Time series applied to finance

Marouane Arab

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Abstract

This document is generated by an R script and a brew file (latex file which embeds R code). All sources are available at https://github.com/arabm/multifractal_model. There are questions about time series:

(i) Is the data relevant ? coherent ?

(ii) Which model is adapted ? How validate or reject it ?

(iii) What is the error commited by using it ?

(iv) How forecast is usefull ?

(v) How much day forecast is relevant ?

(vi) How to validate backtesting ?

I'm counting on your feedback. Do concepts are well introduced, developed and explained ? Do you want more details ? On what ? Are they missing concepts ? Is a concept wrongly explained ? And of course, are there english mistakes ?

Correct english mistakes !

Todo list

Contents

1 Quick start

1.1 Usage

Source The repository github contains two files :

- *script.r* : an R script (official R website is https://cran.r-project.org/)
- *template.brew* : a latex template which embeds R commands. Brew is an R package (see https://cran.r-project.org/web/packages/brew/index.html).

Pre-required Install R, latex

Complete pre-required

First command Once R, latex and every pre-required are done, you can launch the script with *from the clone directory*:

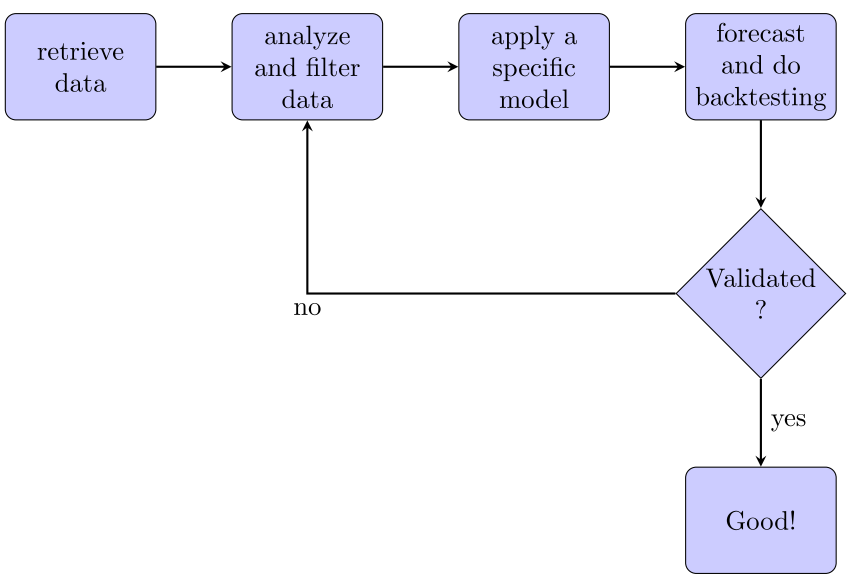
```
R -f script.r
```

It generates *generated_report.pdf* (similar to the one you're reading right now). It also creates a lot of repositories :

```
clone repo
└── output
    ├── report
    ├── example
    └── FCHI
```

Currently there isn't an *example* repository, but rather various example repositories : example_uniform, example_normal,etc. Same apply for *FCHI*, it creates a repository by financial underlying.

1.2 Modelisation process



Workflow The workflow shows linear tasks, while indeed they're strongly linked. Analyzing, filtering and applying a model can be done at the same time. We look at the data. Then apply a model. Readjust the model or the filter applied on data.

Validation The validation is done by analysing residuals. Residuals are errors or noises if we consider that the data follows our model. Backtesting is also important. It's how the model from an extract of the history forecast correctly what happened.

Parameters Model can have various parameters. The model which fits the best the data is the date itself. Of course, that's not the aim of modelisation. We rather prefer to minimize the number of parameters. A good model, is one which has a good ratio between number of paramaters (lower the better) and residuals (lower the better too).

Complete quick start

2 Statistical tests

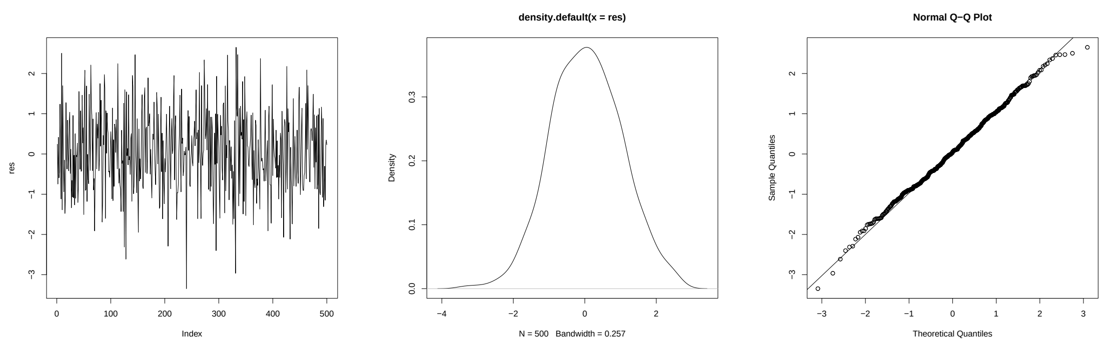
2.1 White noises

Definition Be ϵ_t a white noise. It's independ through time t and got the same law.

white noise definition

Example with $\mathcal{N}(0,1)$ law We consider a white noise which follow $\mathcal{N}(0,1)$, and generate it for 500 dates.

```
>white_noise<-rnorm(500,0,1)
>plot(white_noise,type="l")
>plot(density(white_noise))
>plot(qqnorm(white_noise))
>qqline(white_noise)
```



(a) Normal process (b) Density function (c) QQnorm

Figure 1: Example of $\mathcal{N}(0,1)$

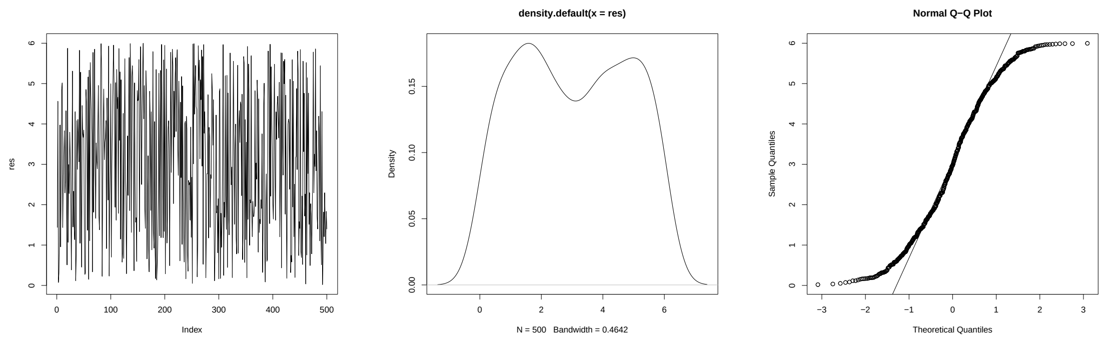
Test of Box-Ljung for independance :

```
>Box.test(white_noise,type='Ljung')
```

Test of Shapiro for Normal law :

```
>shapiro.test(white_noise)

Shapiro-Wilk normality test

data:  res
W = 0.99746, p-value = 0.6467
```

link with Box-Ljung, shapiro,qqnorm



(a) Normal process (b) Density function (c) QQnorm

Figure 2: Example of $\mathcal{N}(0,6)$

Test of Box-Ljung for independance : Test of Shapiro for Normal law :

```
Shapiro-Wilk normality test

data:  res
W = 0.94555, p-value = 1.376e-12
```

Add example for normal, AR/-MA/ARCH/-Garch, normal square

2.2 BIC

do BIC

2.3 AIC

do AIC

2.4 Likelihood

do Likeli-hood

3 Model

introduction Model

3.1 Auto Regressive Moving Average (ARMA)

3.1.1 AR(1)

More verbose AR(1)

Definition Be $c, \beta \in \mathbb{R}^2$, X_t is an AR(1) process if

$$\begin{aligned} X_t &= c + \beta X_{t-1} + \epsilon_t \\ &= c + \sum_{i=1}^{\infty} \beta^i \epsilon_{t-i} + \epsilon_t \end{aligned}$$

Finite variance X_t has a finite variance if $|\beta| < 1$:

$$\begin{aligned} \mathrm{Var}(X_t) &= \mathrm{Var}(\sum_{i=0}^{\infty} \beta^i \epsilon_{t-i}) \\ &= \sum_{i=0}^{\infty} \beta^{2i} \sigma^2 \\ &= \frac{1-\beta^\infty}{1-\beta} \sigma^2 \end{aligned}$$

When $|\beta| > 1$, $\mathrm{Var}(X_t) = \infty$.
When $|\beta| < 1$, $\mathrm{Var}(X_t) = \frac{\sigma^2}{1-\beta}$

Expectation doesn't depend of t It implies $\mathbb{E}\left\{X_t\right\} = \mathbb{E}\left\{X_{t-1}\right\} = \mu$

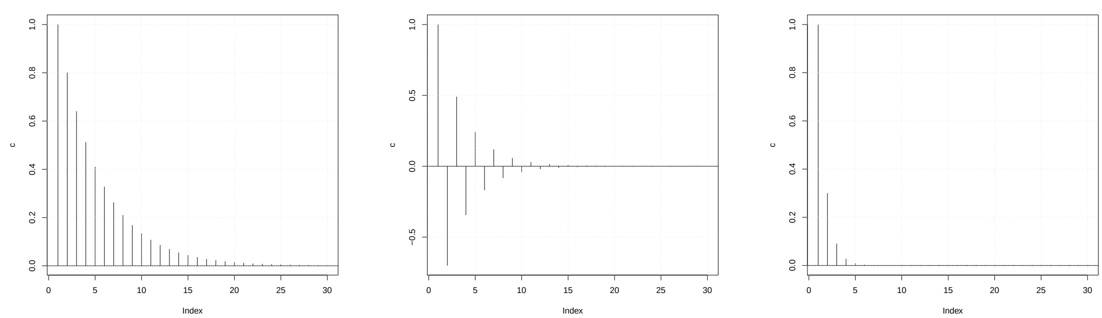
$$\begin{aligned} \mathbb{E}\left\{X_t\right\} &= c + \beta \mathbb{E}\left\{X_{t-1}\right\} \\ \Leftrightarrow \mu &= \frac{c}{1-\beta} \end{aligned}$$

We could rewrite $\mathbb{E}\left\{X_t\right\}$ as :

$$\begin{aligned} \mathbb{E}\left\{X_t\right\} &= c + \mathbb{E}\left\{\beta X_{t-1} + \epsilon_t\right\} \\ &= c + \mathbb{E}\left\{\sum_{i=1}^{\infty} \beta^i (c + \epsilon_{t-i})\right\} \\ &= \frac{c}{1-\beta} \end{aligned}$$

Covariance (ACF) There is a linear relation between lag correlation γ_l.

$$
\begin{aligned}
\gamma_l &= \operatorname{cov}(X_t, X_{t-l}) \\
&= \mathbb{E}\left\{(X_t-\mu)(X_{t-l}-\mu)\right\} \\
&= \mathbb{E}\left\{(\epsilon_t + \sum_{i=1}^{\infty}\beta^i\epsilon_{t-i})(\sum_{i=1}^{\infty}\beta^i\epsilon_{t-l-i})\right\} \\
&= \mathbb{E}\left\{\sum_{i,j=1}^{\infty}\beta^i\epsilon_{t-i}\beta^j\epsilon_{t-l-j}\right\} \\
&= \mathbb{E}\left\{\sum_{i=1}^{\infty}\beta^{i+l}\epsilon_{t-l-i}\beta^i\epsilon_{t-l-i}\right\} \\
&= \sigma^2\sum_{i=1}^{\infty}\beta^{i+l}\beta^i \\
&= \sigma^2\frac{\beta^l}{1-\beta^2} \\
&= \beta\gamma_{l-1}
\end{aligned}
$$



(a) β =0.8 (b) β =-0.7 (c) β =0.3

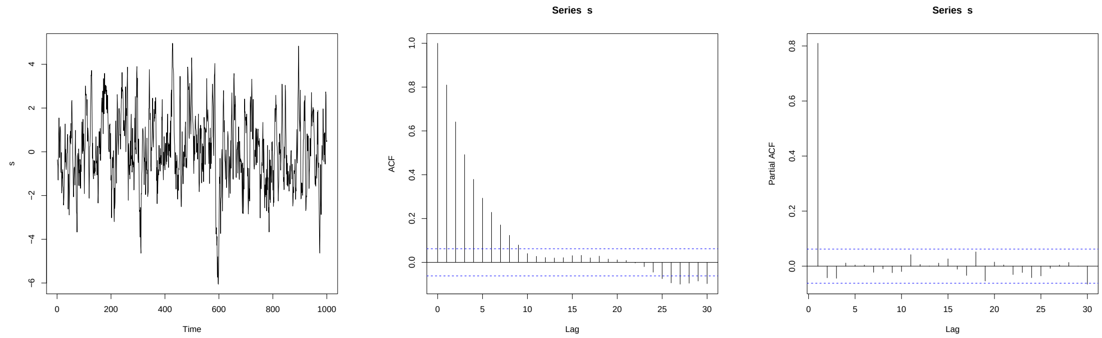
Figure 3: Expected ACF for AR(1) process

In practice, there are noises which deform the theorical linearity relation.

Partial covariance (PACF) TODO

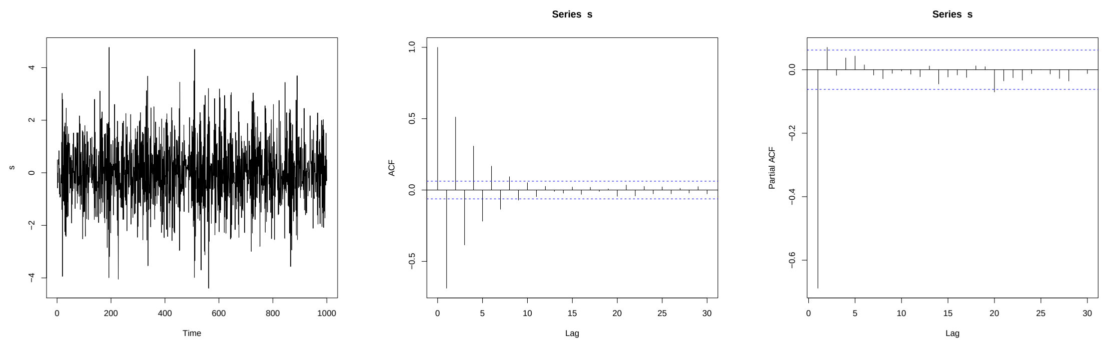
PACF AR(1)

Examples In example below, check that the ACF got the linear relation expected, and that PACF got the correct value for lag 1 and approximatly 0 for others.



