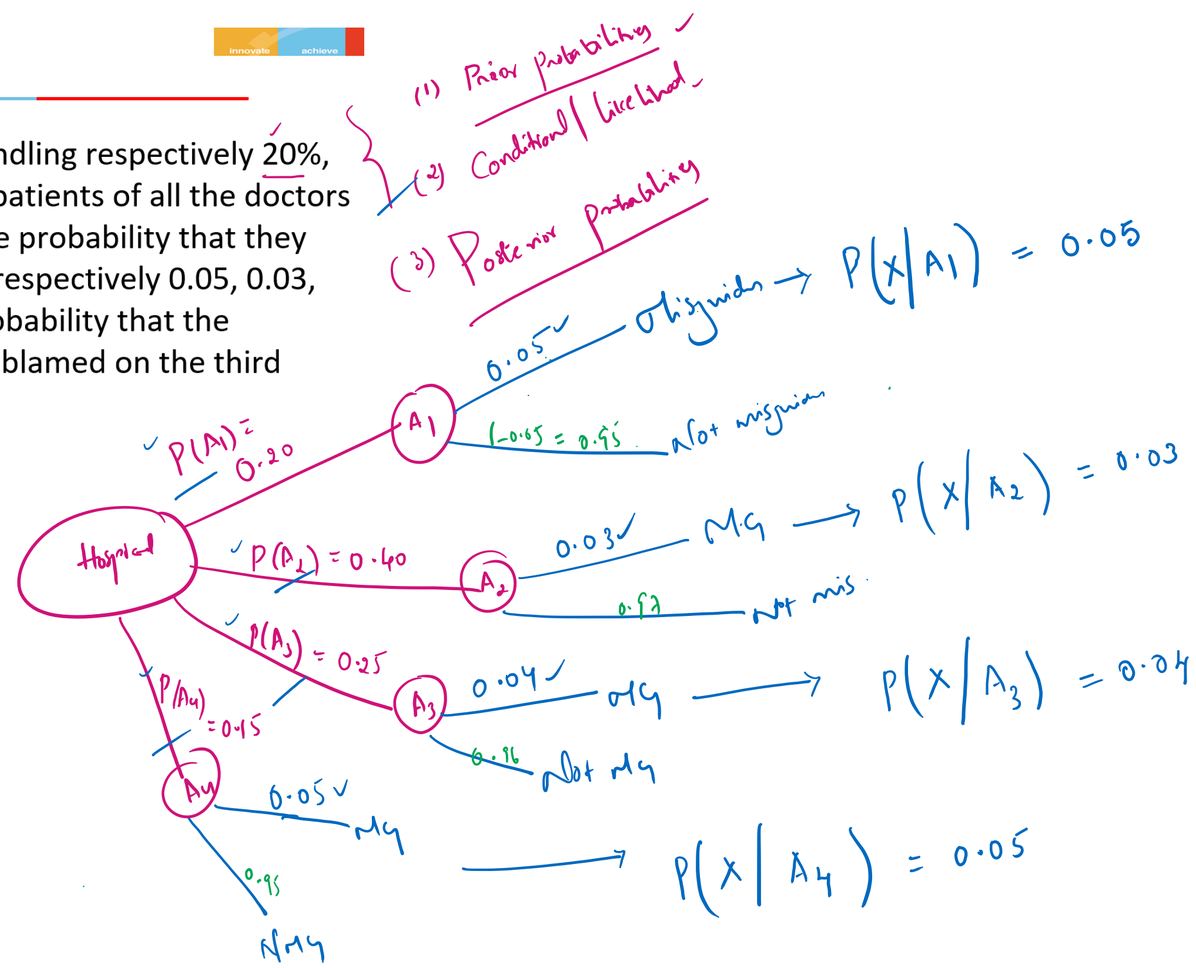
Problem 1

A hospital has 4 nurses handling respectively 20%, 40%, 25% and 15% of the patients of all the doctors coming to the hospital. The probability that they misguide the patients are respectively 0.05, 0.03, 0.04 and 0.05. Find the probability that the misguided incident can be blamed on the third nurse.

(1) Prior probability
(2) Conditional / Likelihood
(3) Posterior probability



X : Patient is misguided

Total Probability:

$$P(X) = P(A_1 \cap X) + P(A_2 \cap X) + P(A_3 \cap X) + P(A_4 \cap X)$$
$$= P(A_1)P(X|A_1) + P(A_2)P(X|A_2) + P(A_3)P(X|A_3) + P(A_4)P(X|A_4)$$
$$= (0.20)(0.05) + (0.40)(0.03) + (0.25)(0.04) + (0.15)(0.05)$$
$$= 0.0395$$

Bayes Law

$$P(A_3|X) = \frac{P(A_3 \cap X)}{P(X)} = \frac{P(A_3)P(X|A_3)}{P(X)} = \frac{(0.25)(0.04)}{0.0395}$$
$$= \frac{0.01}{0.0395} = 0.2532$$

Posterior Probability

≈ 25.32%.

Problem 2

Three machines A, B and C produce respectively 50%, 30% and 20% of the total number of items of a factory. The percentage of defective outputs of these machines are 3% 5% and 2%. An item is selected at random and is found to be defective. (i) Find the probability that the item was produced by machine C? (ii) What is the probability that the item was produced by machine C or B?(iii) What is the probability that an item selected at random is found to be non-defective and also the probability that the item was produced by machine A given that it is non-defective.

$P(A \cup B) = P(A) + P(B)$

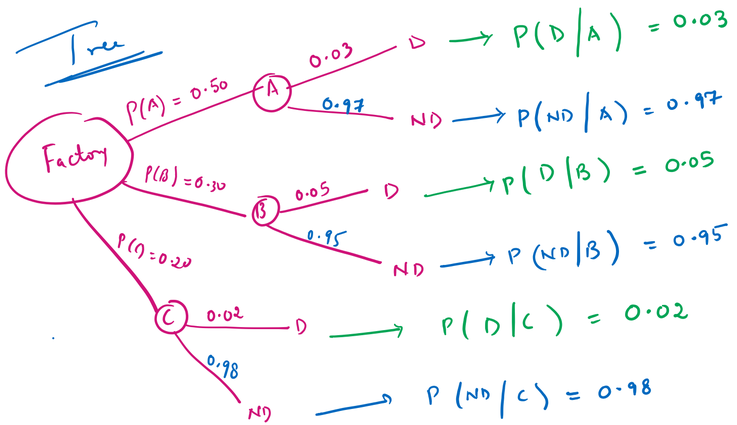
Or → ∪

and → ∩

D : Defective item

ND : Non-defective item

Tree



Law of total probability $P(D) = P(A)P(D|A) + P(B)P(D|B) + P(C)P(D|C)$

$= (0.50)(0.03) + (0.30)(0.05) + (0.20)(0.02) = 0.034$

i) $P(C|D) = \frac{P(C)P(D|C)}{P(D)} = \frac{(0.20)(0.02)}{0.034} = 0.1176$

11.76 %

ii) $P(C \cup B|D) = P(C|D) + P(B|D) = 0.1176 + 0.4412 = 0.5588$

$P(B|D) = \frac{P(B)P(D|B)}{P(D)} = \frac{(0.30)(0.05)}{0.034} = 0.4412$

55.88 %

Law of total probability

$P(ND) = P(A)P(ND|A) + P(B)P(ND|B) + P(C)P(ND|C)$

$= (0.50)(0.97) + (0.30)(0.95) + (0.20)(0.98)$

$= 0.966$

$P(A|ND) = \frac{P(A)P(ND|A)}{P(ND)}$

$= \frac{(0.50)(0.97)}{0.966} = 0.502$

50.2 %

Problem 3

EC2R JULY 2024

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Hardik Pandya, Rishabh Pant and Surya Kumar Yadav are in the race of leading Indian cricket team in the next world cup with probabilities 0.2, 0.5 and 0.3 respectively. The probabilities of getting an increase in the match fee by Hardik, Rishabh and Surya are 0.3, 0.6 and 0.5 respectively if they become the Captain.

If there is an increase in match fee then find the probability

I. that it is because of Hardik
II. that it is because of Rishabh
III. that it is because of Surya Kumar

Event H: HP
Event R: RP
Event S: SKY

F: Increase in the match fee

$P(H) = 0.2$
$P(R) = 0.5$
$P(S) = 0.3$

$P(F|H) = 0.3$
$P(F|R) = 0.6$
$P(F|S) = 0.5$

$$P(F) = P(H)P(F|H) + P(R)P(F|R) + P(S)P(F|S)$$
$$= 0.51$$

$$P(H|F) = \frac{P(H)P(F|H)}{P(F)} = 0.1176 \quad 11.76\%$$

$$P(R|F) = \frac{P(R)P(F|R)}{P(F)} = 0.5882 \quad 58.82\%$$

$$P(S|F) = \frac{P(S)P(F|S)}{P(F)} = 0.2941 \quad 29.41\%$$

Problem 4

EC2R JAN 2024

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In a manufacturing company Machine 1 produces 40% of the items and Machine 2 and Machine 3 produces 25% and 35% of the items respectively. But from the past records it is found that 15%, 20% and 25% of the items they produce are defective.

Then find

i) Total percentage of defective produced

ii) If a defective item is selected randomly, then find the probability that it is produced by Machine 1

iii) If a defective item is selected randomly, then find the probability that it is not produced by Machine 2

D: Claim defective

$P(M_1) = 0.40$ $P(M_2) = 0.25$ $P(M_3) = 0.35$

$P(D|M_1) = 0.15$ $P(D|M_2) = 0.20$ $P(D|M_3) = 0.25$

$P(D) = P(M_1)P(D|M_1) + P(M_2)P(D|M_2) + P(M_3)P(D|M_3)$

$= 0.1975$ 19.75 %

$M_2 \cup \overline{M_2}$

$P(M_2|D) + P(\overline{M_2}|D) = 1$

$P(M_1|D) = 0.3038$

30.38 %

$P(\overline{M_2}|D) = 1 - P(M_2|D)$

$= 1 - \left(\frac{P(M_2)P(D|M_2)}{P(D)}\right)$

$= 1 - 0.2532 = 0.7468$

74.68 %

Problem 5

EC2M JAN 2024

A product is manufactured by four companies A, B,C and D. They produce 20%, 30%, 40% and 10% of the products released in the market. But it is observed that 30%, 20%, 10% and 25% are defective items manufactured by them respectively. Then find

I. Total percentage of defective items manufactured
II. The percentage that the selected defective item is produced by company A
III. The percentage that the selected defective item is not produced by company B
IV. The percentage that the selected defective item is not produced by company D

$P(A) = 0.20$ $\quad P(X|A) = 0.30$

$P(B) = 0.30$ $\quad P(X|B) = 0.20$

$P(C) = 0.40$ $\quad P(X|C) = 0.10$

$P(D) = 0.10$ $\quad P(X|D) = 0.25$

$X \to P(X)$

$P(A|X)$

$P(\bar{B}|X)$

$P(\bar{D}|X)$

$P(E) + P(\bar{E}) = 1$

$P(B|X) + P(\bar{B}|X) = 1$

(1) $P(X) = 0.185$

$18.5\ \%$

(2) $P(A|X) = \dfrac{P(A)P(X|A)}{P(X)} = 0.3243$

$32.43\ \%$

(3) $P(\bar{B}|X) = 1 - P(B|X)$

$= 1 - \left(\dfrac{P(B)\,P(X|B)}{P(X)}\right) = 1 - 0.3243 = 0.6757$

$= 67.57\ \%$

(4) $P(\bar{D}|X) = 1 - P(D|X)$

$= 0.8649 = 86.49\ \%$

Problem 6

Consider the following data set:

Find out whether the object with attribute **Confident = Yes, Sick = No** will Fail or Pass using Bayesian Classification.

Confident	Studied	Sick	Result
Yes	No	No	Fail
Yes	No	Yes	Pass
No	Yes	Yes	Fail
No	Yes	No	Pass
Yes	Yes	Yes	Pass

Data set for classification

$X = \{Confident = yes, Sick = No\}$ Naive Bayes classifier

Result: pass $\frac{3}{5}$, fail $\frac{2}{5}$

Confident: yes $\frac{3}{5}$, NO $\frac{2}{5}$

$$P(Confident = yes \mid pass) = \frac{P(yes \cap pass)}{P(pass)} = \frac{2/5}{3/5} = \frac{2}{3}$$

Studies: yes $\frac{3}{5}$, NO $\frac{2}{5}$

$$P(Confident = yes \mid fail) = \frac{1/5}{2/5} = \frac{1}{2}$$

Sick: yes $\frac{3}{5}$, NO $\frac{2}{5}$

$$P(Sick = NO \mid pass) = \frac{P(Sick = NO \cap pass)}{P(pass)} = \frac{1/3}{3/5} = \frac{1}{3}$$

$$P(Sick = NO \mid fail) = \frac{1/5}{2/5} = \frac{1}{2}$$

Probability — Conditional probability — Bayes theorem

$P(A \cap B) = P(A)P(B)$

$$P(Pass \mid X) = P(Pass \mid Confident = yes, Sick = NO)$$

$$= P(Pass) \times P(Confident = yes, Sick = NO \mid pass)$$

$$= P(pass) \cdot P(Confident = yes \mid pass) \cdot P(Sick = NO \mid pass)$$

$$= \frac{3}{5} \times \frac{2}{3} \times \frac{1}{3} = 0.1333$$

$$P(Fail \mid X) = P(Fail \mid Con = yes, Sick = no)$$

$$= P(Fail) \cdot P(Con = yes \mid fail) \cdot P(Sick = no \mid Fail)$$

$$= \frac{2}{5} \times \frac{1}{2} \times \frac{1}{2} = 0.1$$

$P(Pass \mid X) > P(Fail \mid X)$

Pass class

Problem 7

Example No.	Color	Type	Origin	Stolen?
1	Red	Sports	Domestic	Yes
2	Red	Sports	Domestic	No
3	Red	Sports	Domestic	Yes
4	Yellow	Sports	Domestic	No
5	Yellow	Sports	Imported	Yes
6	Yellow	SUV	Imported	No
7	Yellow	SUV	Imported	Yes
8	Yellow	SUV	Domestic	No
9	Red	SUV	Imported	No
10	Red	Sports	Imported	Yes

***New Instance* = (*Red*, *SUV*, *Domestic*) → (*Yes or No*)**

Input

output

$P(Yes) = \frac{5}{10}$

$P(No) = \frac{5}{10}$

$X = \{Red, SUV, Domestic\}$

Stolen: Yes 5/10, No 5/10

Colour: Yellow — Yes 2/5, No 2/5; Red — Yes 3/5, No 2/5

Type: Sports — Yes 4/5, No 2/5; SUV — Yes 1/5, No 3/5

Origin: Domestic — Yes 2/5, No 3/5; Imported — Yes 3/5, No 2/5

$P(Sports \mid Yes) = \frac{P(Sports \cap Yes)}{P(Yes)} = \frac{4/10}{5/10} = 4/5$

$P(Yes \mid x) = P(Yes \mid Red, SUV, Domestic)$

$= P(Yes) \cdot P(Red, SUV, Domestic \mid Yes)$

$= P(Yes) \cdot P(Red \mid Yes)\, P(SUV \mid Yes)\, P(Domestic \mid Yes) = \frac{5}{10} \times \frac{3}{5} \times \frac{1}{5} \times \frac{2}{5} = \frac{3}{125} = \frac{0.024}{(\ )}$

$P(No \mid x) = P(No \mid Red, SUV, Domestic)$

$= P(No) \cdot P(Red, SUV, Domestic \mid No)$

$= P(No) \cdot P(Red \mid No)\, P(SUV \mid No)\, P(Dom \mid No)$

$= \frac{5}{10} \times \frac{2}{5} \times \frac{3}{5} \times \frac{3}{5} = \frac{9}{125} = \frac{0.072}{(\ )}$

0.024 > 0.072

$P(No \mid x) < P(Yes \mid x)$

Not stolen

Problem 8

EC2M
JULY 2024

Vehicle	Credit card/Debit card	Travel	Employment
BMW	Credit card	Flight	Business
Scooter	Debit card	Bus	Salaried
Bicycle	None	Local train	Business
Verna	Credit card	Vande Bharat	Salaried
Scooter	Credit card	Flight	Salaried
Auto	None	Bus	Business
BMW	Debit card	Vande Bharat	Salaried
No	No	Bus	Salaried

Based on the data is it possible to find whether the following is salaried or business person. If possible find with the help of suitable theorem / algorithm. Show the working, mention assumptions if any.

"Vehicle = scooter; Credit Card = yes, Travel = Bus"

$P(\text{Business}) = \frac{3}{8}$

$P(\text{Salaried}) = \frac{5}{8}$

$X = \{\text{Scooter, Yes, Bus}\}$

$P(\text{Business} \mid X) = P(\text{Business} \mid \text{Scooter, Yes, Bus})$

$= P(\text{Business})\, P(\text{Scooter} \mid \text{Business})\, P(\text{Yes} \mid \text{Business})\, P(\text{Bus} \mid \text{Business})$

$= \frac{3}{8} \times \frac{0+1}{3+6} \times \frac{1}{3} \times \frac{1}{3}$

$= 0$

$P(\text{Salaried} \mid X) = P(\text{Salaried} \mid \text{Scooter, Yes, Bus})$

$= P(\text{Salaried})\, P(\text{Scooter} \mid \text{Salaried})\, P(\text{Yes} \mid \text{Salaried})\, P(\text{Bus} \mid \text{Salaried})$

$= \frac{5}{8} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} = \frac{1}{25} = 0.04$

Discrete Random Variables – Problem 9

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EC2M July 2022

A random variable X can assume five values:0,1,2,3,4. A portion of the probability distribution is shown here:

X	0	1	2	3	4
P(x)	0.1	0.25	0.25	0.2	0.2

i) Find p(3)

ii) Calculate the mean

iii) What is the probability that X is greater than 3?

iv) What is the probability that X is 3 or less?

v) What is the probability that X is in between 1 and 3?

$P(x>3) = P(x=4) = 0.2$

$P(x \leq 3) = P(0) + P(1) + P(2) + P(3)$

$= 0.1 + 0.25 + 0.25 + 0.2$

$= 0.8$

$P(1 < x < 3) = P(x=2) = 0.25$

$\Sigma p(x) = 1$

$0.1 + 0.25 + 0.25 + P(3) + 0.2 = 1$

$\Rightarrow 0.8 + P(3) = 1$

$\Rightarrow P(3) = 1 - 0.8 = 0.2$

ii) $Mean = \Sigma x\, p(x)$

$= 0(0.1) + 1(0.25) + 2(0.25) + 3(0.2) + 4(0.2)$

$= 0 + 0.25 + 0.5 + 0.6 + 0.8$

$= 2.15$

Random

Discrete — Countable

Continuous — Uncountable — Over an interval — $[30, 45]$

x	
$P(x=x)$	

$0 \leq p(x) \leq 1$

$\Sigma p(x=x) = 1$

$Mean = E(x) = \Sigma x\, p(x)$

$Var(x) = E(x^2) - (E(x))^2$

Discrete Random Variables – Problem 10

APR EC2 22

The probabilities that number of times in a day certain computer may malfunction is as follows:

No. of malfunctions in a day	0	1	2	3	4	5	6
Probability	0.17	k	0.27	0.16	0.07	0.03	0.01

Find

(a) the value of k

(b) mean and variance of the malfunction of certain computer

$0.91 + k = 1$

$k = 0.29$

$E(x) = \text{Mean} = \sum x\,p(x) = 0 + 0.29 + 0.54 + 0.48 + 0.28 + 0.15 + 0.06$

$= 1.8$

$\text{Variance} = E(x^2) - (E(x))^2 = 5.04 - (1.8)^2 = 5.04 - 3.24 = 1.8$

$E(x^2) = \sum x^2 p(x) = 0(0.17) + 1(0.29) + 4(0.27) + 9(0.16) + 16(0.07) + 25(0.03) + 36(0.01)$

$= 5.04$

Random Variable – Probability Distribution - Problem 11

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$P(A \cap B) = P(A)P(B)$

A student takes two courses. In each course, the student will earn a B with probability 0.6 or a C with probability 0.4, independent of the other course. To calculate a grade point average (GPA), a B is worth 3 points and a C is worth 2 points. The student's GPA is the sum of the GPA for each course divided by 2. Make a table of the sample space of the experiment and the corresponding values of the student's GPA, G.

$P(B) = 0.6$

$P(C) = 0.4$

B → 3
C → 2

Outcome	Probability	GPA = G
BB	0.36	$\frac{3+3}{2} = 3$
BC	0.24	$\frac{3+2}{2} = 2.5$
CB	0.24	2.5
CC	0.16	2