special accessor functions or macros. For example, to access element `i` of vector `x` you need to use

```
VECTOR_ELT(x, i)
```

and for assigning a new value `v` to this element you would use

```
SET_VECTOR_ELT(x, i, v)
```

These API changes are the main reason that packages need to be recompiled for 1.2. Further details on the current API are available in *"Writing R Extensions"*.

Future Developments

There are many heuristics used in the garbage collection system, both for determining when different generations are collected and when the size of the heap should be adjusted. The current heuristics seem to work quite well, but as we gain further experience with the collector we may be able to improve the heuristics further.

One area actively being pursued by the R core team is interfacing R to other systems. Many of these systems have their own memory management systems that need to cooperate with the R garbage collector. The basic tools for this cooperation are a finalization mechanism and weak references. A preliminary implementation of a finalization mechanism for use at the C level is already part of the collector in R 1.2. This will most likely be augmented with a weak reference mechanism along the lines described by Peyton Jones, Marlow and Elliott (1999).

References

Richard Jones and Rafael Lins (1996). *Garbage Collection*. Wiley. 10

Simon Peyton Jones, Simon Marlow, and Conal Elliott (1999). Stretching the storage manager: weak pointers and stable names in Haskell. http://www.research.microsoft.com/Users/simonpj/Papers/papers.html. 11

Luke Tierney
University of Minnesota, U.S.A.
luke@stat.umn.edu

On Exact Rank Tests in R

by Torsten Hothorn

Linear rank test are of special interest in many fields of statistics. Probably the most popular ones are the Wilcoxon test, the Ansari-Bradley test and the Median test, therefore all implemented in the standard package **ctest**. The distribution of their test statistics under the appropriate hypothesis is needed for the computation of P-values or critical regions. The algorithms currently implemented in R are able to deal with untied samples only. In the presence of ties an approximation is used. Especially in the situation of small and tied samples, where the exact P-value may differ seriously from the approximated one, a gap is to be filled.

The derivation of algorithms for the exact distribution of rank tests has been discussed by several authors in the past 30 years. A popular algorithm is the so called network algorithm, introduced by Mehta and Patel (1983). Another smart and powerful algorithm is the shift algorithm by Streitberg and Röhmel (1986). In this article, we will discuss the package **exactRankTests**, which implements the shift algorithm. The computation of exact P-values and quantiles for many rank statistics is now possible within R.

Using ExactRankTests

The algorithm implemented in package **exactRankTests** is able to deal with statistics of the form

$$T = \sum_{i=1}^{m} a_i$$

where $a = (a_1, \ldots, a_N)$ are positive, integer valued scores assigned to N observations. We are interested in the distribution of T under the hypothesis that all permutations of the scores a are equally likely. Many rank test can be regarded this way, e.g. the Wilcoxon test is a special case with $a_i = i$ and the Ansari-Bradley test has scores $a_i = \min(i, N - i + 1)$. For details about the algorithm we point to the original articles, for example Streitberg and Röhmel (1986) and Streitberg and Röhmel (1987).

Package **exactRankTests** implements the functions `dperm`, `pperm`, and `qperm`. As it is standard in R/S they give the density, the probability function and the quantile function. Additionally, the function `pperm2` computes two-sided P-values. Consider e.g. the situation of the Wilcoxon test. Let `x` and `y` denote two vectors of data, possibly tied. First, we compute the ranks over all observations and second we compute the Wilcoxon statistic, which is the sum over the ranks of the `x` sample.

```
R> ranks <- rank(c(x,y))
```

```
R> W <- sum(ranks[seq(along=x)])
R> pperm(W, ranks, length(x))
```

