MATH 335 S2019
Midterm Exam I

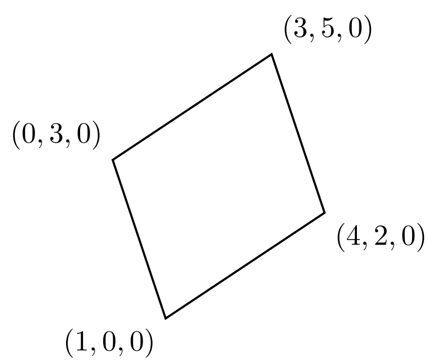
2019-02-19

Read the problems carefully and be sure to show your work. No cell phones or calculators are allowed. Please turn off your phone to avoid any disturbances.

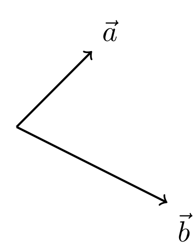
1. Complete the following

   (a) (10 pts) Find the implicit equation for the plane which includes the points $(1, 0, 1)$, $(0, 1, 1)$, and $(-1, 1, 0)$.

   (b) (10 pts) Find the area of the parallelogram in the figure below (drawn in the (x, y) plane) using your preferred method



   (c) (10 pts) Does $\vec{a} \times \vec{b}$ point into the paper or out of the paper?



   (d) (10 pts) Find the component of $(1, 2, 3)$ in the direction $(-3, 0, 4)$.

2. (20 pts) Consider the vector field $\vec{F}(x, y, z) = (-y, x, z)$. Compute the integral

$$\oint_C \vec{F} \cdot d\vec{r} \ , \tag{1}$$

where C is the unit circle $x^2 + y^2 = 1$ with $z = 0$ traced out in the counter-clockwise direction in the (x, y) plane. Is it possible that $\vec{F}$ is a conservative vector field?

3. Consider a point charge with strength q located at the origin so that the electric field at any given point (x, y, z) is

$$\vec{E}(x, y, z) = \frac{q}{4\pi|(x, y, z)|^3} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \ , \tag{2}$$

where $|(x, y, z)|$ is the length of the vector (x, y, z). Consider the outer surface of the cylinder $x^2 + y^2 \leq 1$ with $-1 \leq z \leq 1$. This consists of a tubular part given by $x^2 + y^2 = 1$ with $-1 \leq z \leq 1$ and the two disc-shaped caps $x^2 + y^2 \leq 1$ with $z = -1$ for one and $z = 1$ for the other.

(a) (5 pts) Compute the derivative of $\frac{x}{\sqrt{1+x^2}}$, which may be useful below.

(b) (20 pts) Compute the flux

$$\iint_S \vec{E} \cdot \hat{n} \, dS \ , \tag{3}$$

where S is the tubular part of the cylinder, i.e. the surface $x^2 + y^2 = 1$ with $-1 \leq z \leq 1$. Take the normal vector to be pointing out of the cylinder.

(c) (15 pts) Compute the flux

$$\iint_S \vec{E} \cdot \hat{n} \, dS \ , \tag{4}$$

where S is the top circular cap (a disc), i.e. the surface $x^2 + y^2 \leq 1$ with $z = 1$. Here you may take for granted that the normal vector is $\vec{n} = (0, 0, 1)$. (Hint: $\vec{E} \cdot \hat{n}$ is a scalar function of the disc, so that standard techniques for integrating over a disc may be used).

Using symmetry considerations, the flux through the bottom is the same as the top. What is the total flux through the surface of the cylinder?