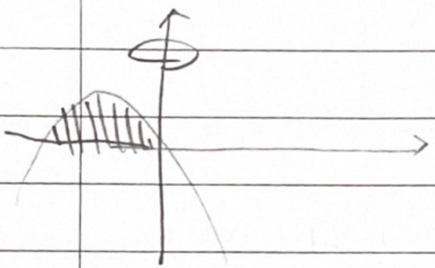
11.18. 土. 대학수 1 과제

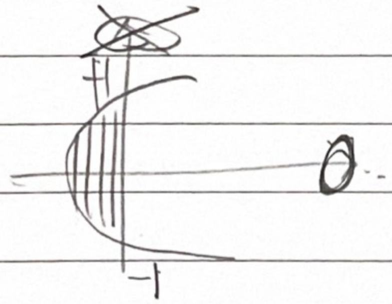
#7.3.1(3)

$y' = 1+2x = 0 \quad x = -\frac{1}{2}$



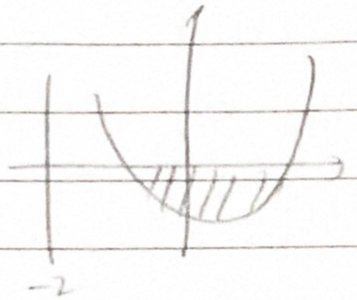
$$
\begin{aligned}
V &= \int_{-1}^{0} 2\pi x y \, dx \\
&= \int_{-1}^{0} 2\pi (x^3 + x^2) \, dx \\
&= 2\pi \left[\frac{1}{4}x^4 + \frac{1}{3}x^3 \right]_{-1}^{0} \\
&= -2\pi \left(\frac{1}{4} - \frac{1}{3} \right) \\
&= \cancel{2\pi} \left(+ \frac{1}{12} \right) \quad = \frac{1}{6}\pi
\end{aligned}
$$

#7.3.2(1)



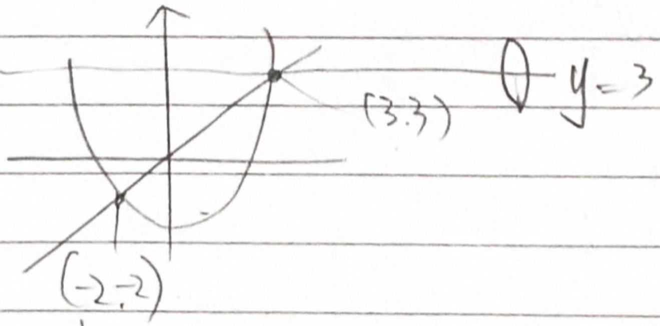
$$
\begin{aligned}
V &= \int_{0}^{1} 2\pi x y \, dy \\
&= \int_{0}^{1} 2\pi (-y^3 + y) \, dy \\
&= 2\pi \left[-\frac{1}{4}y^4 + \frac{1}{2}y^2 \right]_{0}^{1} \\
&= 2\pi \left(-\frac{1}{4} + \frac{1}{2} \right) \\
&= \frac{1}{2}\pi
\end{aligned}
$$

#7.3.3(1)



$$V = \int_{-1}^{2} 2\pi (x+2)(0-x^2+x+2)\,dx$$
$$= \int_{-1}^{2} 2\pi (x+2)(-x^2+x+2)\,dx$$
$$= 2\pi \int_{-1}^{2} (-x^3-x^2+3x+4)\,dx$$
$$= 2\pi \left[-\frac{1}{4}x^4 - \frac{1}{3}x^3 + \frac{3}{2}x^2 + 4x\right]_{-1}^{2}$$
$$= 2\pi\left(-\frac{1}{4}(16+1) - \frac{1}{3}(8+1) + \frac{3}{2}(4-1) + 4(2+1)\right)$$
$$= 2\pi\left(-\frac{1}{4}\cdot 15 - 3 + \frac{9}{2} + 12\right)$$
$$= 2\pi \cdot \cancel{\frac{-15-12+18+48}{4}}$$
$$= \frac{1}{2}\pi(-27+66) = \frac{39}{2}\pi$$

#7.3.3(3)