The one-sided P-value is computed in the last line of the example. In the absence of ties the results of [pq]perm and [pq]wilcox are equal. An exact version of wilcox.test is provided as wilcox.exact. The following example is taken from Mehta and Patel (1998). The diastolic blood pressure (mmHg) was measured on 11 subjects in a control group and 4 subjects in a treatment group. First, we perform the one-sided Wilcoxon rank sum test.

```
R> treat <- c(94, 108, 110, 90)
R> contr <- c(80, 94, 85, 90, 90, 90,
              108, 94, 78, 105, 88)
R> wilcox.exact(contr, treat,
                alternative = "less")

         Exact Wilcoxon rank sum test

data:  contr and treat
W = 9, point prob = 0.019, p-value = 0.05421
alternative hypothesis: true mu is less than 0
```

The one-sided P-value is 0.05421 which coincides with the P-value of 0.0542 given in Mehta and Patel (1998). Additionally, the probability of observing the test statistic itself is reported as point prob.

Usually, the distribution is not symmetric in the presence of ties. Therefore the two-sided P-values need additional effort. StatXact computes the two-sided P-value as 0.0989 and wilcox.exact returns:

```
R> wilcox.exact(contr, treat)

         Exact Wilcoxon rank sum test

data:  contr and treat
W = 9, point prob = 0.019, p-value = 0.0989
alternative hypothesis: true mu is not equal to 0
```

Real or rational scores

The original algorithm is defined for positive, integer valued scores only. Streitberg and Röhmel (1987) suggested to approximate the distribution of a statistic based on real or rational scores by taking the integer part of the appropriately multiplied scores. A bound for the maximal possible error on the quantile scale can be derived (see the documentation of [dpq]perm for more details). As an example, we want to calculate the critical value for a two-sided van der Waerden test for samples of 10 untied observations each and a significance level of $\alpha = 0.05$:

```
R> abs(qperm(0.025, qnorm(1:20/21), 10))
[1] 3.872778
```

By default, the tolerance limit is set to tol=0.01 which means that the computed quantiles does not differ more than 0.01 from the true ones. Due to memory limitations, it might not be possible to calculate a quantile in such a way.

Another approach is to use integer scores with the same shape as the original ones. This can be achieved by mapping the real or rational scores into $\{1, \ldots, N\}$. The idea behind is that one is not interested in approximating the quantiles but to have a test with the same properties as the original one. Additionally, the computational effort is the same as for the Wilcoxon test. This procedure was suggested by my colleague Berthold Lausen during a discussion about the shift algorithm. The two-sided P-value for the van der Waerden test of two samples x and y is now computed as follows:

```
R> N <- length(c(x,y))
R> sc <- qnorm(rank(c(x,y))/(N+1))
R> sc <- sc - min(sc)
R> sc <- round(sc*N/max(sc))
R> X <- sum(sc[seq(along=x)])
R> p <- pperm2(X, sc, length(x))
```

Conclusion

Using the newly introduced package **exactRankTests**, R users are able to compute exact P-values or quantiles of linear rank tests based on the Streitberg-Röhmel shift algorithm. The use the the procedures [dpq]perm is illustrated by many examples in the help files. Additionally, a modified version of wilcox.test using the exact procedures is provided as wilcox.exact. The performance of [dpq]perm is not as good as that of [pq]wilcox but should be sufficient for most applications.

References

Cyrus R. Mehta and Nitin R. Patel. A network algorithm for performing fisher's exact test in $r \times c$ contingency tables. *Journal of the American Statistical Association*, 78(382):427–434, June 1983. 11

Cyrus R. Mehta and R. Patel, Nitin. *StatXact-4 for Windows*. Cytel Software Cooperation, Cambridge, USA, 1998. 12

Bernd Streitberg and Joachim Röhmel. Exact distributions for permutations and rank tests: An introduction to some recently published algorithms. *Statistical Software Newsletters*, 12(1):10–17, 1986. 11

Bernd Streitberg and Joachim Röhmel. Exakte Verteilungen für Rang- und Randomisierungstests im allgemeinen c-Stichprobenfall. *EDV in Medizin und Biologie*, 18(1):12–19, 1987. 11, 12

Torsten Hothorn
Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany
Torsten.Hothorn@rzmail.uni-erlangen.de