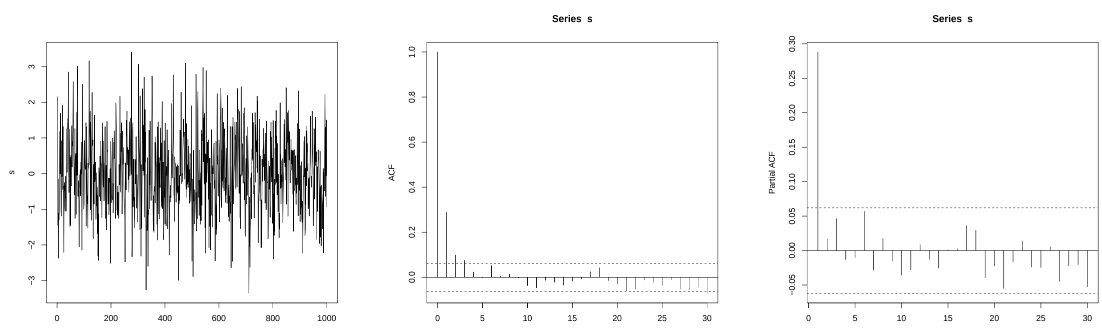
(a) Process (b) ACF (c) PACF

Figure 4: Example of AR(1) process, coefficient AR(0.8) (n=1000)



(a) Process (b) ACF (c) PACF

Figure 5: Example of AR(1) process, coefficient AR(-0.7) (n=1000)



(a) Process (b) ACF (c) PACF

Figure 6: Example of AR(1) process, coefficient AR(0.3) (n=1000)

3.1.2 AR(p)

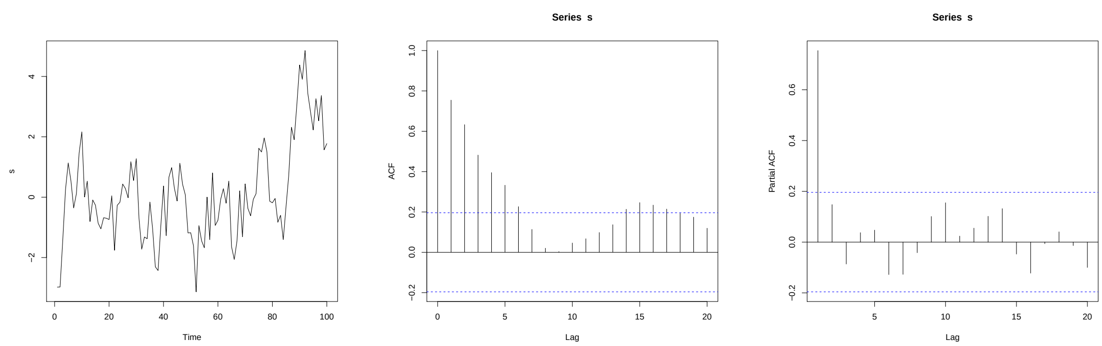
More verbose AR(p)

Definition Be $c, \beta \in \mathbb{R}^2$, X_t is an AR(p) process if

$$
\begin{aligned}
X_t &= c + \sum_{i=1}^{p} \beta_i X_{t-i} + \epsilon_t \\
&= c + \sum_{i=1}^{p} \beta_i
\end{aligned}
$$

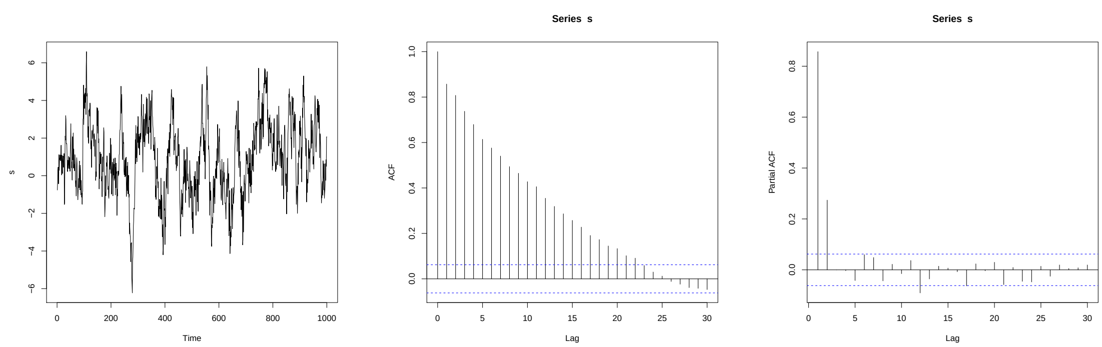
Stationary condition To be stationary, roots of the polynom $z^p - \sum_{i=1}^{p} \beta_i z^{p-i}$ must be within the unit circle, $|z_i| < 1$

Examples See below.



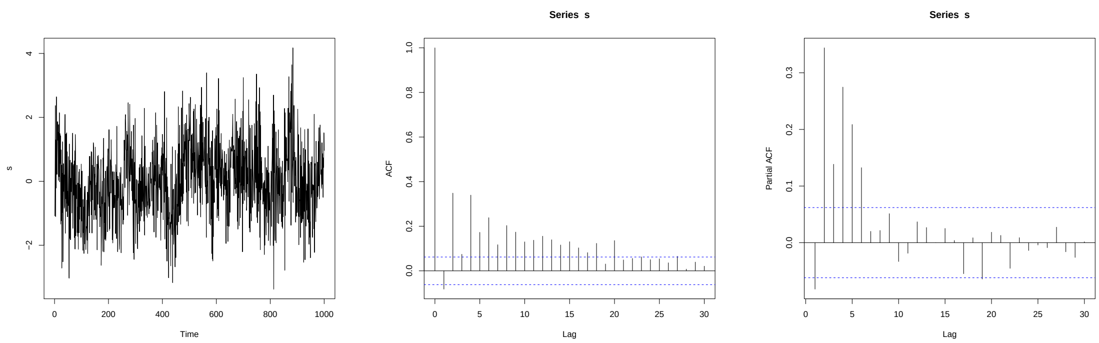
(a) Process (b) ACF (c) PACF

Figure 7: AR(2) process with coefficient AR(0.6,0.3) (n=100)



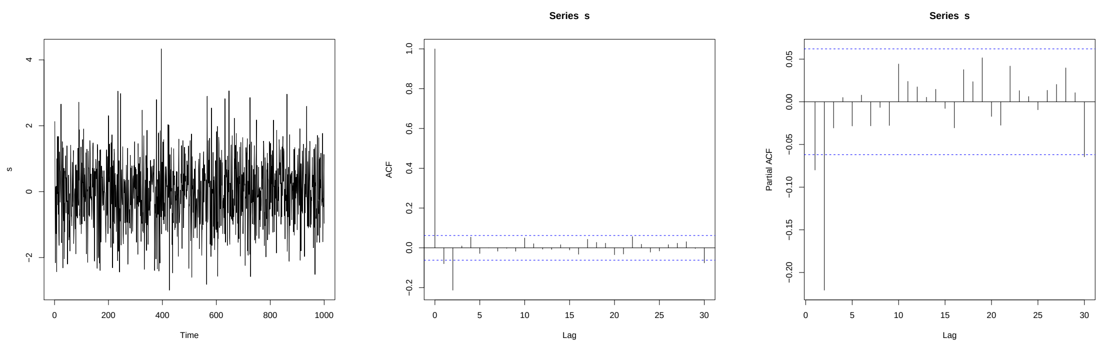
(a) Process (b) ACF (c) PACF

Figure 8: AR(2) process with coefficient AR(0.6,0.3) (n=1000)



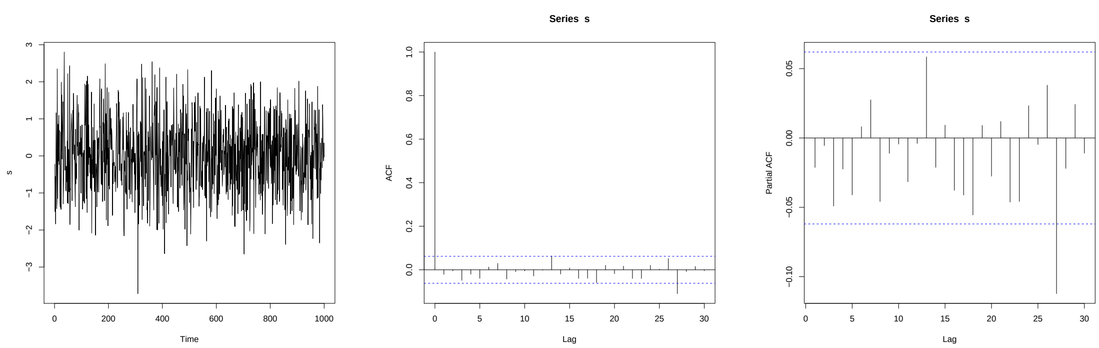
(a) Process (b) ACF (c) PACF

Figure 9: AR(6) process with coefficient AR(-0.2,0.2,0.1,0.3,0.2,0.1) (n=1000)



(a) Process (b) ACF (c) PACF

Figure 10: AR(2) process with coefficient AR(-0.1,-0.2) (n=1000)



(a) Process (b) ACF (c) PACF

Figure 11: AR(2) process with coefficient AR(0.01,0.02) (n=1000)

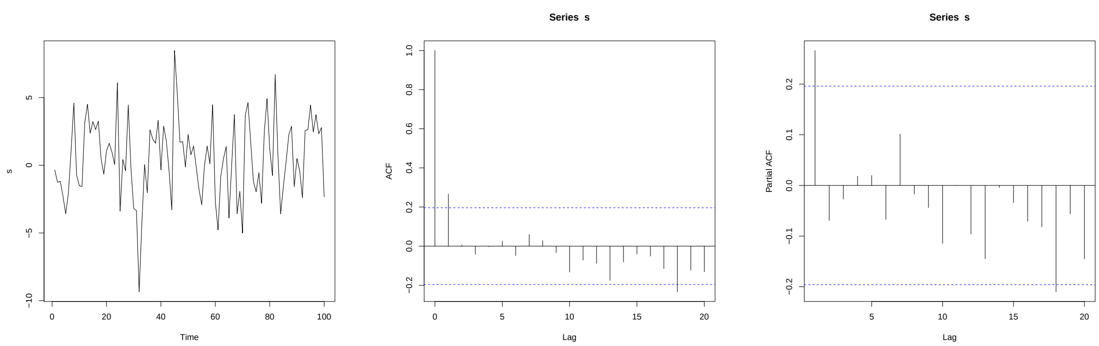
3.1.3 MA(1)

Definition Be $\theta \in \mathbb{R}$, X_t is an MA(1) process if

$$\begin{aligned} X_t &= c + \theta \epsilon_t \\ &= c + \beta^t X_0 + \sum_{i=0}^{t} \beta^i \epsilon_{t-i} \end{aligned}$$

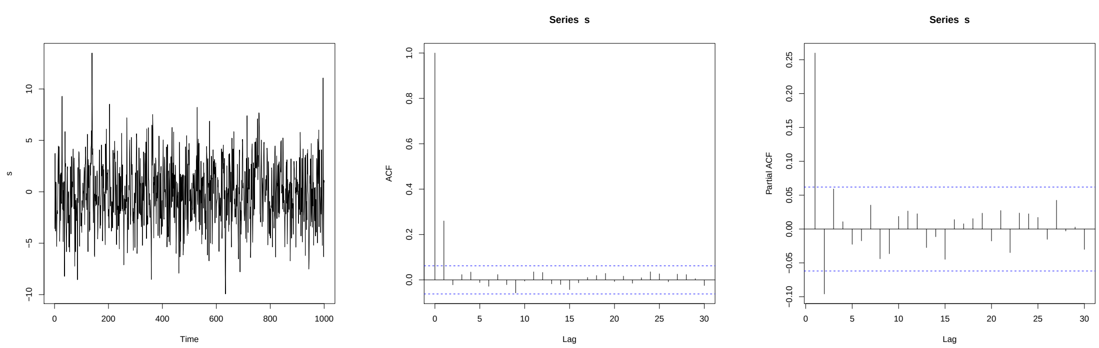
Stationary condition X_t has a finite variance : $\text{Var}(X_t) = \theta^2 \sigma^2$

Examples See below.



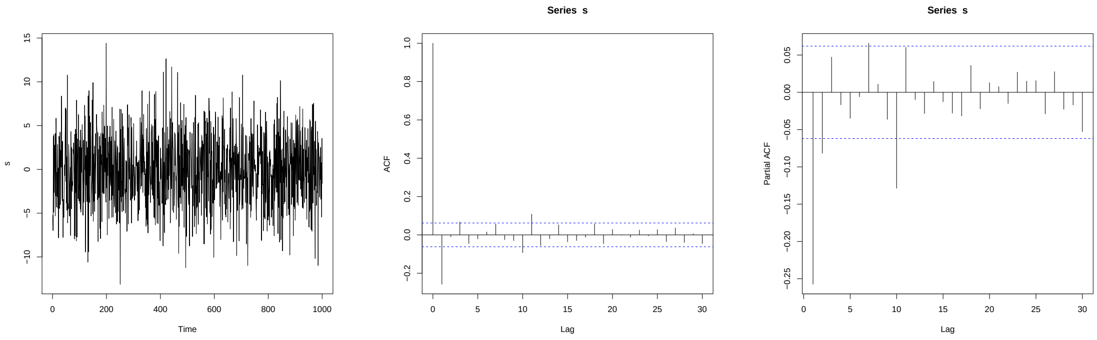
(a) Process (b) ACF (c) PACF

Figure 12: MA(1) process with coefficient MA(3) (n=100)



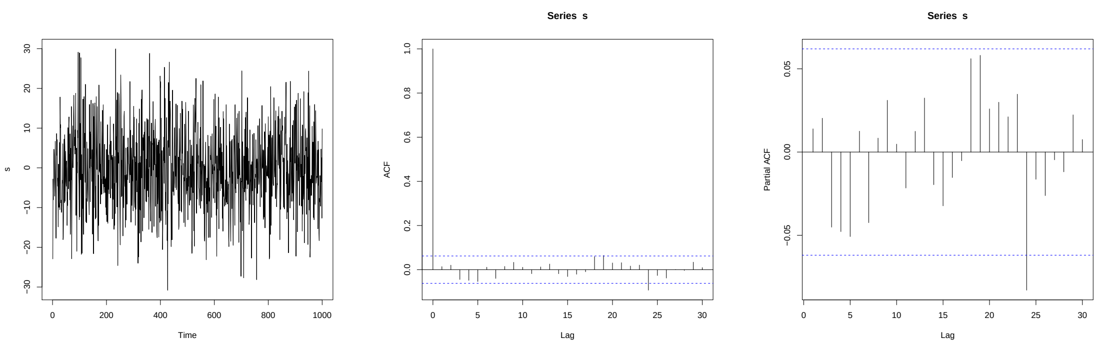
(a) Process (b) ACF (c) PACF

Figure 13: MA(1) process with coefficient MA(3) (n=1000)



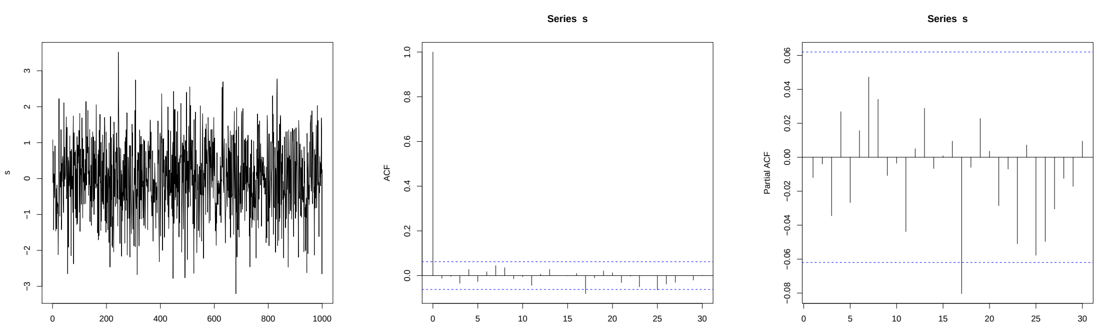
(a) Process (b) ACF (c) PACF

Figure 14: MA(1) process with coefficient MA(-4) (n=1000)



