Volatility Modelisation with R

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March 22, 2016

Abstract

This is some personal research on existing financial modelisation. The aim is to cover :

- ARMA model
- GARCH model
- Multifractal Volatility

Contents

1 ARMA Model

1.1 Stationary process

Be X_t a time series.

X_t is strictly stationary process or strongly stationary process if :

$$\forall h, \forall n, X_{t_1} ... X_{t_n} \text{ has the same law than } X_{t_1+h} ... X_{t_n+h}$$

X_t is a weak or wide-sense stationarity if :

- $\forall t, \mathbb{E}[X_t^2] < \infty$
- $\forall t, \mathbb{E}[X_t] = m$
- $\forall t, \text{cov}(X_t, X_{t+h}) = \gamma(h)$

1.2 AR(1)

Definition Be $\beta \in \mathbb{R}$, X_t is an AR(1) process if

$$\begin{aligned} X_t &= c + \beta X_{t-1} + \epsilon_t \\ &= c + \beta^t X_0 + \sum_{i=0}^{t} \beta^i \epsilon_{t-i} \end{aligned}$$

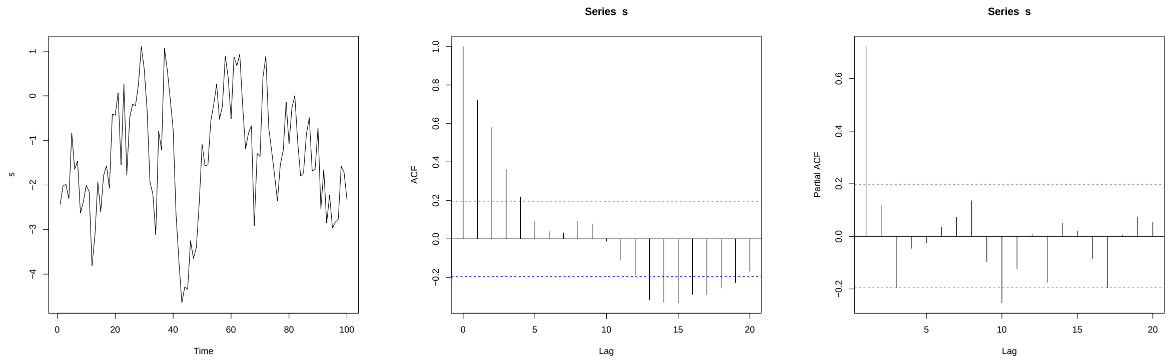
Stationary condition X_t is stationary if $|\beta| < 1$:

$$\begin{aligned} \text{Var}(X_t) &= \text{Var}(\sum_{i=0}^{t} \beta^i \epsilon_{t-i}) \\ &= \sum_{i=0}^{t} \beta^{2i} \sigma^2 \\ &= \frac{1-\beta^{2t}}{1-\beta} \sigma^2 \end{aligned}$$

When $|\beta| > 1$, $\text{Var}(X_t) \xrightarrow[t\to\infty]{} \infty$. When $|\beta| < 1$, $\text{Var}(X_t) \xrightarrow[t\to\infty]{} \frac{1}{1-\beta}\sigma^2$

Examples See below.

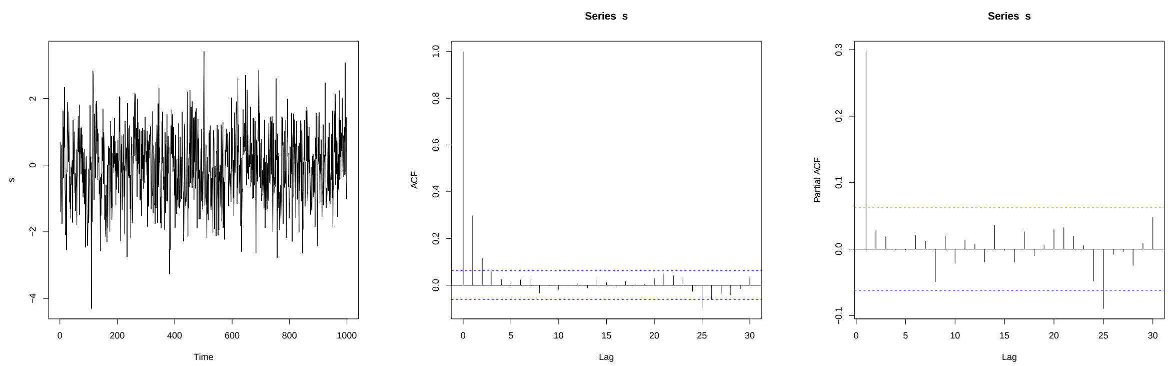
Figure 1 shows an AR(1) model, with AR(0.8), MA().



(a) Process (b) ACF (c) PACF

Figure 1: Example of AR(1) process, coefficient AR(0.8), MA()

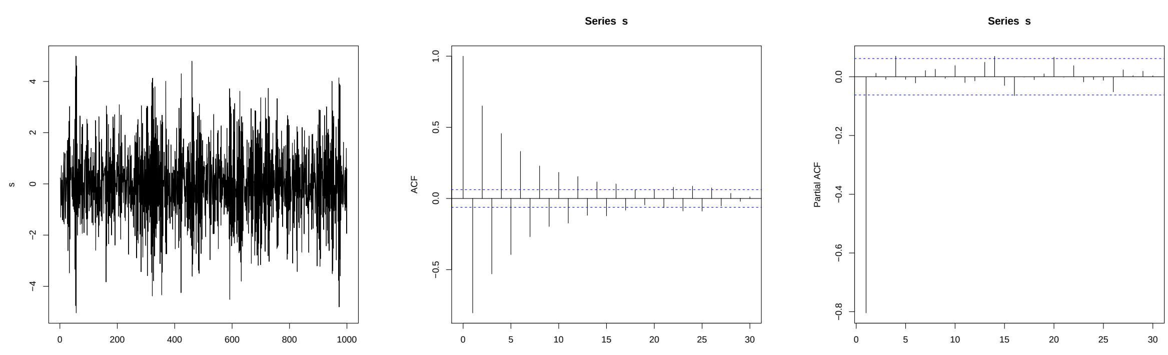
Figure 2 shows an AR(1) model, with AR(0.3), MA().



