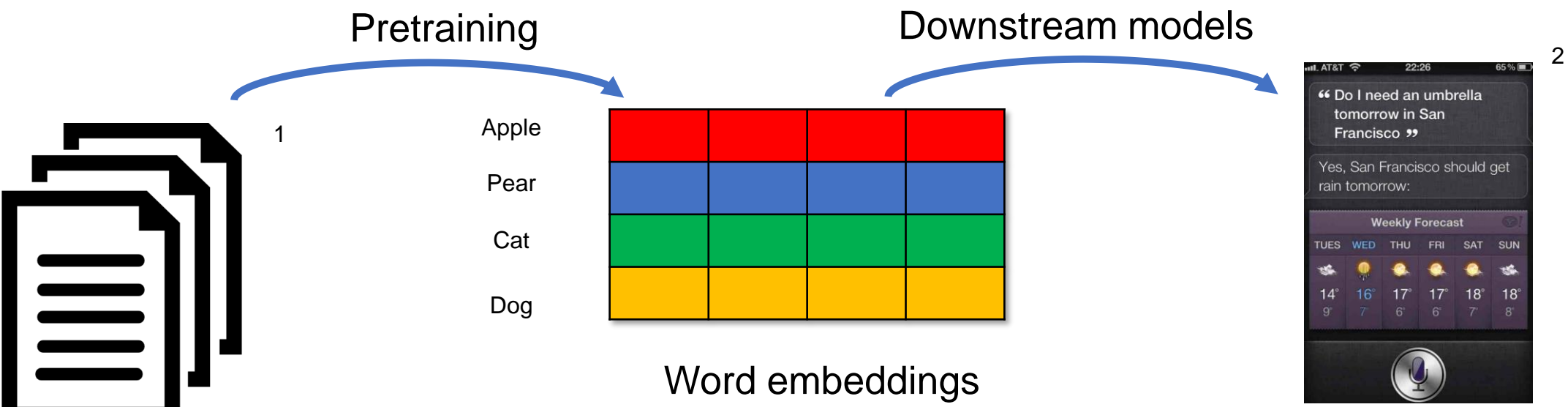


# On the Downstream Performance of Compressed Word Embeddings

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# Word Embeddings

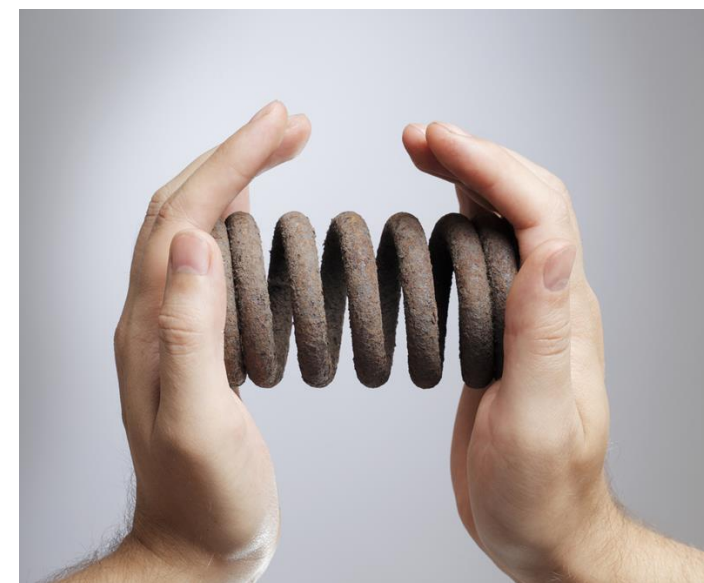


*Word embeddings take a lot of memory*

# Word Embedding Compression

Critical for deployment **under memory constraints**

- Deep compositional code learning (DCCL)<sup>1</sup>
- Kmeans<sup>2</sup>
- Uniform quantization<sup>3</sup>
- Dimension reduction (e.g. PCA)<sup>4</sup>



1. Shu et al. 2017

2. Andrews et al. 2015

3. Gersho et al. 1977

4. Pearson et al. 1901

What determines the ***model accuracy*** attained by different ***compressed word embeddings***?