(a) Process (b) ACF (c) PACF

Figure 15: MA(1) process with coefficient MA(10) (n=1000)



(a) Process (b) ACF (c) PACF

Figure 16: MA(1) process with coefficient MA(0.01) (n=1000)

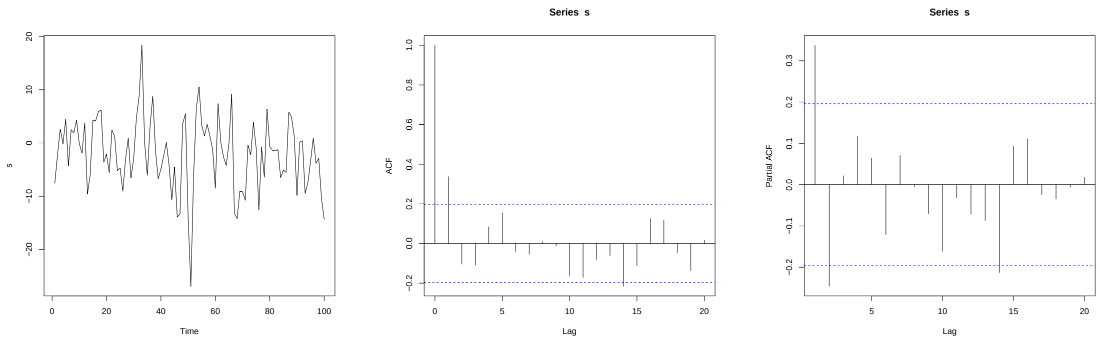
3.1.4 MA(q)

Definition Be $\theta \in \mathbb{R}$, X_t is an MA(q) process if

$$X_t = c + \sum_{i=0}^{q} \theta_i \epsilon_{t-i}$$

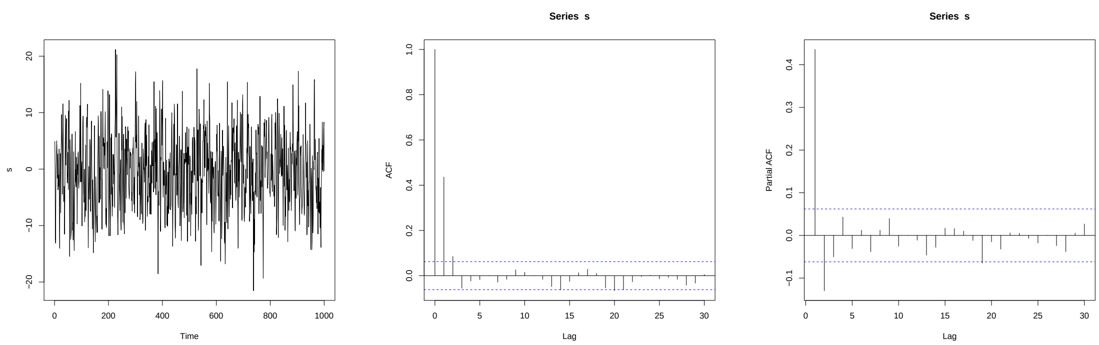
Stationary condition X_t has a finite variance : $\mathrm{Var}(X_t) = \sum_{i=0}^{q} \theta_i^2 \sigma^2$

Examples See below.



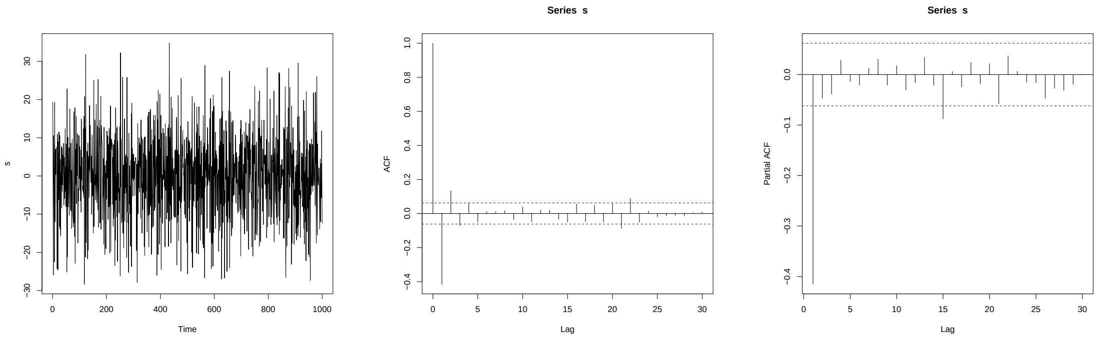
(a) Process (b) ACF (c) PACF

Figure 17: MA(2) process with coefficient MA(3,6) (n=100)



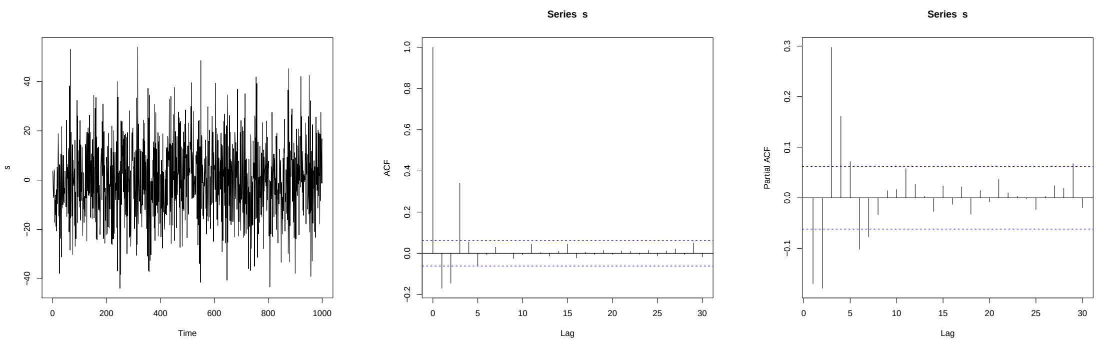
(a) Process (b) ACF (c) PACF

Figure 18: MA(2) process with coefficient MA(3,6) (n=1000)



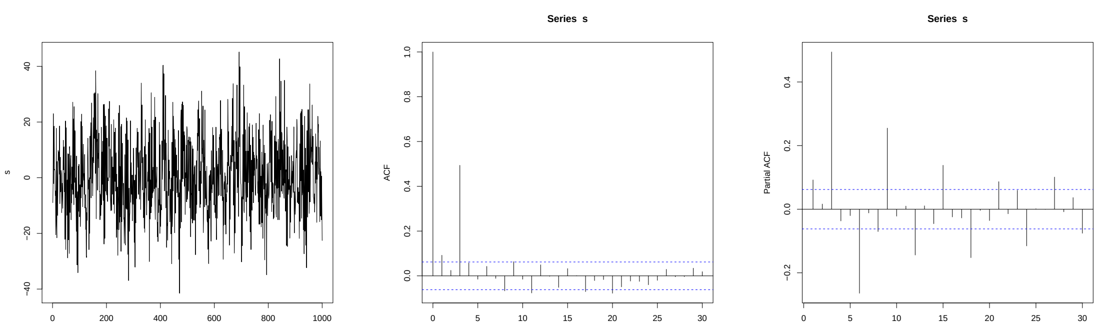
(a) Process (b) ACF (c) PACF

Figure 19: MA(2) process with coefficient MA(-4,10) (n=1000)



(a) Process (b) ACF (c) PACF

Figure 20: MA(4) process with coefficient MA(10,4,-7,9) (n=1000)



(a) Process (b) ACF (c) PACF

Figure 21: MA(4) process with coefficient MA(10,0,0,9) (n=1000)

3.1.5 ARMA(p,q)

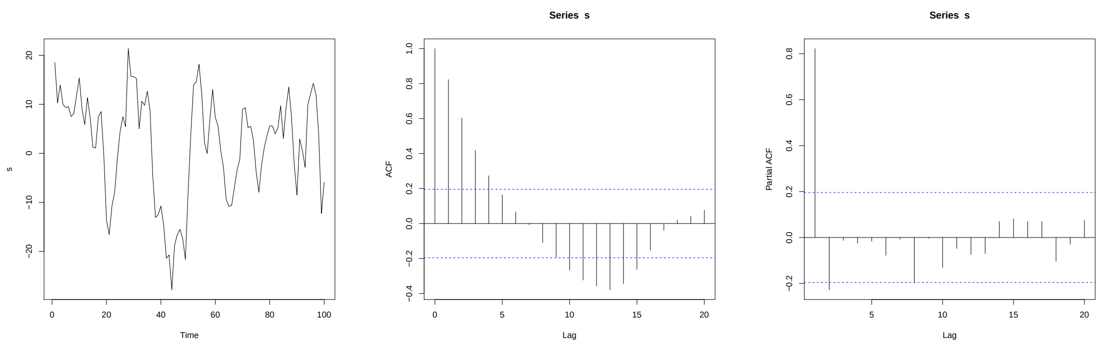
Definition Be $\theta \in \mathbb{R}$, X_t is an MA(q) process if

$$X_t = c + \sum_{i=0}^{q} \theta_i \epsilon_{t-i} + \sum_{i=1}^{p} \beta_i X_{t-i}$$

stationary To see if X_t is stationnary we look at its variance.

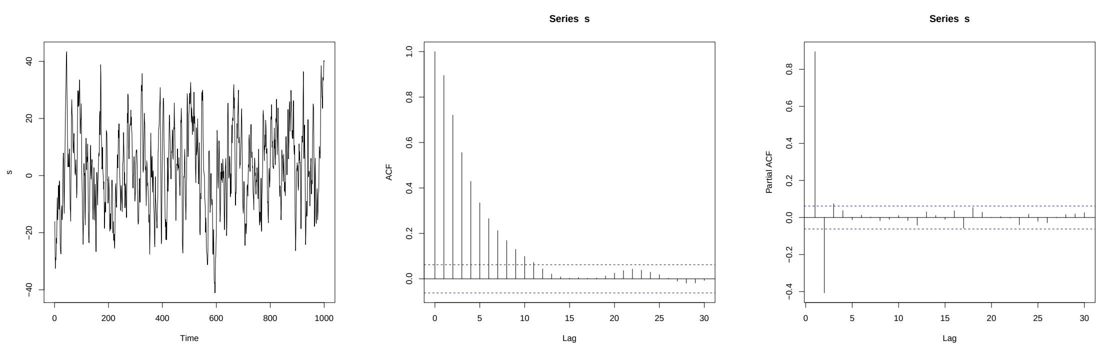
$$\mathrm{Var}(X_t) = \sum_{i=0}^{q} \theta_i^2 \sigma^2$$

Examples See below various example of ARMA(p,q)



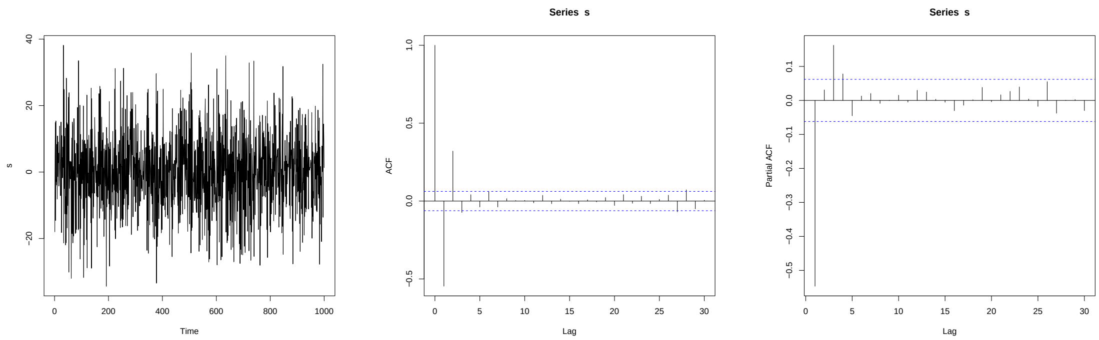
(a) Process (b) ACF (c) PACF

Figure 22: ARMA(1,2) process with coefficient AR(0.8), MA(3,6) (n=100)



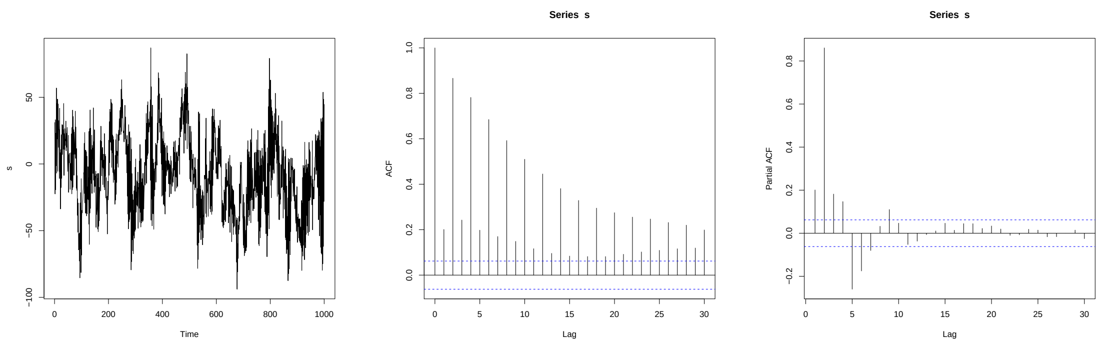
(a) Process (b) ACF (c) PACF

Figure 23: ARMA(1,2) process with coefficient AR(0.8), MA(3,6) (n=1000)



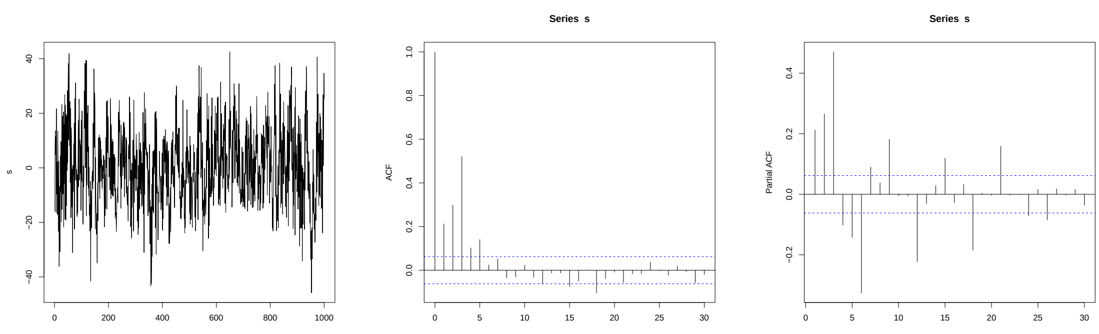
(a) Process (b) ACF (c) PACF

Figure 24: ARMA(3,2) process with coefficient AR(-0.1,0.2,0.1), MA(-4,10) (n=1000)