(a) Process (b) ACF (c) PACF

Figure 2: Example of AR(1) process, coefficient AR(0.3), MA()

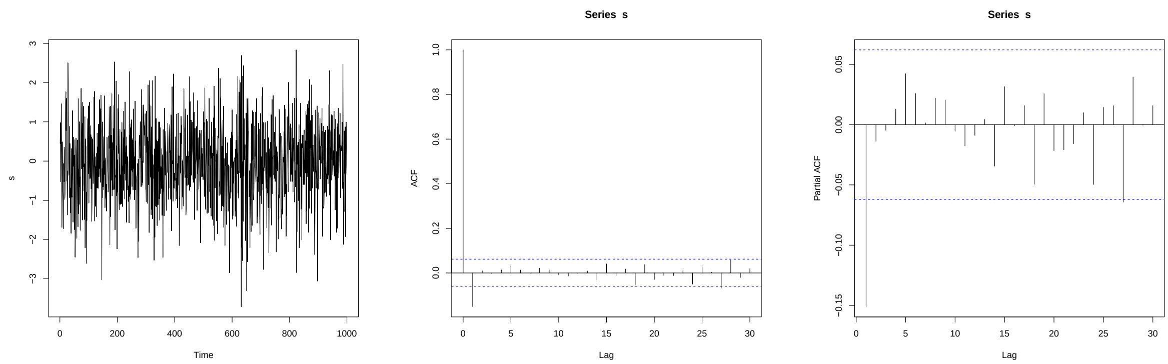
Figure 3 shows an AR(1) model, with AR(-0.8), MA().



(a) Process (b) ACF (c) PACF

Figure 3: Example of AR(1) process, coefficient AR(-0.8), MA()

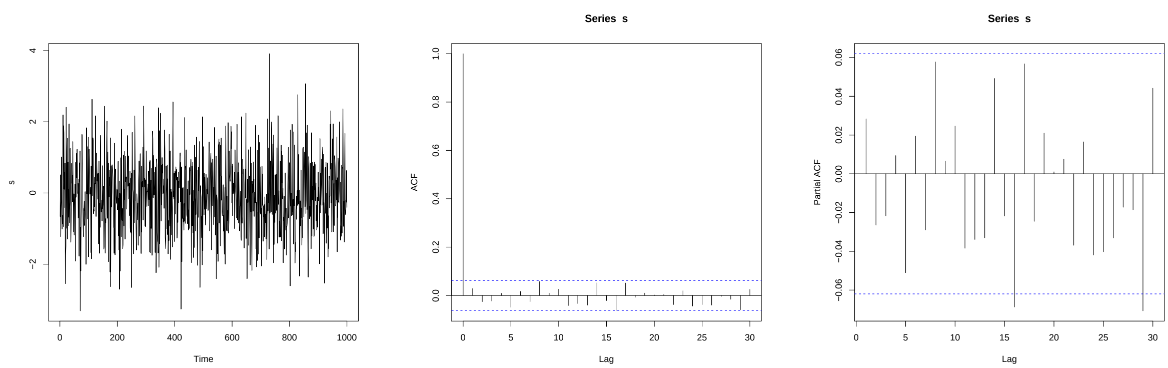
Figure 4 shows an AR(1) model, with AR(-0.1), MA().



(a) Process (b) ACF (c) PACF

Figure 4: Example of AR(1) process, coefficient AR(-0.1), MA()

Figure 5 shows an AR(1) model, with AR(0.01), MA().



(a) Process (b) ACF (c) PACF

Figure 5: Example of AR(1) process, coefficient AR(0.01), MA()

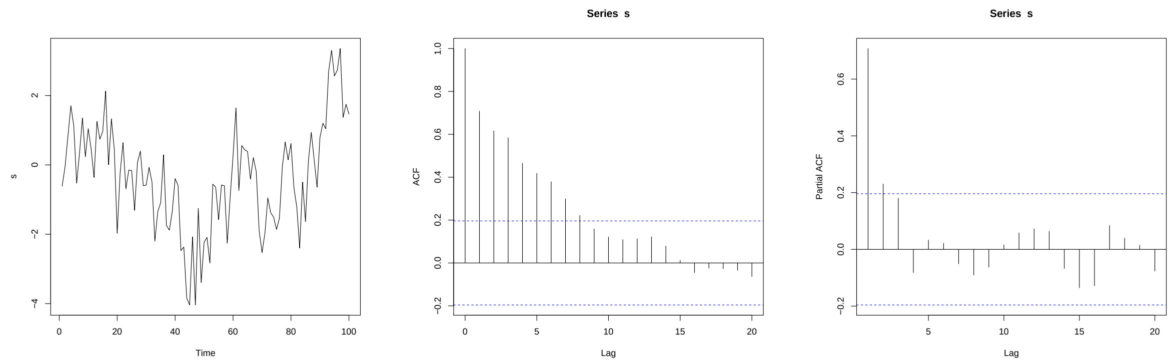
1.3 AR(p)

Definition X_t is an AR(p) process if

$$\begin{aligned} X_t &= c + \sum_{i=1}^{p} \beta_i X_{t-i} + \epsilon_t \\ &= c + \sum_{i=1}^{p} \beta_i \end{aligned}$$

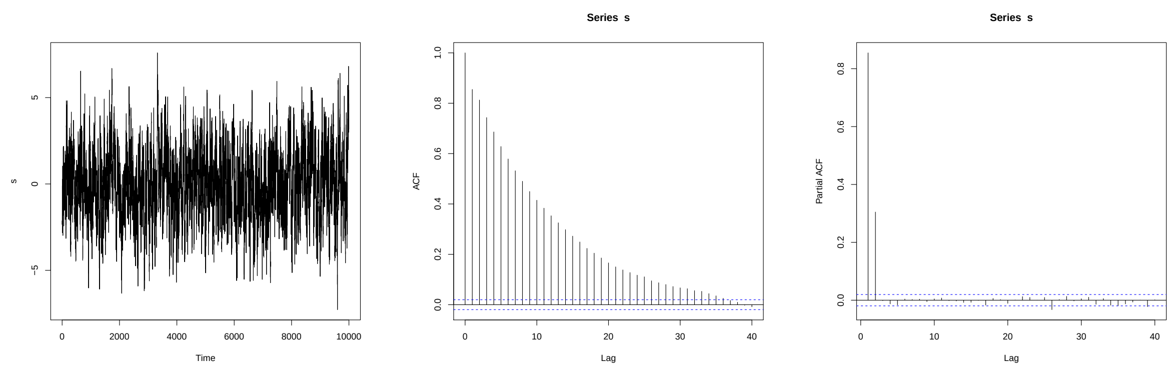
Stationary condition To be stationary, roots of the polynom $z^p - \sum_{i=1}^{p} \beta_i z^{p-i}$ must be within the unit circle, $|z_i| < 1$

Examples See below.