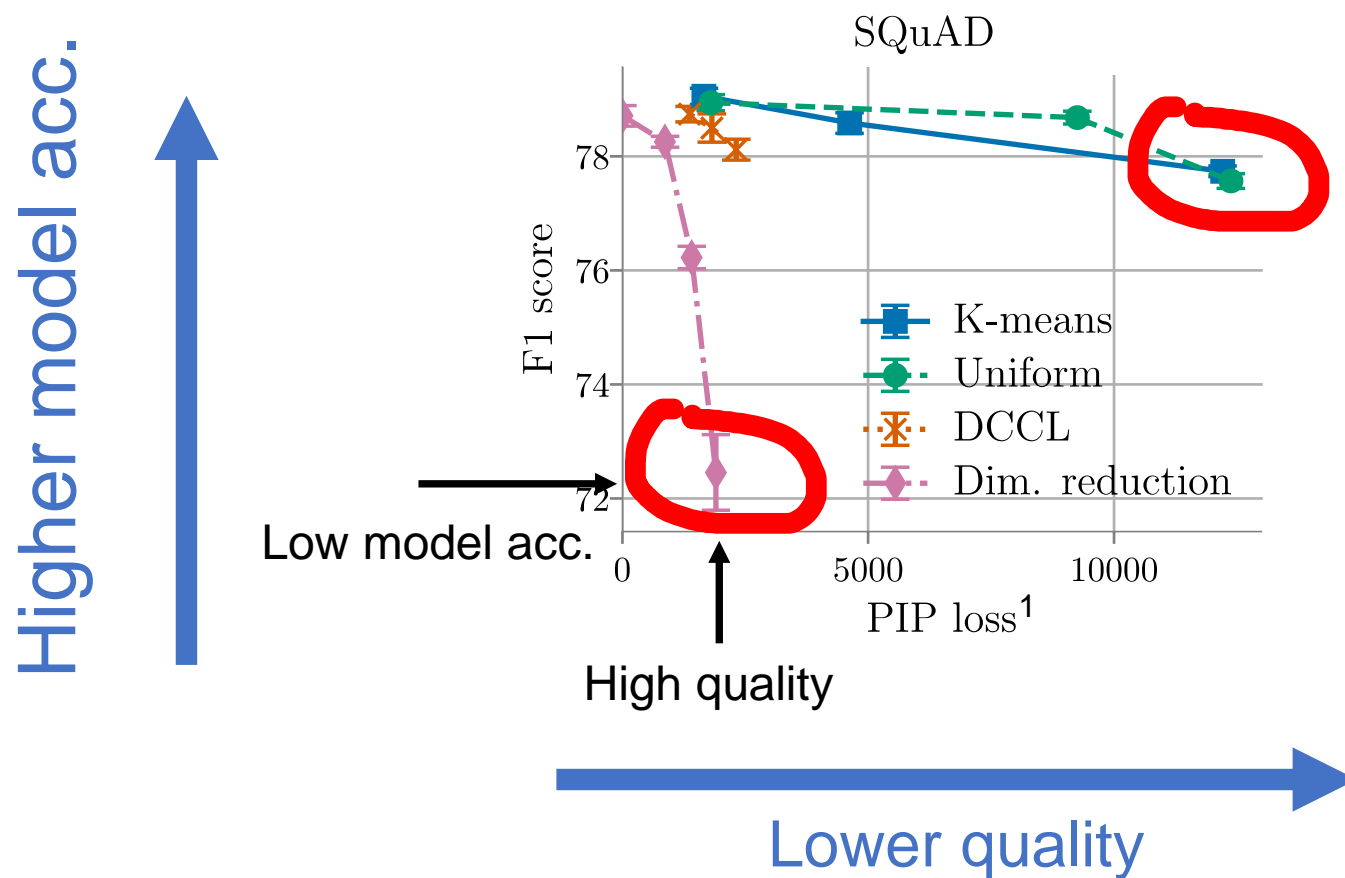
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Can the insights guide the selection of ***compressed word embeddings*** under ***memory constraints***?

