MATH 335 S2019
Quiz II

2019-04-18

Read the problems carefully and be sure to show your work. No cell phones or calculators are allowed. Please turn off your phone to avoid any disturbances.

1. (10 pts) Consider the parabolic coordinate system where the cartesian coordinates (x_1, x_2, x_3) are related to the curvilinear coordinates (u, v, w) by

$$x_1 = 2uv\,, \quad x_2 = u^2 - v^2\,, \quad x_3 = w\,.$$

Compute the scale factors h_u, h_v, h_w and the area element dS in the $w = 0$ plane (where u and v are allowed to vary). Then, use that area element to compute the area of the region defined by $0 \le u \le 1$, $0 \le v \le 1$, and $w = 0$.

Solution
We have

$$h_u = \left|\frac{\partial \vec{x}}{\partial u}\right| = \left|\begin{pmatrix} 2v \\ 2u \\ 0 \end{pmatrix}\right| = 2\sqrt{u^2+v^2}$$
$$h_v = \left|\frac{\partial \vec{x}}{\partial v}\right| = \left|\begin{pmatrix} 2u \\ -2v \\ 0 \end{pmatrix}\right| = 2\sqrt{u^2+v^2}$$
$$h_w = \left|\frac{\partial \vec{x}}{\partial w}\right| = \left|\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right| = 1$$

Thus, in the $w = 0$ plane, $dS = h_u h_v \, du\, dv = 4(u^2+v^2)\, du\, dv$. The area is then

$$\int_0^1 \int_0^1 4(u^2+v^2)\, du\, dv = \frac{8}{3}$$