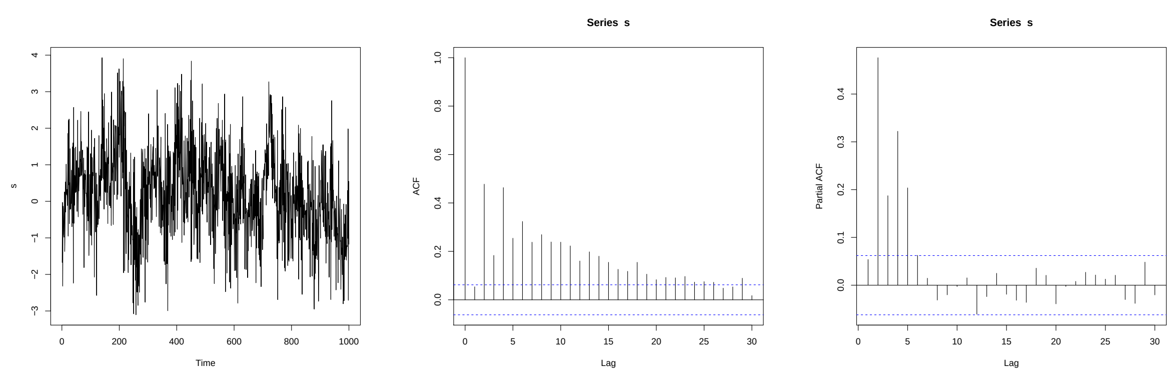
(a) Process (b) ACF (c) PACF

Figure 6: ARMA(2,0) process with coefficient AR(0.6,0.3), MA()



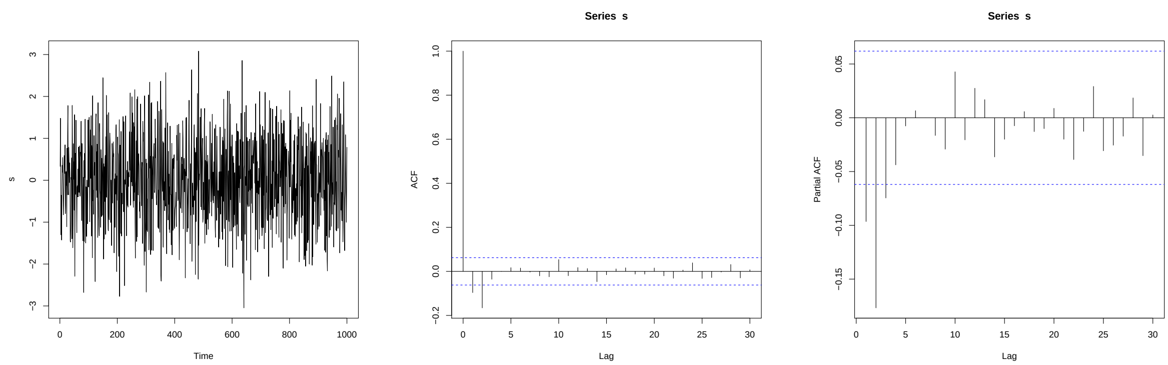
(a) Process (b) ACF (c) PACF

Figure 7: ARMA(2,0) process with coefficient AR(0.6,0.3), MA()



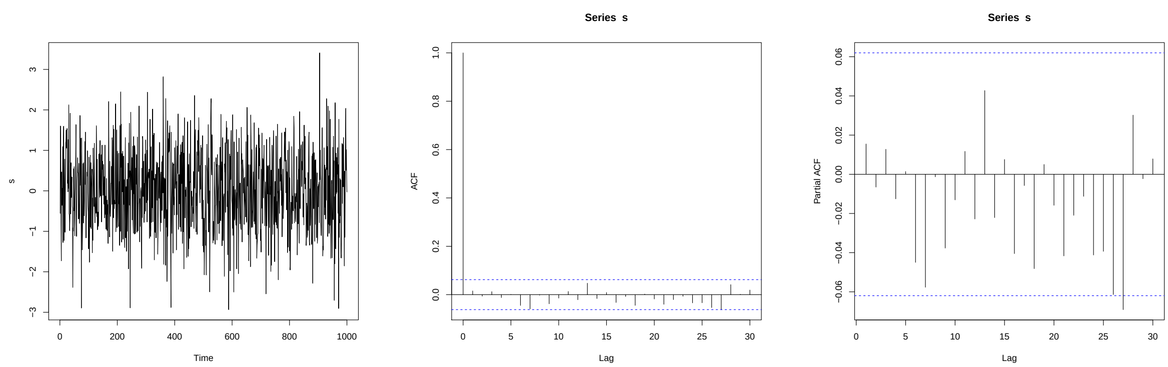
(a) Process (b) ACF (c) PACF

Figure 8: ARMA(6,0) process with coefficient AR(-0.2,0.2,0.1,0.3,0.2,0.1), MA()



(a) Process (b) ACF (c) PACF

Figure 9: ARMA(2,0) process with coefficient AR(-0.1,-0.2), MA()



(a) Process (b) ACF (c) PACF

Figure 10: ARMA(2,0) process with coefficient AR(0.01,0.02), MA()

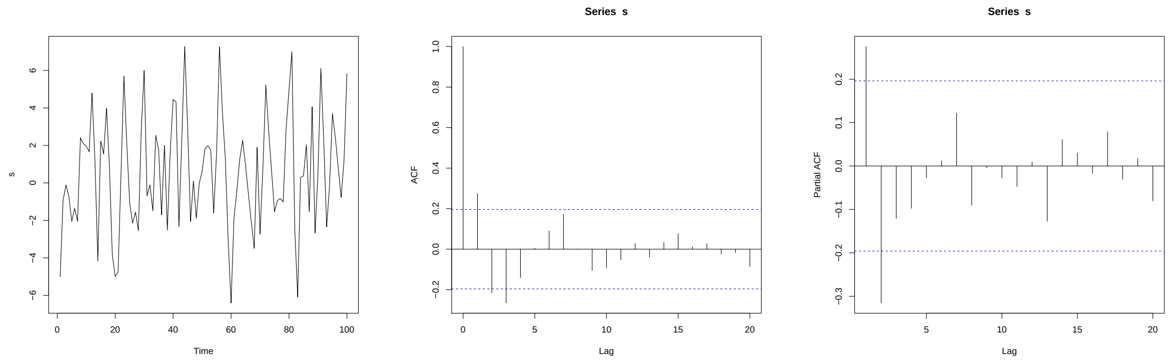
1.4 MA(1)

