Session #2
Gears: Basics, terminology, involute profile

Dan Frey

Today's Agenda

- Distribute homework #1
- Gears
  - Applications
  - Types
  - Terminology / nomenclature
  - Congugate action
  - Involute curve
  - Analysis & design

Applications of Gears

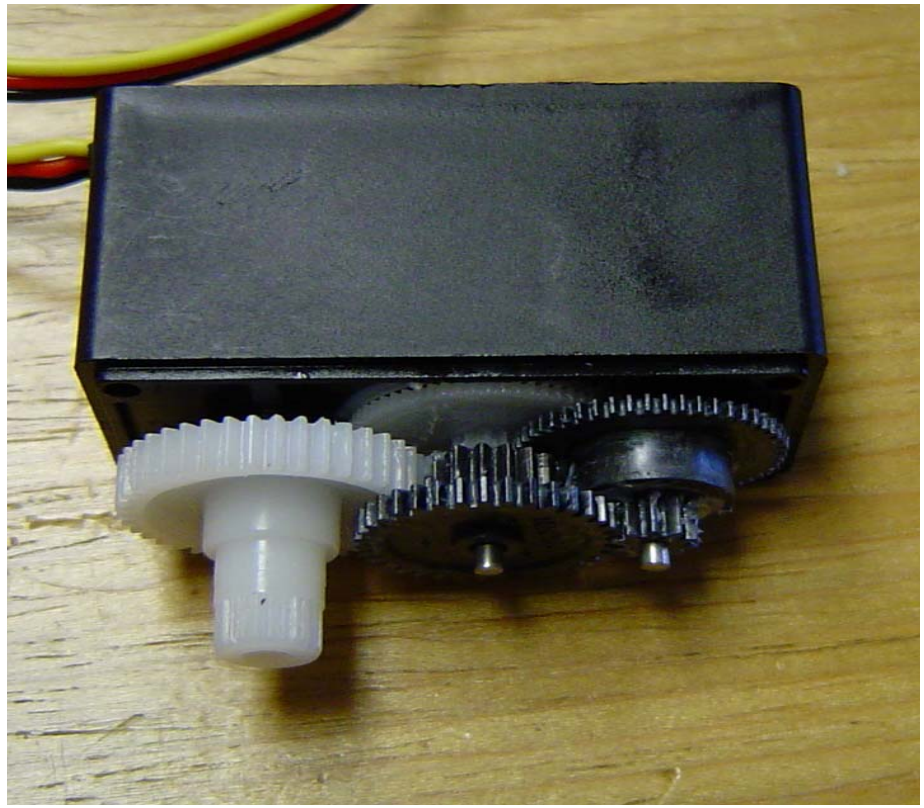
Photo of small electric motor removed for copyright reasons.

Photo of an elaborate chronometer (ref: Dava Sobel's book *Longitude*) removed for copyright reasons.

Diagram removed for copyright reasons.
"AH Sprite/MG Midget Clutch, Transmission and Drive Train, Differential and Axles"

Spur Gears

- Transmit motion between parallel shafts
- Teeth are parallel to the axis of rotation
- This is the simplest kind of gear we'll consider and most of today is dedicated to them



Other Types of Gears

Rack

Courtesy of OSHA.

Bevel

Image removed for Copyright reasons.

Image removed for Copyright reasons.

Worm

Helical

Name That Gear

What type of worm gear set is this?

1) Single-enveloping, single threaded
2) Single-enveloping, multi-threaded
3) Double enveloping single threaded worm gear
4) Double enveloping multi- threaded

Follow up

What is the reduction ratio of this gear set?

1) 10:1
2) 20:1
3) 40:1
4) 80:1

Harmonic Gear Drive

- Not based on rigid body motion!
- Ellipse in center deforms the internal gear

Diagrams removed for copyright reasons.

Early Gears

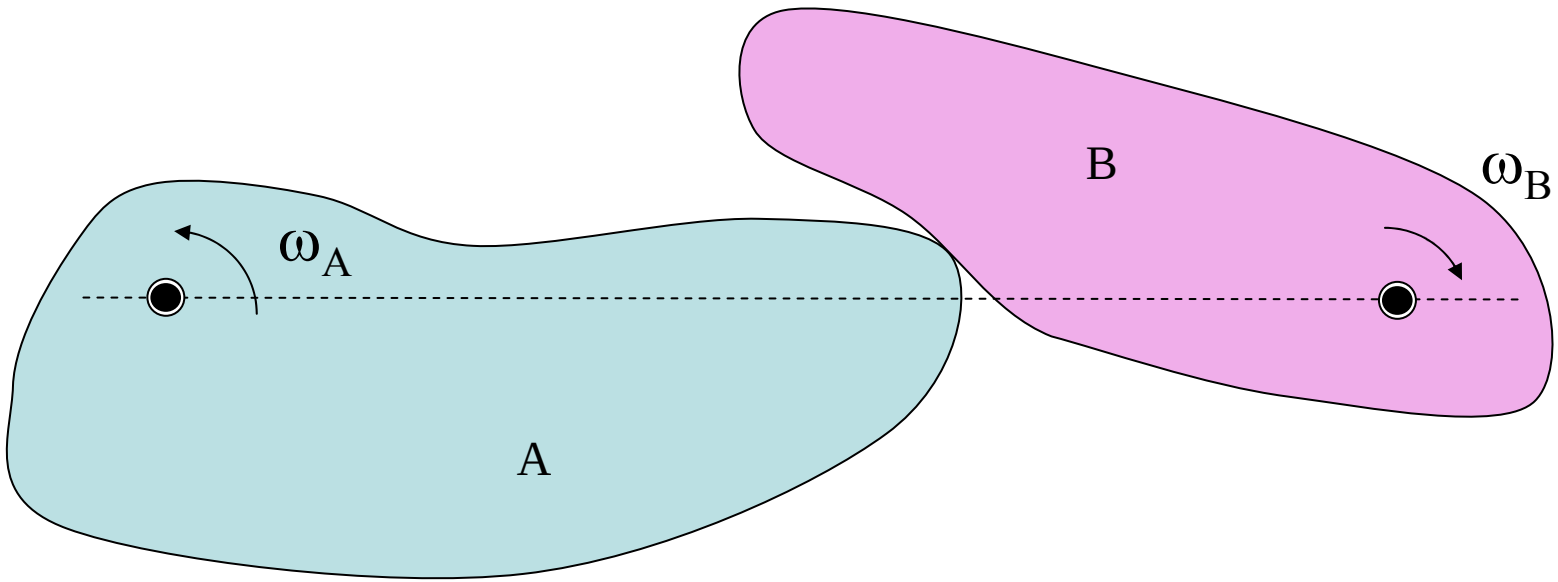
Drawing removed for copyright reasons.

