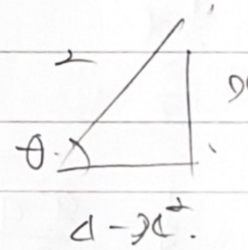
12.01. 金. 개학c 1 마지

#8.4.1.(1)

$x = 2\sin\theta$ $(-\frac{1}{2}\pi \le \theta \le \frac{1}{2}\pi)$라 하자 $\theta = \sin^{-1}(\frac{1}{2}x)$

$dx = 2\cos\theta \cdot d\theta$

$$\int \sqrt{4-x^2}\,dx = \int \sqrt{4-4\sin^2\theta} \cdot 2\cos\theta\, d\theta$$
$$= 4 \int \cos^2\theta\, d\theta$$
$$= 4 \int \frac{1+\cos 2\theta}{2}\, d\theta$$
$$= 2\left(\theta + \frac{1}{2}\sin 2\theta\right) + C$$



$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$= 2 \cdot \frac{x}{2} \cdot \frac{\sqrt{4-x^2}}{2}$$
$$= \frac{x\sqrt{4-x^2}}{2}$$

$2\left(\sin^{-1}(\frac{1}{2}x) + \frac{1}{4}x\sqrt{4-x^2}\right) + C$ (C는 적분상수)

#8.4.1.(3)

$x = 2\sec\theta$라고 하자. $(0 \le \theta < \frac{1}{2}\pi,\ \pi \le \theta < \frac{3}{2}\pi)$

$dx = 2\sec\theta \cdot \tan\theta \cdot d\theta$ $\theta = \sec^{-1}(\frac{1}{2}x)$

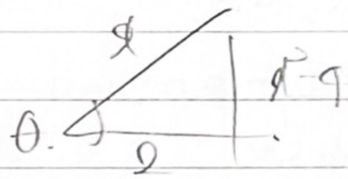
$$\int \frac{1}{x^2\sqrt{x^2-4}}\,dx = \int \frac{1}{\cancel{4}\sec^2\theta\sqrt{4\sec^2\theta-4}}\cdot\cancel{2}\sec\theta\cdot\tan\theta\cdot d\theta$$

$$= \frac{1}{2}\int \frac{1}{\sec\theta\cdot 2\cancel{\tan\theta}}\cancel{\tan\theta}\,d\theta$$

$$= \frac{1}{4}\int \cos\theta\,d\theta = \frac{1}{4}\sin\theta + C$$

$\sec\theta = \frac{1}{2}x$

$\cos\theta = \frac{2}{x}$



$\frac{1}{4}\sin\theta = \frac{1}{4}\cdot\frac{\sqrt{x^2-4}}{x}$

$\frac{\sqrt{x^2-4}}{4x} + C$. (C는 적분상수)

예 4.1.(5)

$x = 3\tan\theta$ 로 하자. $(-\frac{1}{2}\pi \le \theta \le \frac{1}{2}\pi)$

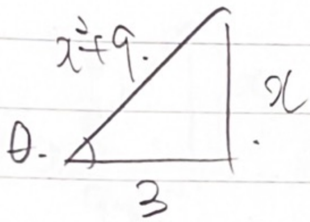
$dx = 3\sec^2\theta\, d\theta$.

$$\int \frac{1}{x^2\sqrt{9+x^2}}\,dx = \int \frac{1}{9\tan^2\theta\cdot\cancel{3}\sqrt{1+\tan^2\theta}}\cancel{3}\sec^2\theta\,d\theta$$

$$= \frac{1}{9}\int \frac{1}{\tan^2\theta\cdot\cancel{\sec\theta}}\sec^2\theta\,d\theta$$

$$= \frac{1}{9}\int \frac{\cos^{\cancel{2}}\theta}{\sin^2\theta}\cdot\frac{1}{\cancel{\cos\theta}}\,d\theta$$

$$= \frac{1}{9}\left(-\frac{1}{\sin\theta}\right) + C$$



$\sin\theta = \frac{x}{x^2+9}$

$\frac{x}{9(x^2+9)} + C$. (~~[illegible]~~ 정답생략)

