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#8.1.1(2)

미	적
x^2	e^x
$2x$	e^x
2	e^x

$$\int x^2 e^x dx = x^2e^x - 2\int x e^x dx$$
$$= x^2e^x - 2\left(xe^x - \int e^x dx\right)$$
$$= x^2e^x - 2xe^x + 2e^x + C$$ (C는 적분상수)

#8.1.1(3)

미	적
x	$\cos x$
1	$\sin x$

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx$$
$$= x\sin x + \cos x + C$$ (C는 적분상수)

#8.1.1.(7)

$$\int x^5\sqrt{1+x^2}\,dx = \frac{1}{3}x^4(1+x^2)^{\frac{3}{2}} - \int \frac{4}{3}x^3(1+x^2)^{\frac{3}{2}}dx$$
$$= \frac{1}{3}x^4(1+x^2)^{\frac{3}{2}} - \frac{4}{3}\cdot\frac{1}{5}x^2(1+x^2)^{\frac{5}{2}}$$
$$- \int \frac{2}{5}x(1+x^2)^{\frac{5}{2}}dx$$
$$= \frac{1}{11}x^{\frac{11}{2}} - \frac{2}{7}x^{\frac{7}{2}} + \frac{1}{3}x^{\frac{3}{2}} + C$$
(C는 적분상수)

#8.1.1.(8)

미	적
$\ln x$	x^2
$\frac{1}{x}$	$\frac{1}{3}x^3$

$$\int x^2\ln x\,dx = \frac{1}{3}x^3\ln x - \int \frac{1}{3}x^2 dx$$
$$= \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C$$ (C는 적분상수)

#8.1.1(11).

미	적		
$\cos 2x$	e^{-x}	$\sin 2x$	e^{-x}
$-2\sin 2x$	$-e^{-x}$	$2\cos 2x$	$-e^{-x}$

$$\int \cos 2x \cdot e^{-x}\,dx = -e^{-x}\cos 2x - \int e^{-2x}\sin 2x\,dx$$

$$= -e^{-x}\cos 2x - \left(-e^{-x}\sin 2x + 2\int e^{-x}\cos 2x\,dx\right)$$

$$3\int \cos 2x\, e^{-x}\,dx = -e^{-x}\cos 2x + e^{-x}\sin 2x.$$

$$\int \cos 2x\, e^{-x}\,dx = \frac{1}{3}e^{-x}(-\cos 2x + \sin 2x) + C \quad (C\text{는 적분상수}).$$

#8.1.1.(13).

미	적
$\frac{1}{2}\ln\lvert x+1\rvert$	1
$\frac{1}{2}\frac{1}{x+1}$	x

$$\int_0^3 \ln\sqrt{x+1}\,dx = \left[\frac{1}{2}x\ln\lvert x+1\rvert\right]_0^3 - \int_0^3 \frac{x}{2\lvert x+1\rvert}\,dx$$

$$= \frac{1}{2}3\ln 4 - 0 - \frac{1}{2}\int_0^3 \left(\frac{x+1}{x+1} - \frac{1}{x+1}\right)dx$$

$$= 3\ln 2 - \frac{1}{2}\left[x - \ln\lvert x+1\rvert\right]_0^3$$

$$= 3\ln 2 - \frac{1}{2}\left(3 - (\ln 4 - 0)\right)$$

$$= 3\ln 2 - \frac{1}{2}(3 - 2\ln 2)$$

$$= \ln 2 - \frac{3}{2}.$$

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#8.2.1(2)

$\frac{1}{x} = t$ 이면

$-\frac{1}{x^2}dx = dt.$ $\frac{1}{x^2}dx = -dt$

$\int -\sin \pi t \, dt.$

$= \frac{1}{\pi}\cos \pi t + C.$

$= \frac{1}{\pi}\cos \pi \frac{1}{x} + C$ (C는 적분상수)

#8.2.1.(4)

$\int \frac{1}{\tan \frac{1}{2}x} dx.$

$= \int \frac{\cos \frac{1}{2}x}{\sin \frac{1}{2}x} dx.$

$= 2\ln|\sin \frac{1}{2}x| + C$ (C는 적분상수)

#8.2.1.(6)

$\int \frac{1+\cos x}{2} \cdot dx.$

$= \frac{1}{2}x + \frac{1}{2}\sin x + C$ (C는 적분상수)

#8.2.1(8)

$\int (1-\sec^2 \frac{3}{4}x)\, dx.$

$= x - \frac{4}{3}\tan \frac{3}{4}x + C$ (C는 적분상수)

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#8.2.1.(12)

$$\int \frac{1}{1-\sin x}\,dx$$

$$= \int \frac{1+\sin x}{1-\sin^2 x}\,dx$$

$$= \int \frac{1+\sin x}{\cos^2 x}\,dx$$

$$= \int (\sec^2 x + \sin x \cdot \cos^{-2} x)\,dx$$

$$= \tan x + \sec x + C$$ (C는 적분 상수)

#8.2.1.(14)

$$\int \frac{\sin x}{1+\sin x}\,dx$$

$$= \int 1 - \frac{1}{1+\sin x}\,dx$$

$$= x + \int \frac{1+\sin x}{1-\sin^2 x}\,dx$$

$$= x - \int \left(\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}\right)dx$$

$$= x - \tan x - \sec x + C$$ (C는 적분 상수)

#8.3.1(1)

$$\int \frac{1}{x(x+2)}\,dx$$

$$= \int \left(\frac{2}{x} + \frac{-\frac{1}{2}}{x+2}\right)dx = 2\ln|x| - \frac{1}{2}\ln|x+2| + C$$ (C는 적분상수)

#8.3.1.(3)

$$\int \frac{3x^2-x+1}{x^2(x-1)}\,dx$$

$$=\int\left(\frac{A_1}{x}+\frac{A_2}{x^2}+\frac{A_3}{x-1}\right)dx$$

$$A_1x(x-1)+A_2(x-1)+A_3x^2=3x^2-x+1$$

$$x^2(A_1+A_3)+x(-A_1+A_2)-A_2=3x^2-x+1$$

$A_2=-1$. $A_1=0$. $A_3=3$.

$$\int\left(\frac{-1}{x^2}+\frac{3}{x-1}\right)dx=\frac{1}{x}+3\ln|x-1|+C$$ (C는 적분상수)

#8.3.1(5)

$$\int\frac{1}{x(1+x^2)}\,dx$$

$$=\int\left(\frac{1}{x}+\frac{f(x)}{1+x^2}\right)dx$$

$1+x^2+x f(x)=1$.. $f(x)=-x$.

$$\int\left(\frac{1}{x}+\frac{-x}{1+x^2}\right)dx$$

$$=\ln|x|-\frac{1}{2}\ln|1+x^2|+C$$

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#8.3.1.(7)

$$\int \left(\frac{A_1}{x-2} + \frac{A_2x+A_3}{x^2+4} \right) dx.$$

$A_1(x^2+4) + (x-2)(A_2x+A_3)$

$= x^2(A_1+A_2) + x(A_3-2A_2) + (4A_1-2A_3) = x^2-4x-4$

$$\begin{cases} A_1+A_2=1 \\ A_3-2A_2=-4. \\ 4A_1-2A_3=-4 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1. & 1. & 0. & 1. \\ 0. & -2. & 1. & -4. \\ \cancel{4}. & \cancel{-4} & \cancel{-2} & \cancel{-4}. \cancel{-8} \ 0. \\ & & -4. & \end{array}\right]$$

$A_3=0.\ A_1=-1.\ A_2=\cancel{8}\ 2.$

$$\int \left(\frac{-1}{x-2} + \frac{2x}{x^2+4} \right) dx.$$

$= -\ln|x-2| + \ln|x^2+4| + C$ (C는 적분 상수)

#8.3.1.(11)

$$\int \left(\frac{-1}{x+1} + \frac{\cancel{1}\,2}{x+2} \right) dx = -\ln|x+1| + 2\ln|x+2| + C.$$

(C는 적분 상수)

#8.3.1.(13)

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$$\int \frac{5x^2-3x+18}{x(3+x)(3-x)}\,dx.$$

45+15+18

45−15+18.

$$=\int\left(\frac{\frac{18}{3\cdot 3}}{x}+\frac{\frac{78}{-3\cdot(6)}}{3+x}+\frac{\frac{48}{3\cdot 6}}{3-x}\right)dx$$

$$=\int\left(\frac{2}{x}-\frac{6}{x+3}-\frac{3}{-3+x}\right)dx$$

$$=2\ln|x|-6|x+3|-3\ln|x-3|+C$$ (C는 적분 상수)

#8.3.1.(15)

$$x^3+1=(x+1)(x^2-x+1)$$

$$\int_0^1 \frac{1}{1+x^3}\,dx.$$

$$=\int_0^1\left(\frac{1}{x+1}\cdot\frac{1}{x^2-x+1}\right)dx$$

$$=\int_0^1\left(\frac{C}{x+1}+\frac{Ax+B}{x^2-x+1}\right)dx.$$

$$C(x^2-x+1)+(x+1)(Ax+B)=1$$

$$x^2(C+A)+x(-C+A+B)+(C+B)=1$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ \cancel{1} & 1 & \cancel{2} & 0 \\ 0 & \cancel{1} & \cancel{3} & 1 \end{array}\right)$$

$C=\frac{1}{3}$.

$B=\frac{2}{3}$

$A=-\frac{1}{3}$

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$$\int_0^1 \left(\frac{1}{3}\cdot\frac{1}{x+1} - \frac{1}{3}\,\frac{x-2}{x^2-x+1}\right) dx$$

$$= \int_0^1 \left(\frac{1}{3}\,\frac{1}{x+1} - \frac{1}{3}\,\frac{(2x-1)\cdot\frac{1}{2}+\frac{1}{2}-2}{x^2-x+1}\right) dx$$

$$= \int_0^1 \frac{1}{3}\,\frac{1}{x+1} - \frac{1}{6}\,\frac{2x-1}{x^2-x+1} + \frac{1}{2}\,\frac{1}{x^2-x+1}) \, dx$$

$$x^2-x+1 = \left(x-\frac{1}{2}\right)^2+\frac{3}{4}$$

$$\left[\frac{1}{3}\ln|x+1| - \frac{1}{6}\ln|x^2-x+1| + \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{\sqrt{3}}\tan^{-1}\frac{x-\frac{1}{2}}{\frac{2}{\sqrt{3}}}\right]_0^1$$

$$= \frac{1}{3}(\ln 2 - 0) - \frac{1}{6}(0-0) + \frac{1}{\sqrt{3}}\left(\tan^{-1}\frac{1}{2}\cdot\frac{\sqrt{3}}{2} - \tan^{-1}\left(-\frac{1}{2}\cdot\frac{\sqrt{3}}{2}\right)\right)$$

$$= \frac{1}{3}\ln 2 + \frac{1}{\sqrt{3}}\left(\tan^{-1}\frac{\sqrt{3}}{4} + \tan^{-1}\frac{\sqrt{3}}{4}\right)$$

$$= \frac{1}{3}\ln 2 + \frac{2}{\sqrt{3}}\tan^{-1}\frac{\sqrt{3}}{4}$$