Definition Be $\theta \in \mathbb{R}$, X_t is an MA(1) process if

$$\begin{aligned} X_t &= c + \theta \epsilon_t \\ &= c + \beta^t X_0 + \sum_{i=0}^{t} \beta^i \epsilon_{t-i} \end{aligned}$$

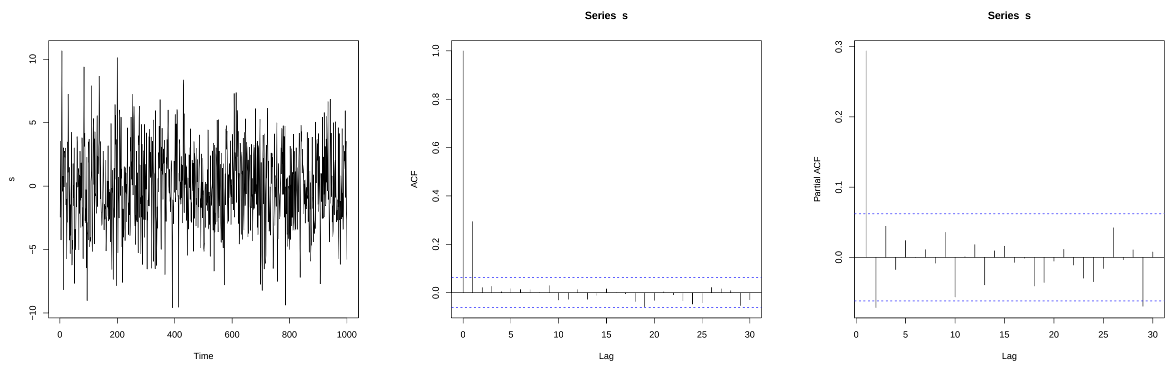
Stationary condition X_t has a finite variance : $\mathrm{Var}(X_t) = \theta^2 \sigma^2$

Examples See below.



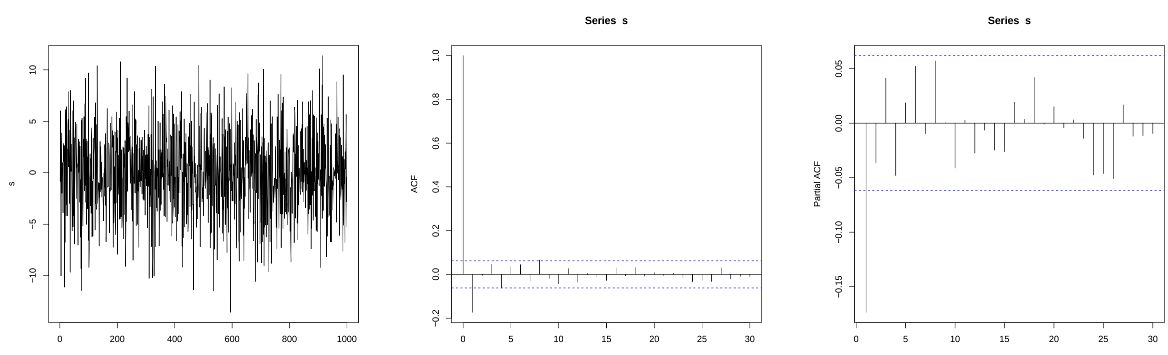
(a) Process (b) ACF (c) PACF

Figure 11: ARMA(0,1) process with coefficient AR(), MA(3)



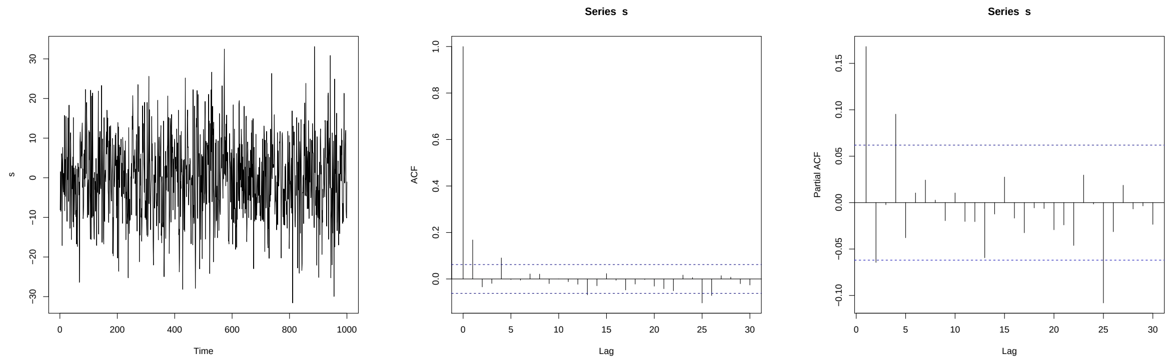
(a) Process (b) ACF (c) PACF

Figure 12: ARMA(0,1) process with coefficient AR(), MA(3)



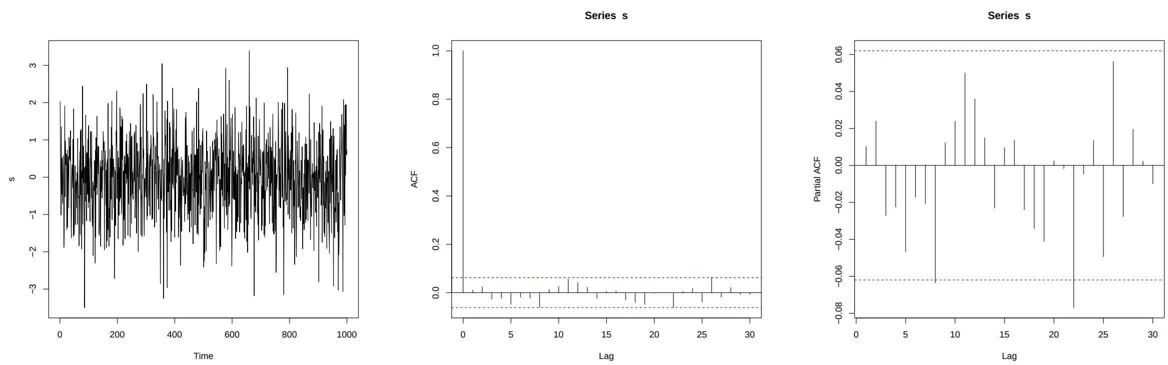
(a) Process (b) ACF (c) PACF

Figure 13: ARMA(0,1) process with coefficient AR(), MA(-4)



(a) Process (b) ACF (c) PACF

Figure 14: ARMA(0,1) process with coefficient AR(), MA(10)



(a) Process (b) ACF (c) PACF

Figure 15: ARMA(0,1) process with coefficient AR(), MA(0.01)

