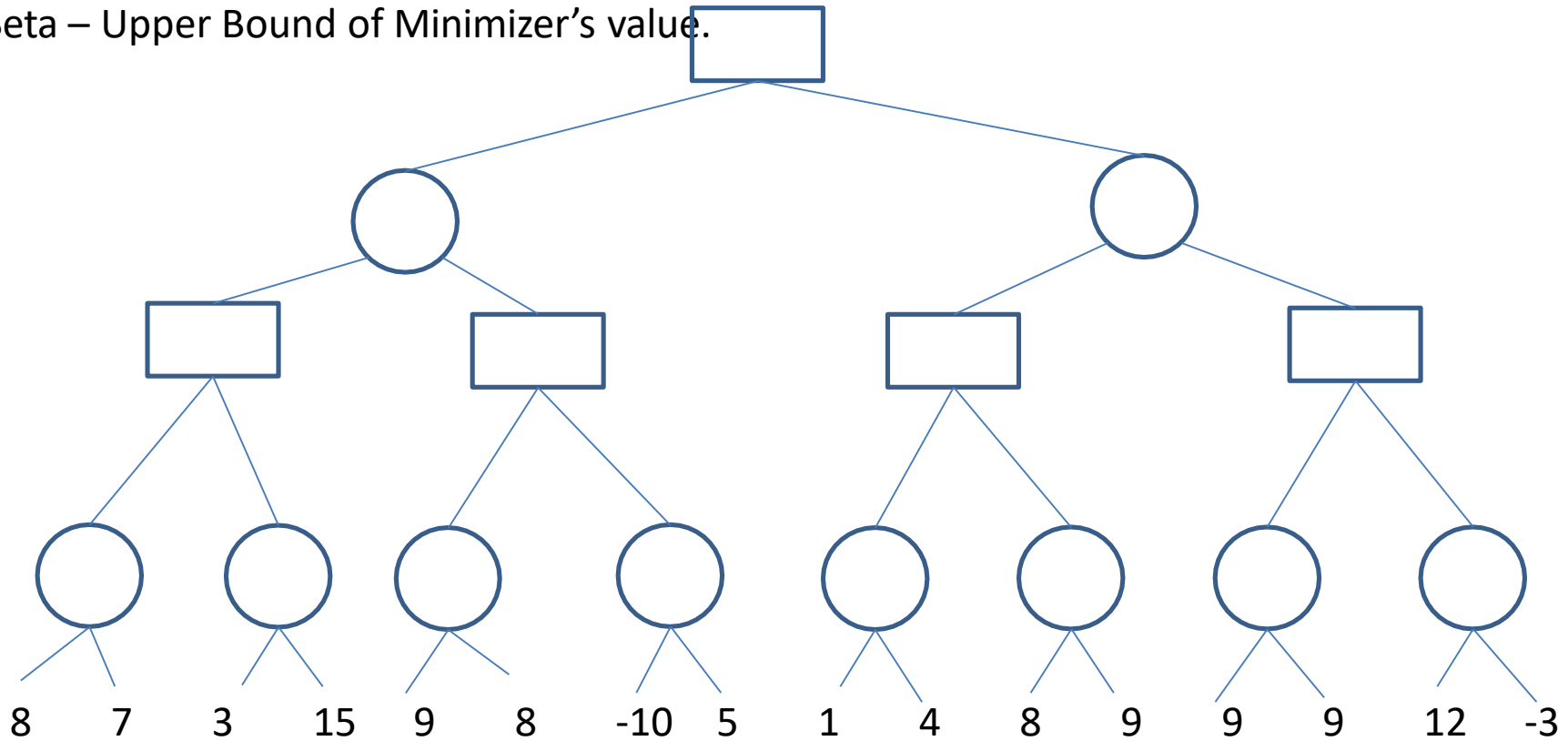
Alpha – beta Pruning – Example -4



Do for practice.