(a) Process (b) ACF (c) PACF

Figure 25: ARMA(2,4) process with coefficient AR(0,0.9), MA(10,4,-7,9) (n=1000)



(a) Process (b) ACF (c) PACF

Figure 26: ARMA(2,4) process with coefficient AR(0,0.3), MA(10,0,0,9) (n=1000)

3.2 Autoregressive conditional heteroskedasticity (ARCH)

ARCH details

Definition X_t is an ARCH(q) model if

$$\begin{aligned} X_t &= \epsilon_t \sigma_t \\ \sigma_t^2 &= \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 \end{aligned}$$

3.3 Generalized autoregressive conditional heteroskedasticity (GARCH)

GARCH details

Definition X_t is an GARCH(p,q) model if

$$\begin{aligned} X_t &= \epsilon_t \sigma_t \\ \sigma_t^2 &= \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \end{aligned}$$

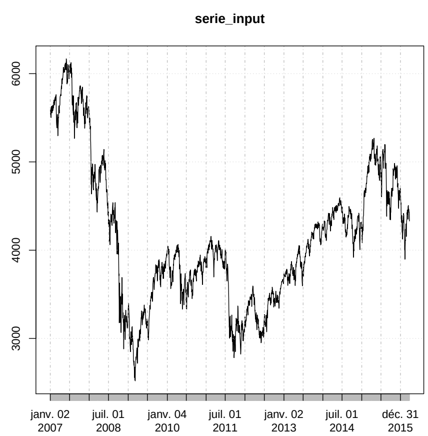
3.4 Markov Switching Model

MSM details

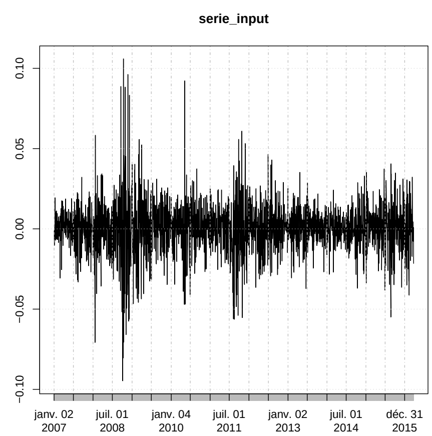
4 Application

4.1 CAC40

This is the historical close quotation for CAC40. Data has been retrieved from yahoo.



(a) Close level of CAC40



(b) log return CAC40

Looking at original

Is it an AR model ? We look for the AR coefficient, and minimize it with the AIC indicator. The AIC is a ratio between a good calibration and the number of parameters. The AR model implies that residual should be white noise (iid normal law). It's tested with the QQnorm. Results:

```
Call:
ar(x = value)

Coefficients:
      1        2        3        4        5        6
 0.9586   0.0052  -0.0014   0.0545  -0.0643   0.0442

Order selected 6  sigma^2 estimated as  4526
```

Explain why we work on log return instead of close level ?

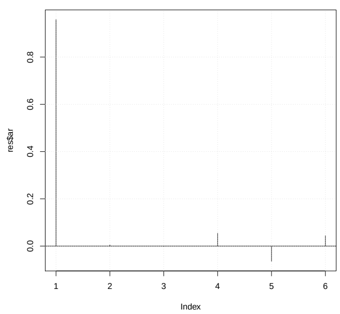
Maybe skip the AR/MA model, and just go to ARMA directly. Move AR/MA into example?

Details in annexe how these tests work

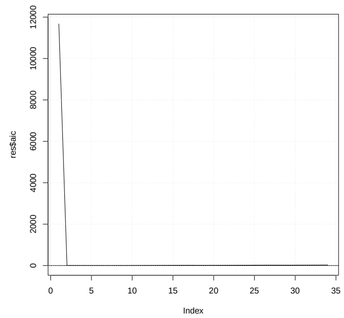
Missing MSM details

Missing Forecast / Backtesting

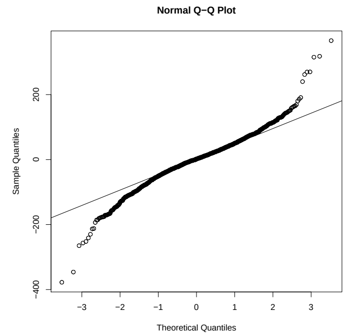
Add references to figure, few introductions ?



(a) AR coefficients



(b) AIC per number of parameters



(c) QQnorm of residual

Test residual independences with ljung:

```
Box-Ljung test

data:  res
X-squared = 0.34186, df = 1, p-value = 0.5588
```

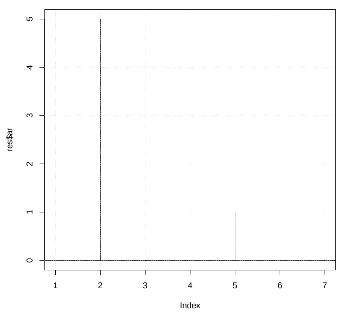
Test residual normal law with Shapiro:

```
Shapiro-Wilk normality test

data:  res
W = 0.9639, p-value < 2.2e-16
```

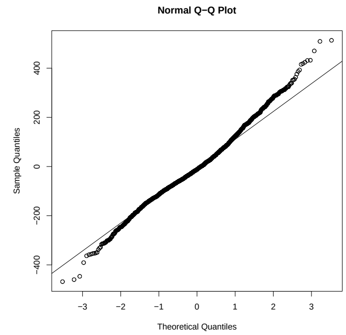
Is it an MA model ? We look for the AR coefficient, and minimize it with the AIC indicator. The AIC is a ratio between a good calibration and the number of parameters. The AR model implies that residual should be white noise (iid normal law). It's tested with the QQnorm.

Results:

```
Series: value
ARIMA(0,0,5) with non-zero mean

Coefficients:
         ma1     ma2     ma3     ma4     ma5  intercept
      1.7514  2.1009  1.9138  1.3178  0.5338  4139.6223
s.e.  0.0265  0.0439  0.0337  0.0234  0.0180    22.3093

sigma^2 estimated as 15867:  log likelihood=-14782.85
AIC=29579.69   AICc=29579.74   BIC=29620.07
```



(a) MA coefficients



(b) QQnorm of residual

Test residual independences with ljung:

```
Box-Ljung test

data:  res
X-squared = 209.85, df = 1, p-value < 2.2e-16
```

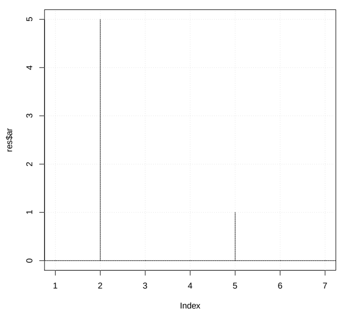
Test residual normal law with Shapiro:

```
Shapiro-Wilk normality test

data:  res
W = 0.98947, p-value = 3.684e-12
```

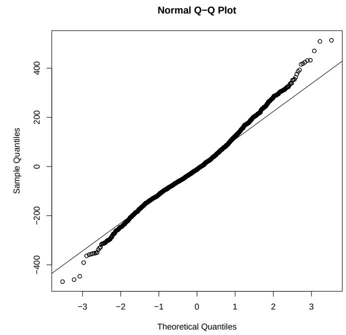
Is it an ARMA model ? We look for the AR coefficient, and minimize it with the AIC indicator. The AIC is a ratio between a good calibration and the number of parameters. The AR model implies that residual should be white noise (iid normal law). It's tested with the QQnorm.

Results:

```
Series: value
ARIMA(0,0,5) with non-zero mean

Coefficients:
         ma1     ma2     ma3     ma4     ma5  intercept
      1.7514  2.1009  1.9138  1.3178  0.5338  4139.6223
s.e.  0.0265  0.0439  0.0337  0.0234  0.0180    22.3093

sigma^2 estimated as 15867:  log likelihood=-14782.85
AIC=29579.69   AICc=29579.74   BIC=29620.07
```



(a) MA coefficients



(b) QQnorm of residual

Test residual independences with ljung:

```
Box-Ljung test

data:  res
X-squared = 209.85, df = 1, p-value < 2.2e-16
```

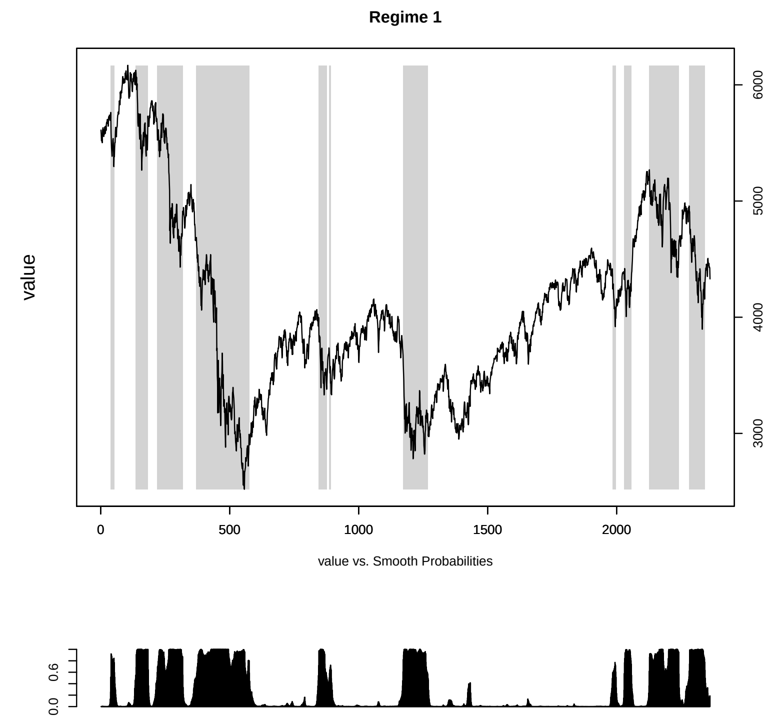
Test residual normal law with Shapiro:

```
Shapiro-Wilk normality test

data:  res
W = 0.98947, p-value = 3.684e-12
```

Markov Switching Model Try with 2 regimes.

Results:

```
Markov Switching Model

Call: msmFit(object = mod, k = 2, sw = c(T, T, T), p = 1, control = list(parallel = F))

       AIC      BIC    logLik
  25609.13 25663.27 -12800.56

Coefficients:
        (Intercept)(S) value_1(S)   Std(S)
Model 1      18.121358  0.9938143 86.46705
Model 2       7.304968  0.9989027 42.64456

Transition probabilities:
          Regime 1  Regime 2
Regime 1 0.9788516 0.0093812
Regime 2 0.0211484 0.9906188
```

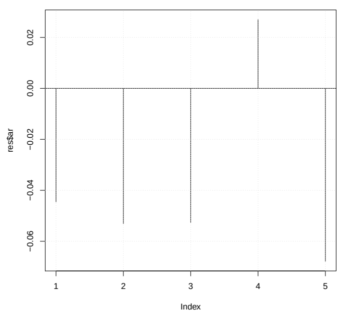


(a) Which 2

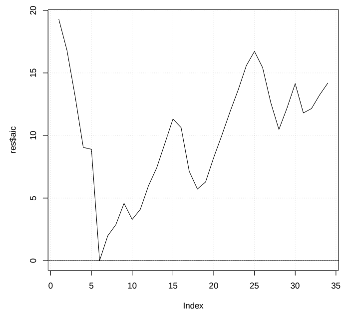
Looking at logret

Is it an AR model ? We look for the AR coefficient, and minimize it with the AIC indicator. The AIC is a ratio between a good calibration and the number of parameters. The AR model implies that residual should be white noise (iid normal law). It's tested with the QQnorm. Results:

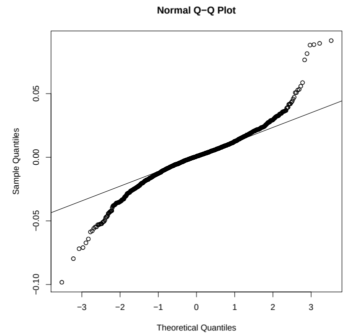
```
Call:
ar(x = value)

Coefficients:
      1        2        3        4        5
-0.0446  -0.0531  -0.0526   0.0270  -0.0679

Order selected 5  sigma^2 estimated as  0.0002385
```



(a) AR coefficients



(b) AIC per number of parameters



(c) QQnorm of residual

Test residual independences with ljung:

```
Box-Ljung test

data:  res
X-squared = 0.00024827, df = 1, p-value = 0.9874
```

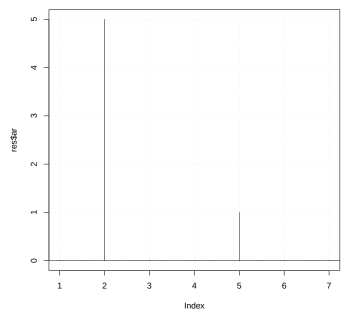
Test residual normal law with Shapiro:

```
Shapiro-Wilk normality test

data:  res
W = 0.94774, p-value < 2.2e-16
```

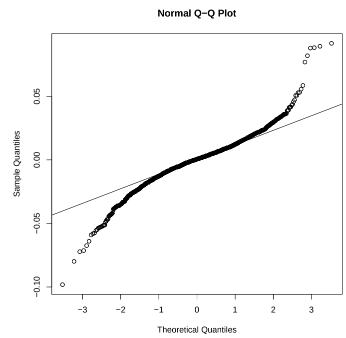
Is it an MA model ? We look for the AR coefficient, and minimize it with the AIC indicator. The AIC is a ratio between a good calibration and the number of parameters. The AR model implies that residual should be white noise (iid normal law). It's tested with the QQnorm.

Results:

```
Series: value
ARIMA(0,0,5) with zero mean

Coefficients:
          ma1      ma2      ma3     ma4      ma5
      -0.0446  -0.0485  -0.0482  0.0307  -0.0627
s.e.   0.0205   0.0207   0.0205  0.0205   0.0211

sigma^2 estimated as 0.000238:  log likelihood=6501.84
AIC=-12991.69   AICc=-12991.65   BIC=-12957.08
```



(a) MA coefficients



(b) QQnorm of residual

Test residual independences with ljung:

```
Box-Ljung test

data:  res
X-squared = 0.00020489, df = 1, p-value = 0.9886
```

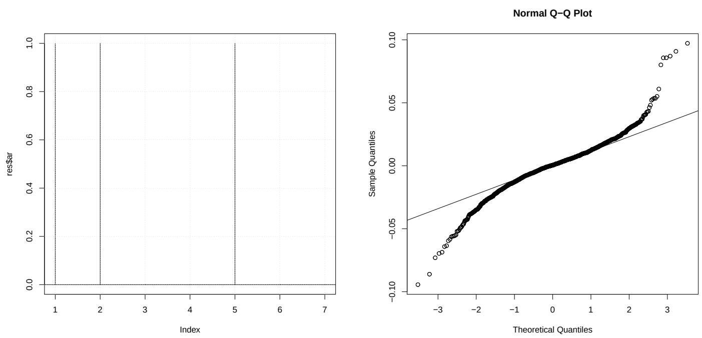
Test residual normal law with Shapiro:

```
Shapiro-Wilk normality test

data:  res
W = 0.94707, p-value < 2.2e-16
```

Is it an ARMA model ? We look for the AR coefficient, and minimize it with the AIC indicator. The AIC is a ratio between a good calibration and the number of parameters. The AR model implies that residual should be white noise (iid normal law). It's tested with the QQnorm.

