Name:

MATH 335-002: Homework #1 Solutions

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Instructions

- This assignment is due in-class Thursday January 31st.
- Please put your full name in the upper right hand corner of each page of your solutions.
- Please show your work and be as neat as possible.
- Submitting typed/ LaTeX-based solutions is encouraged. The LaTeX source for this homework is available on the course website.
- Note that for the exercises from P.C. Matthews' "Vector Calculus" (Corrected Edition), the answers are in the back of the book but you are still expected to write out an answer here. For these problems, it is best to attempt the problem first and then check against the solution.

Outline

In this assignment, we aim to (1) become used to working with vectors, dot products, and cross products and (2) learn the use of these objects in describing geometrical objects (planes, lines, projections).

1 Exercises from Matthews

Please complete exercises 1.2, 1.3, 1.4, and 1.8 from the textbook (2 pts each)

- Exercise 1.2: if $\vec{a} = (2, 0, 3)$ and $\vec{b} = (1, 0, -1)$, find $|\vec{a}|$, $|\vec{b}|$, $\vec{a} + \vec{b}$, $\vec{a} - \vec{b}$, and $\vec{a} \cdot \vec{b}$. What is the angle between vectors $\vec{a}$ and $\vec{b}$?
- Exercise 1.3: if $\vec{u} = (1, 2, 2)$ and $\vec{v} = (-6, 2, 3)$. find the component of $\vec{u}$ in the direction of $\vec{v}$ and the component of $\vec{v}$ in the direction of $\vec{u}$.
- Exercise 1.4: find the (implicit) equation of the plane that is perpendicular to the vector $(1, 1, -1)$ and that passes through the point $x = 1$, $y = 2$, $z = 1$.
- Exercise 1.8: Find the equation of the straight line which passes through the points $(1, 1, 1)$ and $(2, 3, 5)$ (a) in parametric form; (b) in cross-product form.

Answers in back of book.

2 Other Exercises

Name:

Question 1 (5 pts)

Find an implicit equation for the plane containing both of the following lines

$$\vec{l}_1(t) = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} , \quad \vec{l}_2(t) = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} .$$

Solution

Because the lines are parallel, we need to find a second, linearly independent vector which is parallel to the plane in order to find the normal. Note that $(1, 2, 1)$ and $(-1, 0, 1)$ are both points in the plane so that their difference $(2, 2, 0)$ is parallel to the plane. Then, we can obtain a perpendicular vector by taking the cross product of $(2, 2, 0)$ and $(3, -1, 2)$, which gives $\vec{n} = (2, 2, 0) \times (3, -1, 2) = (4, -4, -8)$. The equation of the plane is then $\vec{n} \cdot \vec{x} = c$ for some constant c. Because $(-1, 0, 1)$ is on the plane, $c = -12$. This gives $4x_1 - 4x_2 - 8x_3 = -12$ or $x_1 - x_2 - 2x_3 = -3$.

Question 2 (5 pts)

This question has been edited (mistake is crossed out). Answers to either question will be accepted but the corrected one is more interesting!

Find the line through $\cancel{(0, 1, 0)}$ $(0, 0, 1)$ that intersects and is perpendicular to the line

$$\vec{l}(t) = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} .$$

Hint: The two lines lie in a plane. First, find the plane (in particular, the normal to the plane).

Solution (original problem)

Note that the difference of $(0, 1, 0)$ and $(-1, 2, 1)$ is $(-1, 1, 1)$ which is perpendicular to $(1, 0, 1)$. Therefore, the line which passes through $(0, 1, 0)$ and intersects the other line at $(-1, 2, 1)$ is the line we're looking for. In parametric form we have $\vec{x}(t) = (0, 1, 0) + t(-1, 1, 1)$.

Solution (revised problem)

Note that the difference of $(0, 0, 1)$ and $(-1, 2, 1)$ is $(-1, 2, 0)$ which is not perpendicular to $(1, 0, 1)$ but the two vectors are linearly independent. We can then use the cross product to obtain the normal vector to the plane containing both of these lines, i.e. $\vec{n} = (-1, 2, 0) \times (1, 0, 1) = (2, 1, -2)$. We can then find a vector in the plane which is perpendicular to $(1, 0, 1)$ using another cross product $\vec{n} \times (1, 0, 1) = (1, -4, -1)$. In parametric form, we then have $\vec{x}(t) = (0, 0, 1) + t(1, -4, -1)$.

Question 3 (5 pts)

Find a parametric formula for the plane given by

$$r_1 + r_3 = 4$$

Solution

To write a parametric formula, we need to obtain (1) a point on the plane and (2) two vectors which are parallel to the plane and linearly independent. Because the equation for a plane is so simple, we can obtain a point on the plane by guess and check. For instance, setting $r_1 = r_3 = 2$ and $r_2 = 0$ works, i.e. $(2, 0, 2)$ is on the plane. The normal to the plane is $(1, 0, 1)$ so we are looking for a vector which is perpendicular to this one. Again, by inspection $(1, 0, -1)$ works. As noted in class, we only needed to solve for one of the parallel vectors because we can obtain a second using the cross product $(1, 0, -1) \times (1, 0, 1) = (0, -2, 0)$.

Then, for any scalars α and β we can obtain a point on the plane using

$$\vec{r}(\alpha, \beta) = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix}.$$