22/12/2024

ISM - Assignment 1 Section-9

Qustion 1

a) Compute mean, median, first quartile & third quartile

Given: Heights of 25 students: Heights = {62, 65, 59, 68, 70, 66, 71, 63, 64, 72, 69, 67, 68, 66, 64, 70, 65, 63, 61, 67, 68, 62, 66, 70, 73}

1. MEAN:

$$\text{Mean} = \frac{\sum \text{Heights}}{\text{No. of Students}}$$ formula

$$\text{Mean}: \frac{62+65+59+68+70+66+71+63+64+72+69+67+68+66+64+70+65+63+61+67+68+62+66+70+73}{25}$$

$$: \frac{1659}{25} = \boxed{66.360}$$ Answer

2. MEDIAN: Middle value in ordered data.

Sort the data in acending order: {59, 61, 62, 62, 63, 64, 64, 65, 65, 66, 66, 66, 66, 67, 67, 68, 68, 68, 69, 70, 70, 70, 71, 73}

25 data point, $\left(\frac{n+1}{2}\right)^{th}$ value because of odd

$\left(\frac{26}{2}\right)^{th} \rightarrow 13^{th}$ value in the sorted list

$\boxed{\text{Median} = 66}$

AMULYA GUPTA
2024ab05200

Amulya Gupta
Section 9
2024ab05200

3. FIRST QUARTILE RANGE : The 25^{th} percentile of a data set

Q1 is the median of the lower half of the data {12 value}

{59, 61, 62, 62, 63, 63, 64, 64, 65, 65, 66, 66}

$$Q1 = \left(n+1 \times \frac{1}{4}\right)^{th} = \left(\frac{26}{4}\right)^{th} = (6.5)^{th} \text{ i.e } (6^{th} \,\&\, 7^{th} \text{ value})$$

$$Q_1 = \frac{63+64}{2} = \boxed{64} \text{ Answer}$$

4. THIRD QUARTILE (Q3) : The 75^{th} percentile of a data set

Q3 is the median of the upper half of the data {12 values)

{66, 66, 67, 67, 68, 68, 68, 69, 70, 70, 70, 71, 73}

$$Q3 = \left(n+1 \times \frac{3}{4}\right)^{th} = \left(26 \times \frac{3}{4}\right)^{th} = 19.5 \text{ i.e } \{18^{th} \,\&\, 19^{th} \text{ value})$$

$$Q_3 = \frac{68+69}{2} = \boxed{69}$$

b) Compute Variance, Standard Deviation, Range & IQR Outliers

Mean = 66.36

1. Variance :

$$\text{Variance} = \frac{\sum (\text{Height} - \text{Mean})^2}{n-1}$$

$$\text{Deviations} = \{(-4.36)^2 + (-1.36)^2 + (-7.36)^2 + (1.64)^2 + (3.64)^2 + (-0.36)^2 + (4.64)^2 + (-3.36)^2 + (-2.36)^2 + (5.64)^2 + (2.64)^2 + (0.64)^2 + (1.64)^2 + (1.64)^2 + (-0.36)^2 + (-2.36)^2$$

$$\text{Deviations} = \{ \ldots (3.64)^2 + (-1.36)^2 + (-3.36)^2 + (-5.36)^2 + (0.64)^2 + (1.64)^2 + (-4.36)^2 + (-0.36)^2 + (3.64)^2 + (6.64)^2 \}$$

$$= \{ 19.0096 + 1.8496 + 54.1696 + 2.6896 + 13.2496 + 0.1296 + 21.5296 + 11.2896 + 5.5696 + 31.8096 + 6.9696 + 0.4096 + 2.6896 + 0.1296 + 5.5696 + 13.2496 + 1.8496 + 11.2896 + 28.7296 + 0.4096 + 2.6896 + 19.0096 + 0.1296 + 13.2496 + 44.0896 \}$$

$$= 311.76$$

$$\text{Variance} = \frac{311.76}{24} = \boxed{12.99 = \sigma^2}$$

2. Standard Deviation : $\sqrt{\text{variance}}$

$$s = \sqrt{12.99} \quad \boxed{= 3.6041}$$

3. Range : Maximum − Minimum

$$= 73 - 59 = \boxed{14}$$

4. Interquartile Range (IQR) :

$$IQR = Q_3 - Q_1$$

$$IQR = 69 - 64 = \boxed{5}$$

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5. Outliers: 1.5 * IQR rule.

$$\text{Lower Bound} = Q_1 - 1.5 \times IQR = 64 - 7.5 = 56.5$$

$$\text{Upper Bound} = Q_3 + 1.5 \times IQR = 69 + 7.5 = 76.5$$

Since all data points $(59 \leq \text{Height} \leq 73)$ fall within this range,

$\boxed{\text{No Outliers}}$

c) Skewness:

$$\text{Formula} = \frac{3 \times (\text{Mean} - \text{Median})}{\text{Standard Deviation}}$$

$$= \frac{3 \times (66.36 - 66)}{3.60}$$

$\boxed{\text{Skewness} = 0.3000}$

Thes positive value indicates slight positive skewness that means when the right side of the distribution is longer than the left

Mean value is higher than the median by 0.36 slight +ve

Student with heights below the mean, the data is nearly symmetrical

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d) Result: The heights are centered around 66 inches suggesting most students have similar heights, with only slight deviation

Absence of outliers confirms that there are no exceptionally short or tall students in the classroom