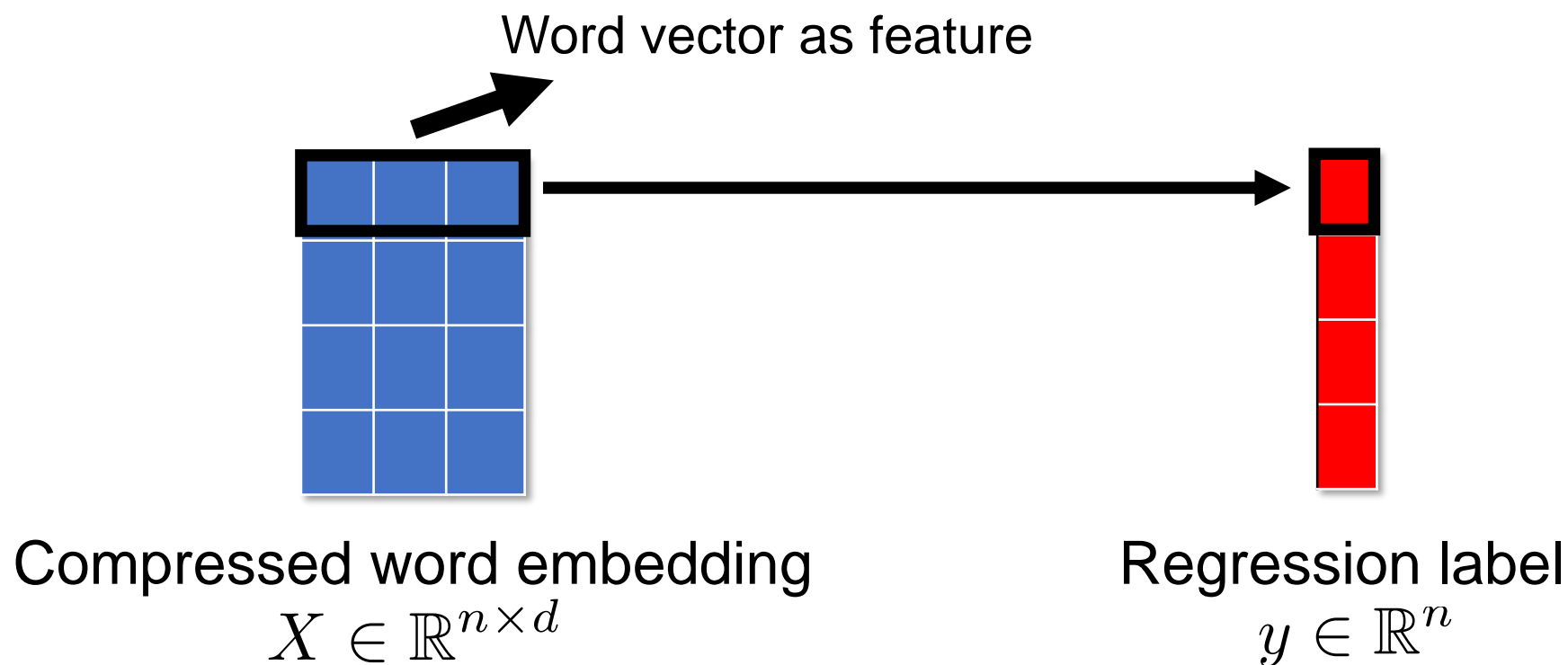


# Existing quality measures

Can't explain the relative model accuracy across compression methods



# Setting to derive a new quality measure



## Model accuracy

**Test mean square error (MSE)** rel. to uncompressed embedding

# In the setting of *linear regression*

**Fixed design linear regression (simple and classic setup):**<sup>1,2,3</sup>

Same set of data points for train and test; noisy training label; noiseless test label