Roman watermills at Barbegal
300AD

Drawing removed for copyright reasons.

Application for powering textile machinery
18th century

Pitch Point

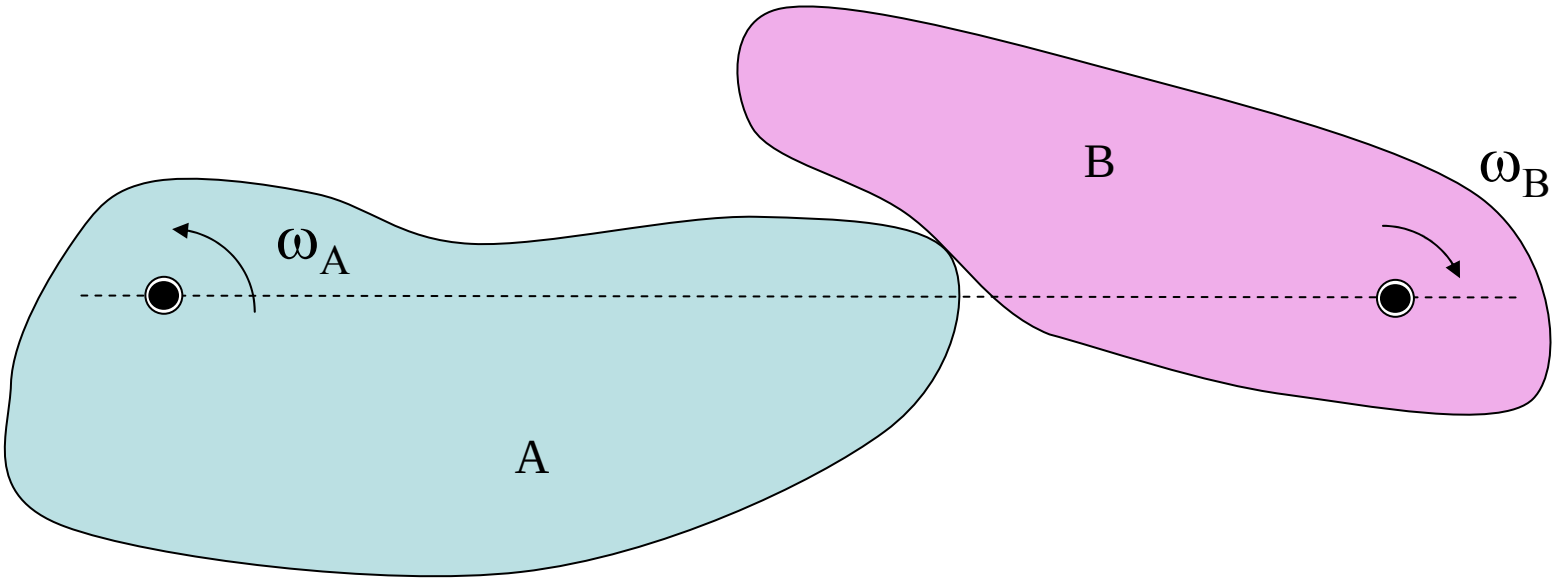


What is the line of action?

What is the pitch point?

What are the relationships among these?