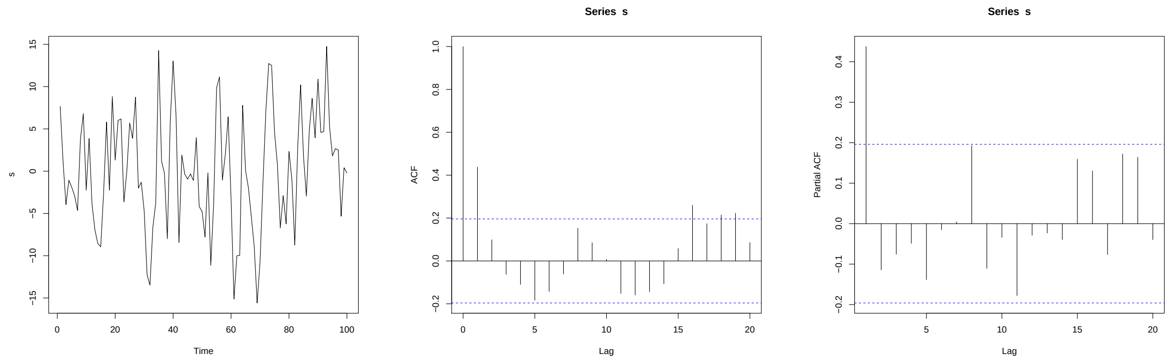
1.5 MA(q)

Definition Be $\theta \in \mathbb{R}$, X_t is an MA(q) process if

$$X_t = c + \sum_{i=0}^{q} \theta_i \epsilon_{t-i}$$

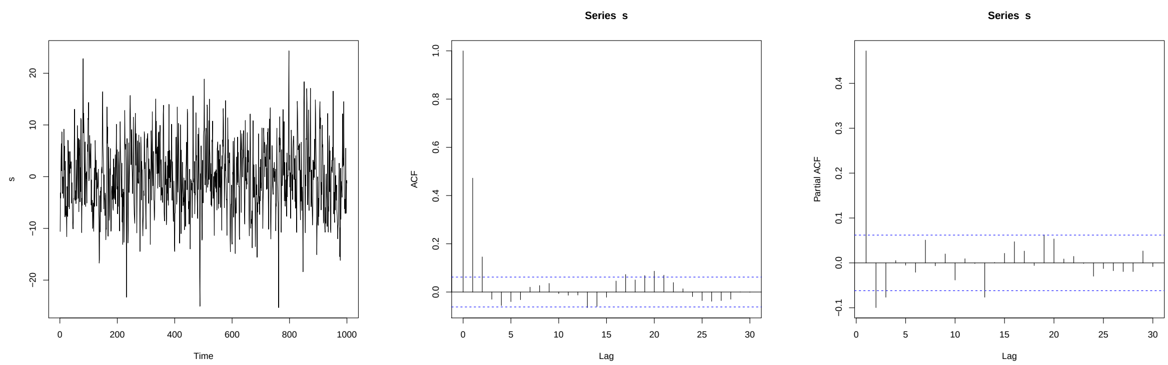
Stationary condition X_t has a finite variance : $\mathrm{Var}(X_t) = \sum_{i=0}^{q} \theta_i^2 \sigma^2$

Examples See below.



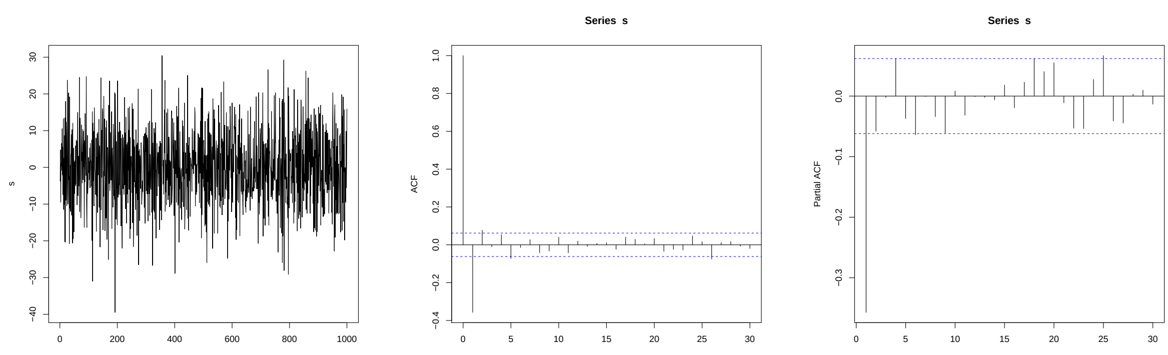
(a) Process (b) ACF (c) PACF

Figure 16: ARMA(0,2) process with coefficient AR(), MA(3,6)



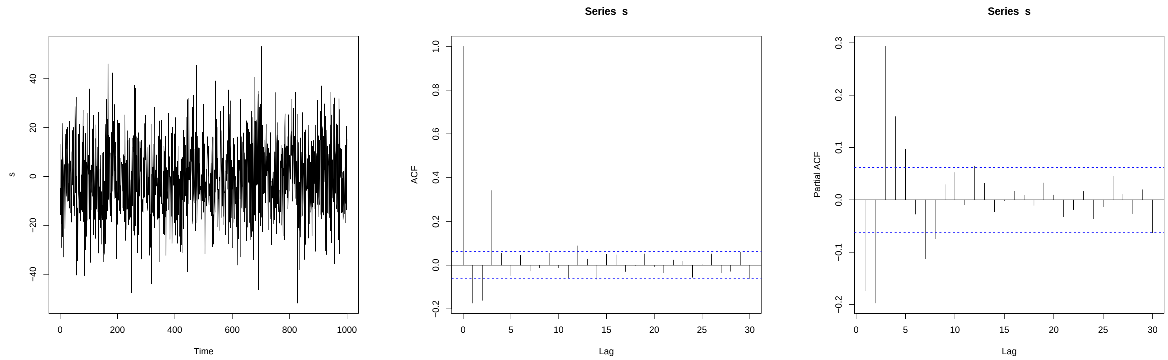
(a) Process (b) ACF (c) PACF

Figure 17: ARMA(0,2) process with coefficient AR(), MA(3,6)



