^{163}Lu - Energy Ellipsoid

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Cuprins

1 Calculations

The initial energy function $\mathscr{H}$ is given in terms of the polar coordinates (θ, φ).

1.1 Cartesian coordinate system - calculations

The maximal MOI is around 1-axis. As a result, quantization axis is the 1-axis. Thus, the coordinate system becomes:

$$\begin{cases} x_1 = I\cos\theta, \\ x_2 = I\sin\theta\cos\varphi, \\ x_3 = I\sin\theta\sin\varphi. \end{cases} \tag{1}$$

The energy function, in Cartesian form, will have the following expression:

$$H = c_0 + c_1(x_1) + c_2(x_1, x_2, x_3) \ . \tag{2}$$

The constant (free-term) can be ignored, so we focus on the analytical expression for the last two terms, namely c_1 and c_2.

$$c_1(x_1) = -2jA_1\sqrt{I^2 - x_1^2} \tag{3}$$

and

$$c_2(x_1, x_2, x_3) = (A_3 - a_3)x_1^2 + (A_1 - a_1)x_2^2 + (A_2 - a_2)x_3^2 \tag{4}$$

with $a_k = \frac{A_k}{2I}$. *Full calculations can be seen in the attached hand-written notes at the end of this document.*

2 Features of the Energy Function - Cartesian form

Based on the expressions given in 3 and 4, one can see that:

- The energy function has an **ellipsoidal** shape, given by the *main* term c_2.
- The contribution of the x_1-dependent term has a negative sign and $c_1 \approx x_1$. This will create a shift in the ellipsoid, along the x_1-direction.
- Since c_0 doesn't have any dependence on the Cartesian coordinates, it can be neglected during the graphical representation procedure.

3 Hand-written notes

$$\mathcal{H} = \frac{I}{2}(A_1 + A_2) + A_3 I^2 + I(I - \frac{1}{2}) \sin^2\theta \left(A_1 \cos^2\varphi + A_2 \sin^2\varphi - A_3\right)$$
$$+ \frac{j}{2}(A_2 + A_3) + A_1 j^2 - 2A_1 I j \sin\theta - V\frac{2j-1}{j+1}\sin\left(\gamma + \frac{\pi}{6}\right).$$

$$H(\theta,\varphi) = h_0 + h_\theta + h_1$$

h_0: const. $\quad h_\theta$: $f(\theta)$ $\quad h_1$: $g(\theta,\varphi)$

$$\begin{cases} x_1 = I\cos\theta \\ x_2 = I\sin\theta\cos\varphi \\ x_3 = I\sin\theta\sin\varphi \end{cases}$$

x_1 – rotation MOI

$$h_0 = \frac{I}{2}(A_1+A_2) + A_3 I^2 + \frac{j}{2}(A_2+A_3) + A_1 j^2 - V\frac{2j-1}{j+1}\sin\left[\gamma + \frac{\pi}{6}\right]$$

$\hookrightarrow$ constant (can be ignored)

$$h_\theta = -2Ij A_1\sin\theta = -2jA_1\sqrt{I^2 - I^2\cos^2\theta}$$
$$= -2jA_1\sqrt{I^2 - x_1^2}$$