Sliding and Rolling

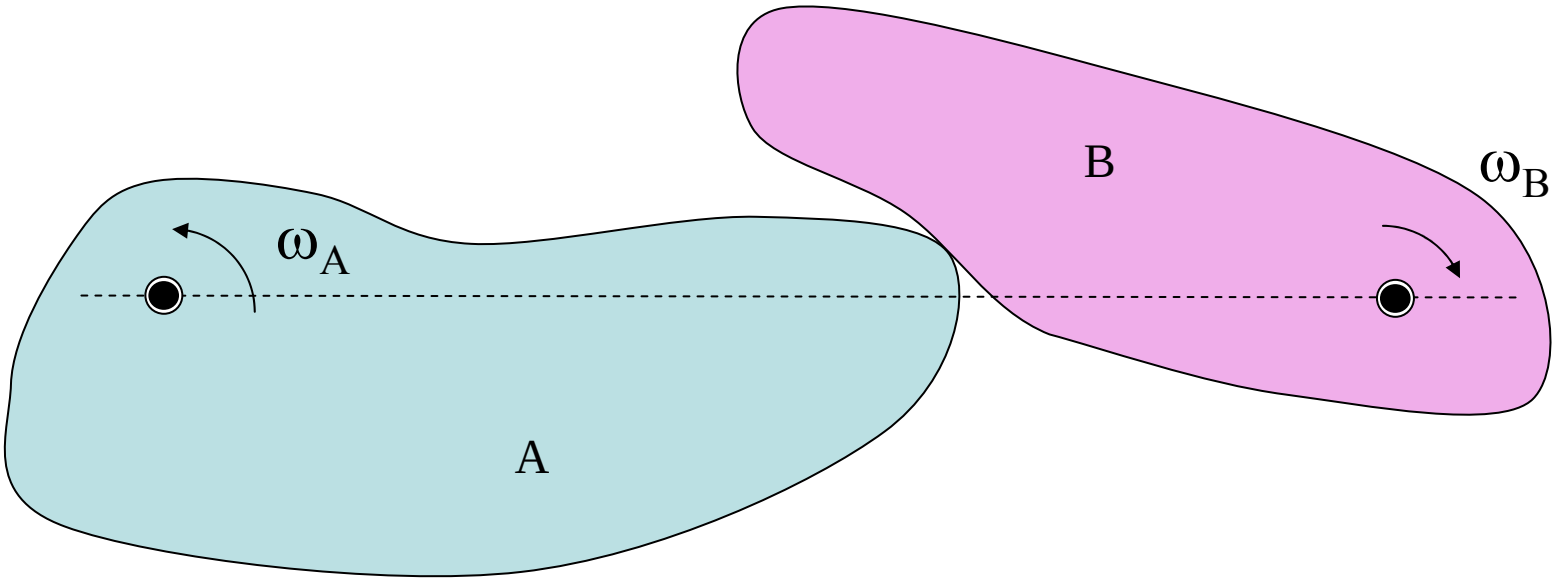


When one body is driving another, do the surfaces slide, roll, or both?

What is the relationship to the pitch circles?

How could you determine this?

Conjugate Action



Let's say ω_A is a known. How can we determine ω_B?

Let's say ω_A is a constant with time. Can we synthesize a shape of body B so that ω_B is also constant with time?

Rack Cutting

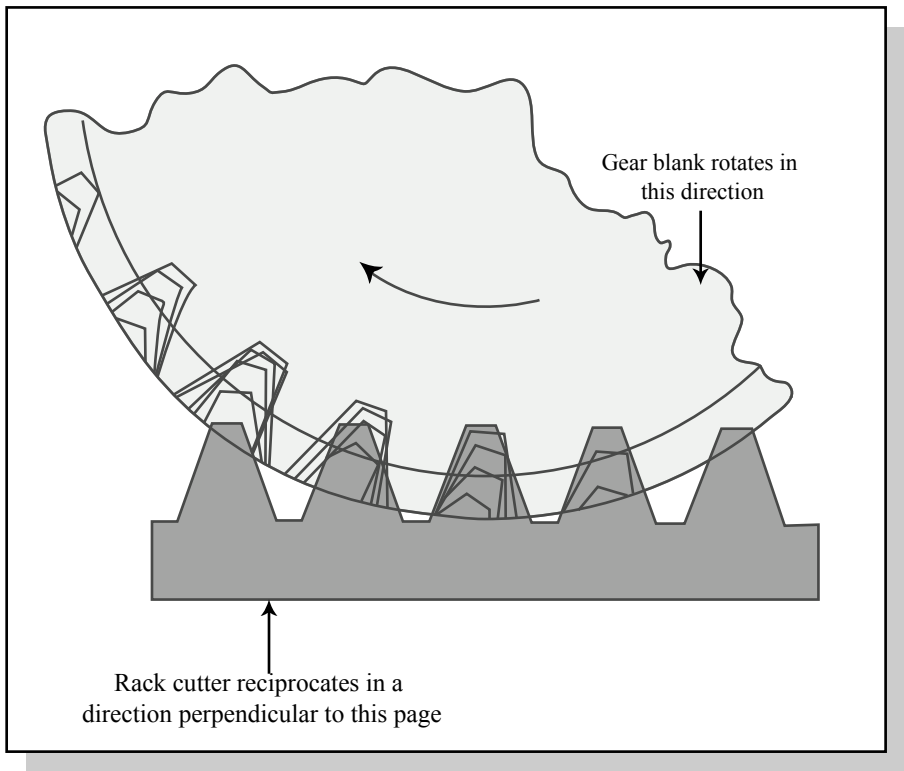
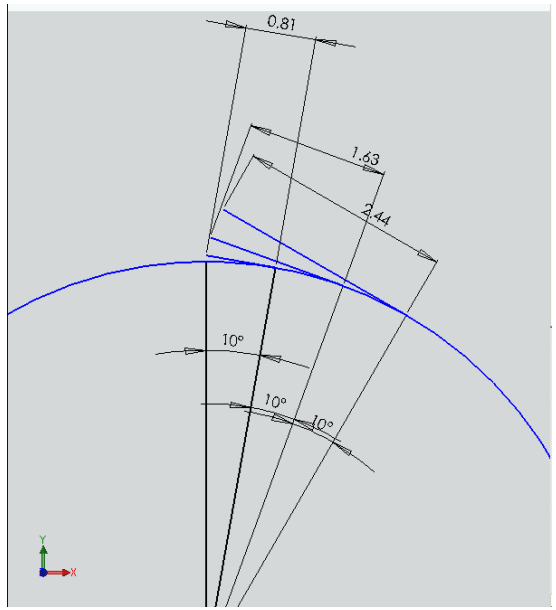


Figure by MIT OCW.