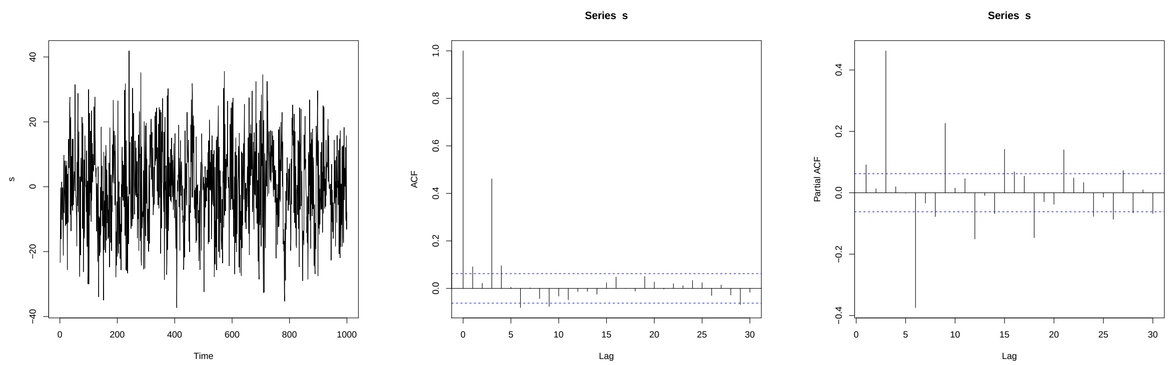
(a) Process (b) ACF (c) PACF

Figure 18: ARMA(0,2) process with coefficient AR(), MA(-4,10)



(a) Process (b) ACF (c) PACF

Figure 19: ARMA(0,4) process with coefficient AR(), MA(10,4,-7,9)



(a) Process (b) ACF (c) PACF

Figure 20: ARMA(0,4) process with coefficient AR(), MA(10,0,0,9)

1.6 ARMA(p,q)

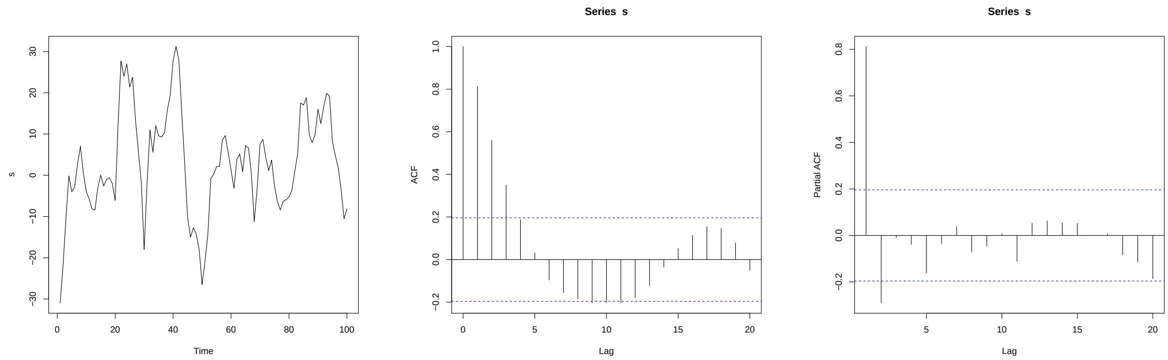
Definition Be $\theta \in \mathbb{R}$, X_t is an MA(q) process if

$$X_t = c + \sum_{i=0}^{q} \theta_i \epsilon_{t-i} + \sum_{i=1}^{p} \beta_i X_{t-i}$$

stationary To see if X_t is stationnary we look at its variance.

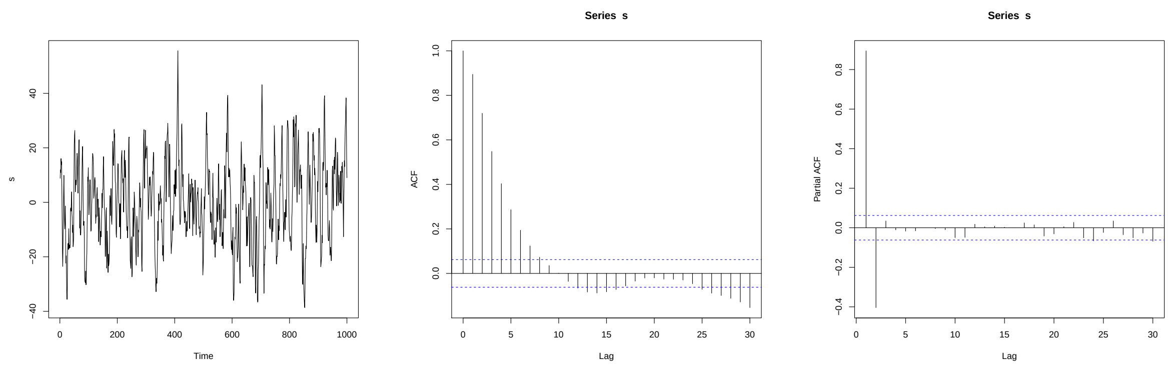
$$\mathrm{Var}(X_t) = \sum_{i=0}^{q} \theta_i^2 \sigma^2$$

Examples See below various example of ARMA(p,q)



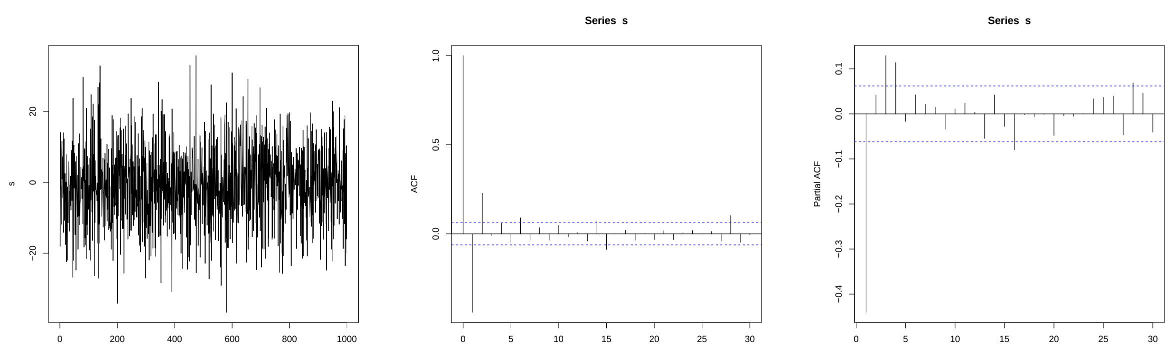
(a) Process (b) ACF (c) PACF

Figure 21: ARMA(1,2) process with coefficient AR(0.8), MA(3,6)