#8.4.1.(8)

$$\int \frac{1}{\sqrt{(x+1)^2+1}}\,dx$$

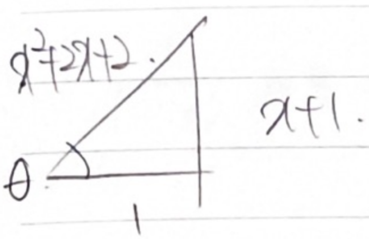
$x+1 = \tan\theta$ 라 하자. $(-\frac{1}{2}\pi \le \theta \le \frac{1}{2}\pi)$

$dx = \sec^2\theta \cdot d\theta$

$$\int \frac{1}{\sqrt{\tan^2\theta+1}} \cdot \sec^2\theta\, d\theta$$

$$= \int \frac{1}{\cancel{\sec\theta}} \cdot \sec^{\cancel{2}}\theta\, d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$



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$\ln\left|x^2+2x+2+x+1\right|+C$

$=\ln\left|x^2+3x+3\right|+C$ (C는 적분상수)

#8.4.1.(10)

$x=3\sin\theta$ 라 하자. $(-\frac{1}{2}\pi\le\theta\le\frac{1}{2}\pi)$.

$dx=3\cos\theta\cdot d\theta$.

$$\int\frac{1}{x^3\sqrt{x^2-9}}\,dx=\int\frac{1}{27\sin^3\theta\cdot\cancel{3}\sqrt{\sin^2\theta-1}}\,\cancel{3}\cos\theta\,d\theta$$

$$=\frac{1}{27}\int\frac{1}{\sin^3\theta\cdot\cancel{\cos\theta}}\,\cancel{\cos\theta}\,d\theta$$

$$=\frac{1}{27}\int\csc^3\theta\,d\theta.$$

$\sin^2\theta+\cos^2\theta=1$.

$1+\cot^2\theta=\csc^2\theta$.

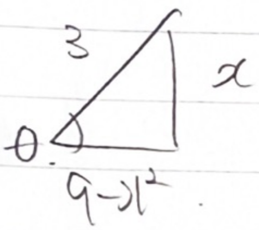
$(\cot\theta)'=-\csc\theta\cdot\cot^2\theta$

$(\csc\theta)'=-\csc^2\theta$

$$\frac{1}{27}\int\csc\theta\cdot(1+\cot^2\theta)\,d\theta.$$

$$=\frac{1}{27}\int(\csc\theta+\csc\theta\cdot\cot^2\theta)\,d\theta.$$

$$=\frac{1}{27}(\ln|\csc\theta-\cot\theta|-\cot\theta)+C.$$



$\frac{1}{3n}\left(\ln\left|\frac{3}{x}-\frac{9-x^2}{x}\right| - \frac{9-x^2}{x}\right)+c$

$=\frac{1}{3n}\left(\ln\left|\frac{x^2-6}{x}\right| - \frac{9}{x}+x\right)+C$ (C는 적분상수)

#8.5.1.(1)

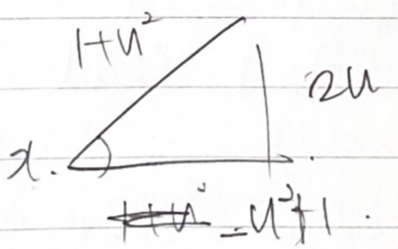
$u=\tan(\frac{1}{2}x)$라 하자.

$\cancel{\frac{1}{2}}x=\tan^{-1}u \times 2$

$dx=\frac{2}{1+u^2}\,du$

$u^4+2u^2+1-4u^2$
$=(u^2-1)^2$

$\sin x=\frac{2u}{1+u^2}$ 이므로



$\cos x=\frac{-u^2+1}{1+u^2}$

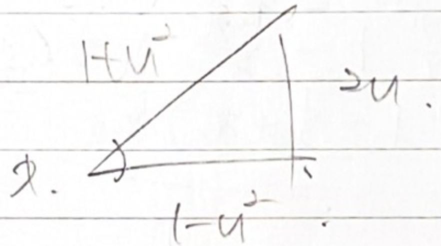
$\int\frac{1}{1-\cos x}\,dx=\int\frac{1}{1+\frac{1-u^2}{1+u^2}}\cdot\frac{2}{1+u^2}\,du$

$=\int\frac{2}{1+\cancel{u^2}+1-\cancel{u^2}}\,du=u+C$

$\tan\frac{1}{2}x + C.$ (C는 적분상수)

#8.5. 1.(5)

$u = \tan(\frac{1}{2}x)$라 하자.



