11.05. 日. 기함수 과제 1.

#6.3.2(2)

$$\int_0^3 (3x^2+1)\,dx$$

$$=\lim_{n\to\infty}\sum_{i=1}^{n}(3x_i^{*2}+1)\cdot\frac{3-0}{n}$$

$$=\lim_{n\to\infty}\sum_{i=1}^{n}3\left(3\cdot\left(\frac{i}{n}\right)^2+1\right)\frac{3}{n}$$

$$9\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\left(\frac{i^2}{n^2}+1\right)$$

$$=9\lim_{n\to\infty}\frac{1}{n}\left(\frac{1}{n^2}\cdot\frac{n(n+1)(2n+1)}{6}+n\right)$$

$$=9\lim_{n\to\infty}\left(\frac{1}{6}\cdot\frac{(n+1)(2n+1)}{n^2}+1\right)$$

$$=9\left(\frac{1}{\cancel{6}_{3}}\cdot\cancel{2}+1\right)=3+9=12$$

#6.3.9(1)

$$\lim_{\|P\|\to 0}\sum_{i=1}^{n}\left[(x_i^*)^2+3x_i^*+2\right]\Delta x_i$$

$$=\int_0^1 \quad \Delta x=\frac{1-0}{n} \qquad \Delta x=\frac{1}{n}$$

$$\int_0^1 (x^2+3x+2)\,dx$$

$$=\left[\frac{1}{3}x^3+\frac{3}{2}x^2+2x\right]_0^1$$

$$=\frac{7}{3}+\frac{3}{2}=\frac{23}{6}$$

11.10. 金. 개학 1 과제

#6.4.1.(4)

$$\frac{dF(x)}{dx} = \frac{d}{dx}\int_{x}^{x} \frac{u}{u^2+1}\,du.$$

$$= \frac{x}{x^2+1}\cdot 1 - \frac{x^2}{x^4+1}\cdot 2x$$

$$= \frac{x}{x^2+x} - \frac{2x^3}{x^4+1}$$

#6.4.2.(3)

$$\int_0^a (\sqrt{a}-\sqrt{x})^2\,dx$$

$$= \int_0^a (a - 2\sqrt{ax} - x)\,dx$$

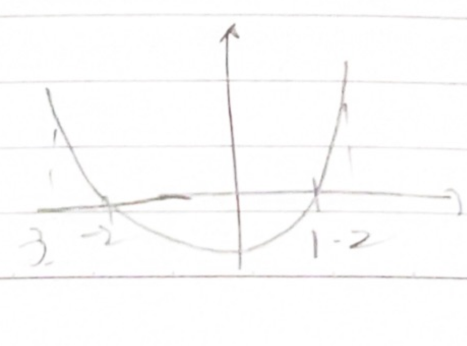
$$= \left[ax - 2\sqrt{a}\cdot\frac{2}{3}(\sqrt{x})^{\frac{3}{2}} - \frac{1}{2}x^2\right]_0^a$$

$$= a^2 - \frac{4}{3}\sqrt{a}\cdot a^{\frac{3}{2}} - \frac{1}{2}a^2$$

$$= \frac{1}{3}a^2 - \frac{4}{3}a$$

#6.4.2(8)

$x^2+x-2 = (x+2)(x-1)$



$\int_{-3}^{2} |x^2+x-2|\, dx$

$= \int_{-3}^{-2} (x^2+x-2)\, dx + \int_{-2}^{1} (-x^2-x+2)\, dx + \int_{1}^{2} (x^2+x-2)\, dx$

$= \left[\frac{1}{3}x^3+\frac{1}{2}x^2-2x \right]_{-3}^{-2} + \left[-\frac{1}{3}x^3-\frac{1}{2}x^2+2x \right]_{-2}^{1} + \left[\frac{1}{3}x^3+\frac{1}{2}x^2-2x \right]_{1}^{2}$

$= \frac{1}{3}(-8+27)+\frac{1}{2}(4-9)-2(-2+3)$

$-\frac{1}{3}(1+8)-\frac{1}{2}(1-4)+2(1+2)$

$+\frac{1}{3}(8-1)+\frac{1}{2}(4-1)-2(2-1)$

$= \frac{1}{3}\cdot 19+\frac{1}{2}(-5)-2-\frac{1}{3}\cdot 9+\frac{1}{2}(+3)+6$
$+\frac{1}{3}(-7)+\frac{1}{2}\cdot 3-2$

$= \frac{1}{3}(19-9-7)+\frac{1}{2}(-5+3+3)+(-2+6-2)$

$= \frac{1}{\cancel{3}}\cdot \cancel{3}+\frac{1}{2}\cdot 1+2 = \frac{7}{2}$

#6.4.9

$\int_0^1 1\, dx = [x]_0^1$
$= 1-0 = 1$

$\int_0^1 (1+x^2)\, dx = \left[\frac{1}{3}x^3+x\right]_0^1$
$= \frac{1}{3}+1 = \frac{4}{3}$

$\sqrt{1+x^2}$ 이 $(0,1)$에서 $\sqrt{1+x^2} \ge 0$ 이므로
$1 \le \sqrt{1+x^2} \le 1+x^2$
$\int_0^1 1\, dx \le \int_0^1 \sqrt{1+x^2}\, dx \le \int_0^1 (1+x^2)\, dx$
$1 \le \int_0^1 \sqrt{1+x^2}\, dx \le \frac{4}{3}$ 이 성립한다.