- A way to get the relative motion you want
- Pick one shape as you wish
- Enforce the motion you want
- Cut away everything that interferes

Involute Profile

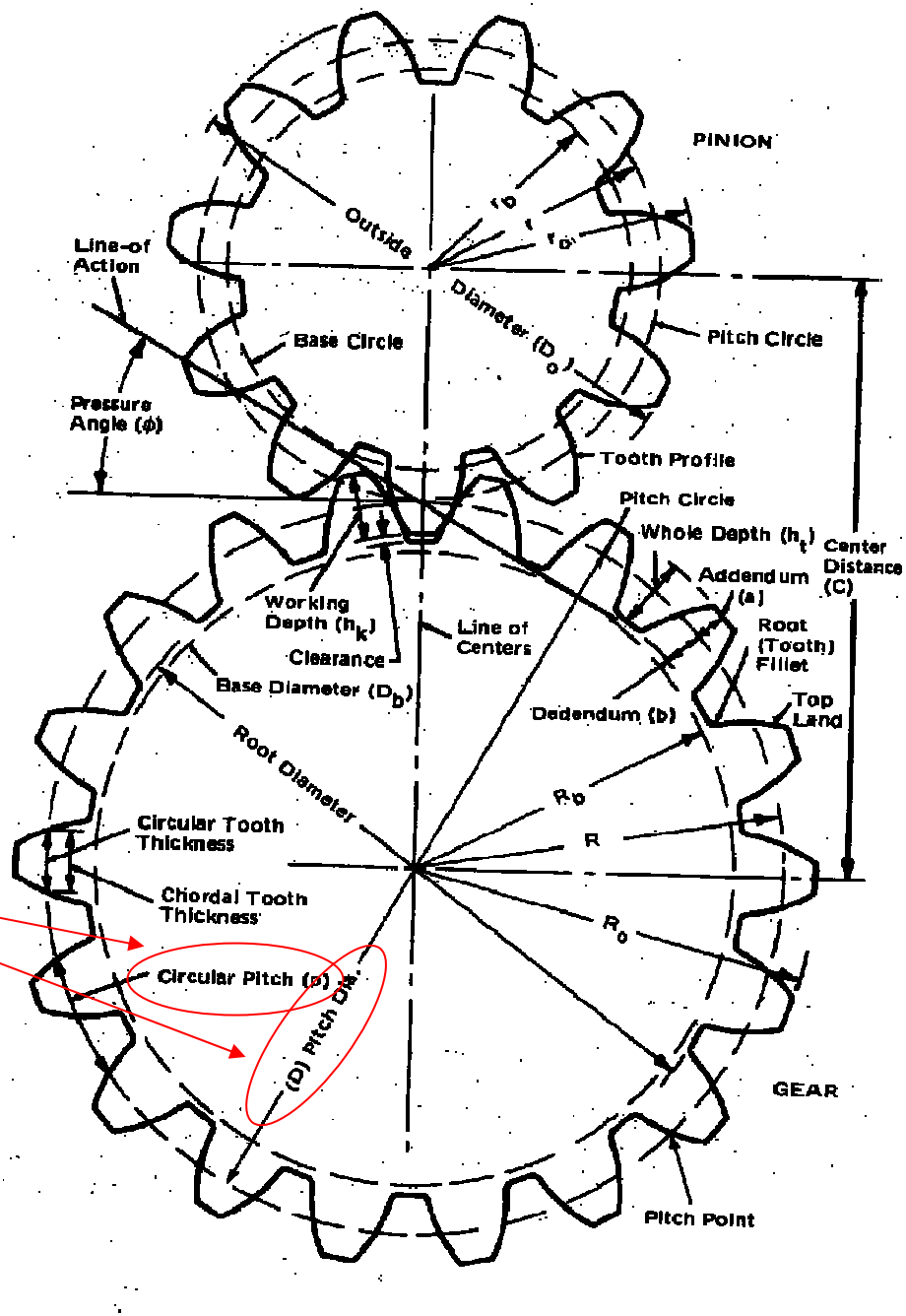
- How it is constructed
  - Demo
- Properties
  - Conjugate action
  - Allows design of whole sets of compatible gears
  - Conjugate action not sensitive to center distance variations



Gear Terminology

Diametral pitch (teeth per inch)
# of teeth on a gear with a
1 inch pitch diameter

Easily confused



Source: Buchsbaum, Frank, *Design and Application of Small Standardized components Data Book 757* Vol. 2, Stock Drive Products, 1983.
See “Handbook of Gears.” http://www.sdp-si.com/D190/D190cat.htm (accessed 28 June 2006.)
Courtesy of Stock Drive Products/Sterling Instrument.

More Gear Terminology

From
Shigley and Mischke

Figure removed for copyright reasons.
Source: Shigley and Mischke, Figure 13-5.

This geometry is not an involute.