$$\text{Test time prediction} = UU^T y$$

Compressed word embedding  $X \in \mathbb{R}^{n \times d}$

$$\text{SVD } X = U\Lambda V^T$$

Training label  $y \in \mathbb{R}^n$

## *Observation*

Prediction highly depends on  *$U$ , the left singular vectors*

1. Avron et al. 2018

2. Bach et al. 2013

3. Cortes et al. 2010



# A new quality measure of compression word embedding

## Eigenspace overlap (EO)

$$\mathcal{E}(X, \tilde{X}) := \frac{1}{\max(d, k)} \|U^T \tilde{U}\|_F^2$$

Compressed  $X \in \mathbb{R}^{n \times d}$  uncompressed  $\tilde{X} \in \mathbb{R}^{n \times k}$

SVD  $X = U\Lambda V^T$ ,  $\tilde{X} = \tilde{U}\tilde{\Lambda}\tilde{V}^T$

## *Intuition*

More *similar left singular vectors*,  
*better model acc.* relative to uncompressed embeddings





# In the setting of *linear regression*

Test MSE rel. to  
uncompressed embedding

$$\mathbb{E}_{\bar{y}} \left[ \mathcal{R}_{\bar{y}}(\tilde{X}) - \mathcal{R}_{\bar{y}}(X) \right] = \mathcal{O} \left( 1 - \mathcal{E}(X, \tilde{X}) \right)$$