$\hookrightarrow$ $x_1^2 \leq I^2$

$|x_1| \leq I$

$$a_k = \frac{A_k}{2I}$$

$$h_1 = I\left(I - \frac{1}{2}\right)\sin^2\theta\left(A_1\cos^2\varphi + A_2\sin^2\varphi - A_3\right)$$
$$= A_1 I^2 \sin^2\theta\cos^2\varphi + A_2 I^2\sin^2\theta\sin^2\varphi - I^2\sin^2\theta A_3 -$$
$$- \frac{A_1}{2} I\sin^2\theta\cos^2\varphi - \frac{A_2}{2} I\sin^2\theta\sin^2\varphi + \frac{A_3 I}{2}\sin^2\theta$$
$$= A_1 x_2^2 + A_2 x_3^2 - A_3 I^2(1-\cos^2\theta) - a_1 x_2^2 - a_2 x_3^2 + a_3 I^2(1-\cos^2\theta)$$
$$= (A_1 - a_1)x_2^2 + (A_2 - a_2)x_3^2 + A_3 x_1^2 - A_3 I^2 + a_3 I^2 - a_3 x_1^2$$
$$= (A_3 - a_3)x_1^2 + (A_1 - a_1)x_2^2 + (A_2 - a_2)x_3^2 - (A_3 - a_3)I^2$$

cartesian form of H

$$H = h_0 - \underbrace{\left(2 j A_1 \sqrt{I^2 - x_1^2}\right)}_{c_1(x_1)} + \underbrace{(A_3 - a_3) x_1^2 + (A_1 - a_1) x_2^2 + (A_2 - a_2) x_3^2}_{c_2(x_1, x_2, x_3)} - (A_3 - a_3) I^2$$

cartesian expression of H

$$H = c_0 + c_1(x_1) + c_2(x_1, x_2, x_3)$$

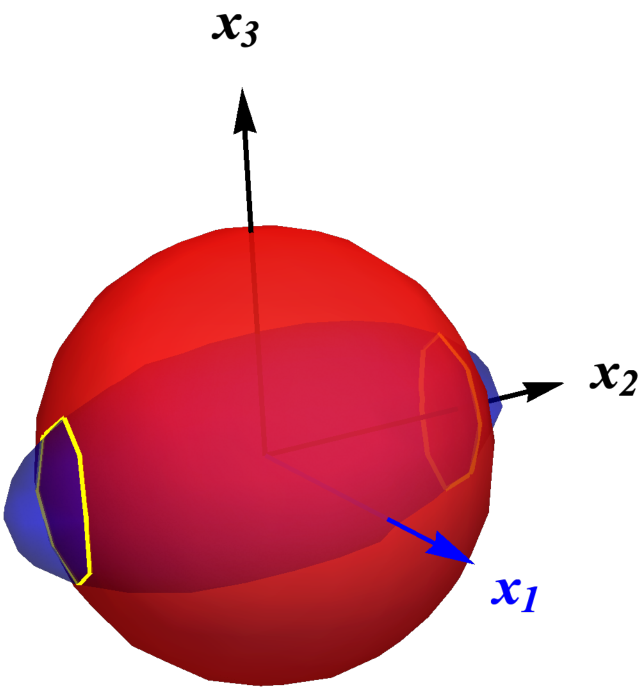

Figura 1: Energy Ellipsoid 1.

4 Energy ellipsoid: 3d representations

For the these calculations, the angular momentum is $I = 17/2$. The moments of inertia are as follows: $\mathscr{I}_1 = 72$, $\mathscr{I}_2 = 15$, $\mathscr{I}_3 = 7$.

The intersection between the angular momentum sphere (of radius $I(I+1)$ and the ellipsoid of energy E) is represented by the yellow contour. By keeping I fixed, and increasing the energy E, the shape of the ellipsoid changes (according to the ratio of the three moments of inertia).

