MATH 335 S2019
Midterm Exam II

2019-03-28

Read the problems carefully and be sure to show your work. No cell phones or calculators are allowed. Please turn off your phone to avoid any disturbances.

Reference

- The following identity may be used in the exam without the need to prove it:

$$\epsilon_{ijk}\epsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$$

Exam

1. (Del and suffix notation)
   (a) (10 pts) Find the unit normal to the surface $z = x^2 + 2y^2$ at the point $(1, 1, 3)$.
   (b) (10 pts) Let $\vec{a}$ and $\vec{b}$ be vectors. Write $\vec{a} \cdot \vec{b}$ and $[\vec{a} \times \vec{b}]_i$ using suffix notation.
   (c) (10 pts) Let $\vec{u}$ be a vector field. Show that
   $$\nabla \times (\nabla \times \vec{u}) = \nabla(\nabla \cdot \vec{u}) - \nabla^2 \vec{u}$$
   using suffix notation.
   (d) (10 pts) Let $\vec{u}$ be a vector field. Show that $\nabla \cdot (\nabla \times \vec{u}) = 0$ using suffix notation.
   (e) (2 pts) Extra credit. Use the physical definition of the divergence to explain why $\nabla \cdot (\nabla \times \vec{u})$ is zero.

2. (30 pts) Evaluate the surface integral
   $$\iint_S \vec{F} \cdot \hat{n}\, dS$$
   where $\vec{F} = (xz, yz, z(x^2+y^2)^2)$ and S is the surface of the cylinder $x^2+y^2 \leq 1$ with $-1 \leq z \leq 1$ (this includes the tubular outer part and disc-shaped caps) using the divergence theorem.

3. (30 pts) Let $\vec{F} = (x^2, 2xy + x, z)$. Let C be the circle $x^2 + y^2 = 1$ with $z = 0$ be oriented counter-clockwise (standard direction) with S the disc $x^2 + y^2 \leq 1$ with $z = 0$ so that the normal vector on S is $\hat{n} = (0, 0, 1)$ (the positive z direction). Validate Stokes' theorem by (1) computing the line integral of $\vec{F}$ around C directly **and** (2) computing the flux of $\nabla \times \vec{F}$ over S.