Results:

```
Series: value
ARIMA(0,0,5) with zero mean

Coefficients:
          ma1      ma2      ma3     ma4      ma5
      -0.0446  -0.0485  -0.0482  0.0307  -0.0627
s.e.   0.0205   0.0207   0.0205  0.0205   0.0211

sigma^2 estimated as 0.000238:  log likelihood=6501.84
AIC=-12991.69   AICc=-12991.65   BIC=-12957.08
```



(a) MA coefficients (b) QQnorm of residual

Test residual independences with ljung:

```
Box-Ljung test

data:  res
X-squared = 0.12362, df = 1, p-value = 0.7251
```

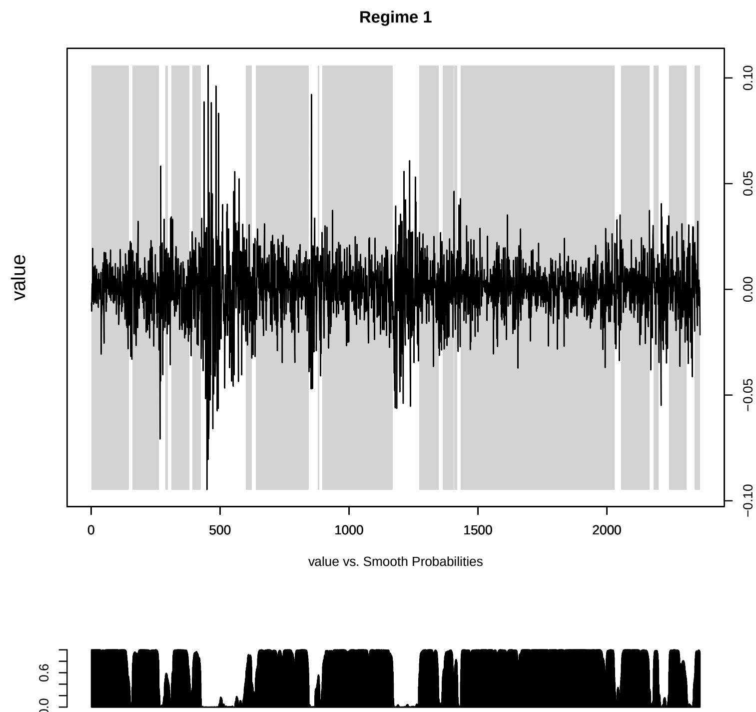
Test residual normal law with Shapiro:

```
Shapiro-Wilk normality test

data:  res
W = 0.94391, p-value < 2.2e-16
```

Markov Switching Model Try with 2 regimes.

Results:

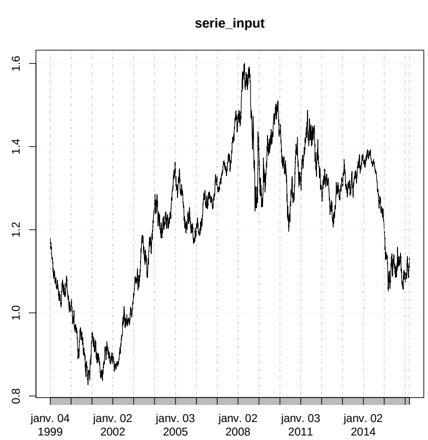
```
Markov Switching Model

Call: msmFit(object = mod, k = 2, sw = c(T, T, T), p = 1, control = list(parallel = F))

        AIC       BIC   logLik
  -13538.51 -13484.38 6773.255

Coefficients:
        (Intercept)(S)  value_1(S)     Std(S)
Model 1   0.0005173361 -0.03039309 0.01084260
Model 2  -0.0021582307 -0.06141297 0.02489934

Transition probabilities:
           Regime 1   Regime 2
Regime 1 0.98836266 0.03654694
Regime 2 0.01163734 0.96345306
```



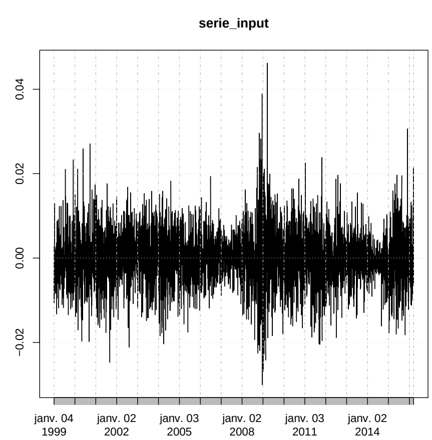
(a) Which 2

4.2 EURUSD

This is the historical close quotation for EURUSD. Data has been retrieved from FRED.



(a) Close level of EURUSD

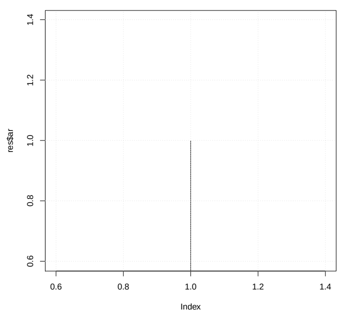


(b) log return EURUSD

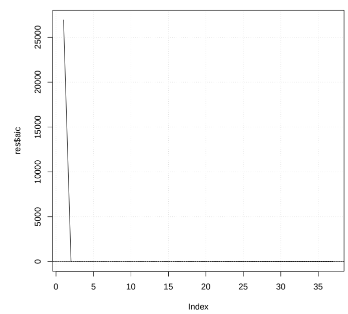
Looking at original

Is it an AR model ? We look for the AR coefficient, and minimize it with the AIC indicator. The AIC is a ratio between a good calibration and the number of parameters. The AR model implies that residual should be white noise (iid normal law). It's tested with the QQnorm. Results:

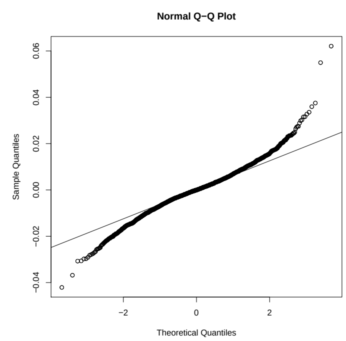
```
Call:
ar(x = value)

Coefficients:
    1
0.999

Order selected 1  sigma^2 estimated as  6.313e-05
```



(a) AR coefficients



(b) AIC per number of parameters



(c) QQnorm of residual

Test residual independences with ljung:

```
Box-Ljung test

data:  res
X-squared = 0.17242, df = 1, p-value = 0.678
```

Test residual normal law with Shapiro:

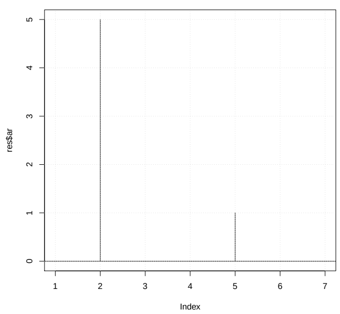
```
Shapiro-Wilk normality test

data:  res
W = 0.97606, p-value < 2.2e-16
```

Is it an MA model ? We look for the AR coefficient, and minimize it with the AIC indicator. The AIC is a ratio between a good calibration and the number of parameters. The AR model implies that residual should be white noise (iid normal law). It's tested with the QQnorm.

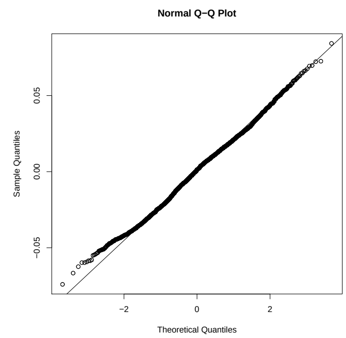
Results:

```
Series: value
ARIMA(0,0,5) with non-zero mean
```

```
Coefficients:
         ma1     ma2     ma3     ma4     ma5  intercept
      2.1483  2.8344  2.6013  1.6428  0.5836     1.2171
s.e.  0.0182  0.0336  0.0296  0.0182  0.0116     0.0036

sigma^2 estimated as 0.0004708:  log likelihood=10429.22
AIC=-20844.43   AICc=-20844.4   BIC=-20799.82
```



(a) MA coefficients



(b) QQnorm of residual

Test residual independences with ljung:

```
Box-Ljung test

data:  res
X-squared = 479.06, df = 1, p-value < 2.2e-16
```

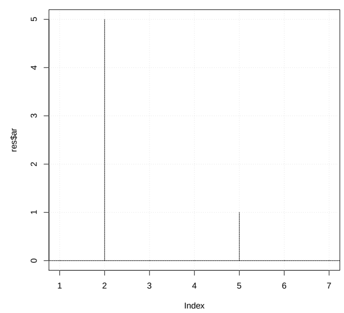
Test residual normal law with Shapiro:

```
Shapiro-Wilk normality test

data:  res
W = 0.99702, p-value = 1.523e-07
```

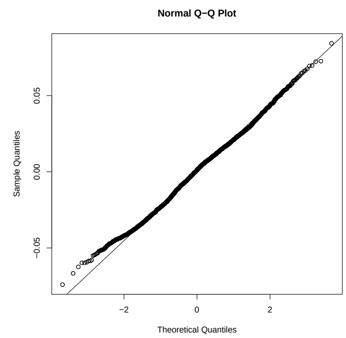
Is it an ARMA model ? We look for the AR coefficient, and minimize it with the AIC indicator. The AIC is a ratio between a good calibration and the number of parameters. The AR model implies that residual should be white noise (iid normal law). It's tested with the QQnorm.

Results:

```
Series: value
ARIMA(0,0,5) with non-zero mean

Coefficients:
         ma1     ma2     ma3     ma4     ma5  intercept
      2.1483  2.8344  2.6013  1.6428  0.5836     1.2171
s.e.  0.0182  0.0336  0.0296  0.0182  0.0116     0.0036

sigma^2 estimated as 0.0004708:  log likelihood=10429.22
AIC=-20844.43   AICc=-20844.4   BIC=-20799.82
```



(a) MA coefficients



(b) QQnorm of residual

Test residual independences with ljung:

```
Box-Ljung test

data:  res
X-squared = 479.06, df = 1, p-value < 2.2e-16
```

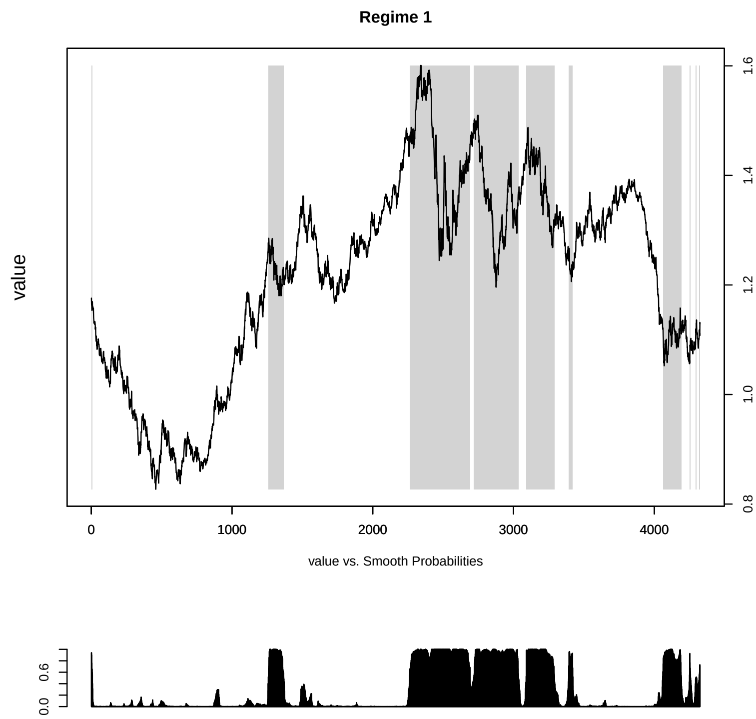
Test residual normal law with Shapiro:

```
Shapiro-Wilk normality test

data:  res
W = 0.99702, p-value = 1.523e-07
```

Markov Switching Model Try with 2 regimes.

Results:

```
Markov Switching Model

Call: msmFit(object = mod, k = 2, sw = c(T, T, T), p = 1, control = list(parallel = F))

        AIC      BIC   logLik
  -30176.57 -30117.6 15092.29

Coefficients:
        (Intercept)(S) value_1(S)      Std(S)
Model 1   0.0051391138  0.9959467 0.010649831
Model 2  -0.0001810602  1.0002282 0.006159484

Transition probabilities:
           Regime 1    Regime 2
Regime 1 0.98867243 0.005043856
Regime 2 0.01132757 0.994956144
```

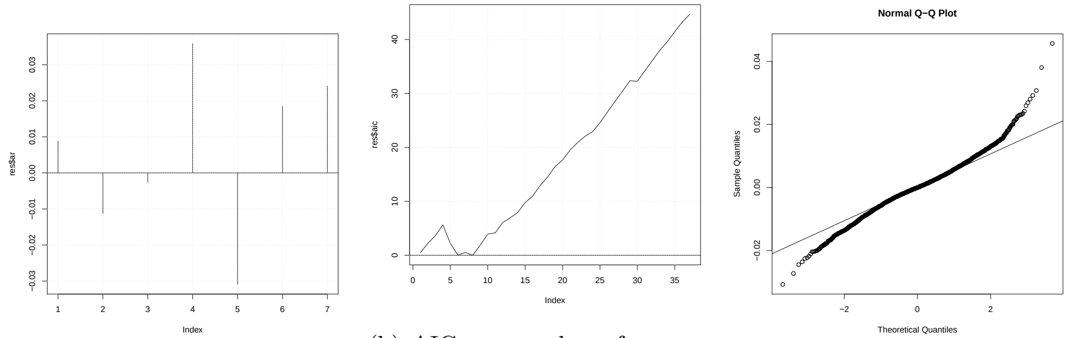


(a) Which 2

Looking at logret

Is it an AR model ? We look for the AR coefficient, and minimize it with the AIC indicator. The AIC is a ratio between a good calibration and the number of parameters. The AR model implies that residual should be white noise (iid normal law). It's tested with the QQnorm. Results:

```
Call:
ar(x = value)

Coefficients:
      1        2        3        4        5        6        7
 0.0088  -0.0113  -0.0026   0.0359  -0.0309   0.0185   0.0241

Order selected 7  sigma^2 estimated as  4.069e-05
```



(a) AR coefficients

(b) AIC per number of parameters

(c) QQnorm of residual

Test residual independences with ljung:

```
Box-Ljung test

data:  res
X-squared = 1.1456e-05, df = 1, p-value = 0.9973
```

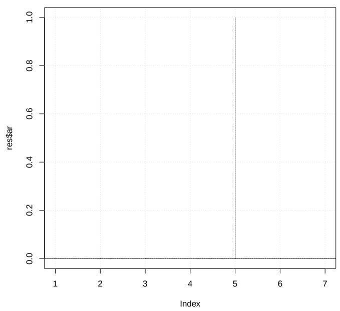
Test residual normal law with Shapiro:

```
Shapiro-Wilk normality test

data:  res
W = 0.98309, p-value < 2.2e-16
```

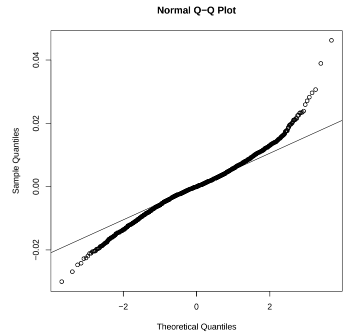
Is it an MA model ? We look for the AR coefficient, and minimize it with the AIC indicator. The AIC is a ratio between a good calibration and the number of parameters. The AR model implies that residual should be white noise (iid normal law). It's tested with the QQnorm.