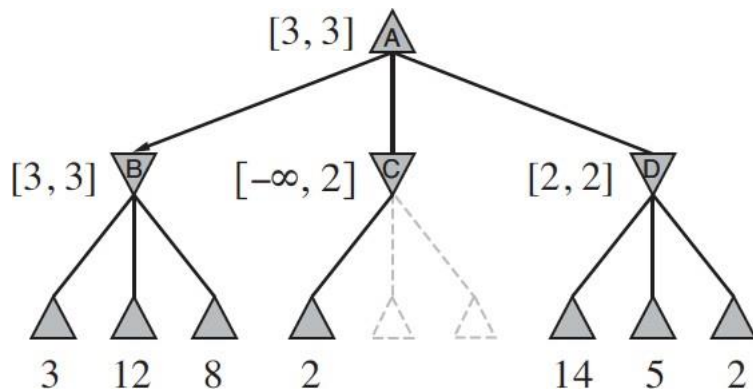
Alpha – Lower bound of Maximizer's value. Perceived value that Maximizer hopes to get with a competitive Minimizer

Beta – Upper Bound of Minimizer's value.



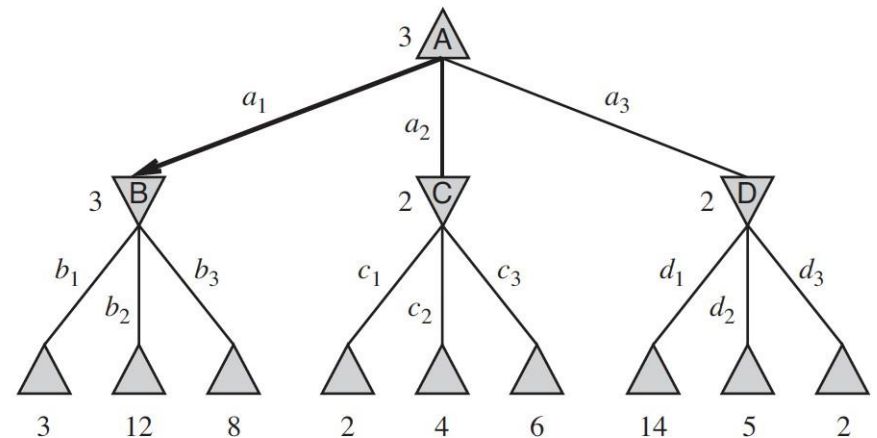
Computational Efficiency

How to reduce the move generations better along while doing Alpha-Beta Pruning?

