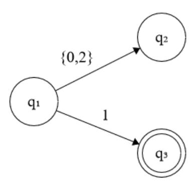
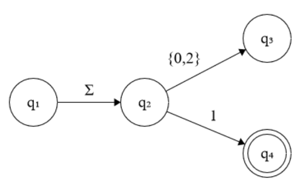
Bhagawat Chapagain – I pledge my honor that I have abided by the Stevens Honor System.

(1) For a language $J_k = \{w \in \{0,1,2\}^* |$ the k^{th} symbol of w is 1.$\}$, the respective machine for each k can be found by observing the first few cases. Namely, if we observe cases of J for small values of k, we can spot a pattern.

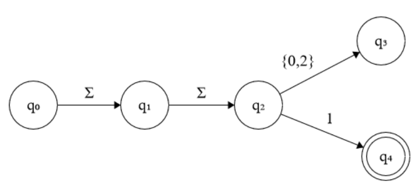
$$k = 1 :$$



$$k = 2 :$$



$$k = 3 :$$



For each case of k, we see that the machine for $J_{k-1}$is present in the the machine diagram. We know that this will always be the case because we cannot use ϵ to remain at the the same position in the input string in in NFAs. Due to this, we know that we need to travel k states, however, we must also include two (2) states for acceptance and rejection. Therefore there must be minimum of $k + 2$ states in J_k's language.

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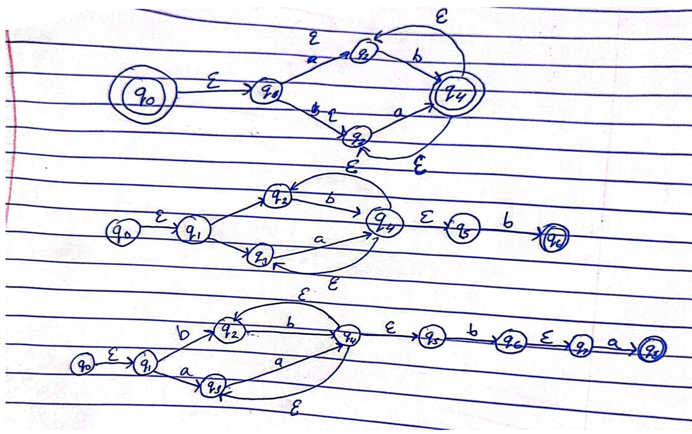
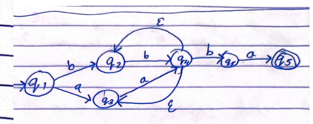
(2)





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(3) As proven in class, if A, B are regular then $A \circ B$ must be regular. If we have $A \star B = \{w \in \Sigma^* | w = a_1 b_1 \ldots a_k b_k$ for some $k \geq 1$ and symbols $a_1, \ldots, a_k, b_k, \ldots, b_k \in \Sigma$ such that $a_1 \ldots a_k \in A$ and $b_1, \ldots, b_k \in B$. If $A \star B$ is found by merging of all pairs of strings in A, B that have the same length, each a_k must be associated with an equally sized b_k but since $\Sigma^* := \{a_1, \ldots, a_k, b_k, \ldots, b_k\}^*$, it is every combination of $\{a, b\}$ generated infinitely. Therefore, there are equally as many a_k, b_k of the same size than those that are not. Therefore $|A \circ B| = |A \star B|$ and $A \star B \subseteq A \circ B$. Since $A \star B$ belongs to the same finite equivalence class as $A \circ B$, it must be regular.

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(4)

$$((00)^*1)^* = \{\{00, 0000, 000000, \ldots\} \circ 1\}^* = \{001, 00001, 0000001, \ldots\}^*$$

$$((000)^*1)^* = \{\{000, 000000, 000000000, \ldots\} \circ 1\}^* = \{0001, 0000001, 0000000001, \ldots\}^*$$

$$\{001,00001,0000001,\ldots\}^* \cap \{0001,0000001,0000000001,\ldots\}^* = (00)^* \cap (000)^* \circ 1^*$$

The intersection between $(00)^*$and $(000)^*$would just be a set of zeroes concatenated by the smallest common factor between two and three zeroes.

$$(000000)^* \circ 1^* = \{000000, 000000000000, 000000000000000000, \ldots\} \circ \{1, 11, 111, \ldots\} = \{0000001\}^*$$

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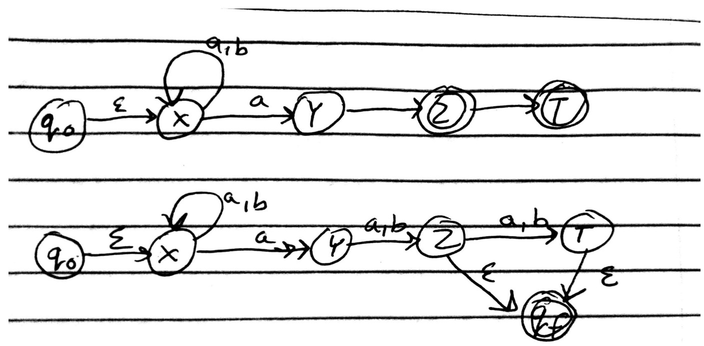
(5) Setting up the State-Elimination method and we get this:



Elim X: $\{(q_0, Y) : (a \cup b)^* a, (Z, T) : (a \cup b), (Z, q_f) : \epsilon, (T, q_f) : \epsilon\}$
Elim Y: $\{(q_0, Z) : (a \cup b)^* a(a \cup b), (Z, T) : (a \cup b), (Z, q_f) : \epsilon, (T, q_f) : \epsilon\}$
Elim Z: $\{(q_0, T) : (a \cup b)^* a(a \cup b)(a \cup b), (q_0, q_f) : (a \cup b)^* a(a \cup b)\}$
Elim T: $\{(q_0, q_f) : (a \cup b)^* a(a \cup b) \cup (a \cup b) \cup (a \cup b)^* a(a \cup b)\}$