$x^2-6=x$

$x^2-x-6=0$

$(x-3)(x+2)=0 \quad x=-2, 3$

$$V = \int_{-2}^{3} 2\pi x(y-3)\,dy$$
$$= \int_{-2}^{3} 2\pi \sqrt{y+6}\,(y-3)\,dy$$
$$= \int_{-2}^{3} 2\pi\left(\sqrt{y+6}\,(y+6) - 9\sqrt{y+6}\right)dy$$
$$= \int_{-2}^{3} 2\pi\left((y+6)^{\frac{3}{2}} - 9(y+6)^{\frac{1}{2}}\right)dy$$

$= 2\pi \left[\frac{2}{5}(y+6)^{\frac{5}{2}} - \cancel{9}\overset{3}{} \cdot \frac{2}{3}(y+6)^{\frac{3}{2}} \right]_{-\cancel{3}}^{\cancel{3}}$ $\cancel{-21}$

$\overset{\Delta}{=} 2\pi \left[\frac{2}{5}(y+6)^{\frac{5}{2}} - 6(y+6)^{\frac{3}{2}} \right]_{-3}^{3}$

$= 2\pi \left(\frac{2}{5}(3^5 - 2^5) - 6(3^3 - 2^3) \right)$

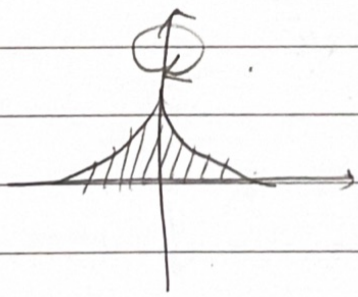
$= 2\pi \left(\frac{2}{5} \cdot (\underbrace{\cancel{243 - 32}}_{=221}) - 6(\underbrace{\cancel{27 - 8}}_{=19}) \right)$

$= 2\pi \left(-\frac{442}{5} + 114 \right)$

$= 2\pi \cdot \frac{-442 + 570}{5}$

$= 2\pi \cdot \frac{128}{5} = \frac{256}{5}\pi$

#133.(6)



$\sqrt{y} = 1 - \sqrt{x}$

$y = 1 - 2\sqrt{x} + x$

$V = \int_0^1 \pi y^2 \cdot dx$

$= \int_0^1 \pi(1 - 2\sqrt{x} + x)\, dx$

$= \pi \left[x - 2 \cdot \frac{2}{3} \cdot x^{\frac{3}{2}} + \frac{1}{2}x^2 \right]_0^1$

$= \pi \left(1 - \frac{4}{3} + \frac{1}{2} \right)$

$= \pi \left(\frac{3}{2} - \frac{4}{3} \right)$

$= \frac{1}{6}\pi(9 - 8) = \frac{1}{6}\pi$

11.24.金. | 기공수1 과제.

#7.4.1(1)

$l = \int_0^{\frac{4}{3}} \sqrt{1+y'^2}\, dx.$

$y = x^{\frac{3}{2}}$

$y' = \frac{3}{2} x^{\frac{1}{2}}$

$y'^2 = \frac{9}{4} x.$

$l = \int_0^{\frac{4}{3}} \sqrt{1+\frac{9}{4}x}\, dx.$

$= \frac{1}{2} \int_0^{\frac{4}{3}} \sqrt{4+9x}\, dx.$

$4+9x = t$ 이면

$9 \cdot dx = dt.$ $dx = \frac{1}{9} dt.$

$l = \frac{1}{2} \int_4^{16} \sqrt{t} \cdot \frac{1}{9} dt$

$= \frac{1}{18} \left[\frac{2}{3} t^{\frac{3}{2}} \right]_4^{16}.$

$= \frac{1}{27} \left(16^{\frac{3}{2}} - 4^{\frac{3}{2}} \right)$

$= \frac{1}{27} (64-8). = \frac{56}{27}$

#7.4.1.(4)