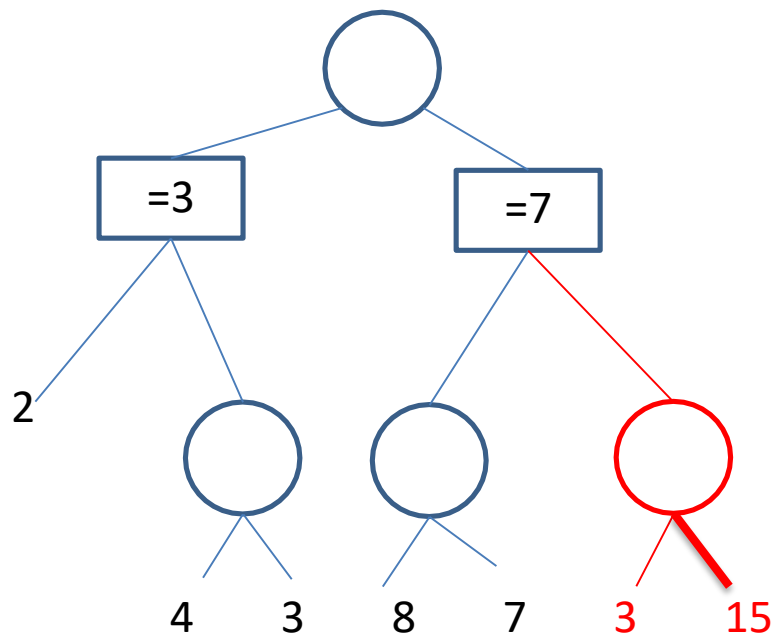


MAX

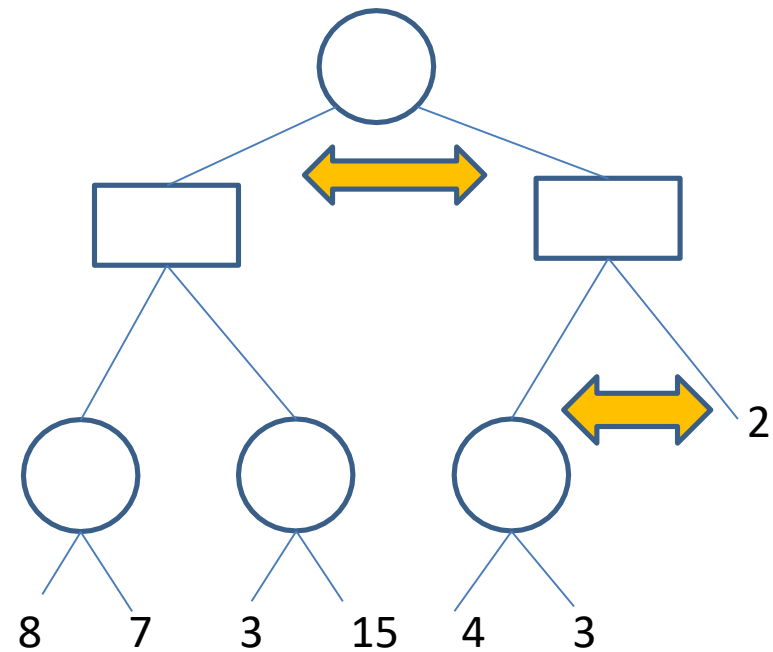
MIN



After Move Ordering



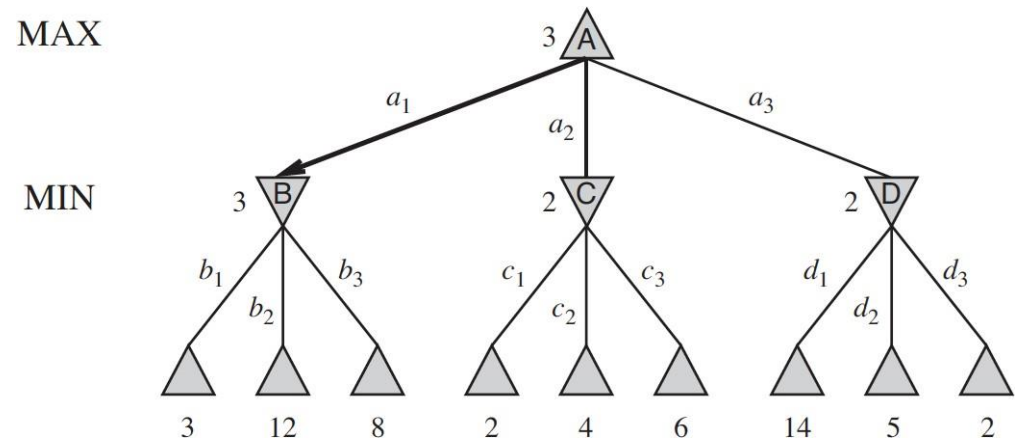
Before Move Ordering



Gaming (Imperfect Decisions)