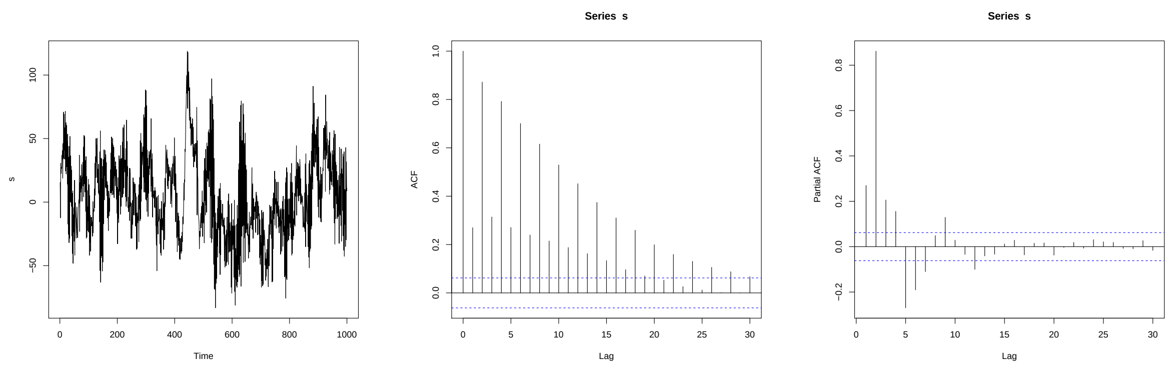
(a) Process (b) ACF (c) PACF

Figure 22: ARMA(1,2) process with coefficient AR(0.8), MA(3,6)



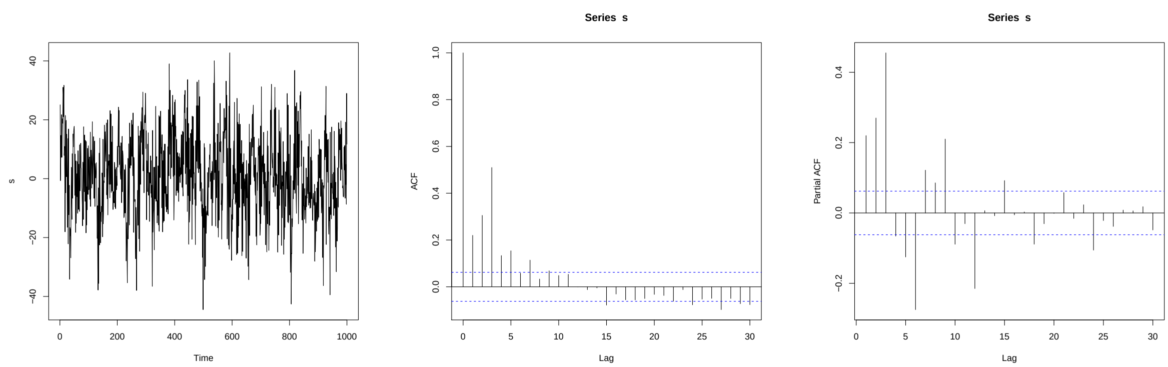
(a) Process (b) ACF (c) PACF

Figure 23: ARMA(3,2) process with coefficient AR(-0.1,0.2,0.1), MA(-4,10)



(a) Process (b) ACF (c) PACF

Figure 24: ARMA(2,4) process with coefficient AR(0,0.9), MA(10,4,-7,9)



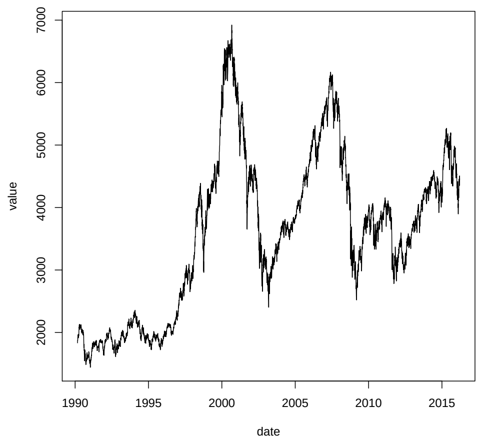
(a) Process (b) ACF (c) PACF

Figure 25: ARMA(2,4) process with coefficient AR(0,0.3), MA(10,0,0,9)

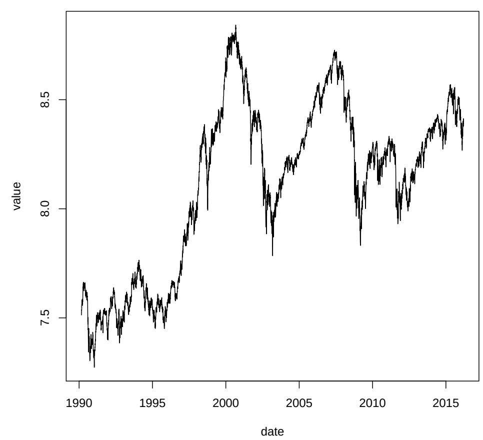
2 Analyse

In this section, there are analysis of some stocks.