Target label vector sampled from  $\text{Span}(U)$

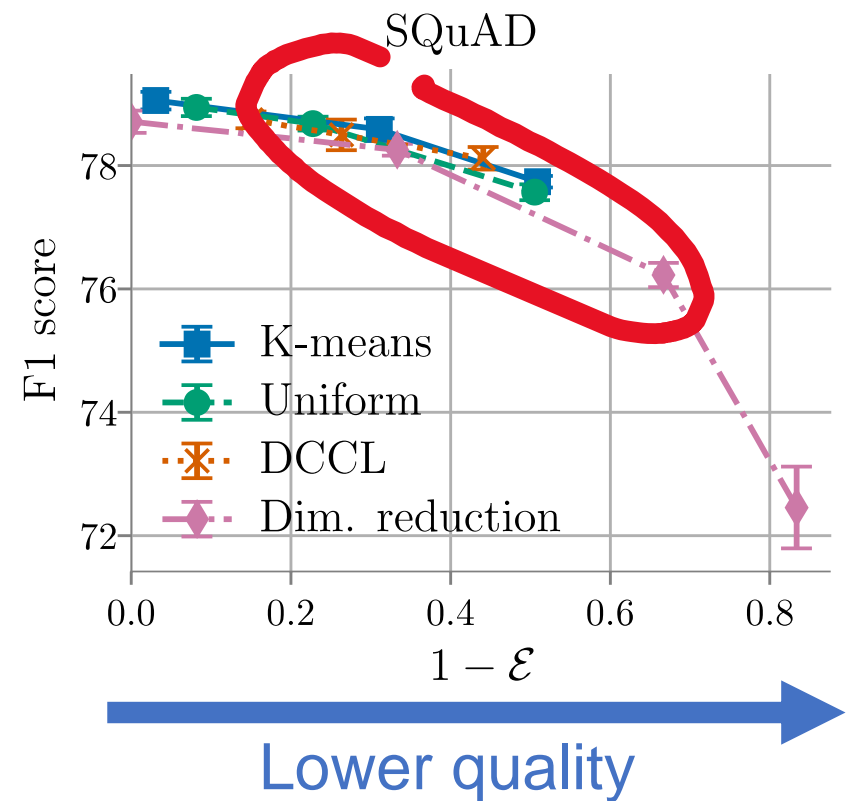
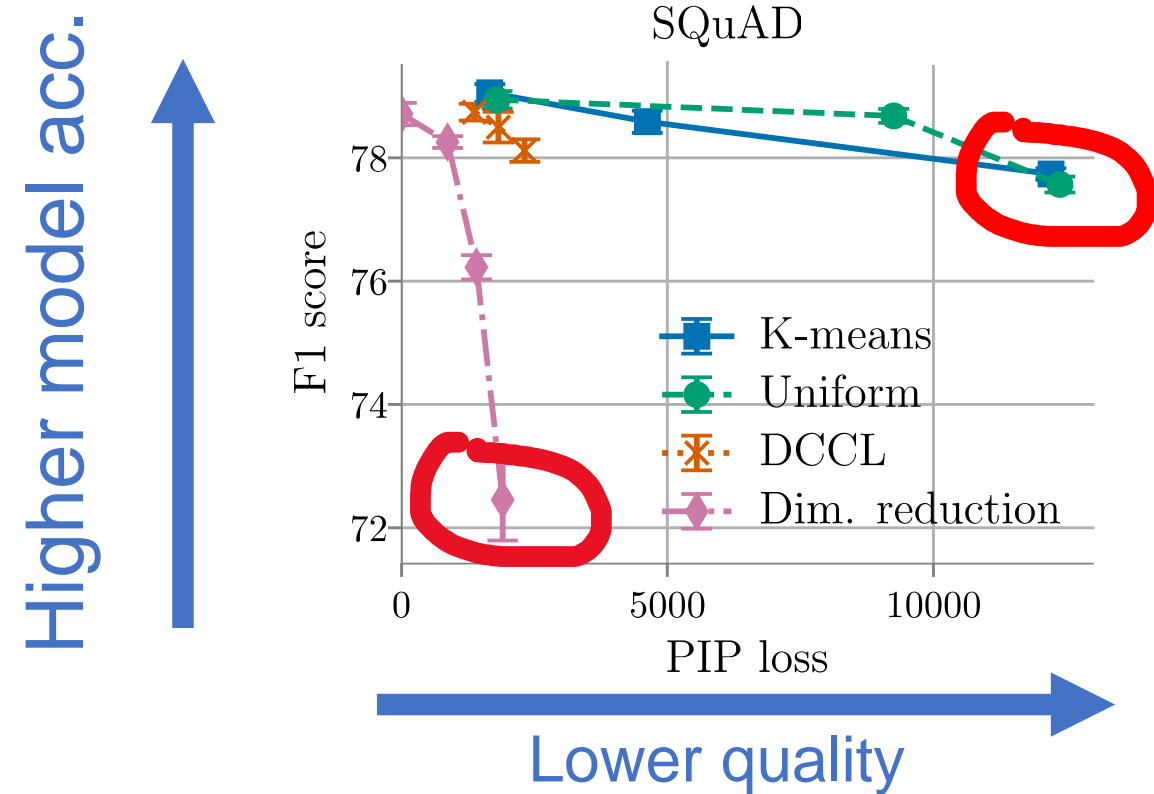
Uncompressed embedding  $X$

Compressed embedding  $\tilde{X}$

## Theory connection (sketch)

Model acc. can be bounded in terms of *eigenspace overlap*

# Empirical correlation beyond *the regression setting*



## Empirical correlation

EO attains *better correlation* with downstream *model acc.*

What determines the *model accuracy* attained by different *compressed word embeddings*?