#6.4.11

$\frac{\infty}{\infty}$ 꼴이므로 로피탈 정리를 이용할 수 있다.

$$\lim_{x\to\infty} \frac{\int_0^{x^2} \sqrt{t^2+1}\, dt}{x^4}$$

$$= \lim_{x\to\infty} \frac{\sqrt{x^4+1} \cdot \cancel{2x}}{2\,\cancel{4}\,x^{\cancel{3}\,2}}$$

$$= \frac{1}{2} \lim_{x\to\infty} \sqrt{\frac{x^4+1}{x^4}}$$

$$= \frac{1}{2} \sqrt{\lim_{x\to\infty} \frac{x^4+1}{x^4}}$$

$$= \frac{1}{2} \cdot 1 = \frac{1}{2}.$$

#6.5.1(2)

$$\int \frac{1}{5-2x}\, dx$$

$$= -\int \frac{1}{2x-5}\, dx.$$

$$= -\frac{1}{2} \int \frac{1}{x-\frac{5}{2}}\, dx.$$

$$= -\frac{1}{2} \ln\left|x - \frac{5}{2}\right| + C$$

#6.5 1(5)

$1+x\sqrt{x} = t$ 라 하자.

$(\sqrt{x} + x\cdot\frac{1}{2\sqrt{x}})dx = dt$

$\frac{3}{2}\sqrt{x}\,dx = dt$

$\sqrt{x}\,dx = \frac{2}{3}dt$

$\int \frac{1}{1+x\sqrt{x}}\,dx = \int \frac{1}{t}\cdot\frac{2}{3}d$

$= \frac{2}{3}\ln|t|$

$= \frac{2}{3}\,|1+x\sqrt{x}| + C.$

#6.5 1.(16)

* $\sqrt[4]{1+2x} = t$ 라 하자

$\frac{1}{4}(1+2x)^{-\frac{3}{4}}\cdot 2\,dx = dt.$

$dx = \sqrt{1+2x}\;dt.$

$dx = \sqrt{t}\,dt$

$\sqrt[4]{1+2x} = t.$

$1+2x = t^4.$

$2x = t^4 - 1.$ $\quad x = \frac{t^4-1}{2}.$

$\int \frac{x}{\sqrt[4]{1+2x}}\,dx \quad \int \frac{\frac{t^4-1}{2}}{t}\cdot\sqrt{t}\,dt.$

$\frac{1}{2}\int \frac{1}{\sqrt{t}}(t^4-1)\,dt$

$\frac{1}{2}\int (t^{\frac{7}{2}} - \frac{1}{\sqrt{t}})\,dt.$

$= \frac{1}{2}\left(\frac{1}{\frac{9}{2}} t^{\frac{9}{2}} - 2\sqrt{t}\right)$

$= \frac{1}{9} t^{\frac{9}{2}} - \sqrt{t}$

$= \frac{1}{9}(1+2x)^{\frac{9}{4}} - (1+2x)^{\frac{1}{4}}$

#6.5.2(3)

$e^{\frac{1}{x}} = t$ 라 하자.

$e^{\frac{1}{x}} \cdot -\frac{1}{x^2} dx = dt$.

$-x^{-2} dx = \frac{1}{t} dt$.

$x^{-2} dx = -\frac{1}{t} dt$.

e^{2} $e^{\frac{1}{2}}$.

$\int_{e^2}^{e^{\frac{1}{2}}} t \cdot \left(-\frac{1}{t}\right) dt$

$= \left[-t\right]_{e^2}^{e^{\frac{1}{2}}}$

$= -\left(e^{\frac{1}{2}} - e^{2}\right) = e^{2} - e^{\frac{1}{2}}$

#6.5.2(7)

$\sqrt{x+1} = t$ 라 하자

$\frac{1}{2\sqrt{x+1}} dx = dt$

$dx = 2t\,dt$

$\sqrt{x+1} = t$

$x+1 = t^2$

$x = t^2-1$

$x^2 = t^4-2t^2+1$

$\sqrt{-1+1} = 0$

$\sqrt{3+1} = 2$

$\int_0^2 (t^4-2t^2+1)t \cdot 2t \, dt$

$= \int_0^2 (2t^6-4t^4+2t^2) \, dt$

$= \left[\frac{2}{7}t^7 - \frac{4}{5}t^5 + \frac{2}{3}t^3 \right]_0^2$

$= \frac{2}{7} \cdot 128 - \frac{4}{5} \cdot 32 + \frac{2}{3} \cdot 8$

$= \frac{256}{7} - \frac{128}{5} + \frac{16}{3}$

$= \frac{3840 - 2688 + 560}{105}$

$= \frac{1152+560}{105}$

$= \frac{1712}{105}$

$$\begin{array}{r} 128 \\ \times\ 21 \\ \hline 128 \\ 256\ \\ \hline 2688 \end{array} \qquad \begin{array}{r} 3840 \\ -2689 \\ \hline 1152 \end{array}$$

$$\begin{array}{r} 256 \\ \times\ 15 \\ \hline 1280 \\ 256\ \\ \hline 3840 \end{array} \qquad \begin{array}{r} 35 \\ \times\ 16 \\ \hline 210 \\ 35\ \\ \hline 560 \end{array}$$