Computational Efficiency

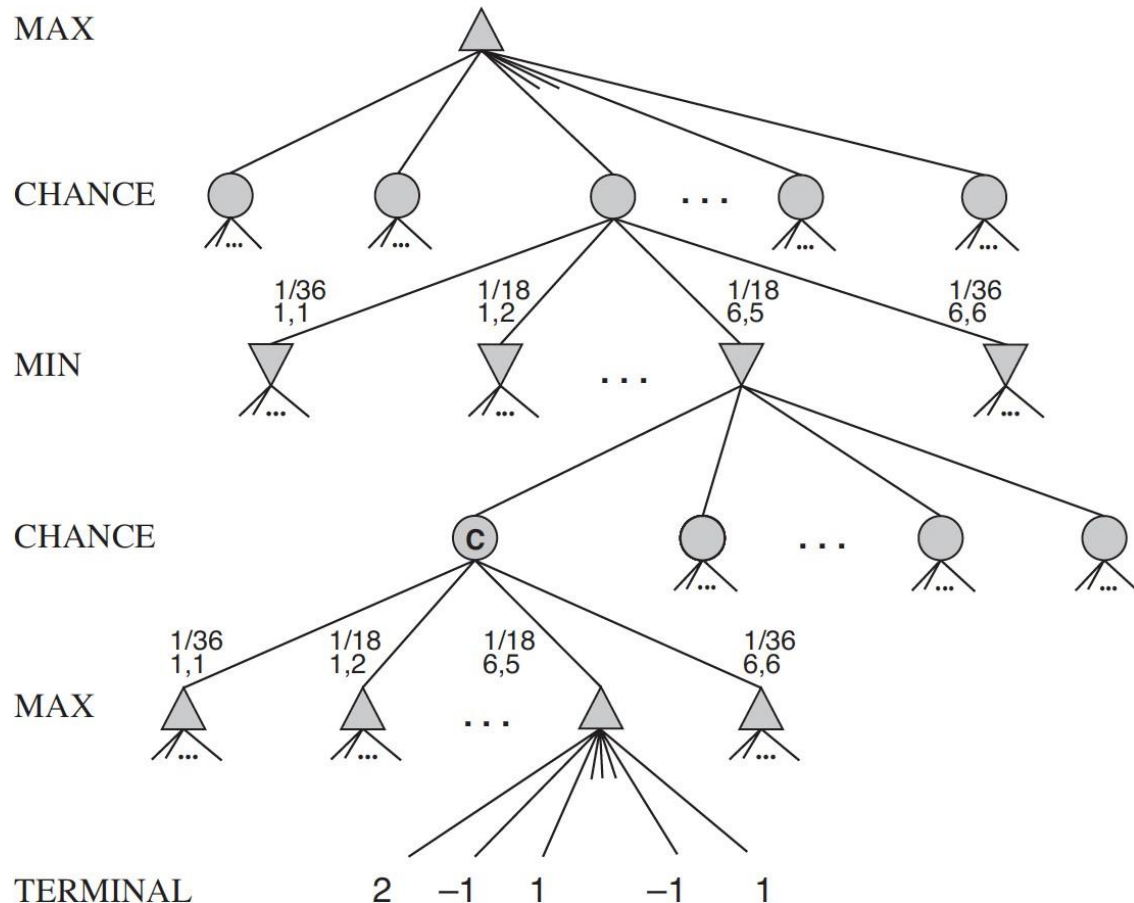
How games can be designed to handle imperfect decisions in real-time?