Results:

```
Series: value
ARIMA(0,0,0) with non-zero mean

Coefficients:
      intercept
          0e+00
s.e.      1e-04

sigma^2 estimated as 4.075e-05:  log likelihood=15721.53
AIC=-31439.06   AICc=-31439.06   BIC=-31426.32
```



(a) MA coefficients



(b) QQnorm of residual

Test residual independences with ljung:

```
Box-Ljung test

data:  res
X-squared = 0.24259, df = 1, p-value = 0.6223
```

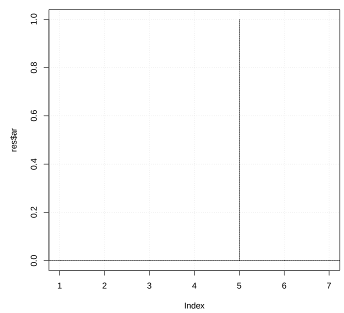
Test residual normal law with Shapiro:

```
Shapiro-Wilk normality test

data:  res
W = 0.98276, p-value < 2.2e-16
```

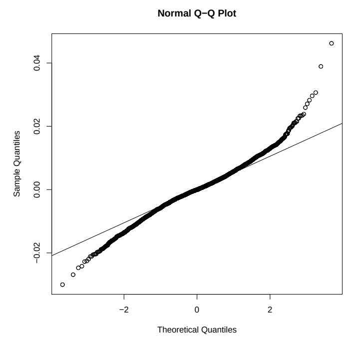
Is it an ARMA model ? We look for the AR coefficient, and minimize it with the AIC indicator. The AIC is a ratio between a good calibration and the number of parameters. The AR model implies that residual should be white noise (iid normal law). It's tested with the QQnorm.

Results:

```
Series: value
ARIMA(0,0,0) with non-zero mean

Coefficients:
      intercept
          0e+00
s.e.      1e-04

sigma^2 estimated as 4.075e-05:  log likelihood=15721.53
AIC=-31439.06   AICc=-31439.06   BIC=-31426.32
```



(a) MA coefficients



(b) QQnorm of residual

Test residual independences with ljung:

```
Box-Ljung test

data:  res
X-squared = 0.24259, df = 1, p-value = 0.6223
```

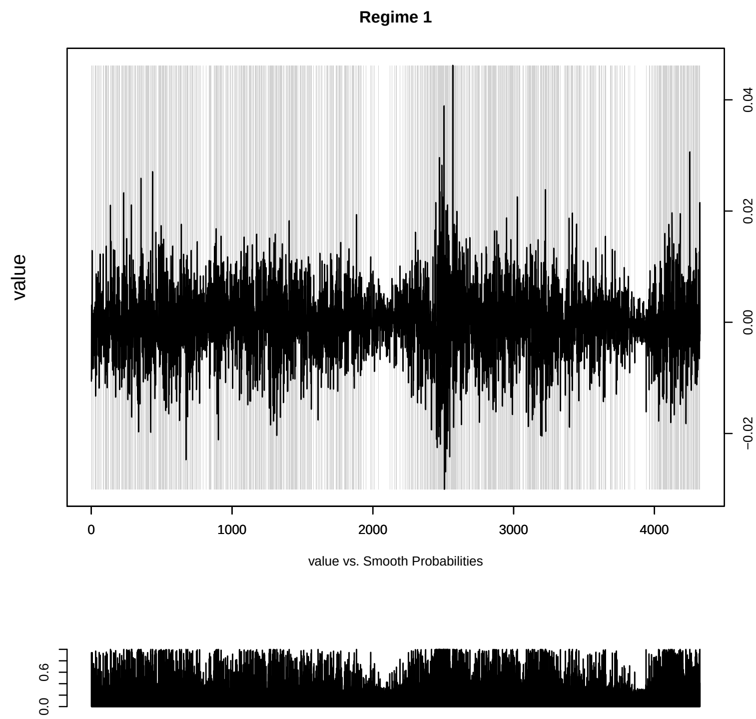
Test residual normal law with Shapiro:

```
Shapiro-Wilk normality test

data:  res
W = 0.98276, p-value < 2.2e-16
```

Markov Switching Model Try with 2 regimes.

Results:

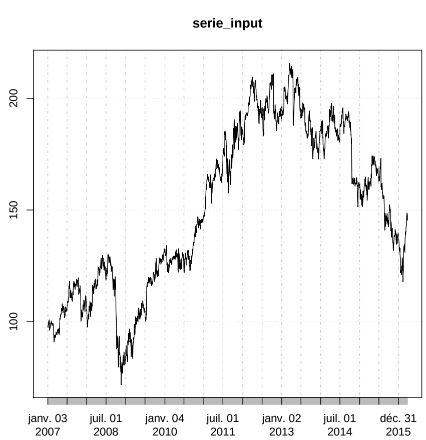
```
Markov Switching Model

Call: msmFit(object = mod, k = 2, sw = c(T, T, T), p = 1, control = list(parallel = F))

        AIC      BIC   logLik
  -31659.28 -31600.3 15833.64

Coefficients:
        (Intercept)(S)  value_1(S)      Std(S)
Model 1  -0.0002184246  0.06645888 0.008243233
Model 2   0.0001969037 -0.06868736 0.003960878

Transition probabilities:
          Regime 1  Regime 2
Regime 1 0.6089057 0.3545759
Regime 2 0.3910943 0.6454241
```



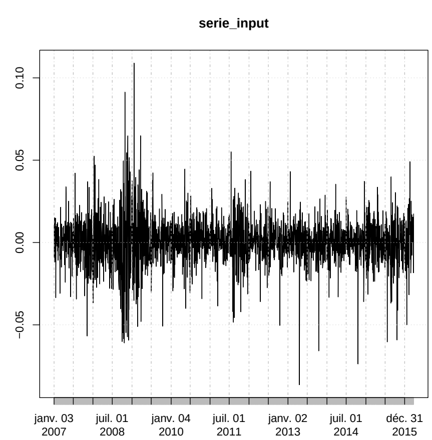
(a) Which 2

4.3 IBM

This is the historical close quotation for IBM. Data has been retrieved from yahoo.



(a) Close level of IBM

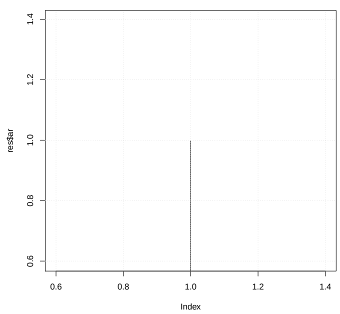


(b) log return IBM

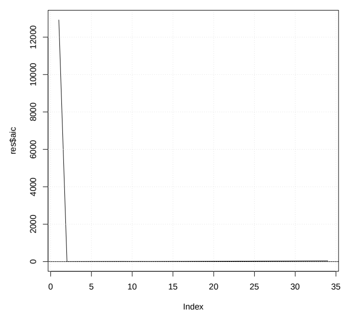
Looking at original

Is it an AR model ? We look for the AR coefficient, and minimize it with the AIC indicator. The AIC is a ratio between a good calibration and the number of parameters. The AR model implies that residual should be white noise (iid normal law). It's tested with the QQnorm. Results:

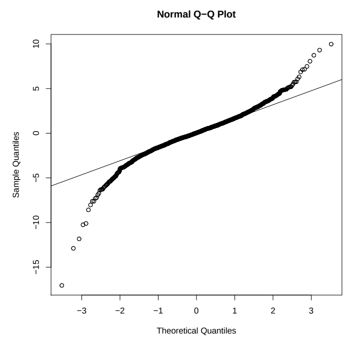
```
Call:
ar(x = value)

Coefficients:
     1
0.9981

Order selected 1  sigma^2 estimated as  5.151
```



(a) AR coefficients



(b) AIC per number of parameters



(c) QQnorm of residual

Test residual independences with ljung:

```
Box-Ljung test

data:  res
X-squared = 0.098621, df = 1, p-value = 0.7535
```

Test residual normal law with Shapiro:

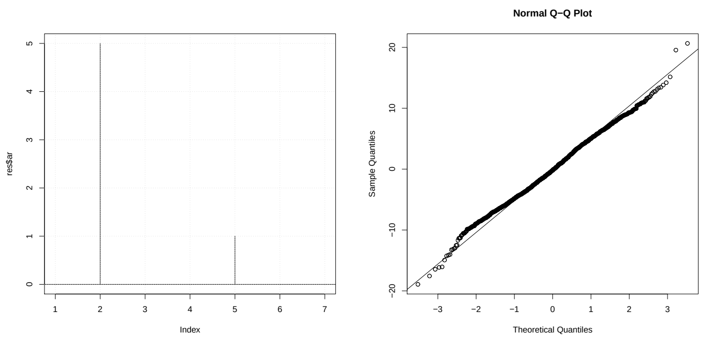
```
Shapiro-Wilk normality test

data:  res
W = 0.9508, p-value < 2.2e-16
```

Is it an MA model ? We look for the AR coefficient, and minimize it with the AIC indicator. The AIC is a ratio between a good calibration and the number of parameters. The AR model implies that residual should be white noise (iid normal law). It's tested with the QQnorm.

Results:

```
Series: value
ARIMA(0,0,5) with non-zero mean
```

```
Coefficients:
         ma1     ma2     ma3     ma4    ma5  intercept
      2.0207  2.6273  2.4116  1.5390  0.568   150.1698
s.e.  0.0216  0.0375  0.0352  0.0242  0.015     1.0180

sigma^2 estimated as 23.34:  log likelihood=-6958.04
AIC=13930.08   AICc=13930.13   BIC=13970.33
```



(a) MA coefficients (b) QQnorm of residual

Test residual independences with ljung:

```
Box-Ljung test

data:  res
X-squared = 213.87, df = 1, p-value < 2.2e-16
```

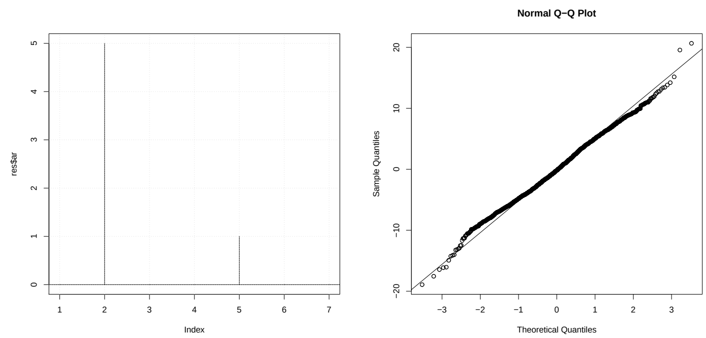
Test residual normal law with Shapiro:

```
Shapiro-Wilk normality test

data:  res
W = 0.99703, p-value = 0.0001803
```

Is it an ARMA model ? We look for the AR coefficient, and minimize it with the AIC indicator. The AIC is a ratio between a good calibration and the number of parameters. The AR model implies that residual should be white noise (iid normal law). It's tested with the QQnorm.

Results:

```
Series: value
ARIMA(0,0,5) with non-zero mean

Coefficients:
         ma1     ma2     ma3     ma4    ma5  intercept
      2.0207  2.6273  2.4116  1.5390  0.568   150.1698
s.e.  0.0216  0.0375  0.0352  0.0242  0.015     1.0180

sigma^2 estimated as 23.34:  log likelihood=-6958.04
AIC=13930.08   AICc=13930.13   BIC=13970.33
```



(a) MA coefficients (b) QQnorm of residual

Test residual independences with ljung:

```
Box-Ljung test

data:  res
X-squared = 213.87, df = 1, p-value < 2.2e-16
```

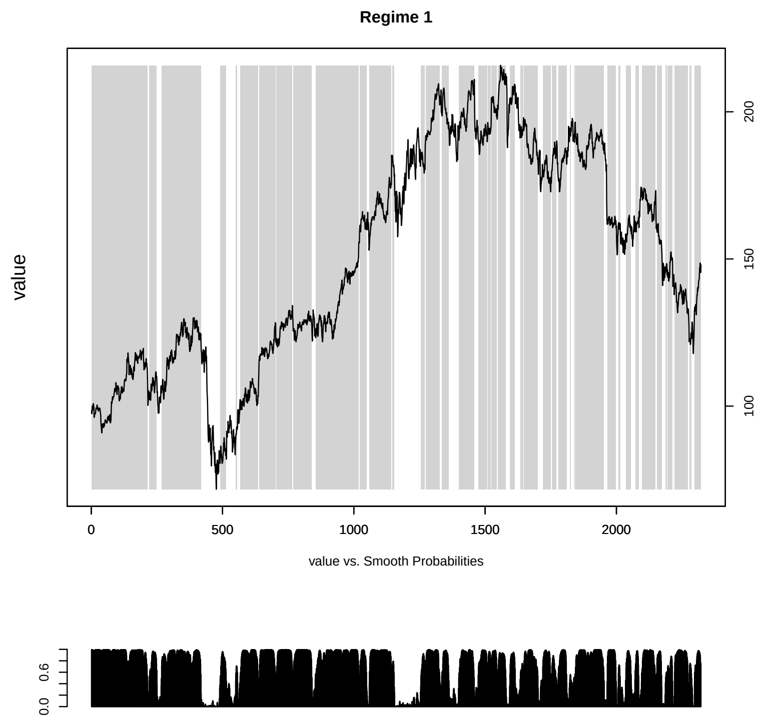
Test residual normal law with Shapiro:

```
Shapiro-Wilk normality test

data:  res
W = 0.99703, p-value = 0.0001803
```

Markov Switching Model Try with 2 regimes.