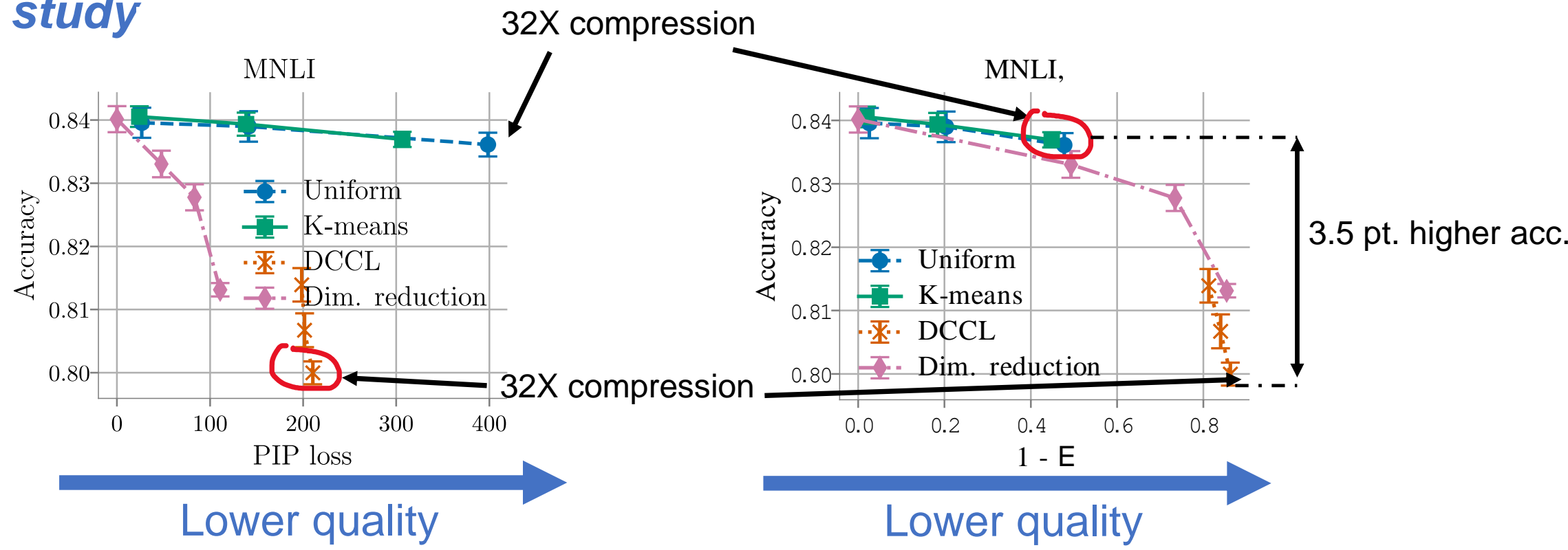
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Can the insights guide the selection of ***compressed word embeddings*** under ***memory constraints***?