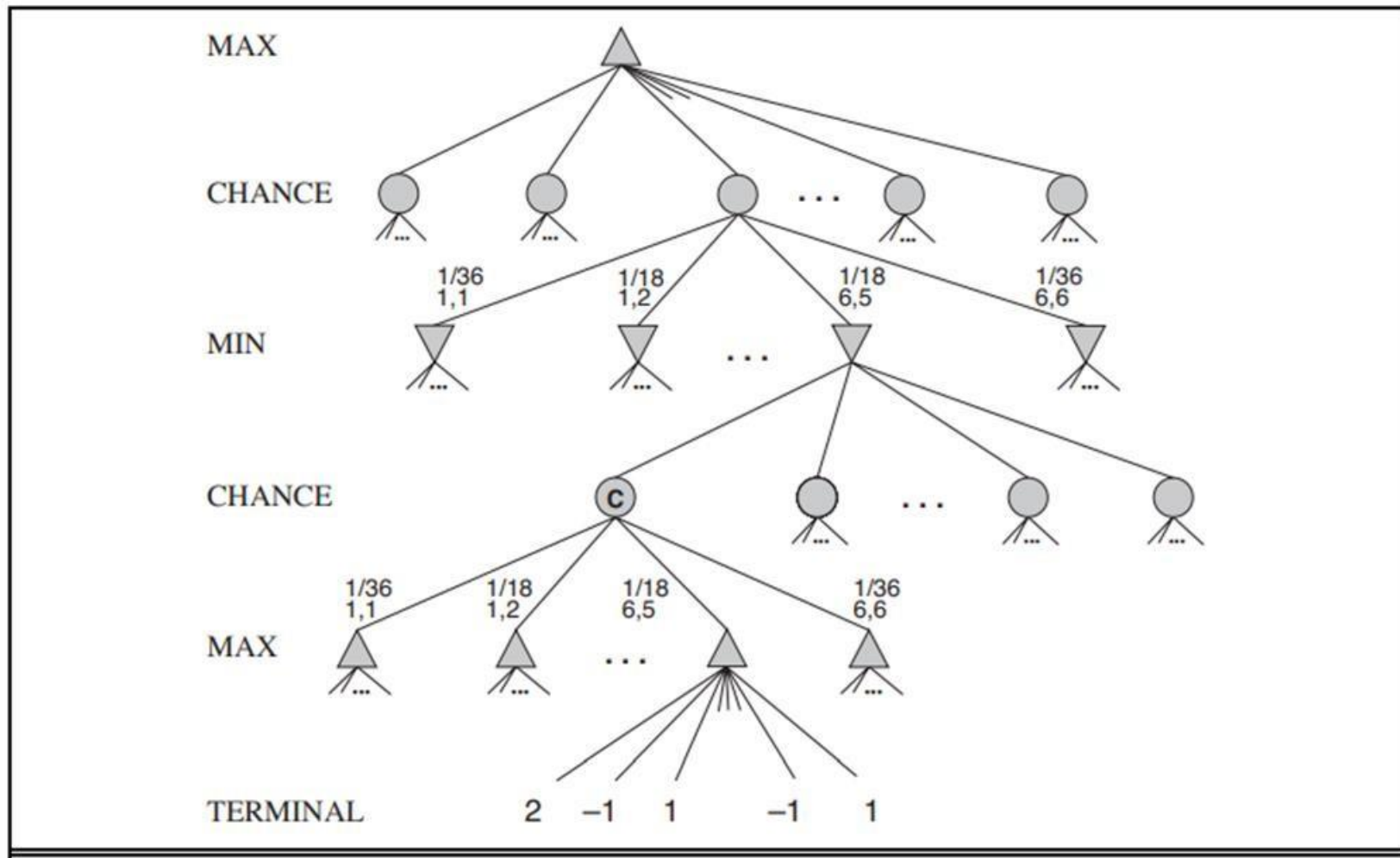
Computational Efficiency

Idea : Chance Node:

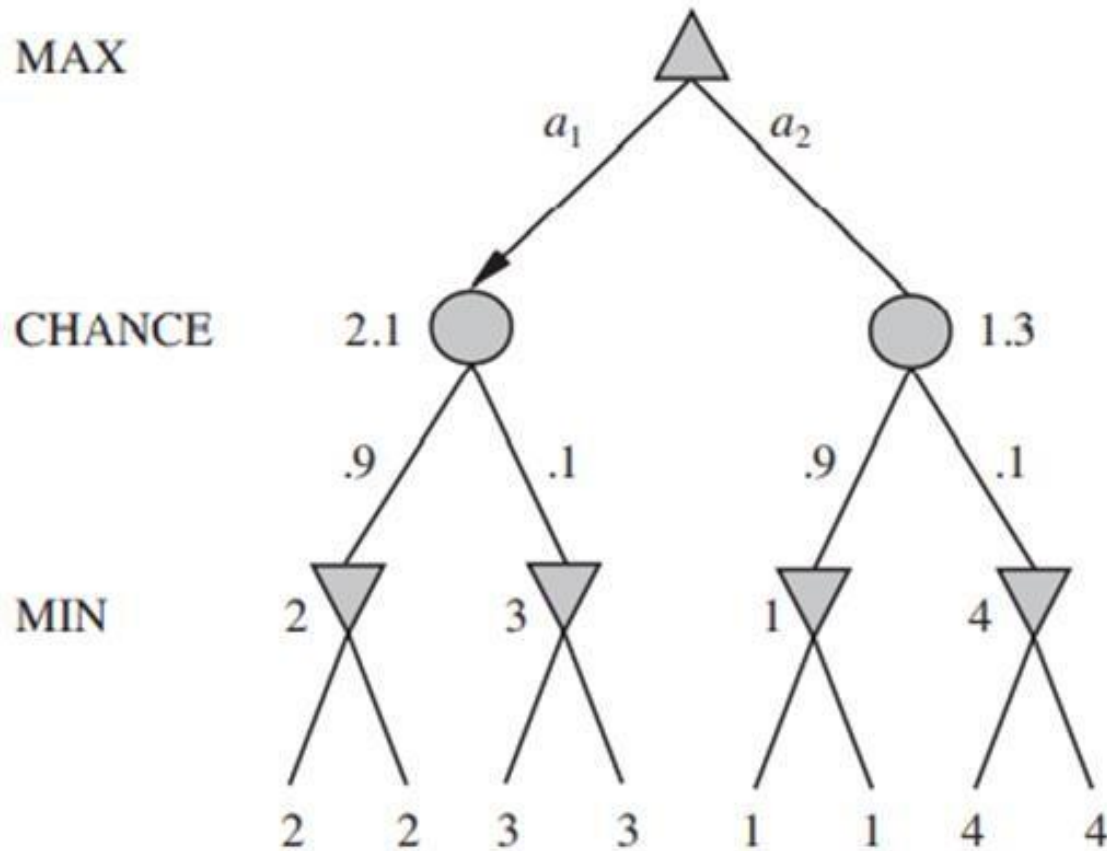
Holds the expected values that are computed as a sum of all outcomes weighted by their probability (of dice roll)