Results:

```
Markov Switching Model

Call: msmFit(object = mod, k = 2, sw = c(T, T, T), p = 1, control = list(parallel = F))

       AIC      BIC    logLik
  9388.168 9442.169 -4690.084

Coefficients:
        (Intercept)(S) value_1(S)   Std(S)
Model 1      0.2648856  0.9988069 1.368896
Model 2      0.3165829  0.9969839 3.033226

Transition probabilities:
           Regime 1   Regime 2
Regime 1 0.96502377 0.09019822
Regime 2 0.03497623 0.90980178
```

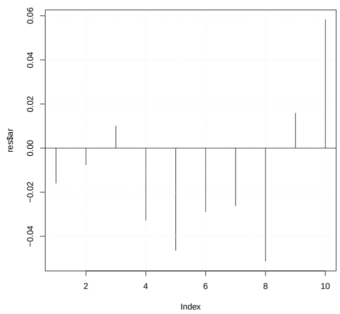


(a) Which 2

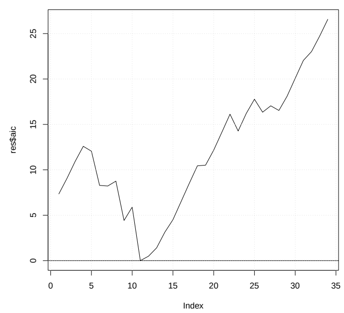
Looking at logret

Is it an AR model ? We look for the AR coefficient, and minimize it with the AIC indicator. The AIC is a ratio between a good calibration and the number of parameters. The AR model implies that residual should be white noise (iid normal law). It's tested with the QQnorm. Results:

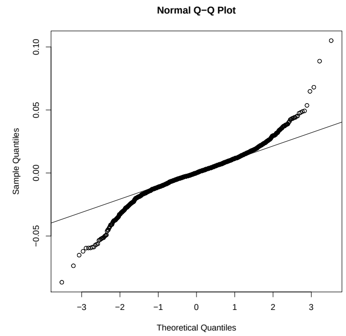
```
Call:
ar(x = value)

Coefficients:
      1        2        3        4        5        6        7        8
-0.0160  -0.0076   0.0100  -0.0327  -0.0464  -0.0288  -0.0262  -0.0513
      9       10
 0.0159   0.0583

Order selected 10  sigma^2 estimated as  0.0002069
```



(a) AR coefficients



(b) AIC per number of parameters



(c) QQnorm of residual

Test residual independences with ljung:

```
Box-Ljung test

data:  res
X-squared = 0.0038622, df = 1, p-value = 0.9504
```

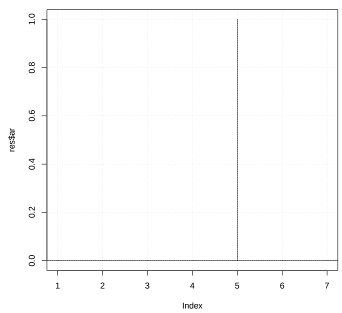
Test residual normal law with Shapiro:

```
Shapiro-Wilk normality test

data:  res
W = 0.94224, p-value < 2.2e-16
```

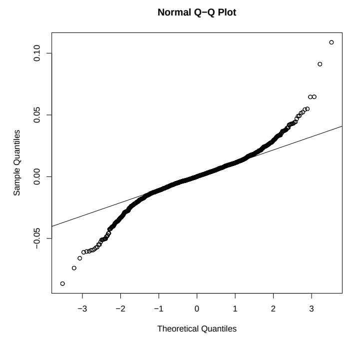
Is it an MA model ? We look for the AR coefficient, and minimize it with the AIC indicator. The AIC is a ratio between a good calibration and the number of parameters. The AR model implies that residual should be white noise (iid normal law). It's tested with the QQnorm.

Results:

```
Series: value
ARIMA(0,0,0) with non-zero mean

Coefficients:
      intercept
          2e-04
s.e.      3e-04

sigma^2 estimated as 0.0002084:  log likelihood=6545.89
AIC=-13087.77   AICc=-13087.77   BIC=-13076.27
```



(a) MA coefficients



(b) QQnorm of residual

Test residual independences with ljung:

```
Box-Ljung test

data:  res
X-squared = 0.28676, df = 1, p-value = 0.5923
```

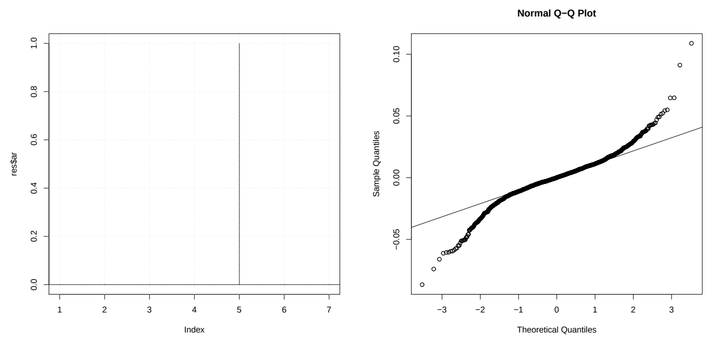
Test residual normal law with Shapiro:

```
Shapiro-Wilk normality test

data:  res
W = 0.94027, p-value < 2.2e-16
```

Is it an ARMA model ? We look for the AR coefficient, and minimize it with the AIC indicator. The AIC is a ratio between a good calibration and the number of parameters. The AR model implies that residual should be white noise (iid normal law). It's tested with the QQnorm.

Results:

```
Series: value
ARIMA(0,0,0) with non-zero mean

Coefficients:
      intercept
          2e-04
s.e.      3e-04

sigma^2 estimated as 0.0002084:  log likelihood=6545.89
AIC=-13087.77   AICc=-13087.77   BIC=-13076.27
```



(a) MA coefficients (b) QQnorm of residual

Test residual independences with ljung:

```
Box-Ljung test

data:  res
X-squared = 0.28676, df = 1, p-value = 0.5923
```

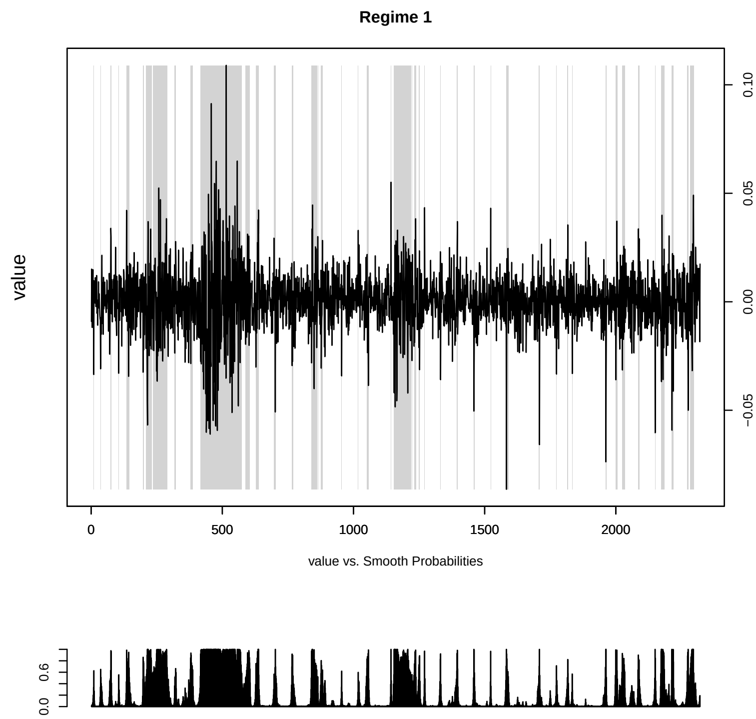
Test residual normal law with Shapiro:

```
Shapiro-Wilk normality test

data:  res
W = 0.94027, p-value < 2.2e-16
```

Markov Switching Model Try with 2 regimes.

Results:

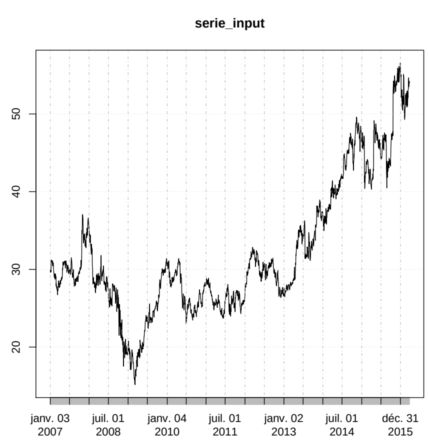
```
Markov Switching Model

Call: msmFit(object = mod, k = 2, sw = c(T, T, T), p = 1, control = list(parallel = F))

        AIC       BIC   logLik
  -13687.27 -13633.27 6847.635

Coefficients:
        (Intercept)(S)  value_1(S)      Std(S)
Model 1  -0.0007904372 -0.00119046 0.023807260
Model 2   0.0005225907 -0.03309368 0.009267872

Transition probabilities:
           Regime 1   Regime 2
Regime 1 0.90554306 0.03205306
Regime 2 0.09445694 0.96794694
```



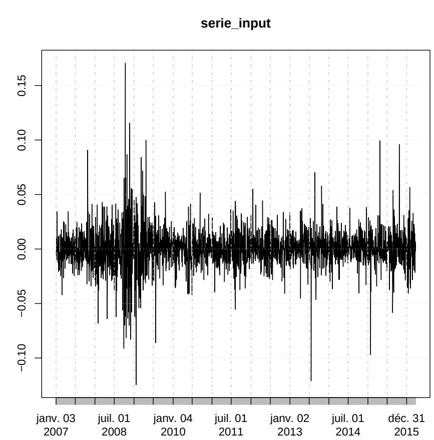
(a) Which 2

4.4 MSFT

This is the historical close quotation for MSFT. Data has been retrieved from yahoo.



(a) Close level of MSFT

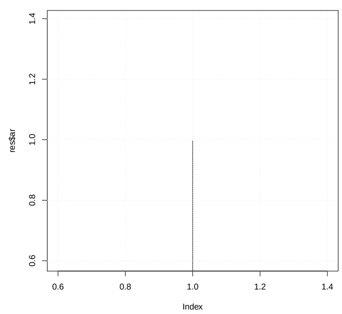


(b) log return MSFT

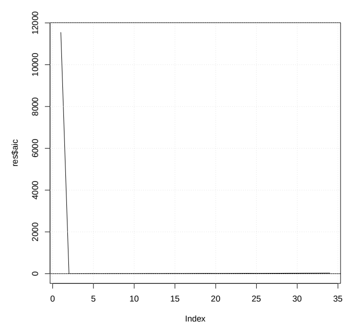
Looking at original

Is it an AR model ? We look for the AR coefficient, and minimize it with the AIC indicator. The AIC is a ratio between a good calibration and the number of parameters. The AR model implies that residual should be white noise (iid normal law). It's tested with the QQnorm. Results:

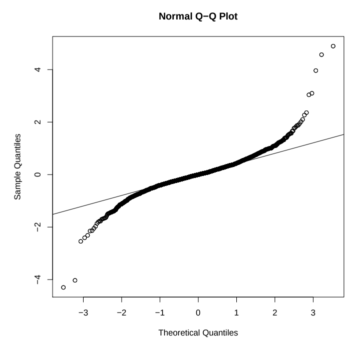
```
Call:
ar(x = value)

Coefficients:
     1
0.9965

Order selected 1  sigma^2 estimated as  0.5186
```



(a) AR coefficients



(b) AIC per number of parameters



(c) QQnorm of residual

Test residual independences with ljung:

```
Box-Ljung test

data:  res
X-squared = 0.23417, df = 1, p-value = 0.6284
```

Test residual normal law with Shapiro:

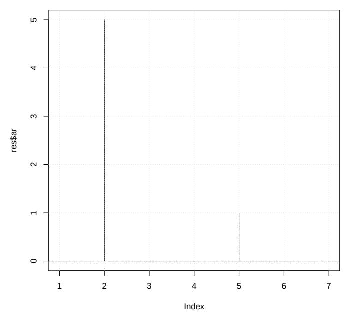
```
Shapiro-Wilk normality test

data:  res
W = 0.91461, p-value < 2.2e-16
```

Is it an MA model ? We look for the AR coefficient, and minimize it with the AIC indicator. The AIC is a ratio between a good calibration and the number of parameters. The AR model implies that residual should be white noise (iid normal law). It's tested with the QQnorm.

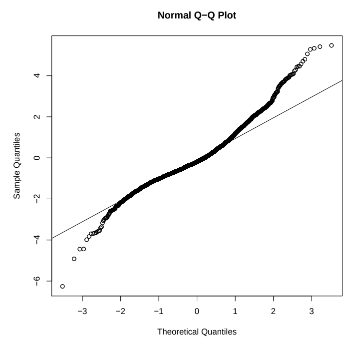
Results:

```
Series: value
ARIMA(0,0,5) with non-zero mean
```

```
Coefficients:
         ma1     ma2     ma3     ma4     ma5  intercept
      1.8856  2.3315  2.1797  1.4642  0.5639    32.1564
s.e.  0.0188  0.0280  0.0285  0.0251  0.0143     0.2431

sigma^2 estimated as 1.549:  log likelihood=-3807.32
AIC=7628.63   AICc=7628.68   BIC=7668.89
```



(a) MA coefficients



(b) QQnorm of residual

Test residual independences with ljung:

```
Box-Ljung test

data:  res
X-squared = 210.06, df = 1, p-value < 2.2e-16
```

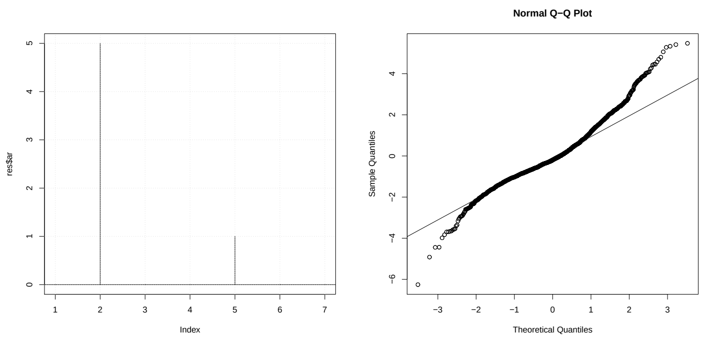
Test residual normal law with Shapiro:

```
Shapiro-Wilk normality test

data:  res
W = 0.96066, p-value < 2.2e-16
```

Is it an ARMA model ? We look for the AR coefficient, and minimize it with the AIC indicator. The AIC is a ratio between a good calibration and the number of parameters. The AR model implies that residual should be white noise (iid normal law). It's tested with the QQnorm.

Results:

```
Series: value
ARIMA(0,0,5) with non-zero mean

Coefficients:
         ma1     ma2     ma3     ma4     ma5  intercept
      1.8856  2.3315  2.1797  1.4642  0.5639    32.1564
s.e.  0.0188  0.0280  0.0285  0.0251  0.0143     0.2431

sigma^2 estimated as 1.549:  log likelihood=-3807.32
AIC=7628.63   AICc=7628.68   BIC=7668.89
```



(a) MA coefficients (b) QQnorm of residual

Test residual independences with ljung:

```
Box-Ljung test

data:  res
X-squared = 210.06, df = 1, p-value < 2.2e-16
```

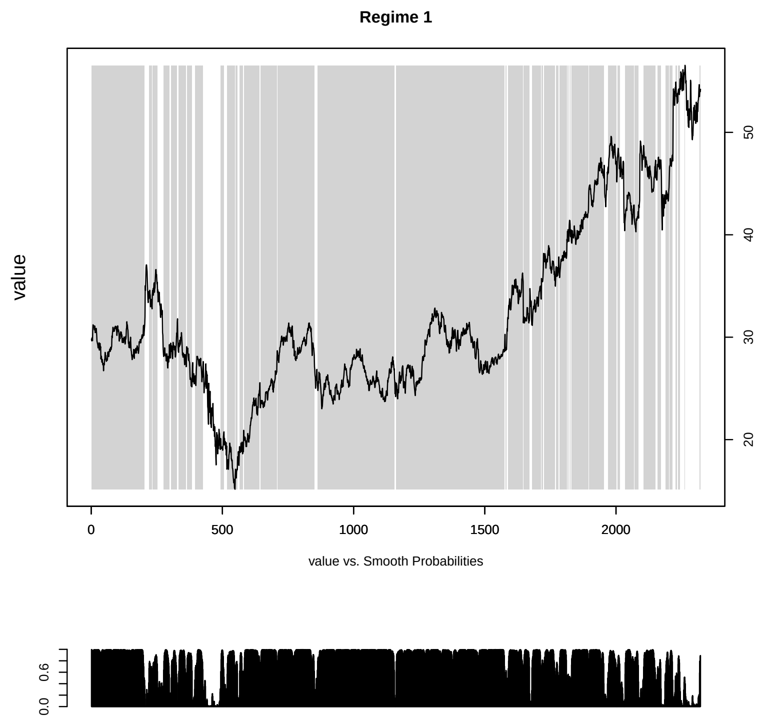
Test residual normal law with Shapiro:

```
Shapiro-Wilk normality test

data:  res
W = 0.96066, p-value < 2.2e-16
```

Markov Switching Model Try with 2 regimes.