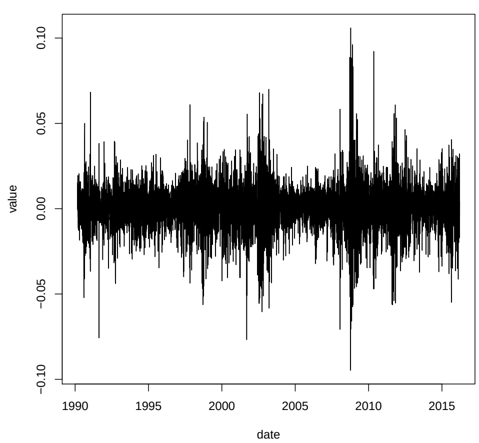
2.1 FCHI



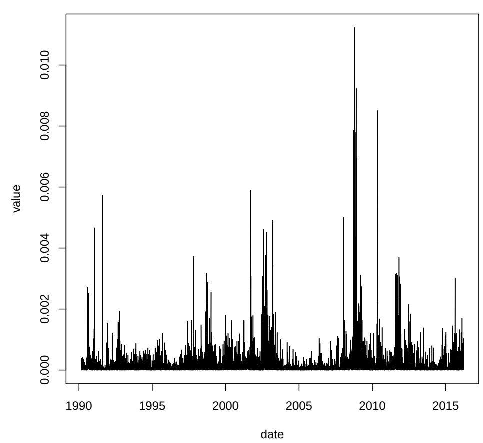
(a) Close level



(b) Log of close level



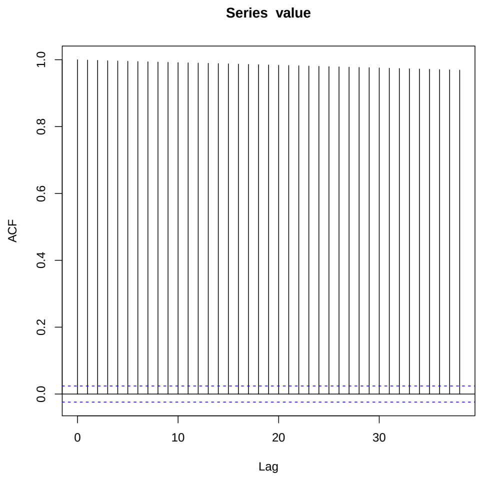
(c) log return



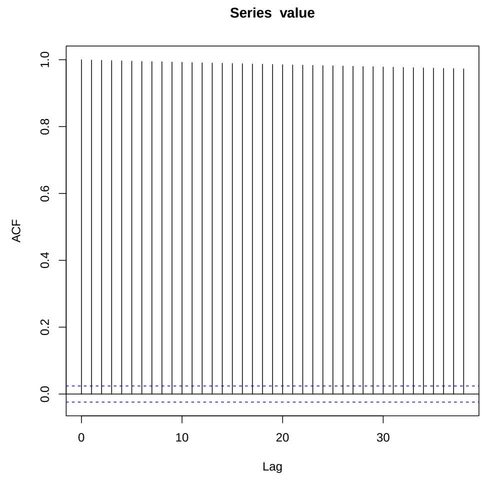
(d) log of squared return

Figure 26: Data set for FCHI

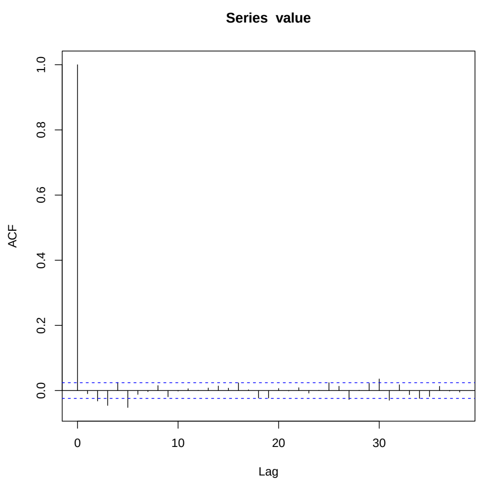
Looking at ACF and PACF



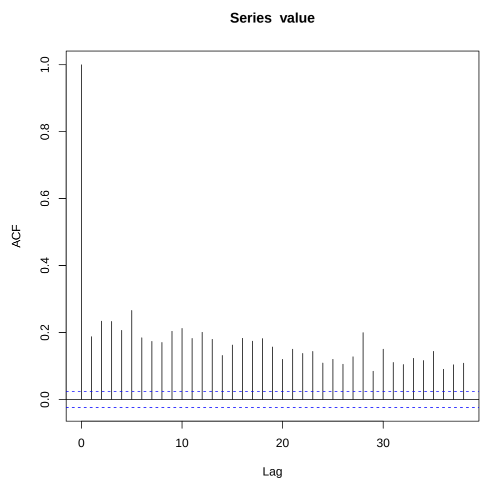
(a) Close level



(b) Log of close level

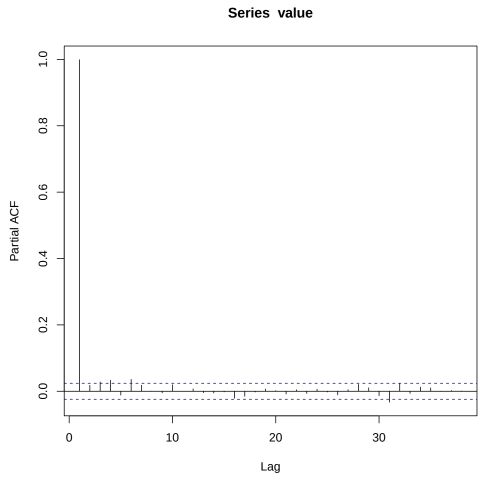


(c) log return

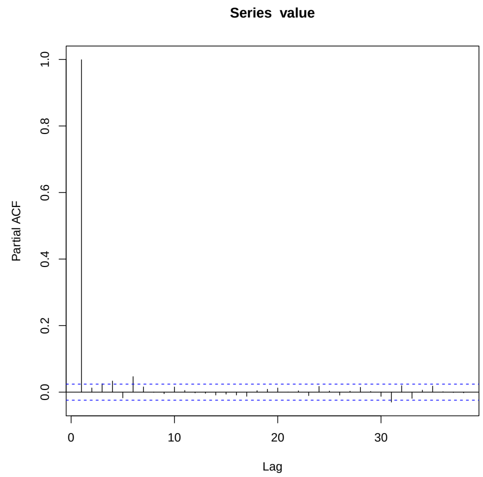


(d) log of squared return

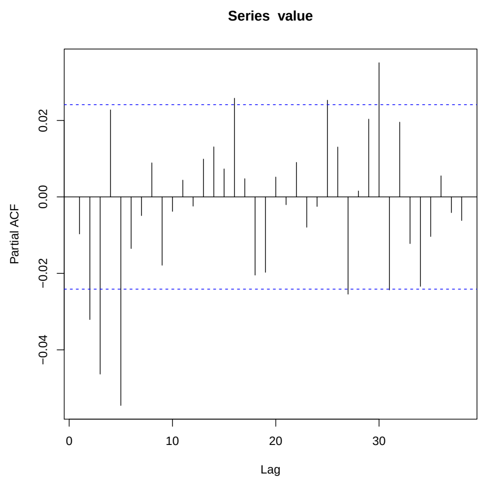
Figure 27: ACF for FCHI



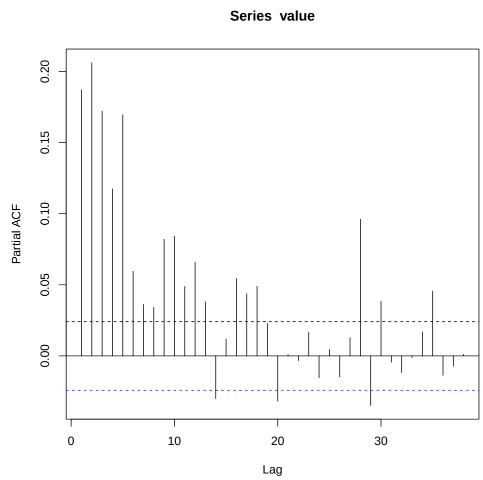
(a) Close level



(b) Log of close level



(c) log return



(d) log of squared return

Figure 28: PACF for FCHI