$\frac{1}{2}x = \tan^{-1}u.$

$x = 2\tan^{-1}u.$

$dx = \frac{2}{1+u^2}du$

$\sin x = \frac{2u}{1+u^2}$ $\cos x = \frac{1-u^2}{1+u^2}$

$0 \sim \frac{1}{2}\pi \to 0 \sim 1.$

$$\int_0^{\frac{1}{2}\pi} \frac{1}{1+\sin x+\cos x}\,dx.$$

$$= \int_0^1 \frac{1}{1+\frac{2u}{1+u^2}+\frac{1-u^2}{1+u^2}} \cdot \frac{2}{1+u^2}\,du$$

$$= \int_0^1 \frac{2}{1+\cancel{u^2}+2u+1-\cancel{u^2}}\,du$$

$$= \int \frac{1}{u+1}\,du.$$

$$= [\ln|u+1|]_0^1.$$

$$= \ln 2 - \cancel{\ln 1}. = \ln 2.$$

DATE. NO.

#8.6.1.(1)

x^{-3}

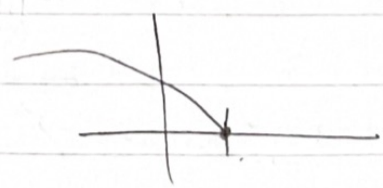
$$\int_1^\infty \frac{1}{x^3}dx = \lim_{t\to\infty}\int_1^t \frac{1}{x^3}dx$$
$$= \lim_{t\to\infty}\left[-\frac{1}{2}x^{-2}\right]_1^t$$
$$= -\frac{1}{2}\lim_{t\to\infty}\left(\frac{1}{t^2}-1\right)$$
$$= -\frac{1}{2}\times(-1) = \frac{1}{2}$$ 수렴

#8.6.1.(4)

$$\int_{-\infty}^\infty \frac{x}{1+x^2}dx = \lim_{t\to-\infty}\int_t^0 \frac{1}{2}\frac{2x}{1+x^2}dx + \lim_{s\to\infty}\int_0^s\frac{x}{1+x^2}dx$$
$$= \lim_{t\to-\infty}\frac{1}{2}\left[\ln|1+x^2|\right]_t^0 + \lim_{s\to\infty}\frac{1}{2}\left[\ln(1+x^2)\right]_0^s$$
$$= \lim_{t\to-\infty}\frac{1}{2}\left(\cancel{\ln 1}-\ln(1+t^2)\right) + \lim_{s\to\infty}\frac{1}{2}\left(\ln(1+s^2)-\cancel{\ln 1}\right)$$
$$= \lim_{t\to-\infty}\frac{1}{2}\left(-\ln(1+t^2)\right) + \frac{1}{2}\lim_{s\to\infty}\left(\ln(1+s^2)\right)$$

$\infty - \infty$ 꼴이므로 발산

#8.6.1.(5)



$$\int_0^1 \frac{1}{\sqrt{1-x}}dx = \lim_{t\to1^-}\int_0^t \frac{2}{2\sqrt{1-x}}dx$$
$$= \lim_{t\to1^-}\left[-2\sqrt{1-x}\right]_0^t$$
$$= \lim_{t\to1^-}\left(-2\sqrt{1-t}+2\cdot1\right)$$
$$= 2+\lim_{t\to1^-}\left(-2\sqrt{1-t}\right)$$
$$= 2+0 = 2$$ 수렴

#8.6.1.(n)

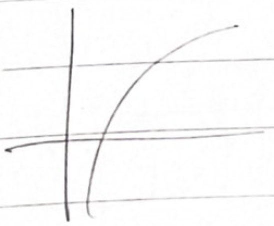
$$\int_0^\infty \frac{1}{(x+1)\sqrt{x}}\,dx = \lim_{a\to 0+}\int_a^{\cancel{\infty}\,1} \frac{1}{(x+1)\sqrt{x}}\,dx + \lim_{b\to\infty}\int_1^b \frac{1}{(x+1)\sqrt{x}}\,dx$$

