11.05. 目 개념 정리

연습 2.2(5)

$$\lim_{x\to 0} \frac{\tan 2x}{\sin^2 \sqrt{x}}$$

$$= \lim_{x\to 0} \frac{\tan 2x}{2x} \cdot \frac{\sqrt{x}\cdot\sqrt{x}}{\sin\sqrt{x}\cdot\sin\sqrt{x}}$$

$$= 1 \cdot 1 \cdot 1 = (1)$$

연습 2.3(1)

$$\lim_{x\to 0} \frac{\cos hx}{1} = (1)$$

연습 2.3(5)

$$\lim_{x\to 0} \frac{1-\cos x}{3x^2}$$

$$= \lim_{x\to 0} \frac{\sin x}{6x}$$

$$= \lim_{x\to 0} \frac{\cos x}{6} = \left(\frac{1}{6}\right)$$

연습 2.4.(3)

$$\lim_{x\to 0} \frac{(\tan h^{-1}x)'}{(\tan^{-1}x)'}$$

$$= \lim_{x \to 0} \frac{\frac{1}{1-x^2}}{\frac{1}{1+x^2}}$$

$$= \lim_{x \to 0} \frac{1+x^2}{1-x^2} = \boxed{1}$$

#5.3.1 (6)

$t = x - \frac{1}{2}\pi$ 라 하자.

$$\lim_{t \to 0} t\{\tan 5(t+\tfrac{1}{2}\pi) - \tan(t+\tfrac{1}{2}\pi)\}$$
$$= \lim_{t \to 0} t\{\tan(5t+\tfrac{5}{2}\pi) + \cot t\}$$
$$= \lim_{t \to 0} t(\cot 5t + \cot t)$$

$$= \lim_{t \to 0} t\left(\frac{\cos 5t}{\sin 5t} + \frac{\cos t}{\sin t}\right)$$

$$= \lim_{t \to 0} \frac{5t}{\sin 5t} \cdot \frac{\cos 5t}{5} + \lim_{t \to 0} \frac{t}{\sin t} \cdot \cos t$$

$$= 1 \cdot \frac{1}{5} + 1 \cdot 1 = \boxed{\frac{6}{5}}$$

#5.3.1 (8) (3)

$0 \times \infty$ 꼴.

$t = \frac{1}{x}$ 라 하자.

$$\lim_{t \to \infty} \frac{1}{t} \cdot e^t$$

$$= \lim_{t\to\infty} \frac{e^t}{t}$$

$$= \lim_{t\to\infty} \frac{e^t}{1} = \infty.$$

#3.1.(12)

$\infty - \infty$ 꼴.

$$\lim_{x\to 0} \frac{x^2 - \sin^2 x}{x^2 \sin^2 x} \quad \left(\frac{0}{0} \text{꼴}\right)$$

$$= \lim_{x\to 0} \frac{2x - 2\sin x \cos x}{2x\sin^2 x + x^2 2\sin x \cos x} \quad \left(\frac{0}{0} \text{꼴}\right)$$

$$= \lim_{x\to 0} \frac{2x - \sin 2x}{2x\sin^2 x + x^2 \sin 2x} \quad \left(\frac{0}{0} \text{꼴}\right)$$

$$= \lim_{x\to 0} \frac{2x - \sin 2x}{2x} \cdot \frac{2x}{2x\sin^2 x + x^2 \sin 2x}$$

$$= \lim_{x\to 0} \left(1 - \frac{\sin 2x}{2x}\right) \cdot \lim_{x\to 0} \frac{2}{2\sin^2 x + x\sin 2x}$$

$$= (1-1) \lim_{x\to 0} \frac{2}{2\sin^2 x + x\sin 2x} = 0.$$

#3.2.(4)

∞^0 꼴.

$$\lim_{x\to\infty} e^{\frac{1}{x}\ln x}$$

$\lim_{x\to\infty} \frac{1}{x}\ln x$

$= \lim_{x\to\infty} \frac{\ln x}{x}$ $\frac{\infty}{\infty}$ 꼴

$\to \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 0$

$e^0 = 1$

#5.3.2(4)

1^∞ 꼴

$\lim_{x\to 0} e^{\frac{1}{x}\ln(1+\tan x)}$

$\lim_{x\to 0} \frac{\ln(1+\tan x)}{x}$ $\frac{0}{0}$ 꼴

$= \lim_{x\to 0} \frac{\frac{1}{1+\tan x}\cdot \sec^2 x}{1}$

$= \lim_{x\to 0} \frac{1}{1+\tan x}\cdot\frac{1}{\cos^2 x} = 1$

$e^1 = e$

#5.3.2.(1)

0^∞ 꼴

$\lim_{x\to\infty} e^{x^2 \ln(\cos(\frac{2}{x}))}$

$\lim_{x\to\infty} x^2 \ln(\cos(\frac{2}{x}))$

$t = \frac{1}{x}$ 라 하자.

$\lim_{t\to 0} \frac{1}{t} \ln(\cos 2t)$ $\quad \frac{0}{0}$ 꼴

$= \lim_{t\to 0} \frac{\frac{1}{\cos 2t} \cdot (-\sin 2t) \cdot 2}{2t}$

$= \lim_{t\to 0} \frac{1}{\cos 2t} \cdot \frac{-\sin 2t}{2t} \cdot 2 = -2$

#6.1.1(8)

$\int \sqrt{x+2}\, dx$

$= \int (x+2)^{\frac{1}{2}} dx$

$= \frac{1}{\frac{1}{2}+1} (x+2)^{\frac{1}{2}+1} + C$

$= \frac{2}{3}(x+2)^{\frac{3}{2}} + C$ (또 정답 상수)

#6.1.1(9)

$\int (\sin x + 2\cos x)\, dx$

$-\cos x + 2\sin x + C$ (또 정답 상수)

#6.1.1.(10)

$\int (\sinh x + \cosh x)\,dx$

$= \cosh x + \sinh x + C$. (C는 적분 상수)

#6.1.3.

$y'' = 6x - 2$

$y' = 3x^2 - 2x + C_1$.

$y = x^3 - x^2 + C_1 x + C_2$ (C_1, C_2는 적분 상수)

(0,1). $1 = C_2$.

$y = x^3 - x^2 + C_1 x + 1$.

(1,3). $3 = 1 - 1 + C_1 + 1$ $C_1 = 2$.

$y = x^3 - x^2 + 2x + 1$

#6.1.9

연속이면 $x=1$ 에서 왼쪽극한 = 오른쪽극한

$$f(x) = \begin{cases} x + C_1 & (x < 1) \\ \frac{1}{2}x^2 + C_2 & (x > 1) \end{cases}$$ (C_1, C_2는 적분상수)

$f(0) = 0 + C_1 = 5 \quad \underline{C_1 = 5}$

$f(1) = 1 + 5 = \frac{3}{2} + C_2$

$6 = \frac{3}{2} + C_2 \quad \underline{C_2 = \frac{9}{2}}$

$$f(x) = \begin{cases} x + 5, & (x < 1) \\ \frac{3}{2}x^2 + \frac{9}{2}, & (x > 1) \end{cases}$$