$l = \int_0^{\frac{1}{2}} \sqrt{1+y'^2}\, dx.$

$y = \ln(1-x^2)$

$y' = \frac{-2x}{1-x^2}$

$y'^2 = \frac{4x^2}{x^4-2x^2+1}.$

$1+y'^2 = \frac{x^4+2x^2+1}{x^4-2x^2+1}.$

$= \left(\frac{x^2+1}{x^2-1} \right)^2.$

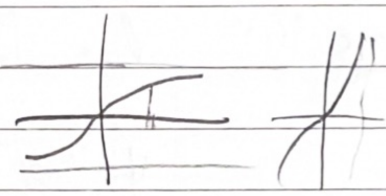
$l = \int_0^{\frac{1}{2}} \left| \frac{x^2+1}{x^2-1} \right| dx.$

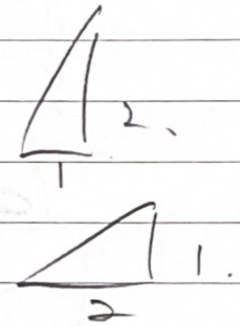
$= \int_0^{\frac{1}{2}} \left| \frac{x^2-1}{x^2-1} + \frac{2}{x^2-1} \right| dx.$

$= \int_0^{\frac{1}{2}} \left| 1 - 2\frac{1}{1-x^2} \right| dx.$

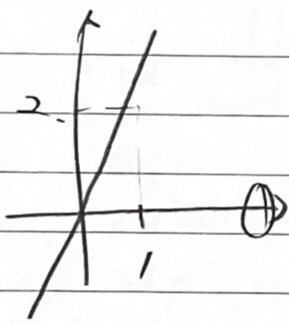
$= \left[x - 2\coth^{-1} x \right]_0^{\frac{1}{2}}.$

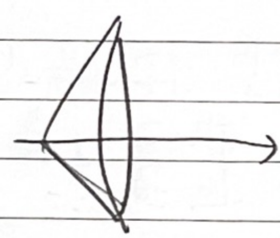
$= \frac{1}{2} - 2\left(\coth^{-1}\frac{1}{2}\right)$





#9.4.3(1)





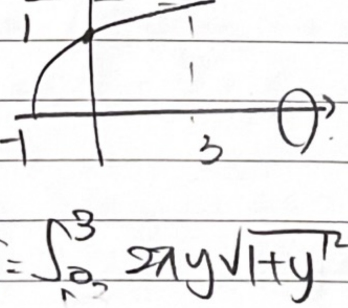
$S = \int_0^1 2\pi y \sqrt{1+y'^2}\, dx.$

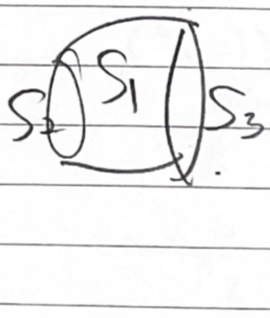
$= \int_0^1 4\pi x \sqrt{1+4}\, dx$

$= 4\sqrt{5}\pi \left[\frac{1}{2}x^2\right]_0^1 = 2\sqrt{5}\pi.$

$2\sqrt{5}\pi + \pi 2^2 = 2(\sqrt{5}+2)\pi$

#9.4.3.(2)





$S_1 = \int_0^3 2\pi y \sqrt{1+y'^2}\, dx.$

$y' = \frac{1}{2\sqrt{x+1}}$

$y'^2 = \frac{1}{4(x+1)}$

$1+y'^2 = \frac{4x+5}{4x+4}$

$2\pi y\sqrt{1+y'^2} = \cancel{2}\pi\,\cancel{\sqrt{x+1}}\cdot\sqrt{\frac{4x+5}{\cancel{4x+4}}}\ \cancel{4}$

$= \pi\sqrt{4x+5}$

$4x+5 = t$ 치환

$4dx = dt \qquad dx = \frac{1}{4}dt$

$S_1 = \frac{1}{4}\pi\int_5^{17}\sqrt{t}\,dt$

$= \frac{1}{4}\pi\left[\frac{2}{3}t^{\frac{3}{2}}\right]_5^{17}$

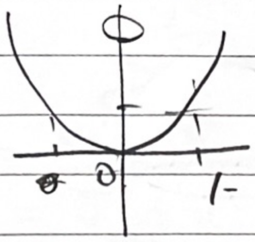
$= \frac{1}{6}\pi(17\sqrt{17}-5\sqrt{5})$