Results:

```
Markov Switching Model

Call: msmFit(object = mod, k = 2, sw = c(T, T, T), p = 1, control = list(parallel = F))

       AIC      BIC    logLik
  3252.634 3306.636 -1622.317

Coefficients:
        (Intercept)(S) value_1(S)    Std(S)
Model 1     0.03245657  0.9993361 0.3726641
Model 2     0.02607107  0.9994247 0.9869924

Transition probabilities:
           Regime 1  Regime 2
Regime 1 0.96872469 0.1242708
Regime 2 0.03127531 0.8757292
```

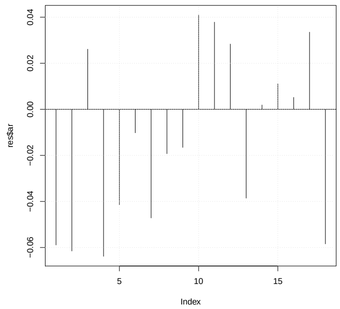


(a) Which 2

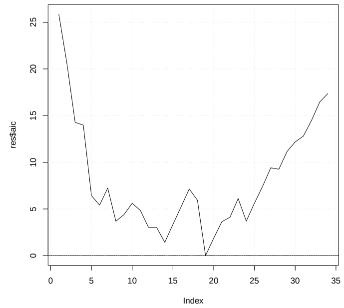
Looking at logret

Is it an AR model ? We look for the AR coefficient, and minimize it with the AIC indicator. The AIC is a ratio between a good calibration and the number of parameters. The AR model implies that residual should be white noise (iid normal law). It's tested with the QQnorm. Results:

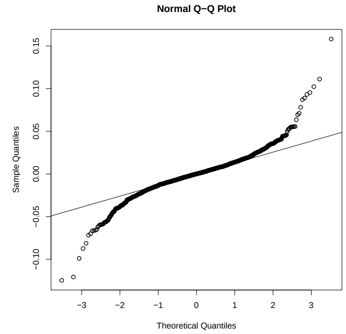
```
Call:
ar(x = value)

Coefficients:
      1        2        3        4        5        6        7        8
-0.0589  -0.0615   0.0261  -0.0638  -0.0414  -0.0102  -0.0472  -0.0193
      9       10       11       12       13       14       15       16
-0.0166   0.0409   0.0378   0.0283  -0.0386   0.0019   0.0110   0.0052
     17       18
 0.0335  -0.0584

Order selected 18  sigma^2 estimated as  0.000324
```



(a) AR coefficients



(b) AIC per number of parameters



(c) QQnorm of residual

Test residual independences with ljung:

```
Box-Ljung test

data:  res
X-squared = 0.00018124, df = 1, p-value = 0.9893
```

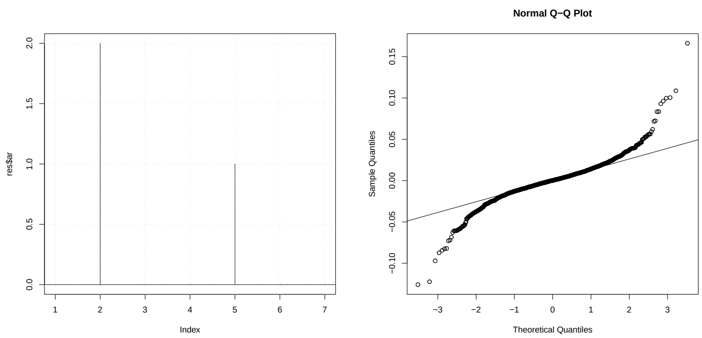
Test residual normal law with Shapiro:

```
Shapiro-Wilk normality test

data:  res
W = 0.91986, p-value < 2.2e-16
```

Is it an MA model ? We look for the AR coefficient, and minimize it with the AIC indicator. The AIC is a ratio between a good calibration and the number of parameters. The AR model implies that residual should be white noise (iid normal law). It's tested with the QQnorm.

Results:

```
Series: value
ARIMA(0,0,2) with zero mean

Coefficients:
          ma1      ma2
      -0.0560  -0.0613
s.e.   0.0209   0.0224

sigma^2 estimated as 0.0003278:  log likelihood=6019.9
AIC=-12033.8   AICc=-12033.79   BIC=-12016.55
```



(a) MA coefficients (b) QQnorm of residual

Test residual independences with ljung:

```
Box-Ljung test

data:  res
X-squared = 0.0083701, df = 1, p-value = 0.9271
```

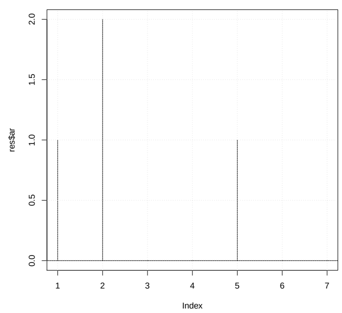
Test residual normal law with Shapiro:

```
Shapiro-Wilk normality test

data:  res
W = 0.91432, p-value < 2.2e-16
```

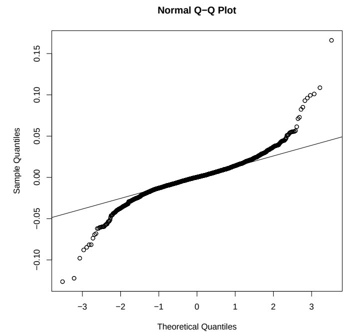
Is it an ARMA model ? We look for the AR coefficient, and minimize it with the AIC indicator. The AIC is a ratio between a good calibration and the number of parameters. The AR model implies that residual should be white noise (iid normal law). It's tested with the QQnorm.

Results:

```
Series: value
ARIMA(0,0,2) with zero mean

Coefficients:
          ma1      ma2
      -0.0560  -0.0613
s.e.   0.0209   0.0224

sigma^2 estimated as 0.0003278:  log likelihood=6019.9
AIC=-12033.8   AICc=-12033.79   BIC=-12016.55
```



(a) MA coefficients



(b) QQnorm of residual

Test residual independences with ljung:

```
Box-Ljung test

data:  res
X-squared = 0.0017872, df = 1, p-value = 0.9663
```

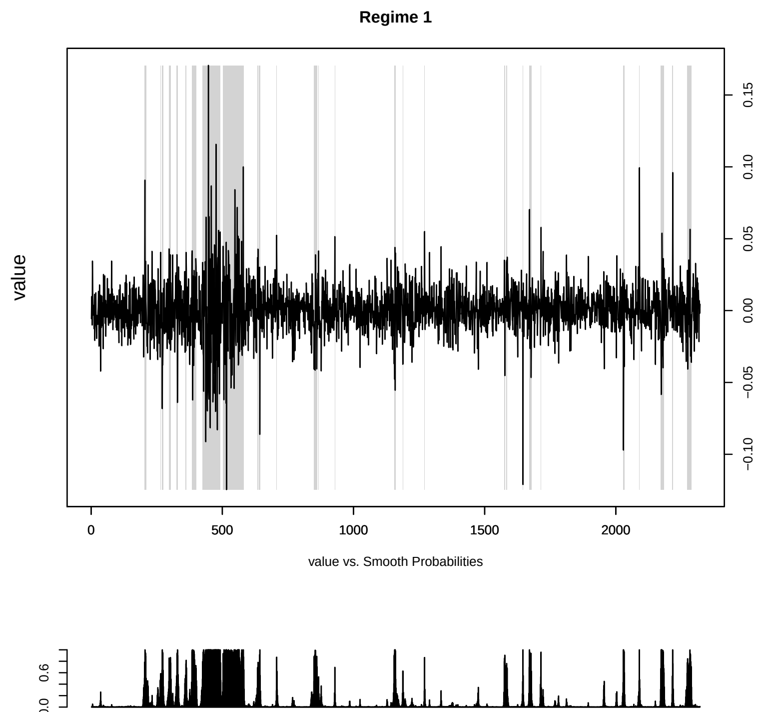
Test residual normal law with Shapiro:

```
Shapiro-Wilk normality test

data:  res
W = 0.91427, p-value < 2.2e-16
```

Markov Switching Model Try with 2 regimes.

Results:

```
Markov Switching Model

Call: msmFit(object = mod, k = 2, sw = c(T, T, T), p = 1, control = list(parallel = F))

        AIC       BIC   logLik
  -12733.08 -12679.08 6370.541

Coefficients:
        (Intercept)(S)  value_1(S)     Std(S)
Model 1   -0.001015905 -0.09460918 0.03640935
Model 2    0.000462657 -0.01414423 0.01256221

Transition probabilities:
          Regime 1   Regime 2
Regime 1 0.8903983 0.01873014
Regime 2 0.1096017 0.98126986
```



Regime 1
value
0.15
0.10
0.05
0.00
−0.05
−0.10
0
500
1000
1500
2000
value vs. Smooth Probabilities
0.6
0.0

(a) Which 2

A Notions

Strongly Stationary Be X_t a time series. X_t is strictly stationary process or strongly stationary process if :

$$\forall h, \forall n, X_{t_1} ... X_{t_n} \text{ has the same law than } X_{t_1+h} ... X_{t_n+h}$$

Weakly stationary X_t is a weak or wide-sense stationary process if :

(i) $\forall t, \mathbb{E}\left\{X_t^2\right\} < \infty$, finite variance

(ii) $\forall t, \mathbb{E}\left\{X_t\right\} = m$, expectation doesn't depend of time t

(iii) $\forall t, \operatorname{cov}(X_t, X_{t+h}) = \gamma(h)$, covariance doesn't depend of time t and only on the lag h.

In practice strongly stationary process are hard to demonstrate. In models below we'll demonstrate weak stationarity.

Covariance Be X, Y two random variables. We note $\operatorname{cov}(X, Y)$ the covariance between X and Y

$$\operatorname{cov}(X, Y) = \mathbb{E}\left\{(X - \mathbb{E}\left\{X\right\})(Y - \mathbb{E}\left\{Y\right\})\right\}$$

Correlation Be X, Y two random variables. We note $\rho_(X, Y)$ the correlation between X and Y

$$\rho_{X,Y} = \frac{\operatorname{cov}(X, Y)}{\sqrt{\operatorname{Var}(X) \operatorname{Var}(Y)}}$$

Autocorrelation Be X_t a time serie. $X_1, X_2, ...$ are random variables. We note γ_l the covariance between X_t and X_{t-l}

$$\gamma_l = \frac{\operatorname{cov}(X_t, X_{t-l})}{\sqrt{\operatorname{Var}(X_t) \operatorname{Var}(X_{t-l})}}$$

Note that $\gamma_0 = 1$

Partial autocorrelation (PACF) We note $\pi(k)$ the partial autocorrelation :

$$\pi(k) = \operatorname{corr}(X_t - \mathbb{E}\left\{X_t | X_{t-1} ... X_{t-k+1}\right\}, X_{t-k} - \mathbb{E}\left\{X_{t-k} | X_{t-1} ... X_{t-k+1}\right\}) =$$

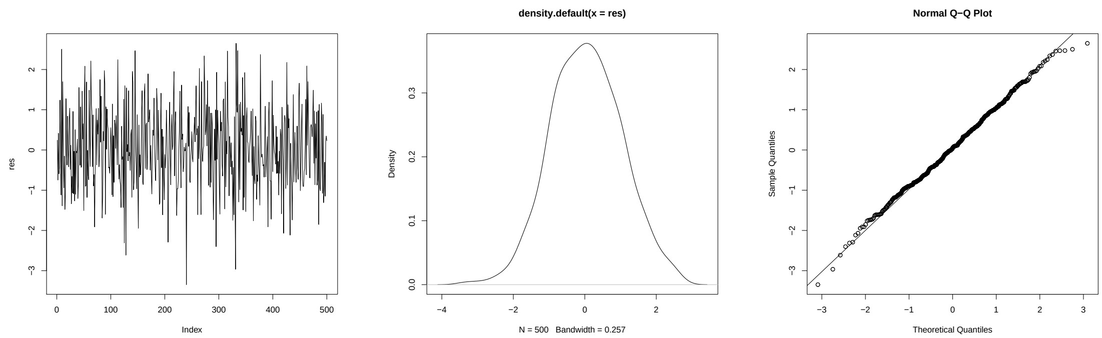
Akaike Information Criterion (AIC)

It misses a real part on AIC, OLS,..

$$AIC = \frac{-2}{T} \ln(\text{likelihood}) + \frac{2}{T}(\text{number of parameters})$$

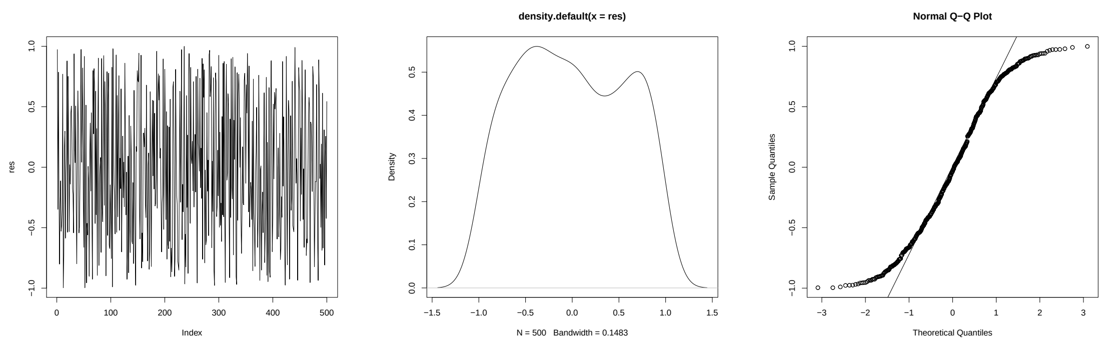
Normal law $\mathcal{N}(\mu, \sigma)$ X follow a normal law $\mathcal{N}(\mu, \sigma)$. The density function is :

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



(a) Normal process (b) Density function (c) QQnorm

Figure 63: Example of $\mathcal{N}(0,1)$



(a) Normal process (b) Density function (c) QQnorm

Figure 64: Example of $\mathcal{U}$(-1,1)