2. Question 2

Given: $P(A) = \frac{3}{4}$ $P(B) = \frac{2}{5}$

i) $\frac{3}{20} \leq P(A \cap B) \leq \frac{2}{5}$

ii) $P(A \cup B) \geq \frac{3}{4}$

a) Showing $\frac{3}{20} \leq P(A \cap B) \leq \frac{2}{5}$

Finding the lower bound of $P(A \cap B)$:

1 From the inclusion-exclusion principle:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\therefore \quad P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

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Properties of probabilities

$$P(A \cup B) \leq 1$$

$\therefore$

$$1 \geq P(A) + P(B) - P(A \cap B)$$

Substituting the values from the given

$$1 \geq \frac{3}{4} + \frac{2}{5} - P(A \cap B)$$

LCM = 20

$$1 \geq \frac{15+8}{20} - P(A \cap B)$$

$$1 \geq \frac{23}{20} - P(A \cap B)$$

$$P(A \cap B) \geq \frac{23}{20} - 1 = \frac{3}{20}$$

therefore,

$$\frac{3}{20} \leq P(A \cap B)$$

2. Finding the upper bound of $P(A \cap B)$:

$$P(A \cap B) \leq P(B)$$ This occurs when $B \subseteq A$

$$P(A \cap B) \leq \frac{2}{5}$$

$$\boxed{\frac{3}{20} \leq P(A \cap B) \leq \frac{2}{5}}$$

Proved

b) Show that $P(A \cap B) \geq \frac{3}{4}$

using principle of inclusion-exclusion:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

substituting value

$$P(A \cup B) \geq \frac{3}{4} + \frac{2}{5} - P(A \cap B)$$

$$P(A \cup B) \geq \frac{15+8}{20} - P(A \cap B)$$

$$P(A \cup B) \geq \frac{23}{20} - P(A \cap B)$$

since we know $P(A \cap B) \leq \frac{2}{5}$

$$P(A \cup B) \geq \frac{23}{20} - \frac{2}{5}$$

$$P(A \cup B) \geq \frac{23-15}{20}$$

$$\boxed{P(A \cup B) \geq \frac{3}{4}}$$

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Question 3

Given :

Bag A : 3 apples + 4 mango = 7 fruits

Bag B : 4 apples + 3 mangos = 7 fruits

Die Toss :

If the die lands on 1 or 3, draw from Bag A
If the die lands on (2,4,5,6) draw from Bag B

$$P(1 \text{ or } 3) = \frac{2}{6} = \frac{1}{3}$$

$$P(2,4,5 \text{ or } 6) = \frac{4}{6} = \frac{2}{3}$$

Define the event :

Let event A : Drawing a fruit from Bag A

$$P(A) = \frac{1}{3}$$

Let event B : Drawing a fruit from Bag B

$$P(B) = \frac{2}{3}$$

Probability of drawing a mango from

From Bag A:

$$P(\text{Mango}|\text{Bag}A) = \frac{\text{Number of mangoes in Bag A}}{\text{Total fruits in Bag A}}$$

$$= \frac{4}{7}$$

From Bag B:

$$P(\text{Mango}|\text{Bag B}) = \frac{\text{Number of mangoes in Bag B}}{\text{Total fruits in Bag B}}$$

$$= \frac{3}{7}$$

Law of Total Probability

$$P(\text{Mango}) = P(\text{Mango}|\text{Bag A}) \cdot P(A) + P(\text{Mango}|\text{Bag B}) \cdot P(B)$$

$$= \frac{4}{7} \cdot \frac{1}{3} + \frac{3}{7} \times \frac{2}{3}$$

$$= \frac{4+6}{21} = \frac{10}{21}$$

$$\boxed{P(\text{Mango}) = \frac{10}{21} = 0.47619 \approx 47.619\%}$$

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Question 4:

Given:

Let A be the event "the person is infected"

Let B be the event "the person tests +ve"

Let $\bar{A}^c$ be the event "the person is not infected"

Let $\bar{B}^c$ be the event "the person tests -ve"

1) $P(A) = \frac{1}{200} = 0.005$

2) $P(\bar{A}^c) = 1 - P(A) = \frac{1 - 200}{200} = 0.995$

3) Test Accuracy:

$P(B|A) = 0.8$ True Positive Rate $P(\text{Positive} | \text{Infected}) = 0.8$

$P(B|+\bar{A}^c) = 0.05$ False Positive Rate $P(\text{Positive} | \text{Not Infected}) = 0.05$

To Find: $P(\text{Not Infected} | \text{Negative})$

Using law of total probablity:

$$P(B) = P(B|A)\,P(A) + P(B|\bar{A}^c)\,P(\bar{A}^c)$$

$$P(B) = (0.8)(0.005) + (0.05)(0.995)$$

$$= 0.004 + 0.04975$$

$$= 0.05375$$

$$P(\bar{B}^c) = 1 - P(B)$$

$$= 1 - 0.05375$$

$$= 0.94625$$

$$P(\bar{B}^c | A) = 1 - P(B|A)$$

$$= 1 - 0.8 \quad = 0.2$$

$$P(\bar{B}^c | \bar{A}^c) = 1 - P(B|\bar{A}^c)$$

$$= 1 - 0.05 \quad = 0.95$$

$$P(\bar{A}^c | \bar{B}^c) = \frac{P(\bar{B}^c | \bar{A}^c) \; P(\bar{A}^c)}{P(\bar{B}^c)}$$ using Bayes's theorem

$$= \frac{(0.95)(0.995)}{0.94625}$$

$$= \frac{0.94525}{0.94625} = 0.998943197$$

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Amulya Gupta
2024ab05200