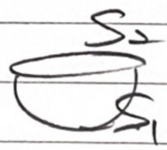
$S = S_1+S_2+S_3$

$= \frac{1}{6}\pi(17\sqrt{17}-5\sqrt{5})+\pi 1+\pi 4$

$= \frac{1}{6}\pi(17\sqrt{17}-5\sqrt{5}+30)$

#7.4.4(1)





$S_1 = \int_0^1 2\pi x\sqrt{1+x'^2}\,dy$

$y = x^2$

$x = \pm\sqrt{y}$

x

$x' = \frac{1}{2\sqrt{y}}$

$x'^2 = \frac{1}{4y}$

$1+x'^2 = \frac{4y+1}{4y}$

$2\pi x\sqrt{1+x'^2} = \cancel{2}\pi \cdot \cancel{x} \cdot \sqrt{\frac{4y+1}{\cancel{4y}}} \cdot \cancel{x}$

$= \pi\sqrt{4y+1}$

$4y+1 = t$ 치환

$4dy = dt$ $\quad dy = \frac{1}{4}dt$

$S_1 = \int_1^5 \frac{1}{4}\pi\sqrt{t}\,dt$

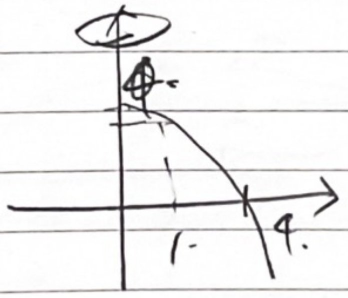
$= \frac{1}{\cancel{4}_2}\pi\left[\frac{\cancel{2}}{3}t^{\frac{3}{2}}\right]_1^5$

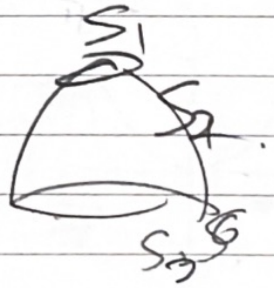
$= \frac{1}{6}\pi(5\sqrt{5}-1)$

$S = S_1 + S_2$

$= \frac{1}{6}\pi(5\sqrt{5}-1) + \pi\cdot 1 = \frac{1}{6}\pi(5\sqrt{5}+5)$

#7.9.9(2)





$x = 2\sqrt{4-\frac{15}{4}}$

$= \cancel{2}\sqrt{\cancel{\tfrac{1}{4}}}\ \cancel{\tfrac{1}{2}}$ $\quad (1, \frac{15}{4})$

$S_2 = \int_0^{\frac{15}{4}} 2\pi x\sqrt{1+x'^2}\,dy$

$x = 2\sqrt{4-y}$

$x' = \cancel{2} \cdot \dfrac{-1}{\cancel{2}\sqrt{4-y}}$

$x'^2 = \dfrac{1}{4-y}$

$1 + x'^2 = \dfrac{5-y}{4-y}$

$2\pi x \sqrt{1+x'^2} = 2\pi \cdot 2\cancel{\sqrt{4-y}} \cdot \sqrt{\dfrac{5-y}{\cancel{4-y}}}$

$= 4\pi\sqrt{5-y}$

$5-y = t$ 치환

$-dy = dt$. $dy = -dt$

$S_2 = \int_5^{\frac{5}{4}} 4\pi\sqrt{t} \cdot (-dt)$

$= 4\pi\left[\dfrac{2}{3} t^{\frac{3}{2}}\right]_{\frac{5}{4}}^{5}$

$= \dfrac{8}{3}\pi\left(5\sqrt{5} - \dfrac{5}{4}\sqrt{\dfrac{5}{4}}\right)$

$= \dfrac{8}{3}\pi\sqrt{5}\left(5 - \dfrac{5}{8}\right)$

$= \dfrac{\cancel{8}}{3}\pi\sqrt{5} \cdot \dfrac{35}{\cancel{8}} = \dfrac{35\sqrt{5}}{3}\pi$

$S = S_1 + S_2 + S_3$

$= \pi + \dfrac{35\sqrt{5}}{3}\pi + 16\pi$

$= \pi\left(\dfrac{35\sqrt{5}}{3} + 17\right)$