К#5

Задача 1) $f = x^4 + x^3 - x^2 + \lambda x + g \quad \in \mathbb{C}[x]$

$x_1 \cdot x_2 = x_3 x_4$

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = -1 \\ x_1x_2 + x_1x_3 + x_1x_4 + x_2x_3 + x_2x_4 + x_3x_4 = -1 \\ x_1x_2x_3 + x_1x_2x_4 + x_2x_3x_4 + x_1x_3x_4 = -\lambda \\ x_1x_2x_3x_4 = g \end{cases}$$

$$\Leftrightarrow \begin{cases} x_3 + x_4 = -(1 + x_1 + x_2) \\ x_1x_2 + (x_1 + x_2)(-(1 + x_1 + x_2)) = -1 \\ 2x_1x_2 + (x_1 + x_2)(-1 - x_1 - x_2) = -1 \\ x_1x_2(-1 - x_1 - x_2) + x_1x_2(x_1 + x_2) = -\lambda \\ x_1^2x_2^2 = g \end{cases}$$

$$\Leftrightarrow \begin{cases} x_3 + x_4 = -(1 + x_1 + x_2) \\ 2x_1x_2 - x_1 - x_1^2 - x_1x_2 - x_2 - x_1x_2 - x_2^2 = -1 \\ x_1x_2(-1 - x_1 - x_2 + x_1 + x_2) = -\lambda \\ x_1x_2 = \pm 3 \end{cases}$$

$$\Rightarrow \begin{cases} x_1^2 + x_2^2 + x_1 + x_2 = -1 \\ x_1x_2 = \lambda \\ x_1x_2 = \pm 3 \end{cases}$$

$$\Rightarrow \lambda = \pm 3$$

$f' = 3x^2 + p, \quad f'' = 6x$

2dg a III) $f = x^3 + px + q$

$$S = \frac{x_1 x_2}{x_3+5} + \frac{x_1 x_3}{x_2+5} + \frac{x_2 x_3}{x_1+5}$$

$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ x_1x_2 + x_1x_3 + x_2x_3 = p \\ x_1 x_2 x_3 = -q \end{cases}$$

- $$\frac{x_1 x_2}{x_3+5} = \frac{x_1x_2(x_1+5)(x_2+5)}{(x_3+5)(x_2+5)(x_1+5)} = \frac{x_1x_2(x_1x_2+5x_1+5x_2+25)}{\prod_{i=1}^{3}(x_i+5)} =$$

$$= \frac{x_1x_2(x_1x_2-5x_3+25)}{\prod_{i=1}^{3}(x_i+5)} = \frac{x_1^2x_2^2+5q+25x_1x_2}{\prod_{i=1}^{3}(x_i+5)}$$

- $$\prod_{i=1}^{3}(x_i+5) = (x_1+5)(x_2+5)(x_3+5) = x_1x_2x_3+5x_1x_2+5x_1x_3+5x_2x_3+25x_1+25x_2+25x_3+125 =$$

$$= -q+5p+125 \overset{\text{podst}}{=} \frac{1}{M}$$

- $$S = M\left(\sum_{1\le i<j\le 3} x_i^2x_j^2 + 5q + 25x_ix_j\right) =$$

$$= M\left(15q + 25p + \sum_{i=1}^{3}\frac{q^2}{x_i^2}\right)$$

$$\sum_{i=1}^{3} \frac{q^2}{x_i^2} = q^2 \sum_{i=1}^{3} \frac{1}{x_i^2} = q^2 \frac{f'(0)f'(0) - f''(0)f(0)}{(f(0))^2} =$$

$$= q^2 \frac{p^2}{q^2} = p^2$$

$$S = \frac{15q + 25p + p^2}{-q + 5p + 125}$$

3. cazul (a 3) $f = x^3 + 2x + 3$

$0 = f(0x_i) = x^3 - 2x + 3 \Rightarrow x^3 = 2x - 3$

$g = ?$ cu $y_i = x_i^3$

$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ x_1x_2 + x_1x_3 + x_2x_3 = -2 \\ x_1x_2x_3 = -3 \end{cases}$$

$\Rightarrow g = y^3 + 9y^2 + 19y + 27$

$y_1 = x_1^3 = 2x - 3$

$g = y^3 + Ay^2 + By + C$

$$\begin{cases} y_1 + y_2 + y_3 = -A \\ y_1y_2 + y_1y_3 + y_2y_3 = B \\ y_1y_2y_3 = -C \end{cases}$$

$$-A = \sum_{i=1}^{3} 2x_i - 3 = -9 + 2\sum_{i=1}^{3} x_i = -9 \Rightarrow \boxed{A = 9}$$

$$B = \sum_{1 \le i < j \le 3} (2x_i - 3)(2x_j - 3) = \sum_{1 \le i < j \le 3} 4x_ix_j - 6x_i - 6x_j + 9 =$$

$$= 4 \cdot (-2) - 12 \cdot 0 + 9 \cdot 3 = -8 + 27 = 19 \Rightarrow \boxed{B = 19}$$

$$-C = \prod_{i=1}^{3} 2x_i - 3 = 8x_1x_2x_3 - 12x_1x_2 - 12x_2x_3 - 12x_1x_3 + 18x_1 + 18x_2 + 18x_3 - 27 =$$

$$= -24 + 24 - 27 = -27 \Rightarrow \boxed{C = +27}$$