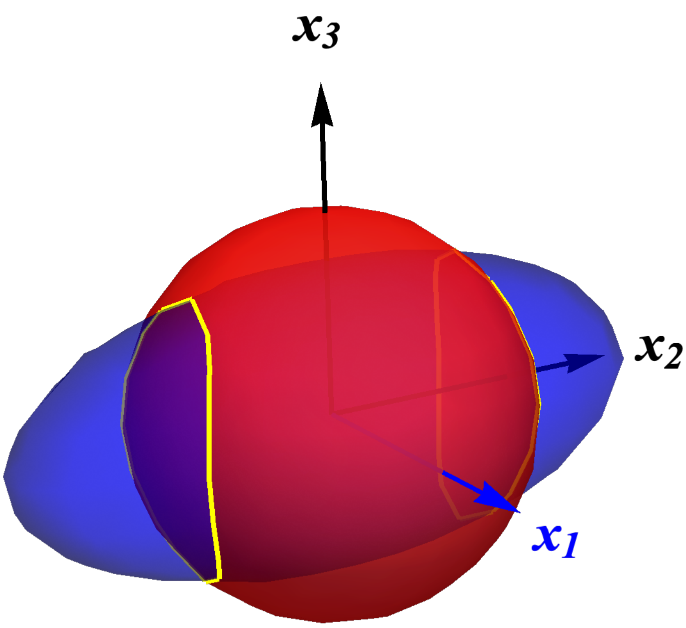

Figura 2: Energy Ellipsoid 2.

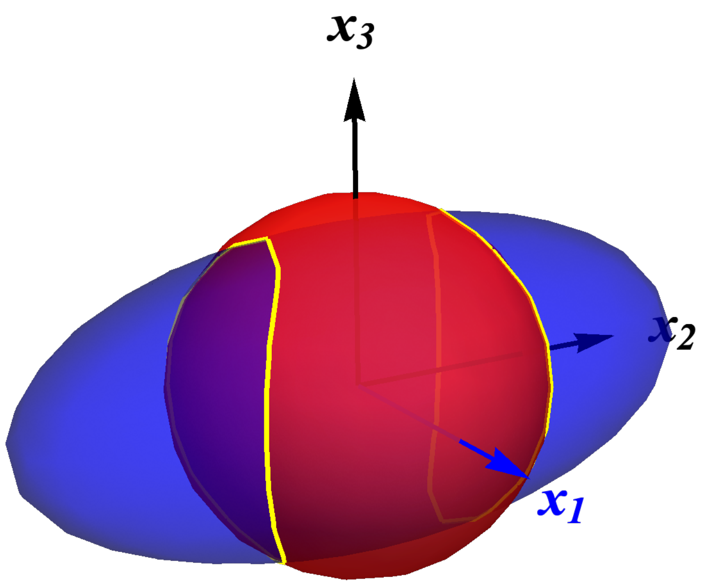

Figura 3: Energy Ellipsoid 3.

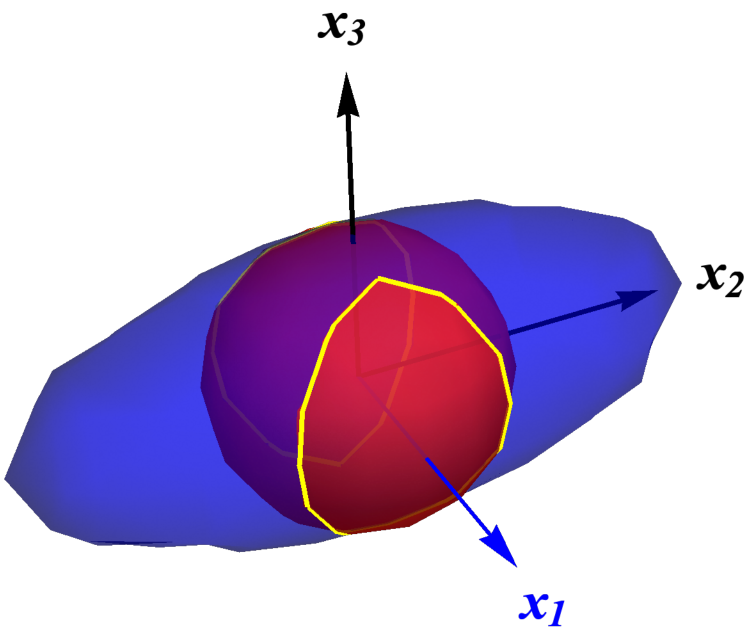

Figura 4: Energy Ellipsoid 4. One can observe that the trajectories are changed: **system starts to rotate around 1-axis.**