$$\frac{1}{(x+1)\sqrt{x}} < \frac{1}{x\sqrt{x}}$$

$$\int_0^\infty \frac{1}{x\sqrt{x}}\,dx = \lim_{a\to 0+}\int_a^1 x^{-\frac{3}{2}}\,dx + \lim_{b\to\infty}\int_1^b x^{-\frac{3}{2}}\,dx$$
$$= \lim_{a\to 0+}\left[-\frac{2}{3}x^{-\frac{1}{2}}\right]_a^1 + \lim_{b\to\infty}\left[-\frac{2}{3}x^{-\frac{1}{2}}\right]_1^b$$
$$= \lim_{a\to 0+}\left(\cancel{-\frac{2}{3}} + \frac{2}{3}\frac{1}{\sqrt{a}}\right) + \lim_{b\to\infty}\left(-\frac{2}{3}\frac{1}{\sqrt{b}} + \cancel{\frac{2}{3}}\right)$$
$$= \lim_{a\to 0+}\frac{2}{3\sqrt{a}} + \text{[illegible]}$$ 발산.

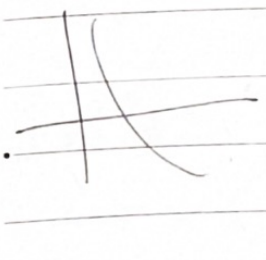
$$\frac{1}{(x+1)x} < \frac{1}{(x+1)\sqrt{x}}$$

$$\int_0^\infty \frac{1}{(x+1)x}\,dx = \lim_{a\to 0}\int_a^1\left(\frac{-1}{x+1}+\frac{1}{x}\right)dx + \lim_{b\to\infty}\int_1^b\left(\frac{-1}{x+1}+\frac{1}{x}\right)dx$$
$$= \lim_{a\to 0}\left[-\ln|x+1|+\ln|x|\right]_a^1 + \lim_{b\to\infty}\left[-\ln|x+1|+\ln|x|\right]_1^b$$
$$= \lim_{a\to 0}\left(\cancel{-\ln|2|}+\cancel{\ln 1}+\ln|a+1|-\ln|a|\right)$$
$$+\lim_{b\to\infty}\left(-\ln|b+1|+\ln|b|+\cancel{\ln 2}-\cancel{\ln 1}\right)$$
$$= \lim_{a\to 0}\left(\cancel{\ln|a+1|}-\ln|a|\right) + \lim_{b\to\infty}\left(-\ln|b+1|+\ln|b|\right)$$
$$= \lim_{a\to 0}\left(-\ln|a|\right) + \lim_{b\to\infty}\left(-\ln|b+1|+\ln|b|\right)$$



$\infty + (-\infty+\infty)$ 꼴 이므로

$\int_0^\infty \frac{1}{x(x+1)}\,dx$는 발산.



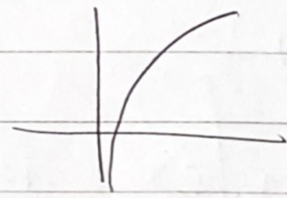
$\frac{1}{(x+1)x} < \frac{1}{(x+1)\sqrt{x}}$ 이고

$\int_0^\infty \frac{1}{x(x+1)}\,dx$은 발산하므로

$\int_0^\infty \frac{1}{(x+1)\sqrt{x}}\,dx$도 발산한다.

~~#8.6 1.(8)~~

$\int_0^1 x\ln x = \lim_{a\to 0-} \int_a^1 x\ln x \, dx$



미 적

$\ln x$. x . $\frac{1}{2}x^2 \ln x - \int \frac{1}{2}x \, dx$

$\frac{1}{x}$. $\frac{1}{2}x^2$ $= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2$

$\lim_{a\to 0-} \left[\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 \right]_a^1$

$= \lim_{a\to 0-} \left\{ \left(\frac{1}{2}\cdot 1 \cdot \ln 1 - \frac{1}{4}\cdot 1 \right) - \left(\frac{1}{2}a^2 \ln a - \cancel{\frac{1}{4}a^2} \right) \right\}$

$= -\frac{1}{4} - \lim_{a\to 0-} \frac{1}{2}a^2 \ln a$

$\lim_{a\to 0-} a^2 \ln a$ 은 $0 \times (-\infty)$ 꼴

$= \lim_{a\to 0-} \frac{\ln a}{\frac{1}{a^2}}$

$= \lim_{a\to 0-} \frac{\frac{1}{a}}{\frac{-2a}{a^4}}$

$= \lim_{a\to 0-} \frac{1}{a} \cdot \frac{a^4}{-2a}$

$= \lim_{a\to 0-} \frac{a^2}{-2a} = 0$

$-\frac{1}{4}$