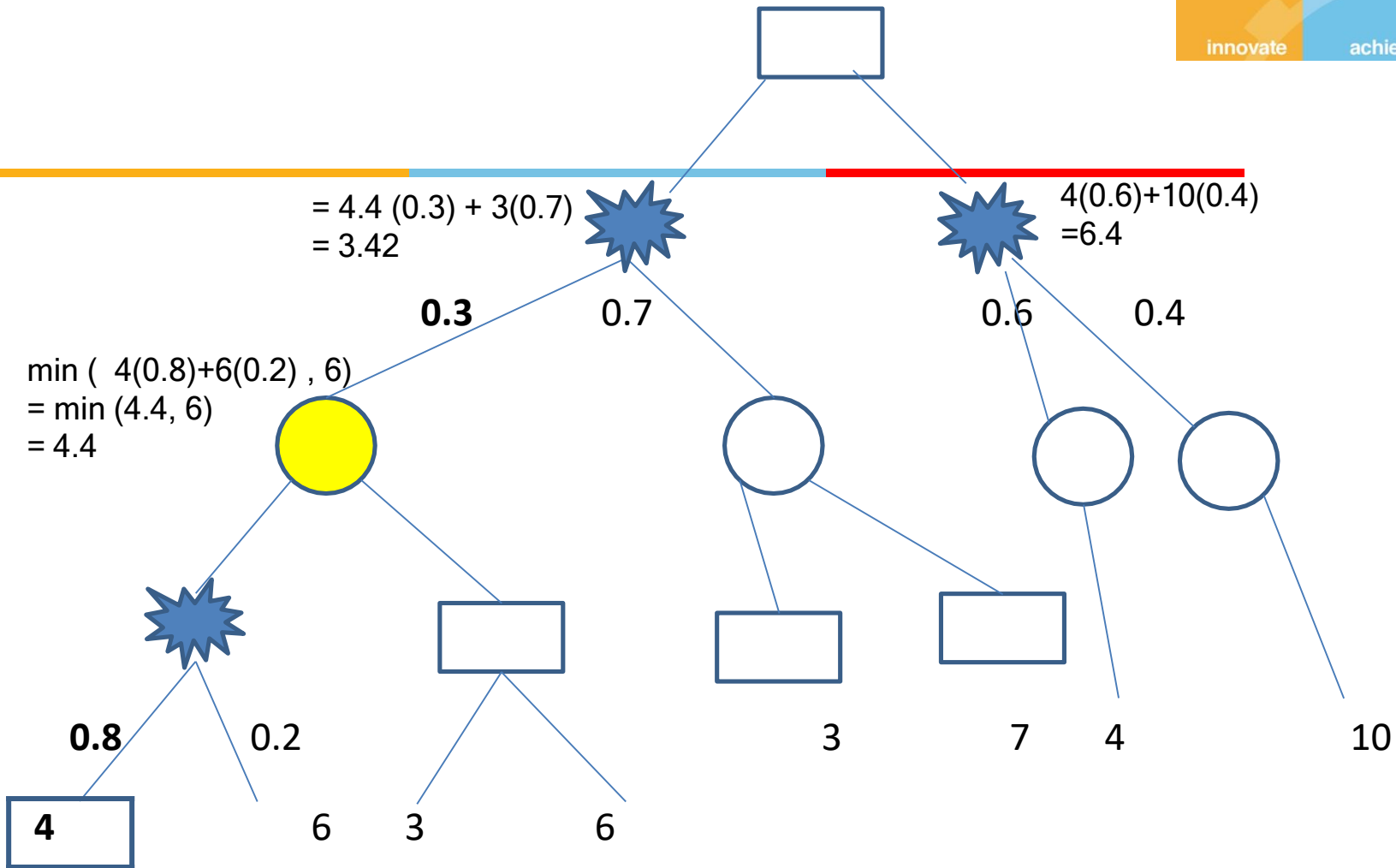


Expecti Mini Max Algorithm



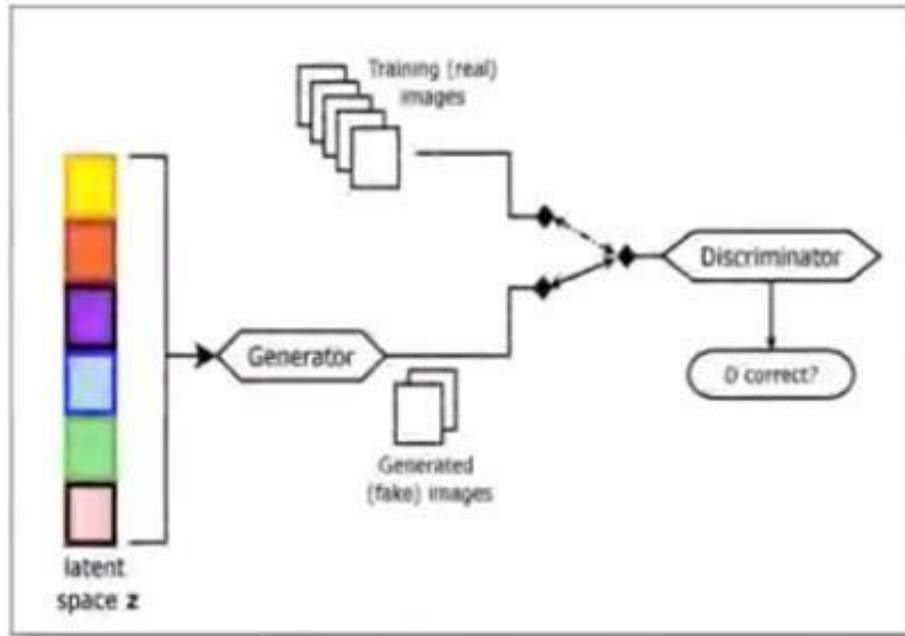
Expecti Mini Max Algorithm





Game Playing (Interesting Case Studies)

Games in Image Processing



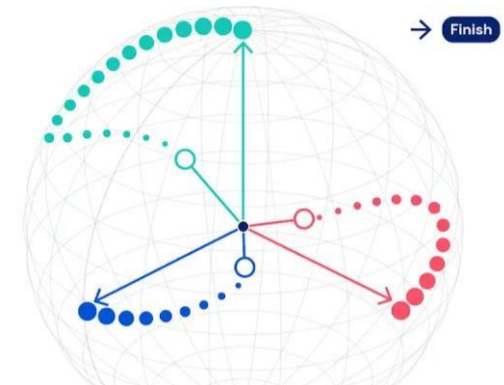
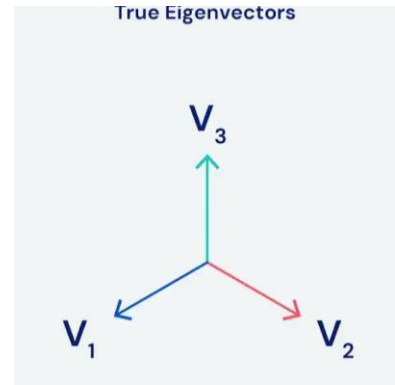
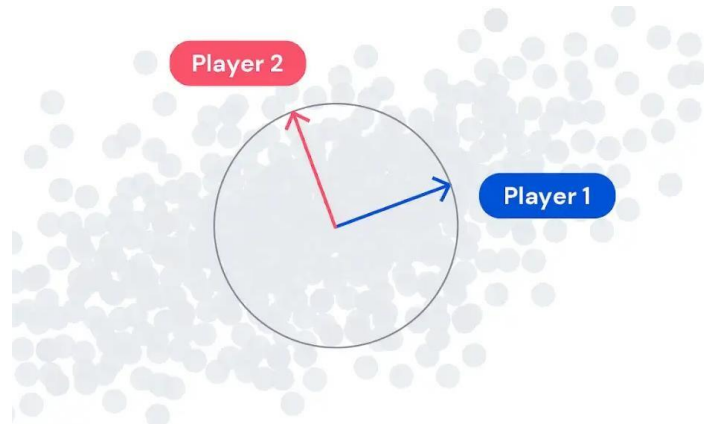
Source Credit:

[2019 - Analyzing and Improving the Image Quality of StyleGAN](#)

[Tero Karras, Samuli Laine, Miika Aittala, Janne Hellsten, Jaakko Lehtinen, Timo Aila](#)

<https://thispersondoesnotexist.com/>

Games in Feature Engineering




Source Credit:

<https://deepmind.com/blog/article/EigenGame>

2021 - EigenGame: PCA as a Nash Equilibrium , Ian Gemp, Brian McWilliams, Claire Vernade, Thore Graepel

Games in Feature Engineering

$$\text{Utility}(v_i | v_{j < i}) = \boxed{\text{Var}(v_i)} - \sum_{j < i} \boxed{\text{Align}(v_i, v_j)}$$


Source Credit:

<https://deepmind.com/blog/article/EigenGame>

2021 - EigenGame: PCA as a Nash Equilibrium , Ian Gemp, Brian McWilliams, Claire Vernade, Thore Graepel

Required Reading: AIMA - Chapter # 4.1, #4.2, #5.1

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials