The vector meson mass in the large N limit of QCD

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Abstract

The vector meson mass is computed as a function of quark mass in the large N limit of QCD. We use continuum reduction and directly compute the vector meson propagator in momentum space. Quark momentum is inserted using the quenched momentum prescription.

Key words:
Large N QCD, Vector meson masses, Low energy constants

Meson masses remain finite in the 't Hooft limit of large N limit of QCD in four dimensions [1]. Chiral symmetry is broken and the value of the chiral condensate has been measured on the lattice for overlap fermions using random matrix theory techniques [2]. The result can be summarized as [3]

$$\frac{\Sigma(b)}{T_c^3(b)} = 0.828 \left[\ln \frac{0.268}{T_c(b)}\right]^{\frac{9}{22}} \tag{1}$$

where $b = \frac{1}{g^2 N}$ is the bare 't Hooft coupling on the lattice. The deconfining temperature, $T_c(b)$, is also known from a lattice calculation [4] and is given by

$$b_I = be(b); \qquad e(b) = \frac{1}{N}\langle TrU_p(x)\rangle; \qquad T_c(b) = 3.85 \left(\frac{48\pi^2 b_I}{11}\right)^{\frac{51}{121}} e^{-\frac{24\pi^2 b_I}{11}}. \tag{2}$$

Continuum reduction holds if $L > \frac{1}{T_c(b)}$ [4] and meson propagators can be directly computed in Euclidean momentum space without any finite volume effects. The pion mass as a function of quark mass, m_o, was computed on the lattice using overlap fermions and the pion decay constant is given by [3]

$$\frac{f_\pi}{\sqrt{N} T_c(b)} = 0.269. \tag{3}$$

In this letter, we present results for the mass of the vector meson, m_ρ, as a function of the quark mass, m_o, using the same technique as the one used for the computation of the pion mass in [3]. The ρ propagator is computed using

$$\mathcal{M}_{\mu\nu}(p, m_o) = \mathrm{Tr}\left[S\gamma_\mu G(U_\mu e^{\frac{ip_\mu}{2}}, m_o) S\gamma_\nu G(U_\mu e^{-\frac{ip_\mu}{2}}, m_o)\right]. \tag{4}$$

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- $G(U_\mu, m_o)$ is the lattice quark propagator computed using overlap fermions in a gauge field background given by U_μ.

- The phase factors, $e^{\pm \frac{ip_\mu}{2}}$, multiplying the gauge fields correspond to the force-fed momentum of the two quarks in the quenched momentum prescription.

- The meson momentum was chosen to be

$$p_\mu = \begin{cases} 0 & \text{if } \mu = 1,2,3 \\ \frac{2\pi k}{NL}; \quad k = 2,3,4,5,6 & \text{if } \mu = 4. \end{cases} \tag{5}$$

- S smears the operator in the zero momentum directions using the inverse of the gauged Laplacian.

The ρ meson is made up of two different quarks (say u and d) with degenerate quark masses. Since the associated vector currents are conserved, the propagator, after averaging over gauge fields, will be of the form

$$\mathcal{M}_{\mu\nu}(p, m_o) = \frac{A\left(p_\mu p_\nu - p^2 \delta\mu\nu\right)}{p^2 + m_\rho^2(m_0)}, \tag{6}$$

assuming the propagator is dominated by the lowest vector meson state. Our numerical result is consistent with the above form. We found all off-diagonal ($\mu \neq \nu$) terms and the $\mu = \nu = 4$ term to be zero within errors for the specific choice of momentum in (5) and we also found the $\mu = \nu = 1,2,3$ terms to be the same within errors in our small test runs. Since the evaluation of the quark propagators is the computationally intensive part, we set $\mu = \nu = 1$ and obtained a value for the ρ meson mass at six different quark masses by fitting it to the form in (6).

Four different couplings were used in [3] for the computation of the meson masses. We found two of those couplings to be too strong for the computation of the ρ mass. We report here, the results for the ρ mass at two couplings, namely, $b = 0.355$ and $b = 0.360$. Chiral perturbation theory for vector mesons [5] suggests that we fit the data to the form

$$M_\rho = \bar{\mathcal{M}}_8 + \Lambda_2 M + \delta M_\rho, \tag{7}$$

where

$$M_\rho = \frac{m_\rho}{T_c(b)}; \quad M = 2\frac{m_0 \Sigma}{T_c^4(b)}; \tag{8}$$

are the mass of the ρ meson and the renormalization group invariant quark mass[1] measured in units of the deconfining temperature. The two coefficients

[1] M denoted the sum of the two quark masses comprising the ρ meson and hence the factor of 2 in the formula for M.

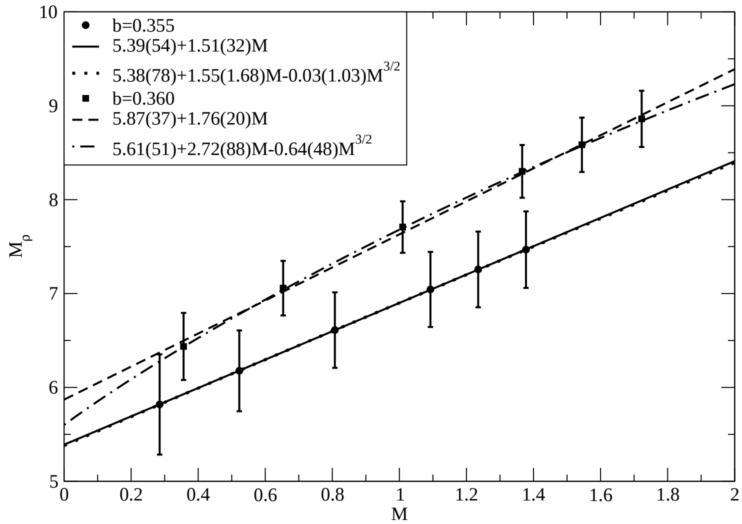

Figure 1: A plot of ρ mass as a function of the pion mass in dimensionless units.

L	N	b	$L_c(b)$	$\Sigma^{1/3}(b)$	$\bar{\mathcal{M}}_8$	Λ_2
10	19	0.355	6.96	0.1265	5.39(54)	1.51(32)
11	17	0.360	8.01	0.1130	5.87(37)	1.76(20)

Table 1: Simulation parameters, critical box size, bare chiral condensate along with the estimates for the two coefficients $\bar{\mathcal{M}}_8$ and Λ_2 in the mass term of the chiral lagrangian.

in (7) are two of the coefficients in the mass term in the chiral lagrangian, namely,

$$\bar{\mathcal{M}}_8 = \frac{\bar{\mu}_8}{T_c(b)}; \qquad \Lambda_2 = \frac{\lambda_2 T_c^3(b)}{\Sigma}. \tag{9}$$

The data is plotted in Fig. 1. The result for the chiral condensate in (1) was obtained such that the pion mass as a function of the quark mass scaled properly in the range of coupling from $b = 0.345$ to $b = 0.360$. It is not necessary that this eliminates finite lattice spacing effects on all quantities and we do see effects of finite lattice spacing effects in Fig. 1. A linear fit performs quite well at both the couplings to yield consistent estimates for $\bar{\mathcal{M}}_8$ and Λ_2. The results are shown both in Fig. 1 and Table 1.

Chiral perturbation theory suggests that δM_ρ in (7) should lead off as $M^{\frac{3}{2}}$ and the coefficient of this leading term should be negative. A fit with a $M^{\frac{3}{2}}$ term is shown in Fig. 1 and we see that the coefficient at $b = 0.360$ is consistent with it being negative. The error in this coefficient is rather large.

Using the result for $\bar{\mathcal{M}}_8$ in Table 1 for $b = 0.360$ and the result for f_π in (3), we have

$$\bar{\mu}_8 = \frac{21.8 \pm 1.4}{\sqrt{N}} f_\pi. \tag{10}$$

If we use $f_\pi = 86$ MeV and $N = 3$, then we get $\bar{\mu}_8 = 1082 \pm 70$ MeV.

The vector meson masses have been computed in the quenched approximation for $N = 2, 3, 4, 6$ in [6, 7]. The couplings used in [6] and in [7] are roughly the same. The strongest and weakest coupling correspond to $b = 0.296$ and $b = 0.353$ respectively in the notation of this paper. There is a bulk transition on the lattice in the large N limit that becomes a cross-over at finite N. The region between $b = 0.34$ and $b = 0.36$ is in the meta-stable region of this transition [4] and we need to be above $b = 0.34$ to be in the continuum phase of the large N theory. Since the vector meson is heavy compared to the pion for small quark masses, finite lattice spacing effects are larger in the case of the vector meson. Our study at $b = 0.350$, not reported in this paper, does yield a value for $\bar{\mathcal{M}}_8$ that is about 25% smaller than the one quoted here at $b = 0.360$ and consistent with the value obtained in [7].

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References

[1] A. V. Manohar, in Les Houches 1997, Probing the standard model of particle interactions, Pt. 2 1091-1169, arXiv:hep-ph/9802419.

[2] R. Narayanan and H. Neuberger, Nucl. Phys. B **696**, 107 (2004) [arXiv:hep-lat/0405025].

[3] R. Narayanan and H. Neuberger, Phys. Lett. B **616**, 76 (2005) [arXiv:hep-lat/0503033].

[4] J. Kiskis, R. Narayanan and H. Neuberger, Phys. Lett. B **574**, 65 (2003) [arXiv:hep-lat/0308033].

[5] E. E. Jenkins, A. V. Manohar and M. B. Wise, Phys. Rev. Lett. **75**, 2272 (1995) [arXiv:hep-ph/9506356].

[6] L. Del Debbio, B. Lucini, A. Patella and C. Pica, JHEP **0803**, 062 (2008) [arXiv:0712.3036 [hep-th]].

[7] G. S. Bali and F. Bursa, JHEP **0809**, 110 (2008) [arXiv:0806.2278 [hep-lat]].