Contact Ratio

Figure removed for copyright reasons.
Source: Shigley and Mischke, Figure 13-16.

contact ratio = length of arc of action / pitch = average number of teeth engaged

Interference

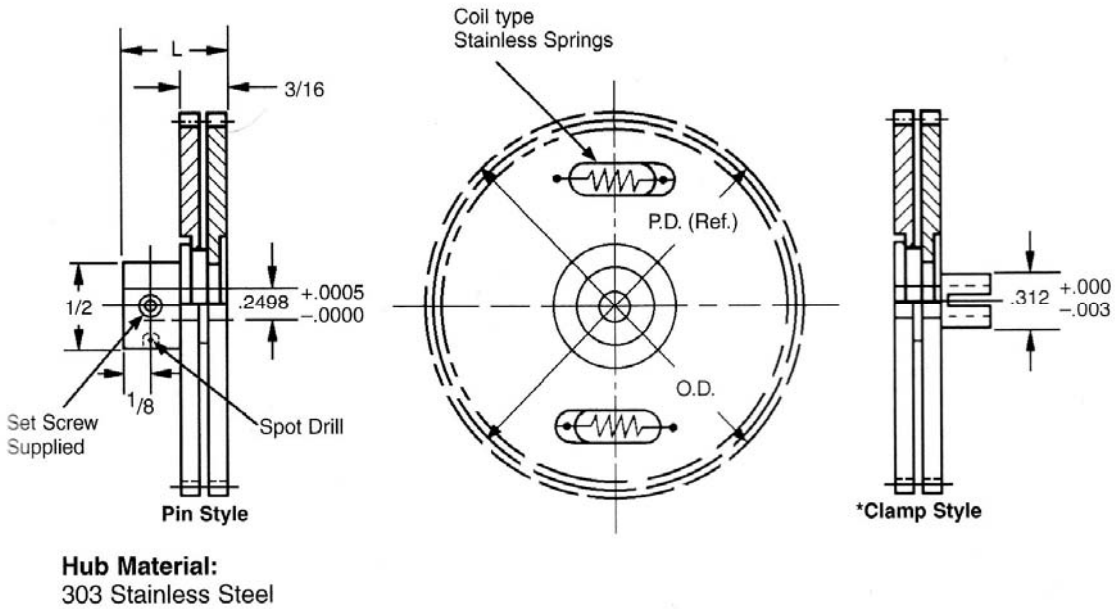
- Part of every tooth is not an involute
- This could in principle cause interference
- Due to the way gears are made, it really causes undercutting

Figure removed for copyright reasons.
Source: Shigley and Mischke, Figure 13-17.

From Shigley and Mischke

Backlash

- If you want any tolerance for
  - Center distance errors
  - Thermal growth
- There will be backlash when gears reverse



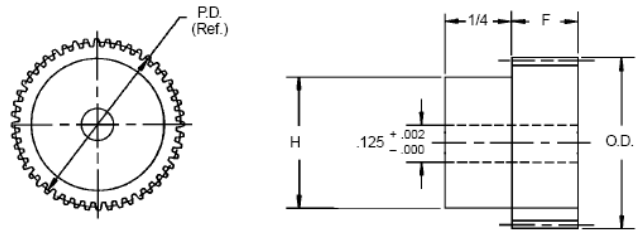
An “anti-backlash” gear

Gear Selection

- Pitch
- Face width
- Material
- Pressure angle
- # of teeth
- Hub style, bore, etc.

Spur Gears

24, 32, 48, and 64 Pitch 1/8" Bore AGMA Quality 4
Cold Rolled Steel and Brass 20° Pressure Angle



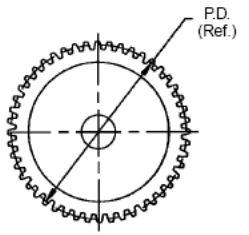
B 9

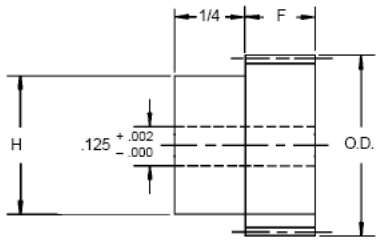
COLD ROLLED STEEL C12L14 OR C12L15 WITH SELENIUM	BRASS ALLOY 360					
STOCK NUMBER	STOCK NUMBER	NO OF. TEETH	PITCH DIA.	OUTSIDE DIA.	H	F
24 PITCH (.1309)						
PX24S-8	PX24B-8	8	.333	.416	.208	1/4
PX24S-9	PX24B-9	9	.375	.458	.250	
PX24S-10	PX24B-10	10	.417	.500	.291	
PX24S-12	PX24B-12	12	.500	.583	.375	
PX24S-16	PX24B-16	16	.666	.750	.542	
PX24S-18	PX24B-18	18	.750	.833	.625	
—	PX24B-22	22	.916	1.000	.792	
32 PITCH (.0981)						
PX32S-10	PX32B-10	10	.312	.375	.218	1/4
PX32S-11	PX32B-11	11	.344	.406	.250	
PX32S-12	PX32B-12	12	.375	.437	.281	
PX32S-14	PX32B-14	14	.438	.500	.343	
PX32S-15	PX32B-15	15	.469	.531	.375	
PX32S-16	PX32B-16	16	.500	.562	.406	
PX32S-18	PX32B-18	18	.563	.625	.468	
PX32S-20	PX32B-20	20	.625	.688	.532	
	PX32B-24	24	.750	.813	.656	
48 PITCH (.0654)						
PX48S-14	PX48B-14	14	.292	.333	.229	1/8
PX48S-15	PX48B-15	15	.312	.353	.250	
PX48S-16	PX48B-16	16	.333	.375	.271	
PX48S-18	PX48B-18	18	.375	.417	.312	
PX48S-24	PX48B-24	24	.500	.542	.437	
PX48S-32	PX48B-32	32	.666	.708	.604	
	PX48B-36	36	.750	.792	.687	
	PX48B-40	40	.833	.875	.770	
64 PITCH (.0490)						
PX64S-15	PX64B-15	15	.234	.265	.187	1/8
PX64S-16	PX64B-16	16	.250	.281	.203	
PX64S-18	PX64B-18	18	.281	.312	.234	
—	PX64B-24	24	.375	.406	.328	
—	PX64B-40	40	.625	.656	.578	
—	PX64B-48	48	.750	.781	.703	

Berg Manufacturing "The Mark of Quality" **1-800-232-BERG**

Spur Gears

24, 32, 48, and 64 Pitch 1/8" Bore AGMA Quality 4
Cold Rolled Steel and Brass 20° Pressure Angle





B 9

COLD ROLLED STEEL C12L14 OR C12L15 WITH SELENIUM	BRASS ALLOY 360					
STOCK NUMBER	STOCK NUMBER	NO OF. TEETH	PITCH DIA.	OUTSIDE DIA.	H	F
24 PITCH (.1309)						
PX24S-8	PX24B-8	8	.333	.416	.208	1/4
PX24S-9	PX24B-9	9	.375	.458	.250	
PX24S-10	PX24B-10	10	.417	.500	.291	
PX24S-12	PX24B-12	12	.500	.583	.375	
PX24S-16	PX24B-16	16	.666	.750	.542	
PX24S-18	PX24B-18	18	.750	.833	.625	
—	PX24B-22	22	.916	1.000	.792	
32 PITCH (.0981)						
PX32S-10	PX32B-10	10	.312	.375	.218	1/4
PX32S-11	PX32B-11	11	.344	.406	.250	
PX32S-12	PX32B-12	12	.375	.437	.281	
PX32S-14	PX32B-14	14	.438	.500	.343	
PX32S-15	PX32B-15	15	.469	.531	.375	
PX32S-16	PX32B-16	16	.500	.562	.406	
PX32S-18	PX32B-18	18	.563	.625	.468	
PX32S-20	PX32B-20	20	.625	.688	.532	
	PX32B-24	24	.750	.813	.656	
48 PITCH (.0654)						
PX48S-14	PX48B-14	14	.292	.333	.229	1/8
PX48S-15	PX48B-15	15	.312	.353	.250	
PX48S-16	PX48B-16	16	.333	.375	.271	
PX48S-18	PX48B-18	18	.375	.417	.312	
PX48S-24	PX48B-24	24	.500	.542	.437	
PX48S-32	PX48B-32	32	.666	.708	.604	
	PX48B-36	36	.750	.792	.687	
	PX48B-40	40	.833	.875	.770	
64 PITCH (.0490)						
PX64S-15	PX64B-15	15	.234	.265	.187	1/8
PX64S-16	PX64B-16	16	.250	.281	.203	
PX64S-18	PX64B-18	18	.281	.312	.234	
—	PX64B-24	24	.375	.406	.328	
—	PX64B-40	40	.625	.656	.578	
—	PX64B-48	48	.750	.781	.703	

Berg Manufacturing "The Mark of Quality" **1-800-232-BERG**

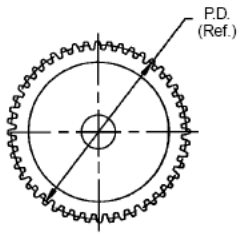
Courtesy of W. M. Berg, Inc. Used with permission.

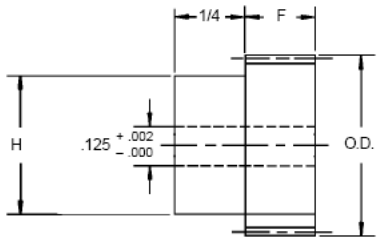
You call up the number 1-800-232-BERG and ask that, for a special application, you want a 48 pitch spur gear, but with a pitch dia of 0.32 inches. They will probably say:

1. OK, no problem
2. OK, but it will cost a lot
3. No, this is not technically possible

Spur Gears

24, 32, 48, and 64 Pitch 1/8" Bore AGMA Quality 4
Cold Rolled Steel and Brass 20° Pressure Angle





B 9

COLD ROLLED STEEL C12L14 OR C12L15 WITH SELENIUM	BRASS ALLOY 360					
STOCK NUMBER	STOCK NUMBER	NO OF. TEETH	PITCH DIA.	OUTSIDE DIA.	H	F
24 PITCH (.1309)						
PX24S-8	PX24B-8	8	.333	.416	.208	1/4
PX24S-9	PX24B-9	9	.375	.458	.250	
PX24S-10	PX24B-10	10	.417	.500	.291	
PX24S-12	PX24B-12	12	.500	.583	.375	
PX24S-16	PX24B-16	16	.666	.750	.542	
PX24S-18	PX24B-18	18	.750	.833	.625	
—	PX24B-22	22	.916	1.000	.792	
32 PITCH (.0981)						
PX32S-10	PX32B-10	10	.312	.375	.218	1/4
PX32S-11	PX32B-11	11	.344	.406	.250	
PX32S-12	PX32B-12	12	.375	.437	.281	
PX32S-14	PX32B-14	14	.438	.500	.343	
PX32S-15	PX32B-15	15	.469	.531	.375	
PX32S-16	PX32B-16	16	.500	.562	.406	
PX32S-18	PX32B-18	18	.563	.625	.468	
PX32S-20	PX32B-20	20	.625	.688	.532	
	PX32B-24	24	.750	.813	.656	
48 PITCH (.0654)						
PX48S-14	PX48B-14	14	.292	.333	.229	1/8
PX48S-15	PX48B-15	15	.312	.353	.250	
PX48S-16	PX48B-16	16	.333	.375	.271	
PX48S-18	PX48B-18	18	.375	.417	.312	
PX48S-24	PX48B-24	24	.500	.542	.437	
PX48S-32	PX48B-32	32	.666	.708	.604	
	PX48B-36	36	.750	.792	.687	
	PX48B-40	40	.833	.875	.770	
64 PITCH (.0490)						
PX64S-15	PX64B-15	15	.234	.265	.187	1/8
PX64S-16	PX64B-16	16	.250	.281	.203	
PX64S-18	PX64B-18	18	.281	.312	.234	
—	PX64B-24	24	.375	.406	.328	
—	PX64B-40	40	.625	.656	.578	
—	PX64B-48	48	.750	.781	.703	

Courtesy of W. M. Berg, Inc. Used with permission.

You call up the number 1-800-232-BERG and ask that, for a special application, you want a 48 pitch spur gear, but with a pitch dia of half the smallest one in the catalog. They will probably say:

1. OK, no problem
2. OK, but it will cost a lot
3. OK, but it will be weak
4. No, this is not technically possible

Strength of Gears

- Any good catalog will have a formula and tables
- What factors must enter the equation?
  -
  -
  -
- Where do the teeth wear the most?

Gear Reference Guide

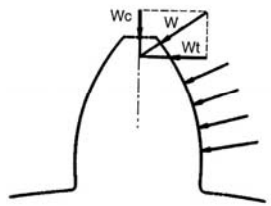
GEAR TOOTH STRENGTH

Many factors must be considered when designing a gear train. The information listed on this pag should be used as a general guideline for your application. If more critical strength calculation required W.M. Berg suggests that you consult our engineering department or any one of the many gea handbooks that are readily available.

When a gear train is transmitting motion, it is save to assume that all of the load is being carried b one tooth. This is because as the load approaches the end of the tooth, where the bending force woul be the greatest, a second tooth comes into mesh to share the load. Simple results can be obtaine from the Lewis bending strength equation.

$$W_t = \frac{SFY}{D.P.}$$

W_t = Maximum transmitted load (lbs or N)
S = Maximum bending tooth stress (taken as 1/3 of the tensile strength) See Table C on Page 5
F = Face width of gear (in. or mm)
D.P. = Diametral Pitch = 1/module (for equation only)
Y = Lewis Factor (See Table)

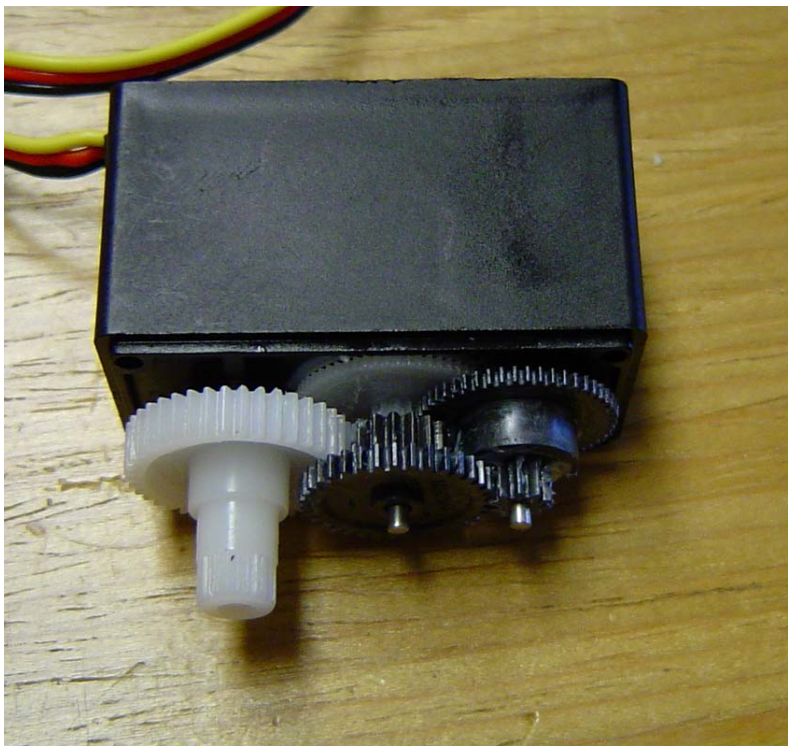


NOTE: The maximum bending tooth stress (S) is valid for well lubricated, low shock applications. For high shock, poorly lubricated applications, the safe stress could be as low as .025S. If your design calls for an unfriendly environment for gears, you might want to lower S to assure a reasonable amount of gear life.

LEWIS FACTOR - Y	NO. OF TEETH	14 1/2° INVOLUTE	20° INVOLUTE
	10	0.176	0.201
	11	0.192	0.226
	12	0.210	0.245
	13	0.223	0.264
	14	0.236	0.276
	15	0.245	0.289
	16	0.255	0.295
	17	0.264	0.302
	18	0.270	0.308
	19	0.277	0.314
	20	0.283	0.320
	22	0.292	0.330
	24	0.302	0.337
	26	0.308	0.344
	28	0.314	0.352
	30	0.318	0.358
	32	0.322	0.364
	34	0.325	0.370
	36	0.329	0.377
	38	0.332	0.383
	40	0.336	0.389
	45	0.340	0.399
	50	0.346	0.408
	55	0.352	0.415
	60	0.355	0.421
	65	0.358	0.425
	70	0.360	0.429
	75	0.361	0.433
	80	0.363	0.436
	90	0.366	0.442
	100	0.368	0.446
	150	0.375	0.458
	200	0.378	0.463
	300	0.382	0.471
	RACK	0.390	0.484

Concept Question

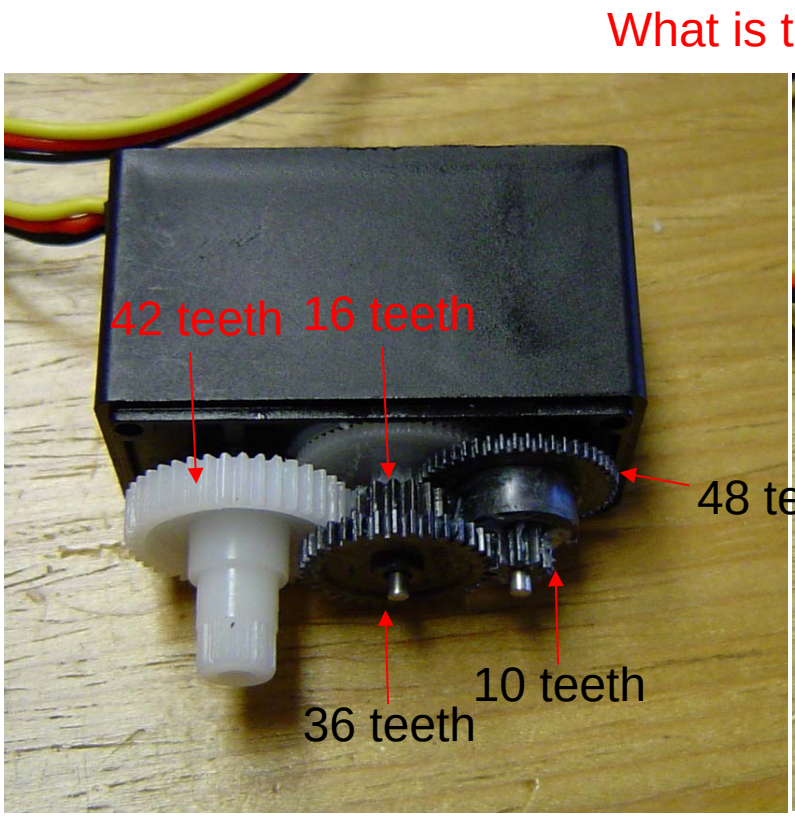
A compound gear train is formed of eight gears. As we proceed from the pinion on the electric motor to the gear on the output shaft, how do the pitch and face width vary?

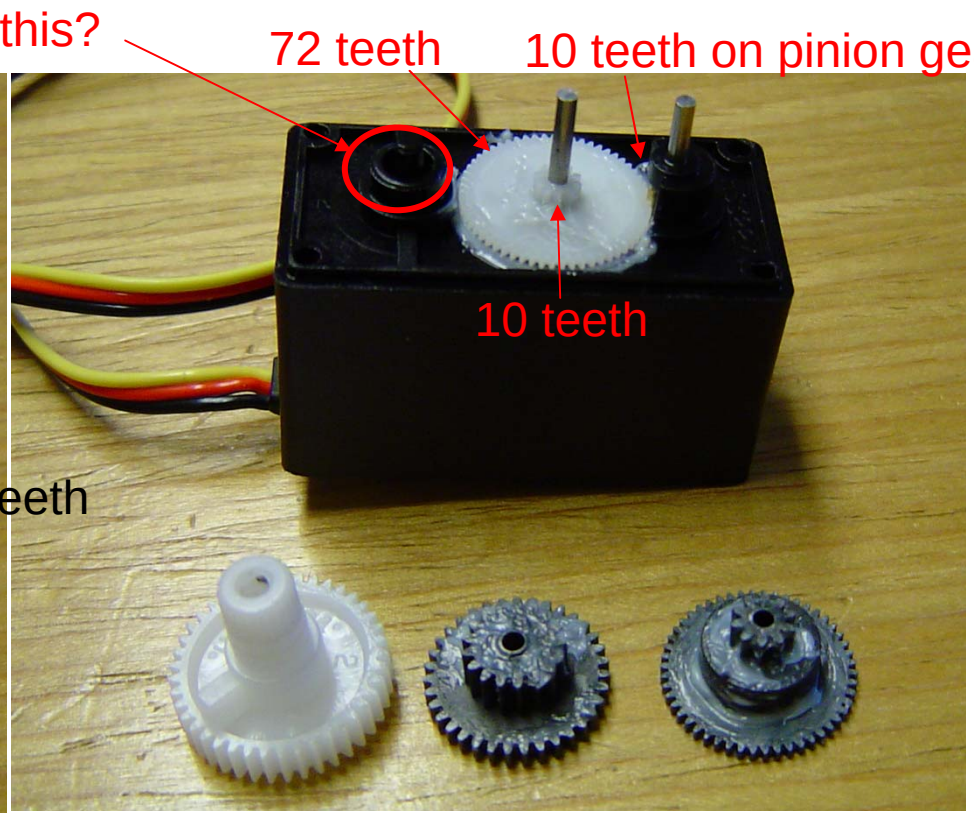


1. Pitch rises,
   face width rises
2. Pitch rises,
   face width falls
3. Pitch falls,
   face width rises
4. Pitch falls,
   face width falls

Homework Question #3

- Given the top output shaft speed, what is the motor shaft speed (in rpm)?





Next Steps

- Begin to develop project ideas
  - Email your slides to TA by Friday noon
- On Friday 10 FEB (right here) you will present
  - A project idea OR what you're looking for in a project
  - Who are you? What do you bring to a project team?
  - Then fill out a project preference form
- On Tues 14 FEB (right here) project teams will be announced
- Start HW#1