# Eigenspace overlap as a selection criterion

Selecting the right embedding → **better model acc.** under **memory budgets**

## Case study

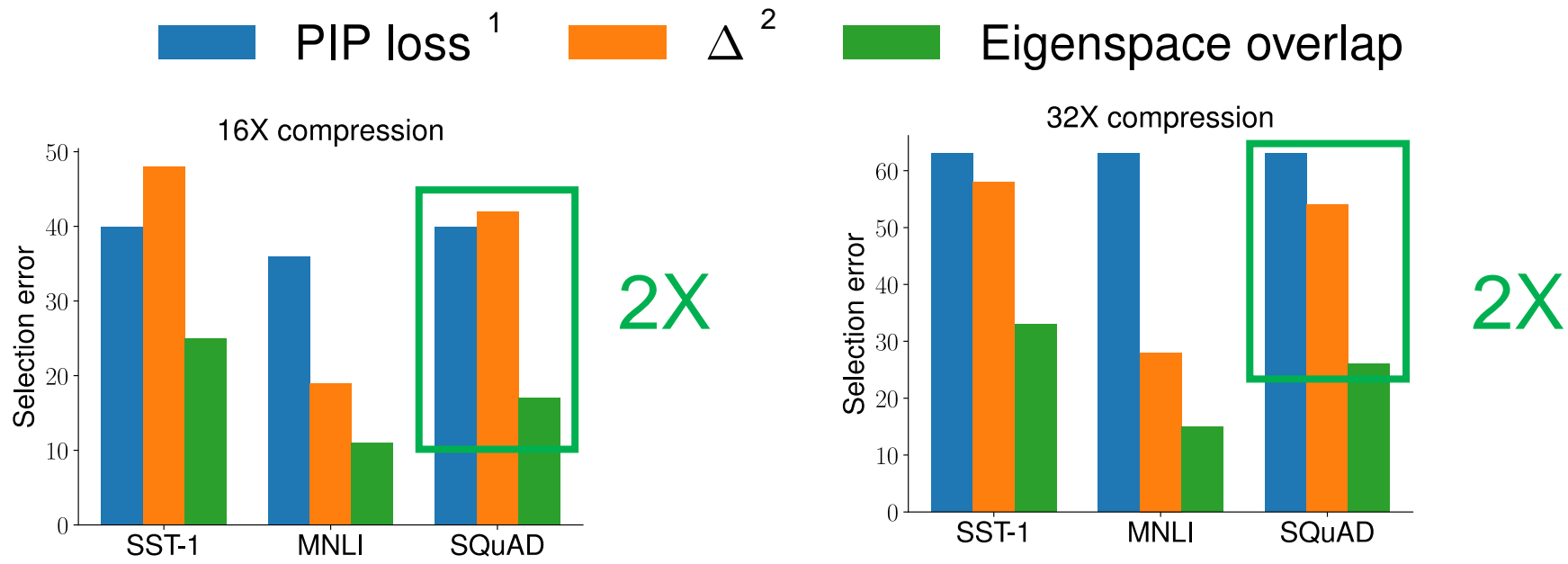


Eigenspace overlap vs. PIP loss → **higher acc. at 32X compression**

# Eigenspace overlap as a selection criterion

## Selection error

Fraction of cases when *failing to select* the embedding with *better model acc.*



## Utility under memory budgets

Up to 2X lower selection error at up to 32X compression

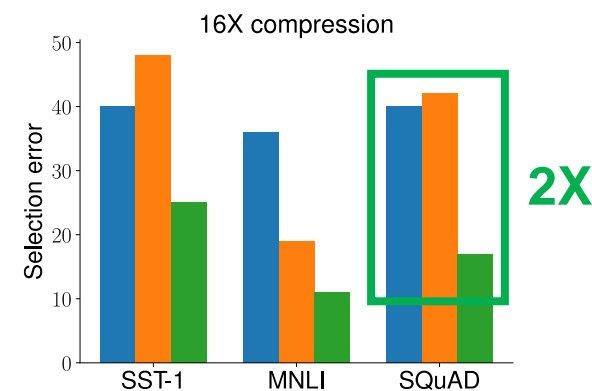
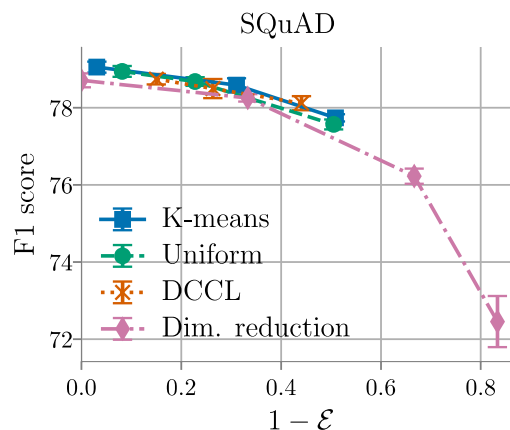
# Summary

Theoretical connection  
*in a regression setting*

Empirical correlations in *a wide range of models / tasks*

Guide the *selection* of compressed word embeddings

$$\mathbb{E}_{\tilde{y}} [\mathcal{R}_{\tilde{y}}(\tilde{X}) - \mathcal{R}_{\tilde{y}}(X)] = \mathcal{O}(1 - \mathcal{E}(X, \tilde{X}))$$



Left singular vector is important, EO captures it

Utility under  
memory constraints



**THANK YOU!**

**Spotlight: Thursday, Dec 12, 4:05 pm**

**Poster: Thursday, Dec 12, 5-7 pm**